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# Ultra-Wideband Antenna Arrays: Systems with Transfer Function and Impulse Response

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**Abstract**—This paper proposes some approaches to model Ultra Wideband (UWB) antenna arrays. Based on the array factor, often stipulated as not adapted for the description of the properties of UWB arrays in the literature, an analytical expression of the beampattern is developed. The achieved results are coherent with other formulations and empiric studies proposed in the literature. Furthermore, a time-frequency modeling of UWB antenna arrays is proposed using the concept of array factor and antenna effective length.

## 1. INTRODUCTION

Even if the concept of antenna arrays is not new with early work in the 1920s [1], Multiple Input Multiple Output (MIMO) communication systems using antenna arrays have recently emerged as a breakthrough for wireless systems of revolutionary importance. All wireless technologies face the challenges of signal fading, multipath, increasing interference, and limited spectrum. Antenna arrays in MIMO systems exploit multipath and associated diversity to provide higher data throughput and simultaneous increase in range and reliability, all without consuming extra radio frequency [2]. They enable the signal-to-interference by suppressing interferers by the use of spatial filtering or spatial diversity [3] and can also present advantages for cryptography [4]. Ultra-Wideband (UWB) technology is a potential candidate in the race of the wireless world since the Federal Communications Commission (FCC) released a report approving its use in the 3.1–10.6 GHz frequency range. However this technology is limited by an extremely low allowable transmitted power, i.e., −41.3 dBm/MHz [5]. To overcome this constraint, the combination of MIMO techniques with UWB technology has been found to be one of the most relevant solutions. Furthermore, it should be noted that the antenna arrays for the UWB systems present the same advantages as in narrowband systems [6].

In this context, a lot of the works concerned the design of UWB MIMO antennas [7–9] as well as the description of their specific properties [10–14]. The direct transposition of narrowband approaches is not adequate and too incomplete for the UWB antenna array descriptors, as is the case of single element [15–17]. The traditional description of antenna arrays from array factors [18] cannot be directly used in UWB arrays because it does not take into account the frequency dependence. As for the single UWB antennas, the modeling of UWB antenna arrays is typically performed in the time domain. Therefore the beampattern has been defined considering the time expressions of the signals [7]. In [12], the properties of short-pulsed sparse transmitting arrays are explored. The array's characterization is carried out via the energy radiation pattern which is decomposed into a set of different types of beam contributions (main beams, grating lobe beams and cross-pulsed lobe beams), and this according to the array's physical and excitation parameters. The properties of UWB arrays are described in [11] which notably highlights that even sparse UWB antenna arrays do not manifest grating lobes. This statement is completed in [13, 14] focusing on the grating lobes and determining the minimum requirements so that an UWB array does not effectively manifest grating lobes. Finally as early as 2006, [10] proposed a study aiming to highlight the signal dispersion due to parameters as the scan angle, the input signal

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duration, the repetition rate of the input pulse train. Guidelines to reduce pulse coupling was introduced from a physical model of the time domain coupling in pulsed antenna arrays.

The objective of this paper is to introduce a time-frequency model of antenna arrays. It extends the system modeling proposed in the case of the single UWB antennas [19] for the UWB arrays. Indeed, it is acknowledged that for evaluating antennas with baseband pulse excitations the most general antenna descriptor is the antenna impulse response, and thus, the UWB antennas can be considered as Linear Time Invariant (LTI) systems characterized by transfer functions or associated impulse responses. The proposed idea is to characterize the UWB antenna arrays using a similar approach. Section 2 shows how the classical approach developed for narrowband antenna arrays relying on the array factor can be generalized for the UWB antenna arrays. Section 3 presents the new system modeling dedicated for UWB antenna arrays. The proposed approach is based on the modeling describing the antennas as systems with transfer function and impulse response and exploits the definition of the array factor. Finally section 4 draws conclusions and outlines future works.

## 2. CHARACTERIZATION OF UWB ANTENNA ARRAYS

### 2.1. Introduction

The antenna arrays are generally described by means of array factors where it is assumed electromagnetic waves at a single frequency [20]. The phase shifts introduced by the array geometry and the eventual amplitude weight and phase shifter of each antenna element define the array factors. For UWB antenna arrays, this approach was abandoned because the signals may be extremely short, and thus the emitted or received signals by individual antenna elements do not always overlap in the time domain [13]. However, the classic method can be exploited and applied in the case of UWB arrays taking into account some precautions. The following parts show how it is possible to describe an antenna array through a system approach based on the array factor and also show the coherence with the used specific time descriptors.

