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Properties and Applications of Error Coefficient Matrix in Linear Antenna Array Design

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Abstract—This paper presents the theoretical framework for a new technique in the field of linear antenna arrays with amplitude control called error coefficient matrix. First of all, the array factor is expressed as a summation of contribution from the elements of the array. It will be shown that for small errors in excitation amplitude, the error in the overall radiation pattern at a given angle is a summation of errors contributed by the individual elements of the array at that angle. An error coefficient matrix is proposed, and its properties are discussed in great detail. The accuracy of the proposed method is investigated for varying levels of errors in weights and for varying number of error elements, using Monte-Carlo simulation. Finally, the applications of this new technique in the field of antenna arrays are presented.

1. INTRODUCTION

Fault analysis is a hot topic of research in the field of antenna arrays. Manufacturing tolerances [1] in the excitation amplitudes result in the deviation of radiation pattern from the desired pattern. In the recent past, several methods have been proposed for detection of such faults. Methods based on Genetic Algorithms (GA) [2], Neural Networks (NN) [3], Bacterial Foraging Optimization (BFO) [4], Bayesian Compressive Sensing [5], Exponentially Weighted Moving Average Scheme (EWMA) [6], etc. were proposed for fault detection, but no method has been proposed for precise location of the error in the array. In [7] a practical approach has been suggested, where the error elements are identified by placing a small number of probes around the array. In [8] failure diagnosis of uniform linear array in the presence of mutual coupling has been presented, but it requires the use of an optimization method to compute the excitation coefficients and also it cannot be used for detection of error elements in the array. An analytical technique which can locate the position and magnitude of error would be highly beneficial for dynamical correction of faults. Analytical techniques [9, 10] have been proposed for radiation pattern tolerance arrays, but no method has been proposed for fault detection and correction.

Radiation pattern synthesis is another area of research in the field of antenna arrays. Several methods are available for design of antenna arrays with specified Side Lobe Level (SLL) and beam width. These approaches are analytical methods such as Taylor's method and Chebyshev's method [11] or are optimization based methods [12–16]. Over the past few decades, many accurate techniques have been developed for antenna pattern synthesis [17–24]. A generalized projection method was proposed in [18] for synthesis of radiation patterns. It is based on a minimization algorithm and requires high computational time. The optimization based techniques [12–16, 20–23] provide the flexibility to design any arbitrary radiation pattern but these techniques require high amount of computational time. An analytical technique which can predict the excitation amplitudes from the given design requirements would greatly reduce the computational time. In [24] an analytical method was proposed, but it is applicable only for symmetric antenna arrays. A simple and general approach is required. In this paper

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an attempt is made to address these issues. The theoretical framework for a new technique called the error coefficient matrix is presented, which focuses on linear antenna arrays with amplitude control. Also, its properties and applications are presented. The simplicity of this method and its effectiveness is demonstrated for null placement in the radiation pattern.

The organization of the paper is as follows. The next section presents the mathematical foundation for the proposed technique. The accuracy of the proposed method is discussed for varying levels of errors in weights and for varying number of error elements, using Monte-Carlo simulation. In Section 3 the properties of the error coefficient matrix with relevant proofs are presented in great detail and its applications are presented in the fourth section. Also an example is presented, where the proposed technique is used for null placement in the radiation pattern. In the last and final section conclusions are presented along with the future scope of work.

2. MATHEMATICAL FOUNDATIONS

The radiation characteristics of an antenna array depend on several factors, such as the amplitude of excitation; inter element spacing and phase of excitation of the elements of the array. The radiation characteristics of the elements of the array are the same for all the elements. The above mentioned factors affect the array factor and so we are interested in the array factor of the antenna array, but not in the radiation characteristics of the individual elements in the array. The array factor of an 'N' element array can be represented as:

$$\mathrm{AF}(\theta) = \sum_{i=0}^{N-1} a_i e^{J\beta d_i \cos(\theta)}. \tag{1}$$

where β is $2\pi/\lambda$ (λ is the wavelength) a_i the excitation amplitude of the $(i+1)$th element, d_i the distance of the $(i+1)$th element from the first element and $J = \mathrm{sqrt}(-1)$. The range of angle 'θ' is taken from 0° to 90°.

