

Propagation below Cutoff in Metallic Waveguide Loaded by Two Slabs with Modified Split Ring Resonator

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Abstract—This paper aims to study the synthesis of negative magnetic permeability and how this leads into some physical phenomena such as the appearance of backward waves and the propagation below cutoff. The extraction of the polarizability tensors of the edge coupled split ring resonator is derived, and the existence of bianisotropic effects of this case is investigated. It is shown how to avoid the bianisotropic effects through using a proposed design. Finally, the backward wave of the proposed design with lower losses than the edge coupled split ring resonator is shown by simulation.

1. INTRODUCTION

Metamaterial is one of the most important methods used to reduce the size of waveguide and to obtain the desired resonant frequency bands. Many terminologies have been suggested for metamaterials, such as “left-handed” “backward-wave media” and “double-negative”, and it is observed that at a certain frequency, metamaterials exhibit both negative permittivity and permeability [1]. One of the most familiar applications of the metamaterial is split ring resonator “SRR” [2–4].

In 1852, Wilhelm Weber formulated the first theory of diamagnetism, discovered by Faraday [5]. Weber assumed the existence of closed circuits at the molecular scale and invoked Faraday’s law [6] to prove that currents would be induced in these circuits when they were under the effect of an external time-varying magnetic field. As the secondary magnetic flux created by such currents was opposite to that created by the external field, this mechanism could explain the diamagnetism reported by Faraday as shown in Fig. 1.

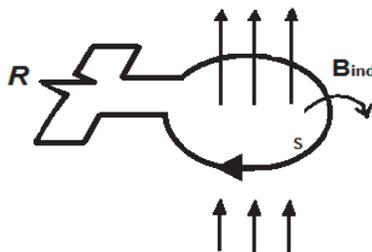


Figure 1. Magnetic field is generated using an Alternating Current (AC) source connected to the coil (Faraday’s law).

However, the diamagnetic effect associated with a closed metallic ring is not strong enough to produce negative values for μ .

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This closed loop with loaded capacitor configuration, proposed by Schelkunoff and Friis [7], provides the following expression for the magnetic polarizability:

$$\alpha_{xx}^{mm} = \frac{\pi^2 r^4}{L} \left(\frac{\omega_o^2}{\omega^2} - 1 \right)^{-1} \quad (1)$$

where ω_o is the resonant frequency of the LC circuit formed by the loop and capacitor, r the radius of the loop, and L the self-inductance of the loop. This expression shows that just above the resonant frequency, the transverse magnetic polarizability becomes negative and very large. Therefore, if there is a regular array of capacitive loaded metallic loops, it will show a high negative magnetic permeability just above the frequency of resonance of the loops, but this configuration may be difficult to manufacture at microwave and higher frequencies [5, 7].

The modification of Shelkunoff's proposal leads to the (SRR) proposed by Pendry et al. [8]. Using this design, it becomes possible to manufacture large series of small loops, with large negative magnetic polarizability, by using standard planar photo-etching techniques (EC-SRR). This EC-SRR can be used in array and inserted in a waveguide, which enables the miniaturization of the waveguide through the phenomena of backward wave that propagates below cutoff [2, 3, 9–11].

2. THE EDGE-COUPLED SPLIT RING RESONATOR (EC-SRR)

The slots in the rings shown in Fig. 2 are used to prevent the conduction current from flowing around any ring of the two rings [8, 9]. However, there are two considerable capacitances. The capacitance between the two rings enables displacement current to flow and the capacitance of the slot for each ring.

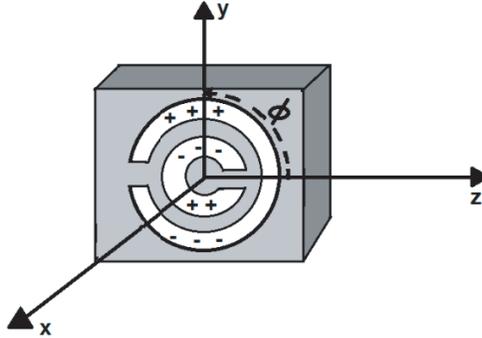


Figure 2. Edge-Coupled split ring resonator.