### 2.2. UWB Array Pattern

#### *2.2.1. Radiation Vector for Antenna Arrays*

Considering a three-dimensional array of $N$ several identical antennas located at positions $\vec{d}_n$ with relative complex feed coefficients $A_n(f)$ $(n = 1, \ldots, N-1)$, the total radiation vector $\vec{F}_{tot}(\vec{k})$ is function of the radiation vector $\vec{F}(\vec{k})$ due to a single antenna element at the origin and the array factor $\mathcal{A}(\vec{k})$ [20]:

$$\vec{F}_{tot}\left(\vec{k}\right) = \mathcal{A}\left(\vec{k}\right)\vec{F}\left(\vec{k}\right) \tag{1}$$

with

$$\mathcal{A}\left(\vec{k}\right) = \sum_{n=0}^{N-1} A_n(f)\exp\left(j\vec{k}\cdot\vec{d}_n\right) \tag{2}$$

$$\vec{k} = k\left(\cos\varphi\sin\theta\hat{x} + \sin\varphi\cos\theta\hat{y} + \cos\theta\hat{z}\right) \tag{3}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c} \tag{4}$$

and where $\lambda$ is the free-space wavelength, $f$ the frequency, $c$ the velocity of light, and $\omega$ the pulsation.

Consequently the far-zone field of an $N$-element array of identical elements is equal to the product of the field of a single element (at a selected reference point, usually the origin) and the array factor of that array. The array factor is a function of the number of elements, their geometrical arrangement, their relative magnitudes and phases and their spacings. This result is general and exact for all types of antennas constituting the array. However, different approaches proposed for UWB arrays take into account the frequency dependence and the fact that the phase information alone is not sufficient as it does not provide information on temporal superposition of the signals [7, 14]. From this remark, without loss of generality, first a simple example is detailed then a generalization is developed in order to bring out the properties in the UWB case.

*2.2.2. Azimuthal Power Pattern for an Array of Two Antennas*

Consider an array of two isotropic antennas at positions $\vec{d_0} = 0$ and $\vec{d_1} = d\hat{x}$ as illustrated in Fig. 1(a).

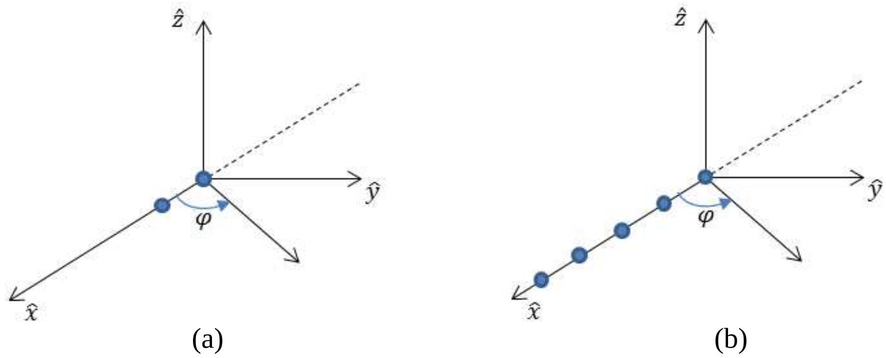


**Figure 1.** Geometry of (a) two-element (b) $N$-element array along $x$-axis.

Assuming the array unit weights and considering the array spacing $d = l\lambda_0$ (where $\lambda_0$ is the central wavelength of the signal and $l$ is a real number) the azimuthal power pattern $g(f, \varphi)$ is:

$$g(f, \varphi) = |\mathcal{A}(f, \varphi)|^2 = \left|1 + \exp\left(j2\pi l \frac{f}{f_0} \cos\varphi\right)\right|^2 \tag{5}$$

can also be rewritten as:

$$g(f, \varphi) = 2\left(1 + \left\{J_0\left(2\pi l \frac{f}{f_0}\right) + 2\sum_{n=1}^{+\infty} (-1)^n J_{2n}\left(2\pi l \frac{f}{f_0}\right) \cos(2n\varphi)\right\}\right) \tag{6}$$

where $J_0(2\pi l \frac{f}{f_0})$ and $J_{2n}(2\pi l \frac{f}{f_0})$ are the coefficients of the Bessel functions of the first kind. These expressions are general and show that it is possible to include the frequency dependence in the analysis of the patterns.

