

Study of the Effects of Eccentric Plasma Coating over Metamaterial Cylinder

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Abstract—A plasma sheath can significantly alter the electromagnetic properties of an object, which leads to many practical applications. In this article, the electromagnetic scattering properties of a DB metamaterial cylinder coated with unmagnetized plasma are presented. The effects of layer thickness, non-uniform cladding (eccentric coating), electron number density, electron-neutral collision frequency and the frequency of incident wave on radar cross-section (RCS) of the object are discussed. It is found that the RCS of the DB metamaterial objects can be reduced or enhanced by appropriate values of plasma parameters, thickness or eccentricity. The anomalous behavior of backscattering cross-section of plasma coated DB cylinder has been observed at frequencies near plasma frequency. The results may serve as a noteworthy reference for experimentalists working in plasma stealth technology for metamaterials.

1. INTRODUCTION

It was known since the first man-made satellite was launched that the bodies traveling with high speed can acquire electrostatic charge while passing through the ionosphere. These charged bodies (satellites, missiles and reentry vehicles) may attract a shell of particles of opposite charge and consequently alter their electromagnetic scattering properties. This alteration becomes significant at frequencies about the plasma resonant frequency. Therefore, the scattering of electromagnetic radiations from objects with a plasma clad or sheath has been a topic of great interest for researchers in the past [1–6]. Similarly, the plasma sheath enveloping a reentry vehicle can result in “communications blackout” during which the performance of on-board electromagnetic systems experience significant degradation [7, 8]. One of the main cause of the “blackout” problem is the failure of on-board antennas which are greatly affected by the reentry plasma sheath [9, 10]. Despite of the fact that the electromagnetic scattering from coated objects has been investigated extensively in past, the study of scattering from plasma coated objects has become very popular in recent years due to its practical applications in RCS reduction (stealth technology), radar camouflage, minimization of scattering by antenna obstacles, development of echo area enhancement devices and many other potential applications [11–15]. Recent progress in artificially created tunable plasmas has set new trends in this particular area of research [16, 17].

Plasma is generally referred to as a highly ionized state of matter; it essentially is a neutral mixture of free ions, electrons and molecules. However, these charged particles are not merely free, because they are strongly affected by each others’ electromagnetic fields. On the other hand, the assembly of these unbounded charged particles exhibit collective motions of immense vitality and complexity. The plasma is generally a dissipative medium and it is mainly because of the brutal collisions of charged particles while they experience thermal agitation. In the absence of applied magnetic field, the plasma can

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be approximated by a lossy dielectric with a complex permittivity. The transmission and reflection coefficients of such medium have been extensively computed and reported [18–24].

Though analysis of EM scattering from various plasma coated geometries can be found in literature, the bodies of cylindrical shape have attained more attraction due to the simplicity in analysis and suitability to model space objects e.g., missiles and rockets. In almost all previous studies, the core cylinder is considered to be a perfectly conducting cylinder. However, the use of metamaterials in engineering applications is increasing rapidly. Researchers are in quest of finding new materials for antennas, communication systems and space applications. The analysis of various metamaterial cylinders coated with ordinary dielectric or other metamaterials has attracted great attention of the researchers in recent past [25–30]. Moreover, the plasma sheath formed around space objects may not be concentric in practical scenarios. Therefore, the analysis of electromagnetic scattering properties of such novel materials when coated eccentrically with plasma may serve as a useful reference for many practical applications.

Recently, a novel surface was proposed and realized by using metamaterials which is known as DB surface [31–33]. An interesting and unusual feature of this novel surface is that the boundary conditions are imposed on normal components of the electric and magnetic flux densities (\mathbf{D} and \mathbf{B}) instead of the tangential components of the electric and magnetic fields (\mathbf{E} and \mathbf{H}). The boundary conditions for a DB surface are given as;

$$\hat{n} \cdot \mathbf{D} = 0 \quad (1a)$$

$$\hat{n} \cdot \mathbf{B} = 0 \quad (1b)$$

where \hat{n} is the unit vector normal to the surface. DB surface exhibits different characteristics (impedance, for example) depending upon the polarization of the incident wave. It has been shown that the DB boundary acts as PEC or PMC boundary for TE or TM polarizations respectively [32]. However, it is worth noting that for a TEM polarized incident plane wave (when there is no normal component, neither of electric nor of magnetic field at the DB surface), it behaves like a transparent surface or medium.

In this paper, the scattering properties of a DB cylinder coated with unmagnetized plasma are studied. Both the concentric (uniform cladding) and eccentric (non-uniform cladding) models are considered. The solutions are obtained by employing superposition of cylindrical wave functions and coordinate transformation. The effects of layer thickness, eccentricity of coating, electron number density, electron-neutral collision frequency and frequency of the incident wave on radar cross-section (RCS) of the DB cylinder are discussed.