The parameters which connect the induced electric and magnetic dipoles (p & m) to the external electric and magnetic fields E^{ext} & H^{ext} are called polarizabilities. These relations may be summarized in the following compact forms [12]:

$$\begin{bmatrix} P \\ m \end{bmatrix} = \begin{bmatrix} \alpha^{ee} & \alpha^{em} \\ \alpha^{me} & \alpha^{mm} \end{bmatrix} \begin{bmatrix} E^{\text{ext}} \\ H^{\text{ext}} \end{bmatrix} \quad (2)$$

Start with the calculation of α_{iy}^{ee} & α_{iy}^{me} where $i = x, y, z$, to determine the particle response (P_i and m_i) in y -directed exciting electric field to the EC-SRR as shown in Fig. 2. Two opposite plane waves are applied, which are travelling in the z -direction with similar polarizations for their electric fields $E_y^{\pm} = E_o e^{j(\omega t \mp kz)}$ where ω & k are the angular frequency and wave number, respectively. Since the two magnetic fields are out of phase, the sum of these two plane waves will copy a standing wave with maximum electric field and zero magnetic field. Assuming that the particle is small compared to the wavelength, the polarizabilities can be easily calculated as:

$$\alpha_{iy}^{ee} = \frac{P_i^+ + P_i^-}{2E_o} \quad (3)$$

$$\alpha_{iy}^{me} = \frac{m_i^+ + m_i^-}{2E_o} \quad (4)$$

In a similar way, subtracting these two plane waves creates a maximum magnetic field with zero electric field. Therefore, one can write:

$$\alpha_{ix}^{em} = \frac{P_i^+ + P_i^-}{2E_o} \eta \tag{5}$$

$$\alpha_{ix}^{mm} = \frac{m_i^+ + m_i^-}{2E_o} \eta \tag{6}$$

where η is the intrinsic impedance, for EC-SRR located perpendicular to x -axis, and it can be shown by the following set of equations:

$$m_x = \alpha_{xx}^{mm} B_x^{ext} - \alpha_{yx}^{em} E_y^{ext} \tag{7}$$

$$P_y = \alpha_{yy}^{ee} E_y^{ext} + \alpha_{yx}^{em} B_x^{ext} \tag{8}$$

The bianisotropy terms α_{yx}^{em} & α_{xy}^{me} occur because SRR acts not only as a magnetic dipole [9, 10], but also as an electric dipole. This behaviour is expected from the fact that quasistatic potentials and charges for $(\pi < \Phi < 2\pi)$ are the electrical images of these quantities for $(0 < \Phi < \pi)$ as sketched in Fig. 2. Two parallel electric dipoles directed along the y -axis are generated at each EC-SRR half, so that the EC-SRR has an electric dipole along the y -direction [13, 14]

It can be noticed that Busch et al. [15] and Smith et al. [16] used modification designs for EC-SRR as BC-SRR and H-shape to produce materials with negative reflective index. Their designs could overcome the bianisotropy problem but still with large losses in the backward wave. The aim of this paper is to eliminate losses of the backward wave, overcoming the bianisotropy problem through using the proposed design.

3. THE PROPOSED DESIGN

For EC-SRR, we will separate the two rings each on a single substrate and with opposite slots as shown in Fig. 3. A regular array of capacitive loaded ring in x -direction will show a large negative magnetic permeability in the direction of the magnetic dipole. In addition to the advantage of avoiding the bianisotropy, where the electric polarization of the upper half side ($y > 0$) must be equal to the opposite electric polarization of the lower half side ($y < 0$) of the rings. By substitute Equation (5) in Equation (6), it is clear that the cross polarization terms α_{yx}^{em} & α_{xy}^{me} will vanish, so that the proposed design is considered to avoid the bianisotropy problem, while the magnetic dipole resulting from the regular array of the rings is approximately doubled. The author used the same proposed design but with single opposite rings which illustrated in [17], and it was shown that the losses strongly decreased with larger bandwidth.