*2.2.3. Beampattern for an Array of N-antennas*

Consider a one-dimensional array of $N$ isotropic antennas at positions $\vec{d_n} = nd\hat{x}$ in Fig. 1(b). Considering the array spacing $d = l\lambda_0$, the array factor is a function of the azimuthal angle $\varphi$ and the frequency variable $f$ as:

$$\mathcal{A}(f, \varphi) = \sum_{n=0}^{N-1} A_n(f) \exp\left(j2\pi l n \frac{f}{f_0} \cos\varphi\right) \tag{7}$$

Assuming the array unit weights, after some calculations the azimuthal power pattern $g(f, \varphi)$ is written:

$$g(f, \varphi) = N + 2\sum_{k=1}^{N-1} (N - k) \cos\left(k 2\pi l \frac{f}{f_0} \cos\varphi\right) \tag{8}$$

The power pattern depends on the azimuthal angle and the frequency variable: the given expression is general and true whatever the shape of the transmitted signals. In order to take into account the spectral characteristics of the signals, it is necessary to complete the approach of modeling. Classically, when one single frequency is considered, the array factor intrinsically includes this frequency. More generally, it is necessary to associate the array factor with the Fourier transform $\mathcal{F}$ of the emitted baseband signal $S(f) = \mathcal{F}[s(t)]$ to find an expression more complete of the radiated signal $X(f)$ by the array:

$$X(f, \varphi) = \mathcal{A}(f, \varphi) \cdot S(f) \tag{9}$$

The normalized beam pattern is defined by:

$$G(\varphi) = \frac{\displaystyle\int_{-\infty}^{+\infty} |X(f,\varphi)|^2 df}{\displaystyle\int_{-\infty}^{+\infty} |S(f)|^2 df} \tag{10}$$

Therefore, after some calculations, its expression is given by:

$$G(\varphi) = N + 2\sum\nolimits_{k=1}^{N-1} (N-k) \frac{\displaystyle\int_{-\infty}^{+\infty} |S(f)|^2 \cos\left(k2\pi l \frac{f}{f_0}\cos\varphi\right) df}{\displaystyle\int_{\infty}^{+\infty} |S(f)|^2 df} \tag{11}$$

or equivalently by

$$G(\varphi) = N + 2\sum\nolimits_{k=1}^{N-1} (N-k) \frac{\mathcal{R}_e\left\{Rs\left(\frac{kl\cos\varphi}{f_0}\right)\right\}}{Rs(0)} \tag{12}$$

where $Rs(n)$ is the autocorrelation function of the signal $s(t)$ and $\mathcal{R}_e\{\cdot\}$ stands for real part.

The expression is the same as that proposed in [13] where the study is realized in the time domain. This result shows that the classical reasoning in the case of a single frequency can be generalized unlike what is suggested by [13]. Here, the objective is not to reproduce the empirical studies showing the properties of beampatterns but rather to propose a general definition and theoretical expressions allowing the characterization of UWB antenna arrays. Illustration examples of simulated beam pattern according to (12) can be found in [13] for a 5-element array.

In conclusion, the main interest of this definition is the consideration of the properties of the transmitted signal (amplitude and phase), and this, regardless of its properties and frequency band. Finally, it should be noted that for this study, it was assumed that the coupling between adjacent radiation elements is not taken into consideration. The following approach proposes a system modeling which includes the eventual distortion of the antennas (always assuming not coupling) and relies on methods proposed for the single UWB antennas.