If all the elements of the array are separated by a uniform spacing of k, the array factor can be expressed as follows:

$$\mathrm{AF}(\theta) = \sum_{i=0}^{N-1} a_i e^{J\beta ik \cos(\theta)}.$$

Expanding the above equation and separating it into real and imaginary term results in

$$\begin{aligned} \mathrm{AF}(\theta) &= \sum_{i=0}^{N-1} a_i e^{J\beta ik \cos(\theta)}. \\ \mathrm{AF}(\theta) &= \sum_{i=0}^{N-1} a_i \cos\left(\beta ik \cos(\theta)\right) + J \sum_{i=0}^{N-1} a_i \sin\left(\beta ik \cos(\theta)\right). \end{aligned} \tag{2}$$

As we are interested in the power radiated by the array, we find the square of the magnitude of the array factor. So finding the absolute value of Equation (2) and then squaring it on both sides results in

$$\begin{aligned} |\mathrm{AF}(\theta)|^2 &= \left(\sum_{i=0}^{N-1} a_i \cos(\beta ik \cos(\theta))\right)^2 + \left(\sum_{i=0}^{N-1} a_i \sin(\beta ik \cos(\theta))\right)^2. \\ |\mathrm{AF}(\theta)|^2 &= \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \cos\left(\beta ik \cos(\theta)\right) \cos\left(\beta jk \cos(\theta)\right) \\ &\quad + \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \sin\left(\beta ik \cos(\theta)\right) \sin\left(\beta jk \cos(\theta)\right). \\ |\mathrm{AF}(\theta)|^2 &= \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \cos\left(\beta k(i-j) \cos(\theta)\right). \end{aligned} \tag{3}$$

From Equation (3) it can be seen that the power radiated by an array is the sum of the contribution from elements of the array. If a single error is introduced in the excitation amplitude at the pth position of the array and the magnitude of the error is δa_p, i.e., $a_p \to a_p + \delta a_p$. The power radiated at the angle 'θ' after the occurrence of error would be:

$$|\mathrm{AF}(\theta)_p|^2 = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \cos\left(\beta k(i-j)\cos(\theta)\right) + \delta a_p \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(p-j)\cos(\theta)\right) + (\delta a_p)^2. \quad (4)$$

The deviation of the resultant radiation pattern from that of the desired radiation pattern at the angle θ is given by the difference between Euations (4) and (3). This error is given by:

$$\xi(\theta) = \delta a_p \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(p-j)\cos(\theta)\right) + (\delta a_p)^2. \quad (5)$$

If the magnitude of the deviation $\xi(\theta)$ is large compared to, the error in excitation amplitude δa_p (i.e., for small errors in the amplitude of excitation) the term $(\delta a_p)^2$ can be neglected. The resulting expression becomes:

$$\xi(\theta) = \delta a_p \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(p-j)\cos(\theta)\right) = \delta a_p u_{p\theta}^N. \quad (6)$$

The term $u_{p\theta}^N$ is called as the error coefficient for pth position measured at the angle θ of an N element array and is given by

$$u_{p\theta}^N = \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(p-j)\cos(\theta)\right). \quad (7)$$

If two elements have errors in their excitation amplitudes, then the array factor becomes:

$$\begin{aligned} |\mathrm{AF}(\theta)_{pq}|^2 \;=\; & \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} a_i a_j \cos\left(\beta k(i-j)\cos(\theta)\right) + \delta a_p \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(p-j)\cos(\theta)\right) + (\delta a_p)^2 + \ldots \\ & + \delta a_q \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(q-j)\cos(\theta)\right) + (\delta a_q)^2 + 2\delta a_p \delta a_q \cos\left(\beta k(p-q)\cos(\theta)\right). \end{aligned} \quad (8)$$