$$M_{XX} = \alpha_{xx}^{mm} B_x^{ext} \tag{9}$$

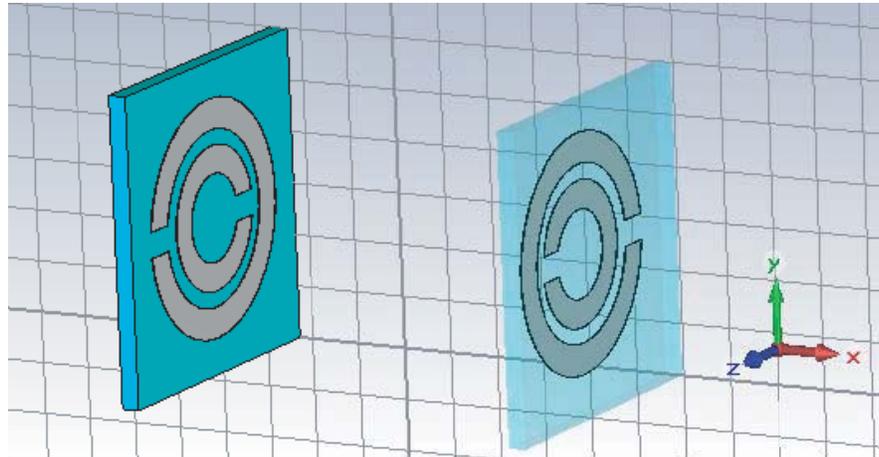


Figure 3. Two separated substrates with opposite edge coupled split ring resonator.

4. RESULTS

Assume a homogeneous medium with effective permittivity and permeability. If a plane wave propagates along the Z -direction inside the medium, the dispersion relation can be shown as:

$$\beta_z^2 = k_o^2 \mu_{\text{eff}} \varepsilon_{\text{eff}} \quad (10)$$

and

$$\mu_{\text{eff}} = \mu_x \quad (11)$$

$$\beta_z = \begin{cases} k_o \sqrt{\mu_{\text{eff}} \varepsilon_{\text{eff}}} & \text{If: } \text{Re}[\mu_{\text{eff}}] \text{ and } \text{Re}[\varepsilon_{\text{eff}}] \text{ are positive} \\ -k_o \sqrt{\mu_{\text{eff}} \varepsilon_{\text{eff}}} & \text{If: } \text{Re}[\mu_{\text{eff}}] \text{ and } \text{Re}[\varepsilon_{\text{eff}}] \text{ are negative} \\ -jk_o \sqrt{\mu_{\text{eff}} \varepsilon_{\text{eff}}} & \text{If: either } \text{Re}[\mu_{\text{eff}}] \text{ or } \text{Re}[\varepsilon_{\text{eff}}] \text{ is negative} \end{cases} \quad (12)$$

In Equation (12), it can be shown that the third condition if “ $\text{Re}[\mu_{\text{eff}}]$ is negative and $\text{Re}[\varepsilon_{\text{eff}}]$ positive” supports the backward wave propagation below the cutoff frequency of an empty waveguide [18].

According to this homogenous medium, to obtain the results of S_{21} and extract the effective permeability and permittivity from it, inclusions in the form of the edge coupled split ring resonator are placed on two thin dielectric slabs of ε_r but with opposite slots of the proposed design as in Fig. 4(a), and on single thin dielectric slab of ε_r as in Fig. 4(b). The results were obtained by using CST MW studio simulation.