## 3. SYSTEM MODEL OF UWB ANTENNA ARRAYS

### 3.1. System Modeling of Antennas

To describe and specify the transient radiation and reception characteristics of antennas, the effective lengths have been considered first [21, 22]. With the emergence of the UWB technology, the transfer function (i.e., frequency response) and the impulse response (i.e., time response), which are derived from the effective length, have been preferred. Therefore the UWB antennas are considered as LTI systems for which the performance affects the overall performance of the wireless systems. In [23], several of the proposed techniques are presented with the objective to compare their approach and to highlight the achieved differences. Fig. 2 illustrates a model of the wireless communication systems. The radio link decomposed into three functional blocs provides a useful modeling: the channel of propagation $H_{ch}(f)$, the TX and RX antennas (which can be single or even multiple as developed later) each described by a transfer function, $\vec{H}_{TX}(f,\theta_{TX},\varphi_{TX})$ and $\vec{H}_{RX}(f,\theta_{RX},\varphi_{RX})$, and the associated impulse response $\vec{h}_{TX}(t,\theta_{TX},\varphi_{TX})$ and $\vec{h}_{RX}(t,\theta_{RX},\varphi_{RX})$ where $f$ is the frequency, $t$ the time, and $\theta$ and $\varphi$ are the polar and azimuth angles. Therefore, the characterization is very complete because it includes the frequency dependence, the phase information, and the polarization and the directional properties. Under far-field propagation conditions, it can be shown that the transfer functions and the impulse responses modeling the antennas present analytical expressions which are functions of the effective length of the antennas expressed in frequency domain or time domain respectively [23].

Moreover, assuming a wireless channel with only one direct path between the transmitter and receiver (i.e., Line-Of-Sight, LOS propagation), the transfer between the output $s$ and $e$ the input can

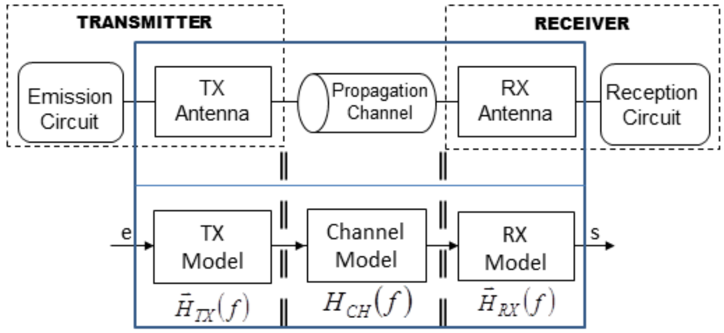


**Figure 2.** Block diagram of wireless communication systems.

also be deduced. The characterization of antennas as LTI systems presents the advantage to achieve time-frequency models, especially suitable for UWB antennas, and for example, allows the determination of the radiated and received transient waveforms of any arbitrary waveform excitation and antenna orientation.

### 3.2. Total Effective Length for an Antenna Array

Using the concept of array factor and conjointly the approaches developed for achieving the system models of UWB antennas, an UWB array system model can be achieved.

Considering the assumptions given in part III.A., the radiated field in transmission $\vec{E}^{rad}$ can be defined in the frequency domain from the effective length $\vec{L}_{e_{TX}}$ of the TX antenna as [18]:

$$\vec{E}^{rad}(f,\theta_{TX},\varphi_{TX}) = j\frac{f}{c}Z_0\frac{\exp\left(-j\omega\frac{dr}{c}\right)}{2r}I(f)\vec{L}_{e_{TX}}(f,\theta_{TX},\varphi_{TX}) \tag{13}$$

where $r$ is the radiation distance, $Z_0$ the free space impedance, and $I$ the excitation current. This expression can be rewritten introducing the transverse part of the radiation vector $\vec{F}_{\perp}$ [20] as follows:

$$\vec{E}^{rad}(f,\theta_{TX},\varphi_{TX}) = -j\frac{f}{c}Z_0\frac{\exp\left(-j\omega\frac{dr}{c}\right)}{2r}\vec{F}_{\perp}(f,\theta_{TX},\varphi_{TX}) \tag{14}$$

The vector $\vec{F}_{\perp}$ includes both the characteristics of the antenna via the effective length $\vec{L}_{e_{TX}}$ and the properties of the emitted signal via the spectral form of the current $I(f)$.