The errors in the magnitude of excitation are δa_p and δa_q, and the positions at which they occur are given by p and q, respectively. The deviation of the radiation pattern in this case from that of the desired radiation pattern can be given by the difference between Equation (8) and Equation (3):

$$\begin{aligned} \xi(\theta) \;=\; & \delta a_p \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(p-j)\cos(\theta)\right) + (\delta a_p)^2 + \delta a_q \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(q-j)\cos(\theta)\right) + (\delta a_q)^2 + \ldots \\ & + 2\delta a_p \delta a_q \cos\left(\beta k(p-q)\cos(\theta)\right). \end{aligned} \quad (9)$$

We can safely neglect the terms δa_p^2, δa_q^2 and $\delta a_p \delta a_q$, by properly choosing the angle θ (in the case of pattern synthesis) to be within the main lobe range (to be discussed later) or by limiting the values of the errors in excitation amplitudes (this is automatically fulfilled in the case of fault detection due to manufacturing tolerances as the tolerance limits are usually small). The expression of error then becomes:

$$\xi(\theta) = \delta a_p u_{p\theta}^N + \delta a_q u_{q\theta}^N. \quad (10)$$

$u_{p\theta}^N$ is the error coefficient for pth position and $u_{q\theta}^N$ the error coefficient for qth position, both of them measured at the angle θ. From the above equation, it can be seen that the total error in the radiation pattern is a summation of errors contributed by the individual elements. Generalizing this principle in the case of n errors, the total deviation in the radiation pattern at a given angle θ is given as:

$$\xi(\theta) = \delta a_{i_1} u_{i_1\theta}^N + \delta a_{i_2} u_{i_2\theta}^N + \ldots + \delta a_{i_{n-1}} u_{i_{n-1}\theta}^N + \delta a_{i_n} u_{i_n\theta}^N. \quad (11)$$

where $i_1, i_2, \ldots, i_n$ are the positions at which the errors occurs and $\delta a_{i_1}, \delta a_{i_2}, \ldots, \delta a_{i_n}$ are the magnitude of errors in excitation at $i_1, i_2, \ldots$ and i_n respectively. Since the remaining elements have no error in their excitation amplitude, their contribution to the overall error in the radiation pattern would be zero. Hence Equation (11) can also be written as:

$$\xi(\theta) = \delta a_1 u_{1\theta}^N + \delta a_2 u_{2\theta}^N + \ldots + \delta a_{N-1} u_{(N-1)\theta}^N + \delta a_N u_{N\theta}^N. \tag{12}$$

In the above equation, there are N unknowns of which there are $N-n$ zeros, and the remaining are the errors in the excitation amplitudes. So, if we have at least N equations, then it is possible to find the errors in the excitation amplitudes of all the elements. These equations can be obtained easily by choosing N different angles and measuring the overall error in the radiation pattern. If $\theta_1, \theta_2, \ldots, \theta_N$ are the angles at which the errors are measured, and $\xi(\theta_1), \xi(\theta_2), \ldots, \xi(\theta_N)$ are the overall errors calculated at these angles, the error equations can be written in terms of matrix notation as given below:

$$\begin{bmatrix} \xi(\theta_1) \\ \xi(\theta_2) \\ \vdots \\ \xi(\theta_N) \end{bmatrix} = \begin{bmatrix} u_{1\theta_1}^N & u_{2\theta_1}^N & \cdots & u_{N\theta_1}^N \\ u_{1\theta_2}^N & u_{2\theta_2}^N & \cdots & u_{N\theta_2}^N \\ \vdots & \vdots & & \vdots \\ u_{1\theta_N}^N & u_{2\theta_N}^N & \cdots & u_{N\theta_N}^N \end{bmatrix} \begin{bmatrix} \delta a_1 \\ \delta a_2 \\ \vdots \\ \delta a_N \end{bmatrix}. \tag{13}$$

$$[E(\theta)] = \left[U_\theta^N\right][\delta A].$$

$[E(\theta)]$ is called the error matrix, $[U_\theta^N]$ called the error coefficient matrix of an N element array, and $[\delta A]$ is the matrix containing the magnitude of errors of individual antenna elements. Since the error coefficient matrix and the error matrix can be determined, we can know the errors in the excitation amplitudes.