2. PROBLEM FORMULATION

2.1. DB Cylinder with Uniform Plasma Coating (Concentric Model)

Let us consider a circular cylinder made from DB metamaterial coated with a uniform layer of unmagnetized plasma as shown in Fig. 1(a). The radii of the core and coated cylinders are denoted by

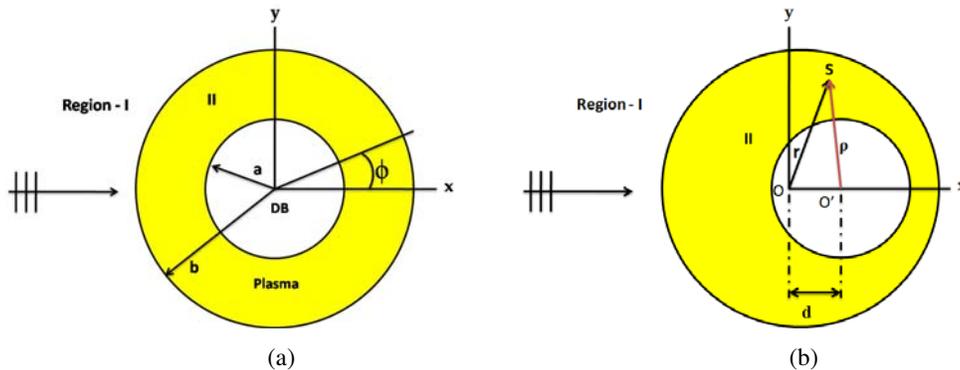


Figure 1. (a) Concentric model. (b) Eccentric model.

a and b respectively. The region-I ($\rho > b$) is considered to be free space ($\varepsilon_1 = \varepsilon_0$ and $\mu_1 = \mu_0$), while the region-II ($a \leq \rho \leq b$) is a layer of homogeneous plasma characterized by the constitutive parameters ε_2 and μ_2 . It should be noted that in case of unmagnetized plasma, the permeability μ_2 is equal to that of free space.

Suppose a TM^Z polarized plane wave traveling in x direction is normally incident to the coated structure. Then the incident and scattered electric field vectors in region-I can be expressed in cylindrical coordinates as:

$$\mathbf{E}_I^i = \hat{z}E_0 \sum_{n=-\infty}^{n=\infty} j^{-n} J_n(k_1\rho) e^{jn\phi} \tag{2}$$

$$\mathbf{E}_I^s = \hat{z}E_0 \sum_{n=-\infty}^{n=\infty} a_n j^{-n} H_n^{(2)}(k_1\rho) e^{jn\phi} \tag{3}$$

The corresponding magnetic field vectors can be obtained by Maxwell equations as follows:

$$\mathbf{H}_I^i = -\hat{\rho} \frac{E_0}{\omega\mu_1\rho} \sum_{n=-\infty}^{n=\infty} n j^{-n} J_n(k_1\rho) e^{jn\phi} + \hat{\phi} \frac{E_0 k_1}{j\omega\mu_1} \sum_{n=-\infty}^{n=\infty} j^{-n} J'_n(k_1\rho) e^{jn\phi} \tag{4}$$

$$\mathbf{H}_I^s = -\hat{\rho} \frac{E_0}{\omega\mu_1\rho} \sum_{n=-\infty}^{n=\infty} n a_n j^{-n} H_n^{(2)}(k_1\rho) e^{jn\phi} + \hat{\phi} \frac{E_0 k_1}{j\omega\mu_1} \sum_{n=-\infty}^{n=\infty} a_n j^{-n} H_n^{(2)'}(k_1\rho) e^{jn\phi} \tag{5}$$

Similarly, the electric and magnetic field vectors in region-II can be written as;

$$\mathbf{E}_{II} = \hat{z}E_0 \sum_{n=-\infty}^{n=\infty} j^{-n} \left[b_n H_n^{(1)}(k_2\rho) + c_n H_n^{(2)}(k_2\rho) \right] e^{jn\phi} \tag{6}$$

$$\begin{aligned} \mathbf{H}_{II} = & -\hat{\rho} \frac{E_0}{\omega\mu_2\rho} \sum_{n=-\infty}^{n=\infty} n j^{-n} \left[b_n H_n^{(1)}(k_2\rho) + c_n H_n^{(2)}(k_2\rho) \right] e^{jn\phi} \\ & + \hat{\phi} \frac{E_0 k_2}{j\omega\mu_2} \sum_{n=-\infty}^{n=\infty} j^{-n} \left[b_n H_n^{(1)'}(k_2\rho) + c_n H_n^{(2)'}(k_2\rho) \right] e^{jn\phi} \end{aligned} \tag{7}$$

In above equations, ω , k_1 and k_2 are the angular frequency, wave numbers in free space and wave number in plasma layer, respectively. a_n, b_n and c_n are the unknown coefficients to be determined by appropriate boundary conditions. $J_n(\circ)$ denotes the n^{th} order Bessel function of first kind, and $H_n^{(p)}(\circ)$ denotes the p^{th} kind Hankel function of order n . The prime ($'$) denotes the derivative with respect to whole argument.