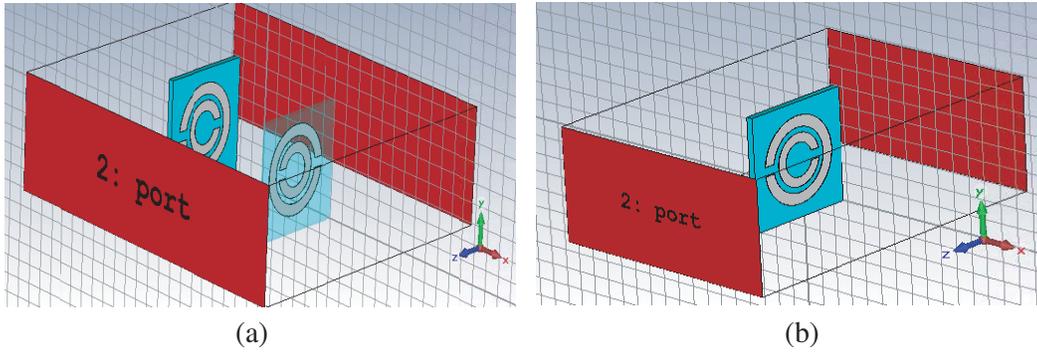


Figure 4. (a) Proposed design. (b) Edge coupled split ring resonator.

Single edge coupled rings are designed as shown in Fig. 4(b) at resonant frequency $f_o = 7.8$ GHz, where $r_i = 0.8$ mm, $r_o = 1.3$ mm, $R_i = 2$ mm, and $R_o = 2.3$ mm. The slot of the rings is 0.5 mm and etched on a copper cladding substrate of thickness 0.7 mm and copper thickness 0.02 mm with dielectric permittivity $\varepsilon_r = 2.6$. The result of S_{21} is shown in Fig. 5.

The same EC-SRR inclusions at resonant frequency $f_o = 7.8$ GHz are formed in two slabs but with opposite slots as shown in Fig. 4(a). The result of S_{21} is shown in Fig. 6 which is near -42 db, much greater than the single EC-SRR by 20 db approximately.

The results of the effective permeability and permittivity for both cases are extracted from the transmission coefficient S_{21} and shown clearly in [Figs. 7(a), (b)] and [Figs. 8(a), (b)].

It can be observed that the real part of the effective permeability is negative near 7.8 GHz for both cases as shown in Fig. 7(a) and Fig. 8(a), while the imaginary part is negative in the single edge coupled case [Fig. 7(a)] in contrast to that of the proposed design which is positive [Fig. 8(a)]. It is expected that the total loss leading to the negative $\text{Im}(\mu_{\text{eff}})$ for the EC-SRR includes the major loss “radiation loss” and dissipation loss which can be neglected. So it can be concluded that the positive imaginary part in the proposed design results from the improvement of decreasing the radiation loss by the new design; also it is a proof of overcoming the bianisotropy problem.

The next step is to insert a single slab loaded by ten EC-SRRs at the same resonant frequency inside a waveguide operating below cutoff of dimensions (12 mm \times 12 mm) and 60 mm along. The waveguides loaded by these inclusions were introduced directly between two feeding waveguides of dimensions (45 mm \times 15 mm) as in Fig. 9 [5] to verify the reported results.

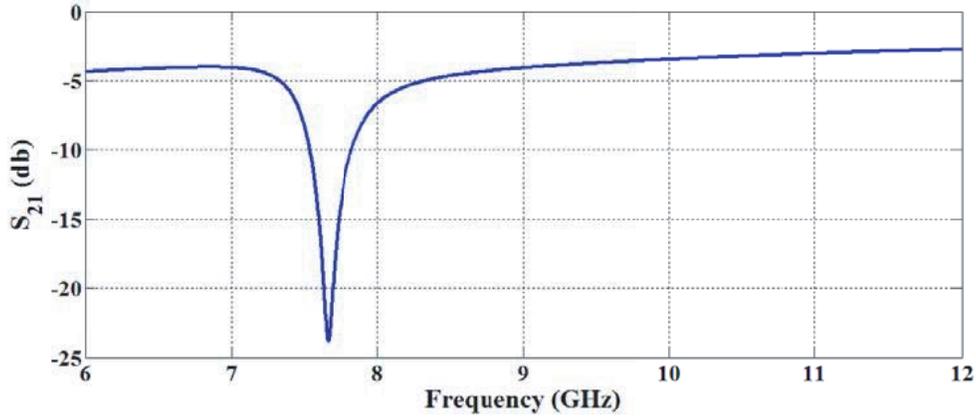


Figure 5. Result of S_{21} for a Single Edge coupled split ring resonator at $f_o = 7.8$ GHz.