Before discussing the properties of the error coefficient matrix, the accuracy of the proposed method has to be investigated. Monte-Carlo simulation analysis is performed to investigate the reliability of the proposed technique. Monte-Carlo analysis is performed for varying levels of error in amplitudes and for varying number of error elements. A 12-element array was used, and the Monte-Carlo simulation was performed for 2000 generations. The R.M.S value of the difference between the actual error in radiation pattern and that of the error obtained from the simplified expression in Equation (12) is calculated for varying number of errors in amplitudes and varying number of elements. The results are shown in Table 1. The plot showing the variation in R.M.S value with respect to the angle from 0° to 90° is shown in Fig. 1.

From Table 1, it can be observed that in a given array the R.M.S value of error increases with increase in error magnitude and also with the increase in the number of error elements. However, the value of error is small, so the proposed method can be used for calculating the errors in radiation pattern for arrays having small errors in excitation amplitude (i.e., for ±10% error limit) and having a small number of error elements.

An interesting observation can be made from Fig. 1. It can be observed that the R.M.S value of error is almost the same at all angles except for the angles within the main lobe range, and for the angles in the main lobe range, the R.M.S value of error is small. This is quite helpful to improve the accuracy of the proposed method for varying levels of error weights and for varying numbers of error elements.

Table 1. R.M.S values of difference between actual error and the error calculated using the proposed method, modeled by Monte-Carlo technique.

Number of error elements	Error % in excitation amplitudes		
	±10%	±5%	±1%
2	2.65	1.31	0.264
5	6.463	3.47	0.63
7	10.341	5.064	1