The following matrix equations are to be solved by using appropriate boundary conditions.

$$AX = B \tag{8}$$

$$A = \begin{bmatrix} 0 & H_n^{(1)}(k_2a) & H_n^{(2)}(k_2a) \\ H_n^{(2)}(k_1b) & -H_n^{(1)}(k_2b) & -H_n^{(2)}(k_2b) \\ H_n^{(2)'}(k_1b) & -\frac{\mu_1 k_2}{k_1 \mu_2} H_n^{(1)'}(k_2b) & -\frac{\mu_1 k_2}{k_1 \mu_2} H_n^{(2)'}(k_2b) \end{bmatrix} \tag{9a}$$

$$X = \begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix} \tag{9b}$$

$$B = \begin{bmatrix} 0 \\ -J_n(k_1b) \\ -J'_n(k_1b) \end{bmatrix} \tag{9c}$$

2.2. DB Cylinder with Non-Uniform Plasma Coating (Eccentric Model)

Let us now consider the case of circular DB cylinder coated with non-uniform layer of unmagnetized plasma as depicted in Fig. 1(b). The origin of the core cylinder has been shifted to O' which is off-set with the original coordinates by a distance d , we will refer d as *eccentricity* parameter. It can be seen that the transformation between the two cylindrical coordinate systems O and O' is given by:

$$\mathbf{r} = \rho + \mathbf{d} \quad (10)$$

The Helmholtz equations in region-II will remain the same, while the cylindrical functions will be modified in Equations (6) and (7) as prescribed by Graf's addition formulas given as;

$$Z_n(k\rho)e^{jn\phi} = \sum_{p=-\infty}^{\infty} [Z_p(kd)Z_{n+p}(kr)] e^{j(n+p)\phi} \quad (11a)$$

$$Z'_n(k\rho)e^{jn\phi} = \sum_{p=-\infty}^{\infty} [Z_p(kd)Z'_{n+p}(kr)] e^{j(n+p)\phi} \quad (11b)$$

Consequently, the electric and magnetic field vectors in region-II will be modified as;

$$\mathbf{E}_{II} = \hat{z}E_0 \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} j^{-n} J_p(k_2d) (b_n H_{n+p}^{(1)}(k_2r) + c_n H_{n+p}^{(2)}(k_2r)) e^{j(n+p)\phi} \quad (12)$$

$$\begin{aligned} \mathbf{H}_{II} = & -\hat{\rho} \frac{E_0}{\omega\mu_2\rho} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} n j^{-n} J_p(k_2d) \left[b_n H_{n+p}^{(1)}(k_2r) + c_n H_{n+p}^{(2)}(k_2r) \right] e^{j(n+p)\phi} \\ & + \hat{\phi} \frac{E_0 k_2}{j\omega\mu_2} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} j^{-n} J_p(k_2d) \left[b_n H_{n+p}^{(1)'}(k_2r) + c_n H_{n+p}^{(2)'}(k_2r) \right] e^{j(n+p)\phi} \end{aligned} \quad (13)$$

The a_n scattering coefficient in region-I is then determined by solving the modified system of equations. In general, the total scattering cross-section is given by the ratio of scattered power to the incident power per unit area [34], i.e.,

$$\sigma = \frac{4}{k_1} \left| \sum_{n=-\infty}^{\infty} a_n e^{jn\phi} \right|^2 \quad (14)$$

The backscattering cross-section is indispensable for many practical applications, which can be given by setting the $\phi = \pi$ in Equation (14) as

$$\sigma_b = \frac{4}{k_1} \left| \sum_{n=-\infty}^{\infty} (-1)^n a_n \right|^2 \quad (15)$$

3. NUMERICAL RESULTS AND DISCUSSION

Numerical results for the scattering properties of a DB cylinder coated with unmagnetized plasma are presented in this section. The term Normalized RCS will refer to the quantity $\frac{\sigma}{\lambda_0^2}$ in this discussion. The value of electron neutral collision frequency ν is taken 0.1 GHz or specified otherwise. The expressions for scattering coefficients were derived and implemented through a computer code. The obtained results were first validated by reducing the problem to special cases and comparing with relevant published literature [25–28]. For instance, it is reported in [25] that the uncoated DB cylinder behaves like a PEC when illuminated by a TM^Z plane wave. Hence, for such incidence the results of the DB cylinder coated with ordinary dielectric material and metamaterial (concentric model i.e., $d = 0$) were supposed to be similar to that of reported in [26]. Fig. 2 shows that our results are in complete agreement with results of [26] for both type of material coatings. This example was chosen because the value of relative permittivity of plasma varies from negative to positive over a frequency range used in this analysis.

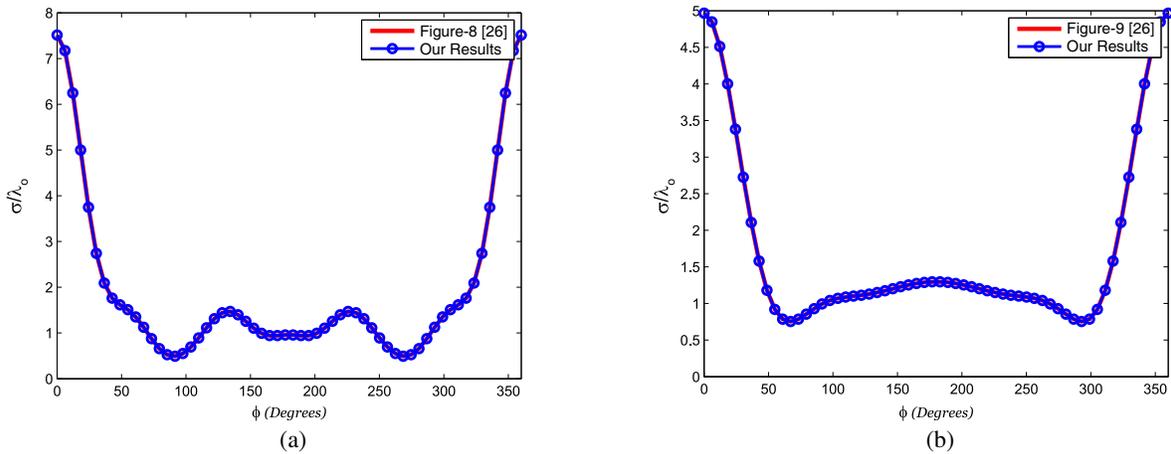


Figure 2. For the same input parameters, our results are in complete agreement with the results reported in [26] $a = 50$ mm, $b = 100$ mm, $f = 1$ GHz (a) $\epsilon_r = 9.8$, $\mu_r = 1$, (b) $\epsilon_r = -9.8$, $\mu_r = -1$.