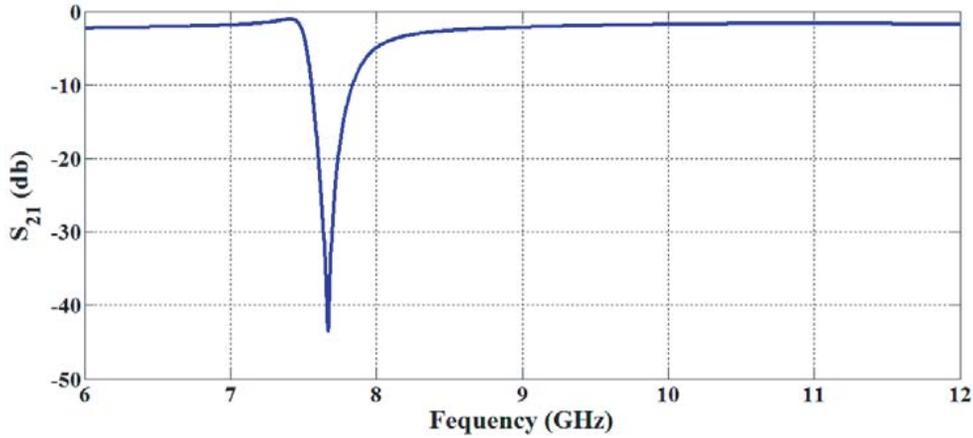
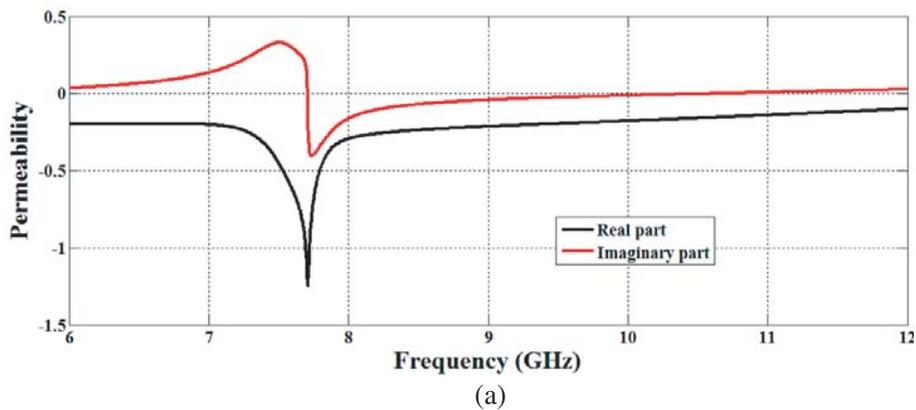


Figure 6. Result of S_{21} for the proposed design at $f_o = 7.8$ GHz.

To compare the proposed design result with the reported one, two slabs, each loaded by ten EC-SRRs of same resonant frequency, were inserted face to face with opposite slots in the same waveguide, where the distance between the two slabs $d = 7$ mm. The results of S_{21} of the two cases are shown in Fig. 10.