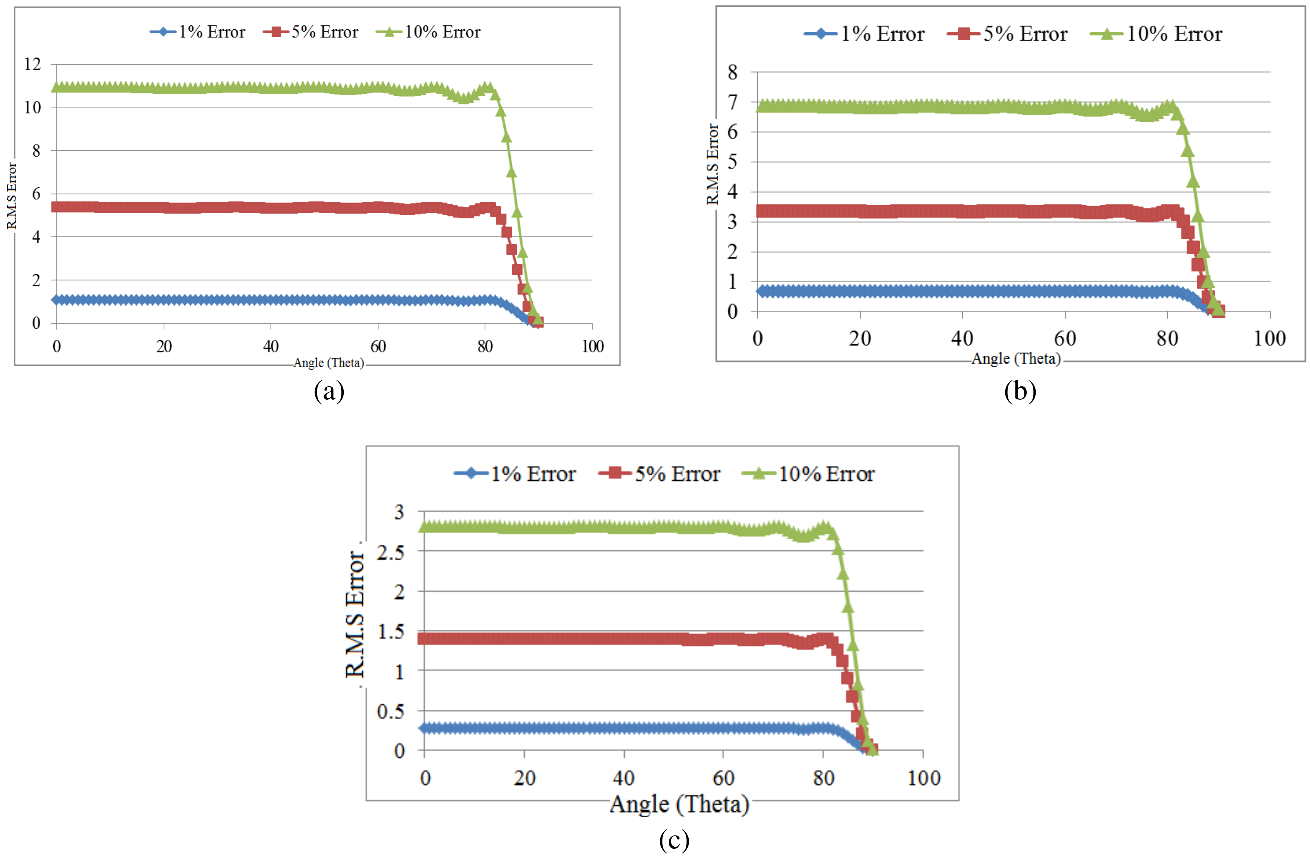

Figure 1. R.M.S error vs. angle when number of errors are (a) 7, (b) 5, (c) 2.

This observation indicates that the error calculated using Equation (12) is approximately the same as the original error in the radiation pattern, at these angles. So in Equation (12), by selecting the angles $\theta_1, \theta_2, \ldots, \theta_N$ within the main lobe range, the accuracy of the results obtained using Equation (13) can be improved. The properties of the error coefficient matrix are discussed in the next section.

3. PROPERTIES OF ERROR COEFFICIENT MATRIX

Each element of an error coefficient matrix is given by Equation (7)

$$u_{i\theta}^{N} = \sum_{j=0}^{N-1} 2a_j \cos\left(\beta k(i-j)\cos(\theta)\right).$$

Writing the above equation in more general terms as an element of an array results in

$$u_{ij} = \sum_{p=0}^{N-1} 2a_p \cos\left(\beta k(j-p)\cos(\theta_i)\right). \tag{14}$$

where j is the position at which we are calculating the error coefficient and is equivalent to the term i in Equation (7). Similarly, p is equivalent to the term j in Equation (7).

There are several properties of this error coefficient matrix which are discussed in this section. The properties are as follows:

1. *Angle symmetry*: From Equation (14) we can clearly see that if we replace θ_i with $180 - \theta_i$, the elements of the error coefficient matrix will remain the same. So, we can choose the values of θ

with the range 0° to 90°. This property arises due to the symmetry of the radiation pattern with respect to 90°.

Proof:

$$u_{ij} = \sum_{p=0}^{N-1} 2a_p \cos\left(\beta k(j-p)\cos(180-\theta_i)\right) = \sum_{p=0}^{N-1} 2a_p \cos\left(\beta k(j-p)\cos(\theta_i)\right).$$

2. *Excitation symmetry*: If the elements of the array have symmetrical amplitude distribution, such as Chebyshev's distribution, then the elements of the error coefficient matrix would also be symmetrical.

   Proof:

   Since the amplitude distribution is symmetrical

$$a_p = a_{N-1-p}.$$
$$u_{ij} = \sum_{p=0}^{N-1} 2a_p \cos\left(\beta k(j-p)\cos(\theta_i)\right). \quad (15)$$

   The element of the error coefficient matrix located at the symmetrical position to the element u_{ij} is $u_{i(N-1-j)}$. From (14) $u_{i(N-1-j)}$ is given by

$$u_{i(N-1-j)} = \sum_{p=0}^{N-1} 2a_p \cos\left(\beta k(N-1-j-p)\cos(\theta_i)\right).$$