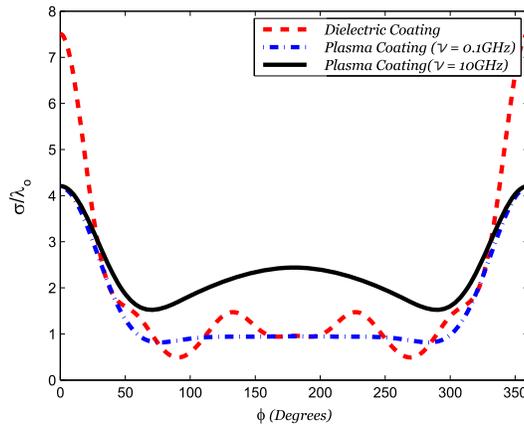


Figure 3. Comparison of ordinary dielectric coating with plasma coating over DB metamaterial $a = 50$ mm, $b = 100$ mm, $f = 1$ GHz, $\epsilon_r = 9.8$, $\mu_r = 1$.

The normalized RCS from a DB cylinder coated with unmagnetized plasma in contrast with ordinary dielectric is presented in Fig. 3. It can be seen that the forward scattering is reduced significantly and backscattering is uniform for a wide range of observation angles. This is what we can expect from an unmagnetized plasma sheath (equivalence to ENG material) at operating frequency less than the plasma frequency. However, an increase in backscattering cross section is observed when the electron-neutral collision frequency (ν) is increased. The electron-neutral collision frequency affects the imaginary part of the permittivity and hence contributes towards losses.

The effects of eccentricity are illustrated in Fig. 4. If the core is displaced from center, it is observed that significant variations occur in forward scattering cross-section while the backscattering cross-section remains steady. It is evident in Figs. 4(a), 4(b) and 4(c) that the RCS increases when the core is moved away from the source ($d > 0$) and decreases when the core is moved in opposite direction ($d < 0$). The eccentricity has negligible effects at operating frequencies less than the plasma frequency as shown in Fig. 4(d).

Figure 5(a) shows the RCS of the DB cylinder coated with unmagnetized plasma when operating frequency is equal to, above and below the plasma frequency. At conditions when the electron-neutral collision are very low (i.e, $\nu \rightarrow 0$), the resonance occurs at operating frequency equal to plasma frequency as shown in Fig. 5(b). It should be noted that in given circumstances (when the imaginary and real part of the permittivity simultaneously approaches to zero) the unusual behavior is expected as indicated