It can be concluded that there is a large negative magnetic permeability in the direction of the magnetic dipole at ($f_o = 8$ GHz), just above the resonant frequency of the rings ($f_o = 7.8$ GHz) when



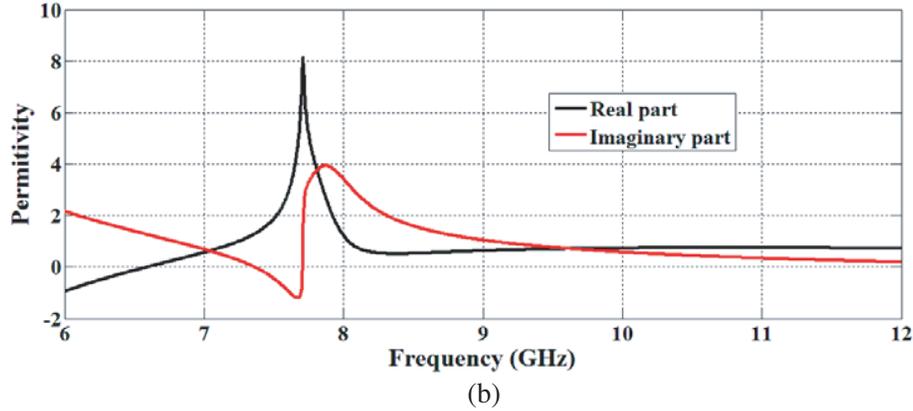


Figure 7. (a) The extracted real part and imaginary part of the effective permeability for Single Edge coupled split ring resonator. (b) The extracted real part and imaginary part of the effective permittivity for Single Edge coupled split ring resonator.

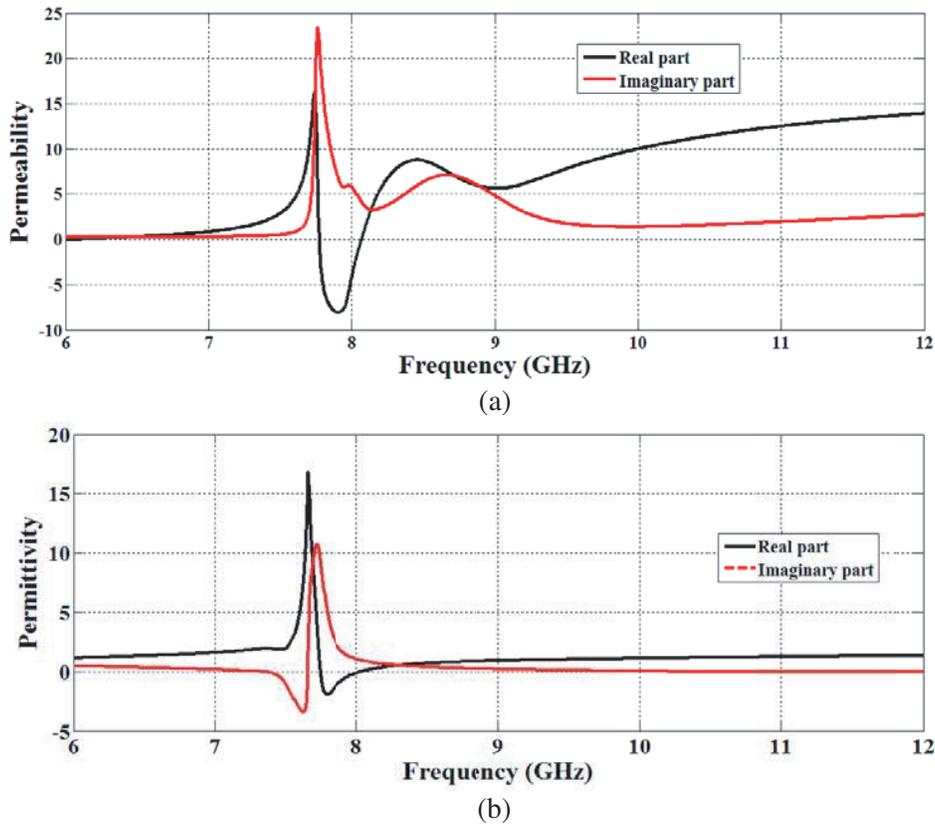


Figure 8. (a) The extracted real part and imaginary part of the effective permeability for the proposed design. (b) The extracted real part and imaginary part of the effective permittivity for the proposed design.

a regular array of capacitive loaded ring is designed with two rings with opposite slots on two slabs in the transversal direction of propagation (x -direction) and 10 rings on each slab along the direction of propagation (z -direction), which leads S_{21} to reach 0 db, and it can overcome the bianisotropy problem of the EC-SRR. By comparing the result of S_{21} for EC-SRR of the single slab to the result of the proposed design of the two slabs in Fig. 10, it is found that the backward wave is increased by 30 db.