   Substituting Equation (15) in the above equation we get

$$u_{i(N-1-j)} = \sum_{p=0}^{N-1} 2a_{N-1-p} \cos\left(\beta k(j-(N-1-p))\cos(\theta_i)\right).$$
$$u_{i(N-1-j)} = \sum_{k=0}^{N-1} 2a_k \cos\left(\beta k(j-k)\cos(\theta_i)\right).$$
$$u_{i(N-1-j)} = u_{ij}.$$

   So arrays having symmetrical amplitude distributions have a symmetrical error coefficient matrix. This is demonstrated with the help of a 12-element array having Chebyshev's distribution. The excitation amplitude of each element and the angles at which the error is measured are shown in Table 2 and Table 3, respectively. The error coefficient matrix is calculated at these angles using MATLAB® and is shown below in Table 3. It can be seen that the error coefficient matrix is symmetrical about its center.

3. If the error coefficient matrix is singular, then the errors in the excitation amplitude cannot be determined uniquely using Equation (13). Several solutions are possible, and all of them may not satisfy the design requirements. Those solutions which do not satisfy the requirements can be discarded.

Table 2. Excitation amplitude and errors in excitation amplitude of a 12 element array.

Excitation Amplitude	0.008	0.053	0.192	0.452	0.772	1.000	1.000	0.772	0.452	0.192	0.053	0.008
Errors	0.025	0.025	0	0.011	0.027	0.035	0	0	0	0	0	0

Table 3. Angle at which the errors are measured and the corresponding errors in the radiation pattern.

Angle	26.14	55.54	23.88	74.19	88.44	65.72	30.95	52.57	9.70	81.57	79.17	73.60
Error	0.001	0.003	0.001	0.039	1.164	0.021	0.001	0.002	0.000	0.188	0.027	0.046

4. APPLICATIONS

The technique presented in this paper has many applications in the field of antenna arrays. Some of these applications are listed below:

1. For fault location and fault correction, one should know the location of the occurrence of fault and the magnitude of the fault. Faults occur due to manufacturing tolerances in antenna arrays. The tolerance limit is small and hence, errors in the excitation amplitudes are also small. Using the proposed technique, it is possible to detect the location of faults and the magnitude of error in the excitation amplitude. If the error coefficient matrix is non-singular, then from Equation (13) it is possible to detect and correct the fault. But for arrays having symmetrical distributions, the error coefficient matrix is singular, and hence there will be more than one solution for Equation (13). In this case, the final solution has to be selected from all the possible solutions of Equation (13) after testing them for the actual occurrence of faults.
2. For the design of antenna arrays, having the desired radiation pattern, one should know the amplitude distribution of the elements of the array. For designing the arrays having the desired pattern, the deviation of the desired radiation pattern from that of a uniformly excited linear array is calculated at various angles (these angles should be selected in such a way that the deviation is large). The error coefficient matrix is then determined for these angles, and from Equation (13) the errors in the excitation amplitudes are calculated. These errors when added to the uniform distribution give the amplitude distribution needed to generate the desired radiation pattern.
3. This technique can be used for reduction of Side Lobe Level (SLL) and for null placement (or placement of minima).

The simplicity and effectiveness of the proposed technique can be demonstrated using the problem of a null placement. Consider a uniformly excited antenna array consisting of 12 elements, i.e., $N = 12$. The design problem is to place a null point at 25°, keeping the radiation pattern unchanged at other angles. First, we generate the error matrix by taking the error at 25° to be −100 and the error at remaining angles is taken as zero. The error coefficient matrix is calculated for 100 angles. By increasing the number of angles, in effect, we are increasing the number of observation points, and hence the accuracy of the result improves. The errors in the excitation magnitudes needed for null placement at 25° are obtained from Equation (13) using MATLAB®. These values are shown in Table 4. The final excitation amplitudes are shown in Table 5. The final excitation amplitudes are obtained by the adding the values of errors obtained from solving Equation (13) to the uniform amplitude distribution. These tables also depict the values needed for null placement at 40° and 55°. The average simulation time was found

Table 4. Errors in excitation amplitude.