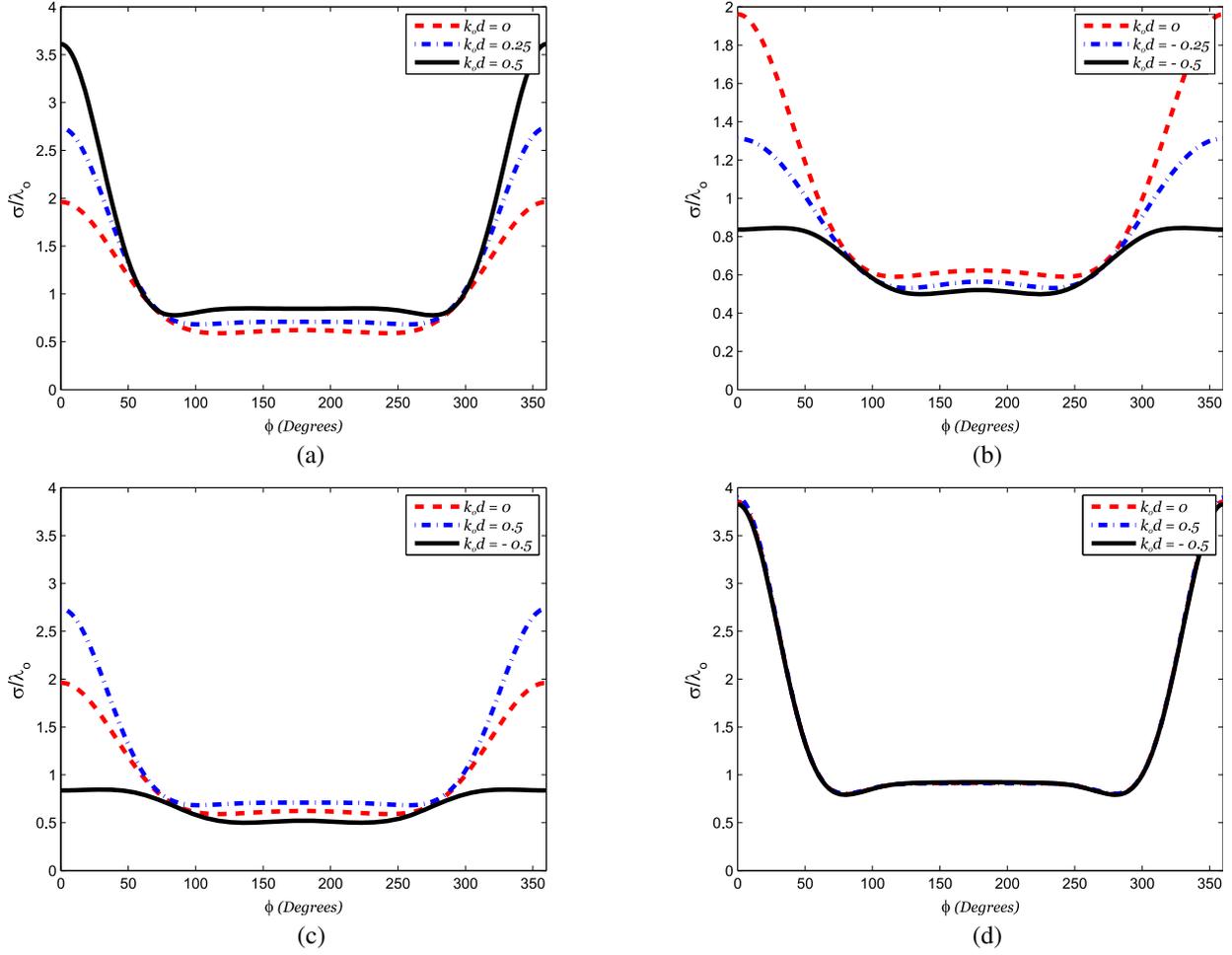


Figure 4. The effects of eccentric coating of plasma over DB metamaterial cylinder: $k_0a = 1.0$, $k_0b = 2.0$ (a) $f = 10$ GHz, (b) $f = 10$ GHz, (c) $f = 10$ GHz, (d) $f = 1$ GHz.

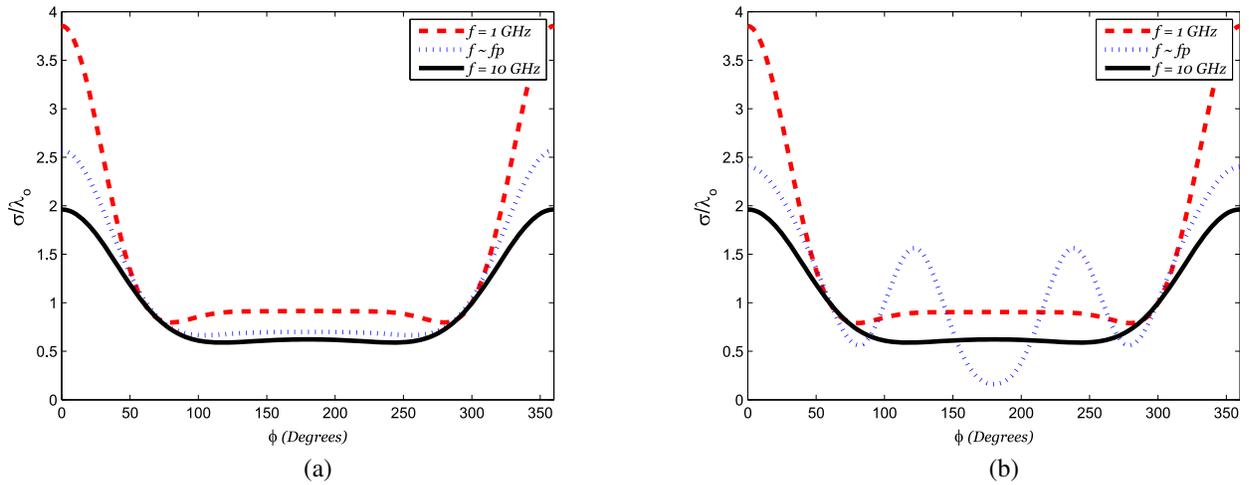


Figure 5. RCS of the plasma coated cylinder when the operating frequency f' is below, equal or above the plasma frequency f'_p : $k_0a = 1.0$, $k_0b = 2.0$ (a) $\nu = 0.1$ GHz, (b) $\nu \rightarrow 0$.