Under the same assumptions and considering an antenna array (with $N$ elements), the total radiated field $\vec{E}^{tot}_{rad}$ is:

$$\vec{E}^{rad}_{tot}(f,\theta_{TX},\varphi_{TX}) = -j\frac{f}{c}Z_0\frac{\exp\left(-j\omega\frac{dr}{c}\right)}{2r}\vec{F}_{tot,\perp}(f,\theta_{TX},\varphi_{TX}) \tag{15}$$

with

$$\vec{F}_{tot,\perp}(f,\theta_{TX},\varphi_{TX}) = \mathcal{A}(f,\theta_{TX},\varphi_{TX})\cdot\vec{F}_{\perp}(f,\theta_{TX},\varphi_{TX}) \tag{16}$$

Consequently, an equivalent effective length $\vec{L}_{e_{tot,TX}}$, called total effective length by analogy with the total radiation vector, for the antenna array can be introduced as:

$$\vec{L}_{e_{tot,TX}}(f,\theta_{TX},\varphi_{TX}) = \mathcal{A}(f,\theta_{TX},\varphi_{TX})\cdot\vec{L}_{e_{TX}}(f,\theta_{TX},\varphi_{TX}) \tag{17}$$

The array factor $\mathcal{A}(f,\theta_{TX},\varphi_{TX})$ is easily defined by the traditional approaches; for example, in the azimuthal plan and for the antenna array represented by Fig. 1(a), it is expressed by (7). Furthermore, the study remains general as shown in the previous section; the antenna array can be constituted of all types of the similar antennas, narrowband or UWB antennas.

As in the case of a single antenna, the total effective length is a very complete representation taking into account the main characteristics and allowing the description of the antenna array through descriptors such as impulse response or function transfer.

### 3.3. UWB Antenna Array System Modeling

An extension of the system modeling for the UWB antenna arrays can now be deduced. As presented in [22], several formulations are possible according to the chosen way for the modeling. For illustrating the concept, the Fig. 2 being considered, a TX model can be established. The TX antenna assumed to be an antenna array, the transfer function $\vec{H}_{TX}(f, \theta_{TX}, \varphi_{TX})$ and the associated impulse response $\vec{h}_{TX}(f, \theta_{TX}, \varphi_{TX})$ can be expressed in function of the total effective length as shown below. Therefore, the function transfer of an antenna array in transmission mode can be written in a very general form as:

$$\vec{H}_{TX}(f, \theta_{TX}, \varphi_{TX}) = \alpha \vec{L}_{e_{tot},TX}(f, \theta_{TX}, \varphi_{TX}) \tag{18}$$

where the coefficient $\alpha$ is a scalar, frequency dependent, which includes the modeling approach and the generator and antenna impedances (for more details about $\alpha$, see for example Equations (16) to (18) in [23]).

Therefore, the corresponding impulse response is:

$$\vec{h}_{TX}(t, \theta_{TX}, \varphi_{TX}) = \mathcal{F}^{-1}[\alpha] * \vec{l}_{e_{tot},TX}(t, \theta_{TX}, \varphi_{TX}) \tag{19}$$

with

$$\vec{l}_{e_{tot},TX}(t, \theta_{TX}, \varphi_{TX}) = \mathcal{A}(t, \theta_{TX}, \varphi_{TX}) * \vec{l}_{e_{TX}}(t, \theta_{TX}, \varphi_{TX}) \tag{20}$$

The array factor appears in its time form, and it can be simply written according to the array which it represents. For example, the time expression equivalent to (7) is calculated using the inverse Fourier transform as:

$$\mathcal{A}(t, \theta_{TX}, \varphi_{TX}) = \sum\nolimits_{n=0}^{N-1} a_n \left( t - \frac{nl \cos\varphi}{f_0} \right) \tag{21}$$

This study shows that the concepts of the system modeling developed for the case of a single antenna can be generalized for the antenna arrays. Moreover the developed principle is very general and can be applied to UWB antennas but also in the case of a single frequency because the different expressions are simplified and the classical equations are obtained. Another remark is that the influence of transmitted signals (i.e., input signals) can be also taken into account independently. The proposed description with the array factor leads to a useful time/frequency system modeling.

## 4. CONCLUSION

This paper emphasizes views for modeling the antenna arrays, and more particularly the UWB antenna arrays. The array factor has been used in order to establish an analytical expression of the beampattern of an array of $N$-antennas. Moreover, from the classical approach describing the arrays through the array factor and using a system modeling of antennas, a time-frequency model of TX antenna arrays has been proposed. The proposed concept leads to general and elegant models, including the cases of single antennas and antenna arrays, which are valuable for narrowband and UWB communication systems.

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