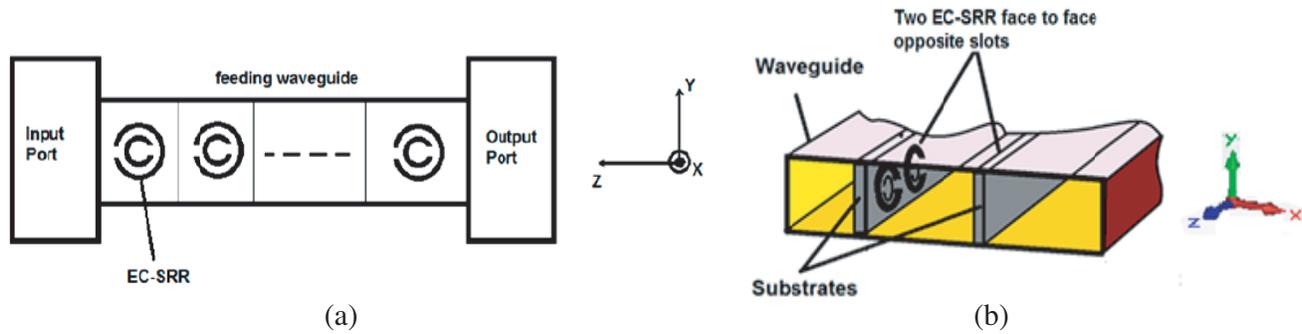


Figure 9. (a) Side view of the simulation system. (b) 3D view of the waveguide loaded by two slabs.

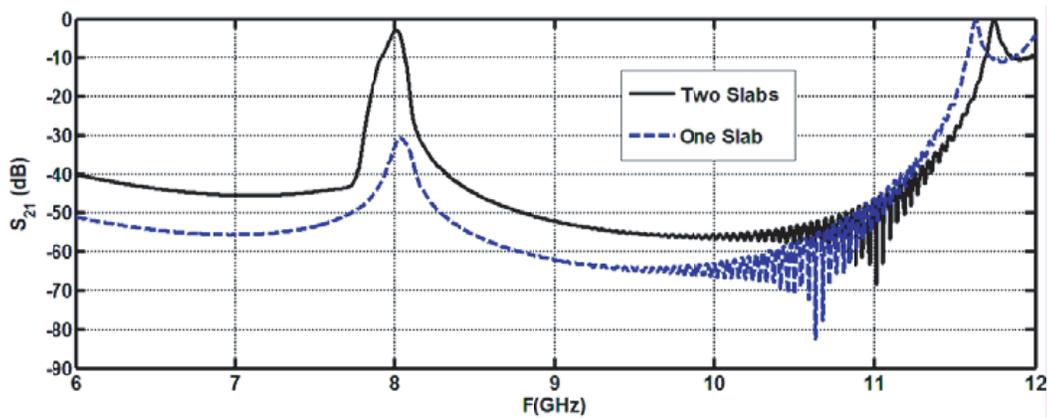


Figure 10. Result of S_{21} for a waveguide of $a = 12$ mm where $f_c > f_o$: Solid line for the case of the proposed design, dotted line for the case of single slab.

5. CONCLUSION

Electric and magnetic dipoles of the edge coupled split ring resonator are extracted in details, and the appearance of the bianisotropy in the edge coupled split ring resonator is shown clearly. It is shown how to avoid the bianisotropic effect of the edge coupled split ring resonator through using the proposed design. Finally, the increase of the negative permeability and the decrease of losses of the backward wave for the proposed design are shown by simulation compared to the single slab of EC-SRR.

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