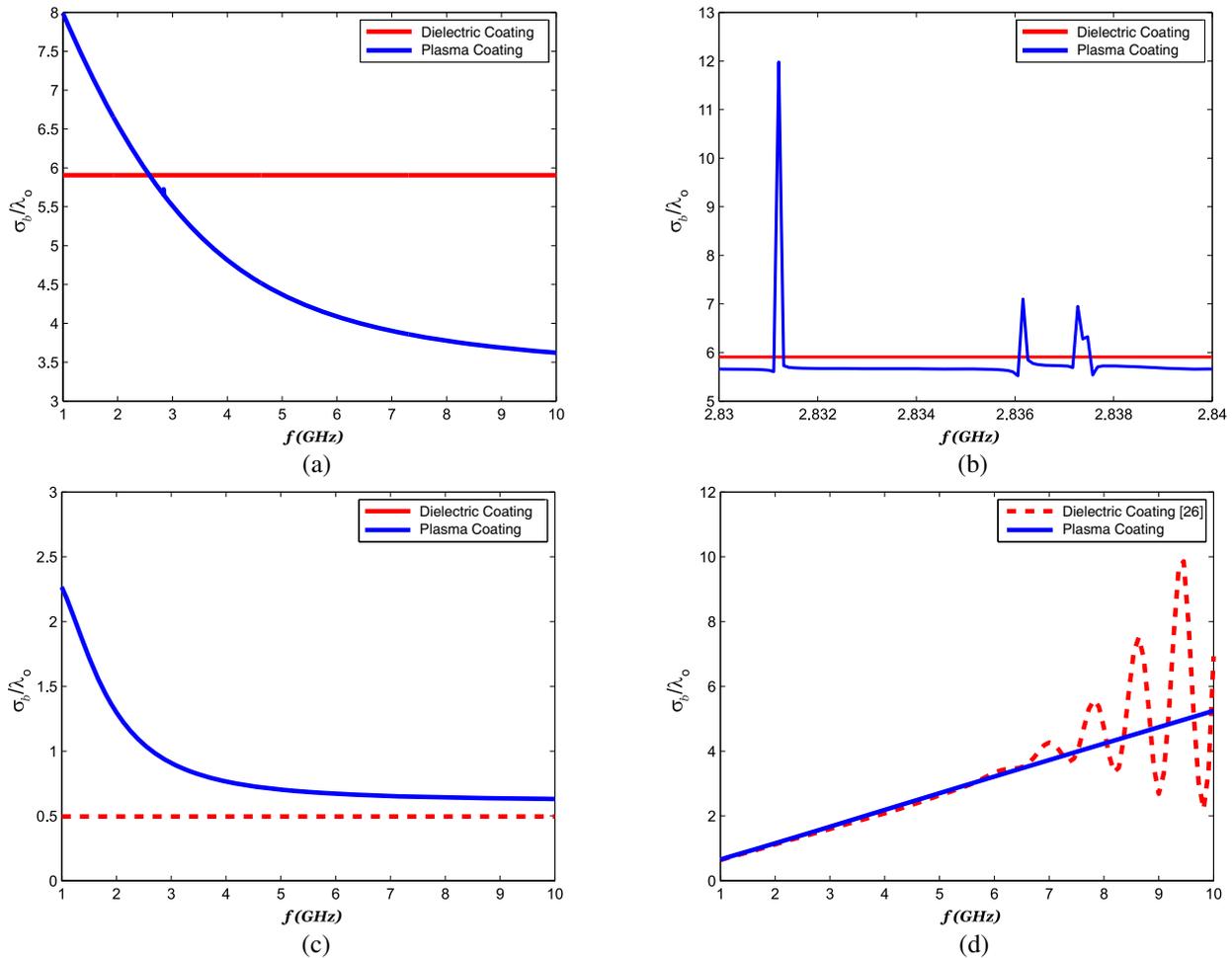


Figure 6. Monostatic RCS of plasma coated cylinder in comparison with dielectric coated cylinder: (a) $k_0a = 1.5, k_0b = 3.0, \epsilon_r = 9.8, \mu_r = 1.0, \nu \rightarrow 0$, (b) $k_0a = 1.5, k_0b = 3.0, \epsilon_r = 9.8, \mu_r = 1.0, \nu \rightarrow 0$, (c) $k_0a = 1.0, k_0b = 2.0, \epsilon_r = 9.8, \mu_r = 1.0, \nu = 10 \text{ GHz}$, (d) $a = 50 \text{ mm}, b = 1.2a, \epsilon_r = 2.2, \mu_r = 1.0, \nu \rightarrow 0$.

by [29].

The backscattering (monostatic) RCS of the plasma coated DB cylinder in comparison with that of dielectric is presented in Fig. 6(a). The backscattering cross-section decreases with increase in operating frequency while it tends to attain a constant value when operating frequency is above 10 GHz. This is obvious as the relative permittivity approaches to unity for such high frequencies. One can observe the unusual surge in monostatic RCS at operating frequencies closer to the plasma frequency. Fig. 6(b) presents a close up analysis of the situation near plasma frequencies. It is observed that for higher electron-neutral collision frequency values, this surge is suppressed as illustrated in Fig. 6(c). It is indicating that the dissipation effects are significant for such values of ν , because in this case the imaginary part of permittivity is not zero. An interesting feature is revealed when the physical size of the object is fixed and operating frequencies are varied, as illustrated in Fig. 6(d). The monostatic RCS of the plasma coated cylinder has a linear observed over a range of frequencies while it is in case of a dielectric coating, the object starts oscillating after a certain frequency.

The effects of layer thickness are illustrated in Fig. 7. It is evident from Fig. 7(a) that the plasma layer thickness has considerable effects on RCS at low operating frequencies. However, the effect of layer thickness seems negligible in case of high operating frequencies as shown in Fig. 7(b). This is what we can expect when relative dielectric constant of the medium tends to unity.

