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Improving Efficiency of the Secondary Sources Method for Modeling of the Three-Dimensional Electromagnetic Field

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Abstract—A mathematical model is constructed for calculating a three-dimensional quasistationary electromagnetic field in a piecewise-homogeneous medium containing massive conductors which is excited by a variable magnetic field. The field is varying in time according to an arbitrary law. It is proposed to use the integral relation instead of the boundary condition written at a point, which allows one to get away from the problem of collocation points and at the same time increase the computational efficiency of the numerical model. The magnetic field is calculated for the case of the excitation of eddy currents in a conducting sample containing a cut of finite size. The results obtained are confirmed by natural experiments.

1. INTRODUCTION

Mathematical modeling of the distribution of eddy currents in conductive bodies of various geometries has a wide range of practical applications [1–5]. From the point of view of automation, as well as the possibility of studying both surface and subsurface defects, one of the most effective methods of nondestructive testing of conducting non-ferromagnetic samples is eddy current testing (ECT) [6, 7]. The calculation of eddy currents can be carried out analytically only in the simplest cases [8, 9], when the geometry of the computational domain is balls, cylinders, etc. For the majority of practically significant cases, the mathematical model requires the use of numerical methods.

Finite element method (FEM) is the most common numerical method for solving boundary problems of electrodynamics [2, 5]. The key problem of FEM is that the computational domain includes the amount of empty space surrounding the conductive body under consideration [10], which affects the amount of computational work. In the tasks of ECT this is especially significant, since the air gaps between the conductors in this case can be a few microns. In [10], a method is presented in which the introduction of a special-type computational grid allows one to avoid the key problem of the FEM. The problem in [10] is solved under the condition of sinusoidal electromagnetic (EM) field in the approximation of a thin metal sheet. In [11], a model oriented on the calculation of an eddy current signal in ECT problems is presented. In the framework of this approach, the influence of an oscillating charge on the surface of a conductor is neglected [12]. In [13, 14], an efficient method is presented for simulating eddy currents in multiply connected thin conductors using the boundary element method.

Among the methods of numerical simulation of field problems, the secondary sources method is also known [15]. This method consists in solving integral (or integro-differential) equations for the density of secondary sources. The advantage of this method over FEM is that only the investigated conducting bodies are subjected to discretization. In accordance with the above, in this work a mathematical model of the EM eddy current field is developed based on the secondary sources method.

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2. FEATURES OF THE SOLUTION OF THE SYSTEM OF INTEGRO-DIFFERENTIAL EQUATIONS BY THE COLLOCATION METHOD

In [16], a numerical method was described for solving a system of integro-differential equations for the density of secondary sources which arise in the volumes and on the surfaces of conductors under the action of a quasistationary electromagnetic field of variable currents arbitrarily varying in time or moving permanent magnets. This numerical method is widely known as the collocation method. Its essence is that the densities of secondary sources (surface charges and eddy currents) are replaced by a set of piecewise constant functions, and the observation point is fixed within the region splitting element. Such a replacement allows instead of a system of integro-differential equations to write down a system of linear algebraic equations (SLAE). We give this system of equations [16]:

$$\sigma_{k,i} - \frac{1}{2\pi}\sum_{m=1}^{N_S}\sigma_{m,i}\int\limits_{\Delta S_m}\frac{(\mathbf{r}_{PQ},\mathbf{n}_Q)}{r_{PQ}^3}dS_P = -\frac{\varepsilon_0\mu_0}{2\pi}\sum_{m=1}^{N_V}\frac{\partial\boldsymbol{\delta}_{m,i}}{\partial t}\mathbf{n}_Q\int\limits_{\Delta V_m}\frac{dV_N}{r_{QN}} - 2\varepsilon_0\frac{\partial}{\partial t}\left(\mathbf{A}_0\left(Q_k,t_i\right),\mathbf{n}_Q\right),$$
$$k = 1,2,\ldots,N_S, \quad i = 0,1,\ldots,N_T-1; \tag{1}$$

$$\boldsymbol{\delta}_{k,i} = -\frac{\gamma\mu_0}{4\pi}\sum_{m=1}^{N_V}\frac{\partial\boldsymbol{\delta}_{m,i}}{\partial t}\int\limits_{\Delta V_m}\frac{dV_N}{r_{MN}} + \frac{\gamma}{4\pi\varepsilon_0}\sum_{m=1}^{N_S}\sigma_{m,i}\int\limits_{\Delta S_m}\frac{\mathbf{r}_{PM}}{r_{PM}^3}dS_P - \gamma\partial\mathbf{A}_0\left(M_k,t_i\right)/\partial t,$$
$$k = 1,2,\ldots,N_V, \quad i = 0,1,\ldots,N_T-1, \tag{2}$$