Errors at 25°	Errors at 40°	Errors at 55°
−0.1722	0	0
0	0	−0.0527
0	0	0.4101
0.1784	0	−0.0889
0	0	−0.3986
0.0351	−0.1192	0
0	0	0.2715
−0.1107	0.2948	0
0	−0.3154	0
−0.2313	0.1573	0
0.2890	0.1198	0
0	−0.1548	−0.1695

Table 5. Excitation amplitudes for null placement.

At 25°	At 40°	At 55°
0.8278	1.0000	1.0000
1.0000	1.0000	0.9473
1.0000	1.0000	1.4101
1.1784	1.0000	0.9111
1.0000	1.0000	0.6014
1.0351	0.8808	1.0000
1.0000	1.0000	1.2715
0.8893	1.2948	1.0000
1.0000	0.6846	1.0000
0.7687	1.1573	1.0000
1.2890	1.1198	1.0000
1.0000	0.8452	0.8305

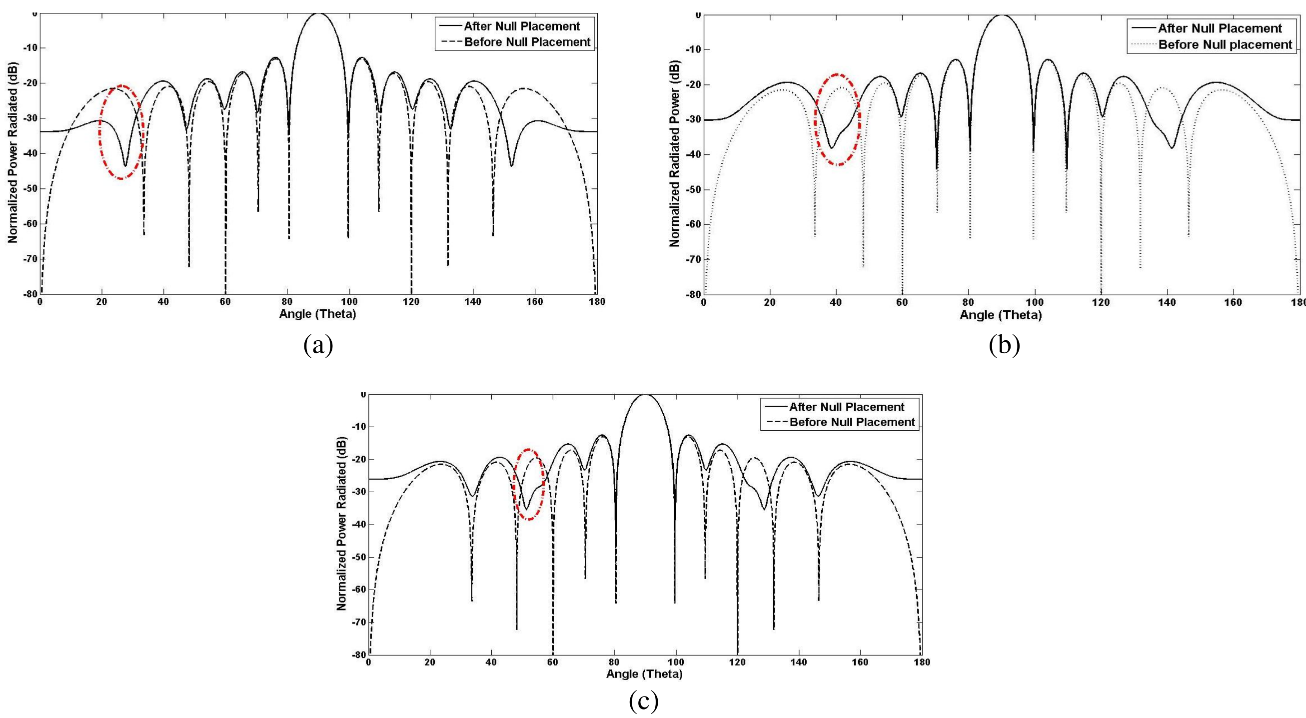

Figure 2. Radiation pattern after null placement at (a) 25°, (b) 40° and (c) 55°.

to be 4.30 Sec. The simulations were performed on a Pentium Core Duo Processor having a processor speed of 2 GHz and 1 GB RAM. The computational time indicates that the proposed technique is simple and can be used for real time implementation.