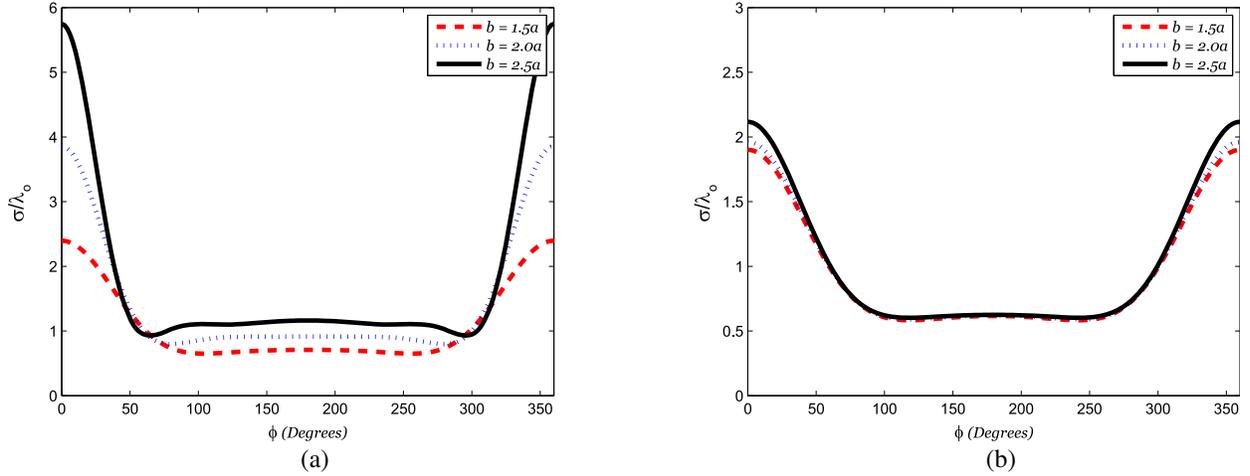


Figure 7. Effects of the thickness of plasma layer are shown in this figure: (a) $k_0a = 1.0$, $f = 1$ GHz, (b) $k_0a = 1.0$, $f = 10$ GHz.

Since the plasma frequency is given by the expression $f_p = \sqrt{\frac{n_e e^2}{4\pi^2 \epsilon_0 m_e}}$, therefore, electron number density (n_e) is the key parameter to alter the plasma frequency. Consequently, we may achieve the phenomenon discussed above at desired operating frequencies.

4. CONCLUSION

The electromagnetic scattering properties of cylindrical objects made from DB metamaterial covered with uniform and non-uniform plasma sheath are studied. The effects of eccentricity, electron-neutral collision frequency, electron number density, layer thickness, object size and operating frequencies on radar cross-section of the object have been investigated. It is observed that a plasma coating can alter the RCS of the object significantly. Many interesting and unusual features that could find engineering, space and military applications are revealed in this study.

It is found that the RCS (forward and backward) of an object can be modified (increased or decreased) by choosing appropriate values of plasma parameters. The enhancement and reduction in RCS of the object is desirable in distinct applications. For example, the RCS of the commercial planes, buried conduits and pipes are required to have large value for easy detection while low RCS values are demanded in case of military applications.

The monostatic RCS of the plasma coated object remains low for wide range of frequencies. However, anomalous peaks are observed at some specific frequencies. The plasma coated DB metamaterial with such fascinating attributes can be a good candidate for defense applications. For example, it can be used in land mines where easy detection characteristics are required in demining phase along with the general requirement of camouflage.

Besides the space plasmas, the artificial and tunable metamaterial plasmas have also been achieved that can be used for cladding of different objects. Hence, this study may serve as a useful reference for researchers working in plasma stealth technology, metamaterial plasmas, inverse scattering from objects having plasma layer over the surface.

REFERENCES

1. Kraus, J. D., *Detection of Sputnik-i and Sputnik-ii by CW Reflection*, 1958.
2. Jastrow, R. and C. A. Pearse, *Atmospheric Drag on a Satellite*, 1957.
3. Dolph, C. L. and H. Weil, "On the change in radar cross-section of a spherical satellite caused by a plasma sheath," *Planetary and Space Science*, Vol. 6, 123–132, 1961.