where $\sigma_{k,i}$ is the value of the surface density of electric charge on the k-th element of the conductor surface partitioning S at the i-th instant of time; N_S is the number of elements of the surface partitioning S; $\mathbf{r}_{PQ}$ is the radius vector drawn from point $P \in S$ to point $Q_k \in \Delta S_k$; $\mathbf{n}_Q$ is the outward normal to point Q_k; ΔS_k is the area on the surface S of the conductor occupied by the k-th splitting element; ε_0 and μ_0 are the electric and magnetic constants, respectively; N_V is the number of partitioning elements of the volume V of the conductor; $\boldsymbol{\delta}_{k,i}$ is the vector of the volume density of eddy currents in the k-th element ΔV_k of the partitioning of the conductor volume V at the i-th moment of time; r_{QN} is the distance between point $Q_k \in \Delta S_k$ and point $N \in \Delta V_k$; $\mathbf{A}_0$ is the vector potential of free field sources; M_k is a point located in the geometric center of the element ΔV_k; N_T is the number of moments of time into which the period under consideration is divided.

An iterative procedure was proposed in [16], with the help of which the system in Eqs. (1), (2) can be solved.

The magnetic field at a point in space M at time t_i, after finding the density of eddy currents, is calculated by the formula:

$$\mathbf{H}\left(M,t_i\right) = \mathbf{H}_0\left(M,t_i\right) + \frac{1}{4\pi}\sum_{k=1}^{N_V}\frac{\left[\boldsymbol{\delta}_{m,i},\mathbf{r}_{MN_k}\right]}{r_{MN_k}^3}\Delta V_k,$$

where $\mathbf{H}_0$ is the magnetic field of free sources; N_k is the point in the center of the region ΔV_k.

Equation (1) approximate the boundary integral equation, which is derived from the boundary condition [16]:

$$(\boldsymbol{\delta},\mathbf{n}_Q) = 0. \tag{3}$$

The normal component of the density vector of eddy currents at point $Q \in S$ is zero.

In accordance with the collocation method, when calculating the coefficients included in Equation (1), point Q is fixed within the limits of the partitioning elements. Therefore, from the continuous spectrum of $Q \in S$ values, a transition is made to a discrete set of $Q_k \in \Delta S_k$ $(k = 1,2,\ldots,N_S)$ values. This means that the solution of Equations (1), (2) will satisfy the boundary condition in Equation (3) only at a given set of points Q_k $(k = 1,2,\ldots,N_S)$.

As a rule, the collocation points are specified in the centers of the discretization elements. This method is not always optimal in terms of computational efficiency. However, they continue to use this method, since a mathematically rigorous method, which makes it possible to determine the optimal position of the collocation points, has not been proposed to date.

In practice, when solving problems, especially with a complex geometry of the computational domain, the collocation method requires an unjustified large splitting. The authors associate this phenomenon with the fact that the points $Q_k \in \Delta S_k$ $(k = 1, 2, \dots, N_S)$ are fixed in the centers of the elements of the partition. This is the main disadvantage of the collocation method.

This article proposes a new method to eliminate the problem of collocation points, while significantly increasing the computational efficiency of the secondary sources method.

3. MODIFICATION OF THE SECONDARY SOURCES METHOD

3.1. Derivation of a New System of Linear Algebraic Equations

Consider an arbitrary conducting non-ferromagnetic body with magnetic permeability μ_0 and specific electrical conductivity γ (Figure 1).

Divide the surface of the body S into elements ΔS_k $(k = 1, 2, \dots, N_S)$. We assume that within each element, the density of the electric charge at the i-th moment of time is $\sigma_{k,i}$. Each partitioning element is covered by a closed surface ΔS_k^* in the form of an irregular cylinder, which tightens this element (Figure 2).

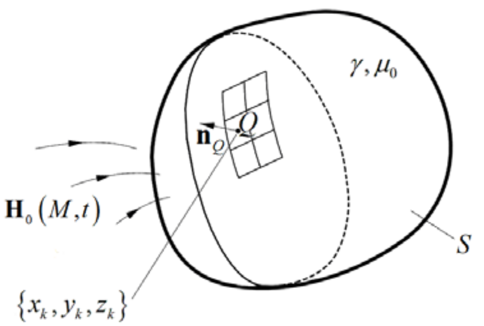

Figure 1. An arbitrary conducting body in an alternating magnetic field.