Finally, the radiation pattern obtained after null placement (or placement of minima) along with the radiation pattern before null placement (i.e., uniformly excited array) is shown in Fig. 2. For all cases. The placement of null point is indicated by the dotted circle.

$$\left[U_\theta^N\right] = \begin{bmatrix} 0.004 & -0.004 & 0.004 & -0.003 & 0.002 & -0.001 & -0.001 & 0.002 & -0.003 & 0.004 & -0.004 & 0.004 \\ -0.002 & 0.000 & 0.002 & -0.001 & -0.002 & 0.001 & 0.001 & -0.002 & -0.001 & 0.002 & 0.000 & -0.002 \\ 0.004 & -0.004 & 0.003 & -0.003 & 0.002 & -0.001 & -0.001 & 0.002 & -0.003 & 0.003 & -0.004 & 0.004 \\ -0.016 & -1.988 & -2.592 & -1.411 & 0.742 & 2.384 & 2.384 & 0.742 & -1.411 & -2.592 & -1.988 & -0.016 \\ 8.722 & 9.069 & 9.349 & 9.562 & 9.704 & 9.776 & 9.776 & 9.704 & 9.562 & 9.349 & 9.069 & 8.722 \\ 0.239 & 0.313 & -0.067 & -0.349 & -0.126 & 0.280 & 0.280 & -0.126 & -0.349 & -0.067 & 0.313 & 0.239 \\ 0.002 & -0.003 & 0.003 & -0.003 & 0.002 & -0.001 & -0.001 & 0.002 & -0.003 & 0.003 & -0.003 & 0.002 \\ -0.001 & -0.001 & 0.001 & 0.000 & -0.002 & 0.001 & 0.001 & -0.002 & 0.000 & 0.001 & -0.001 & -0.001 \\ 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\ -5.623 & -3.297 & -0.285 & 2.787 & 5.278 & 6.668 & 6.668 & 5.278 & 2.787 & -0.285 & -3.297 & -5.623 \\ -5.338 & -4.749 & -2.552 & 0.509 & 3.397 & 5.136 & 5.136 & 3.397 & 0.509 & -2.552 & -4.749 & -5.338 \\ 0.391 & -1.554 & -2.354 & -1.420 & 0.560 & 2.128 & 2.128 & 0.560 & -1.420 & -2.354 & -1.554 & 0.391 \end{bmatrix}$$

From Fig. 2. it can be observed that using the proposed technique we can place the null point at any desired location. The computational cost involved in finding the excitation amplitudes and needed for generating the desired radiation pattern is considerably lower than the optimization methods. From this simple example of null placement, we can understand the ability of the proposed method in designing of antenna arrays.

5. CONCLUSION

The theoretical framework for a novel technique in the field of antenna arrays is presented. Through detailed mathematical derivations, it is shown that for small errors in the excitation amplitudes (i.e., for $\pm 10\%$), the deviation of the radiation pattern from that of the desired radiation pattern can be expressed as a summation of errors contributed by the individual elements of the array. This approximation is justified by performing a Monte-Carlo simulation for varying number of error elements and for different values of tolerance limits. It is also observed that by calculating the error coefficients at angles within the main lobe range of the antenna array, the accuracy can be improved in all cases. An error coefficient matrix is proposed, and its properties are discussed. The application of this matrix in the field of fault detection, fault correction and radiation pattern synthesis is presented. An application of the proposed method of placement of the null point in the radiation pattern is also presented. The proposed technique is found effective in generating the desired radiation pattern, and the computational cost involved is much less than that of optimization techniques. A simple and general analytical method is proposed for generating any desired radiation pattern. There are innumerable applications of this method in the field of antenna arrays, and it is beyond the scope of this paper to discuss all the applications with relevant examples. The author intends to use the theoretical framework presented in this paper for fault analysis (which also includes mathematical details of what to do with singular cases) and radiation pattern synthesis of antenna arrays, in his future work.

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