4. Rusch, W. V. T., "Radiation from a plasma-clad axially-slotted cylinder," Electrical Engineering Department, University of Southern California, 1962.
5. Swarner, W. G. and L. Peters, Jr., "Radar cross sections of dielectric or plasma coated conducting spheres and circular cylinders," *IEEE Transactions on Antennas and Propagation*, Vol. 11, No. 5, 558–569, 1963.
6. Chen, H. C. and D. K. Cheng, "Scattering of electromagnetic waves by an anisotropic plasma-coated conducting cylinder," *IEEE Transactions on Antennas and Propagation*, Vol. 12, NO. 3, 348–353, 1964.
7. Rybak, J. P. and R. J. Churchill, "Progress in reentry communications," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 5, 879–894, 1971.
8. Zheng, L., Q. Zhao, and X. J. Xing, "Effect of plasma on electromagnetic wave propagation and THZ communications for reentry flight," *Applied Computational Electromagnetics Society Journal*, Vol. 30, No. 11, 2015.
9. Shi, L., B. Guo, Y. Liu, and Ji. Li, "Characteristic of plasma sheath channel and its effect on communication," *Progress In Electromagnetics Research*, Vol. 123, 321–336, 2012.
10. He, G., Y. Zhan, N. Ge, Y. Pei, B. Wu, and Y. Zhao, "Channel characterization and finite-state Markov channel modeling for time-varying plasma sheath surrounding hypersonic vehicles," *Progress In Electromagnetics Research*, Vol. 145, 299–308, 2014.
11. Shan, Z.-X., "Electromagnetic scattering by an impedance cylinder coated eccentrically with a chiroplasma cylinder," *IEE Proceedings — Microwaves, Antennas and Propagation*, Vol. 141, No. 4, 279–284, 1994.
12. Yin, B., F. Yang, H. Hao, and C. Li, "Analysis of scattering wave for a conducting cylinder coated with eccentric plasma," *Radiation Effects and Defects in Solids*, Vol. 168, No. 5, 344–351, 2013.
13. Li, J., L.-X. Guo, Y.-C. Jiao, and R. Wang, "Composite scattering of a plasma-coated target above dispersive sea surface by the ADE-FDTD method," *IEEE Geoscience and Remote Sensing Letters*, Vol. 10, No. 1, 4–8, 2013.
14. Li, J., L.-X. Guo, Q. He, and B. Wei, "Investigation on scattering from a plasma-coated target over a rough sea surface using a multi-hybrid method," *Waves in Random and Complex Media*, Vol. 22, No. 3, 344–355, 2012.
15. Liu, S. and S. Zhong, "FDTD study on scattering for conducting target coated with magnetized plasma of time-varying parabolic density distribution," *Progress In Electromagnetics Research M*, Vol. 22, 13–25, 2012.
16. Sakai, O. and K. Tachibana, "Plasmas as metamaterials: A review," *Plasma Sources Science and Technology*, Vol. 21, No. 1, 013001, 2012.
17. Sakai, O., "Emerging aspects in a plasma-metamaterial composite," *2011 XXXth URSI IEEE General Assembly and Scientific Symposium*, 1–4, 2011.
18. De Ridder, C.-M. and L. G. Peterson, "Scattering from a homogeneous plasma cylinder of infinite length," Technical report, DTIC Document, 1962.
19. Geng, Y., X. Wu, and L.-W. Li, "Analysis of electromagnetic scattering by a plasma anisotropic sphere," *Radio Science*, Vol. 38, No. 6, 2003.
20. Wu, X. P., J.-M. Shi, Z. S. Chen, and B. Xu, "A new plasma antenna of beam-forming," *Progress In Electromagnetics Research*, Vol. 126, 539–553, 2012.
21. Bulanov, S. V., T. Z. Esirkepov, J. Koga, and T. Tajima, "Interaction of electromagnetic waves with plasma in the radiation-dominated regime," *Plasma Physics Reports*, Vol. 30, No. 3, 196–213, 2004.
22. Umeda, T., *Electromagnetic Waves in Plasma*, INTECH Open Access Publisher, 2011.
23. Batchelor, D. B., R. C. Goldfinger, and H. Weitzner, "Propagation and absorption of electromagnetic waves in fully relativistic plasmas," *Physics of Fluids (1958–1988)*, Vol. 27, No. 12, 2835–2846, 1984.
24. Ivanov, S. T. and E. G. Alexov, "Electromagnetic waves in a plasma waveguide," *Journal of plasma Physics*, Vol. 43, No. 1, 51–67, 1990.

25. Khalid, M., S. Ahmed, A. A. Syed, and Q. A. Naqvi, "Electromagnetic response of a circular db cylinder in the presence of chiral and chiral nihility metamaterials," *Progress In Electromagnetics Research M*, Vol. 21, 253–266, 2011.
26. Li, C. and Z Shen, "Electromagnetic scattering by a conducting cylinder coated with metamaterials," *Progress In Electromagnetics Research*, Vol. 42, 91–105, 2003.
27. Kishk, A., R. P. Parrikar, A. Z. Elsherbeni, et al., "Electromagnetic scattering from an eccentric multilayered circular cylinder," *IEEE Transactions on Antennas and Propagation*, Vol. 40, No. 3, 295–303, 1992.
28. Roumeliotis, J. A., J. G. Fikioris, and G. P. Gounaris, "Electromagnetic scattering from an eccentrically coated infinite metallic cylinder," *Journal of Applied Physics*, Vol. 51, No. 8, 4488–4493, 1980.
29. Lakhtakia, A. and J. B. Geddes, III, "Scattering by a nihility cylinder," *AEU-International Journal of Electronics and Communications*, Vol. 61, No. 1, 62–65, 2007.
30. Ahmed, S. and Q. A. Naqvi, "Electromagnetic scattering from a chiral-coated nihility cylinder," *Progress In Electromagnetics Research Letters*, Vol. 18, 41–50, 2010.
31. Lindell, I. V. and A. H. Sihvola, "Electromagnetic boundary conditions defined in terms of normal field components," *IEEE Transactions on Antennas and Propagation*, Vol. 58, No. 4, 1128–1135, 2010.
32. Lindell, I. V. and A. H. Sihvola, "Electromagnetic boundary and its realization with anisotropic metamaterial," *Physical Review E*, Vol. 79, No. 2, 026604, 2009.
33. Sihvola, A., H. Walien, P. Ylä-Oijala, J. Markkanen, and I. V. Lindell, "Material realizations of extreme electromagnetic boundary conditions and metasurfaces," *2011 XXXth URSI IEEE General Assembly and Scientific Symposium*, 1–4, 2011.
34. Balanis, C. A., *Advanced Engineering Electromagnetics*, 1989.