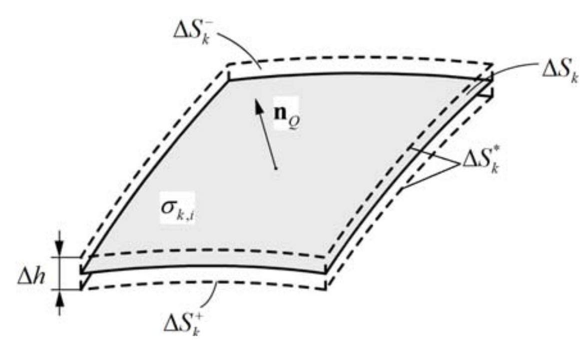

Figure 2. The element of the partition ΔS_k, covered by a closed surface (cylinder).

We denote the base of the cylinder located outside the conductor by ΔS_k^- and denote the base of the cylinder located inside the conductor by ΔS_k^+ (Figure 2). The bases of the cylinder ΔS_k^- and ΔS_k^+ are infinitely close to the element ΔS_k; therefore, the size Δh tends to zero.

We write for the closed surface ΔS_k^* Maxwell's postulate:

$$\oint_{\Delta S_k^*} \mathbf{D} d\mathbf{S} = \int_{\Delta S_k} \sigma_{k,i} dS. \tag{4}$$

In Eq. (4) $\mathbf{D}$ is the electric displacement vector.

If we take into account that the integration over the lateral surface of the vector $\mathbf{D}$ will give zero in view of the condition $\Delta h \to 0$, then the integral over the closed surface ΔS_k^*, in the left-hand side of Eq. (4) can be represented in the form of the sum of two integrals:

$$\oint_{\Delta S_k^*} \mathbf{D} d\mathbf{S} = \varepsilon_0 \oint_{\Delta S_k^*} \mathbf{E} d\mathbf{S} = -\varepsilon_0 \int_{\Delta S_k^+} E_n^+ dS + \varepsilon_0 \int_{\Delta S_k^-} E_n^- dS. \tag{5}$$

In Eq. (5) E_n^+ and E_n^- are the projections of the intensity vector on the normal $\mathbf{n}_Q$ to the surface ΔS_k as the conductor tends to it from inside and outside, respectively.

Since $E_n^+ = 0$ (this follows from the condition in Eq. (3)), Equation (4) takes the form:

$$\varepsilon_0 \int_{\Delta S_k^-} E_n^- dS = \int_{\Delta S_k} \sigma_{k,i}\, dS. \tag{6}$$

The intensity E_n^- included in Eq. (6) can be represented as a superposition of the fields:

$$E_n^- = E_{nk}^- + E_{n\Sigma} + E_{n0}, \tag{7}$$

where E_{nk}^- is the normal component of the vector of the electric field strength at points of the surface ΔS_k^- created by the charge distributed on the element ΔS_k; $E_{n\Sigma}$ is a normal component of the vector of the electric field strength generated by charges distributed over all other elements of the surface partitioning; E_{n0} is a normal component of the vector of the electric field strength generated by third-party sources (including eddy currents).

The values included in Eq. (7) are respectively equal to [16]:

$$E_{nk}^- = \frac{\sigma_{k,i}}{2\varepsilon_0}; \tag{8}$$

$$E_{n\Sigma} = \frac{1}{4\pi\varepsilon_0} \sum_{\substack{j=1 \\ j\neq k}}^{N_S} \sigma_{j,i} \int\limits_{\Delta S_j} \frac{\mathbf{r}_{PQ}\mathbf{n}_Q}{r_{PQ}^3} dS_P; \tag{9}$$

$$E_{n0} = -\frac{\mu_0}{4\pi} \sum_{m=1}^{N_V} \frac{\partial \boldsymbol{\delta}_{m,i}}{\partial t} \mathbf{n}_Q \int\limits_{\Delta V_m} \frac{dV_N}{r_{QN}} - \frac{\partial}{\partial t} \left(\mathbf{A}_0(Q,t_i), \mathbf{n}_Q\right). \tag{10}$$

When writing Eq. (10), it is assumed that the volume V of the conductor (Figure 1) is divided into N_V elements, and the density of eddy currents within the k-th element at the i-th moment of time is equal to $\boldsymbol{\delta}_{k,i}$ [16].

Taking into account Eqs. (7)–(10) and also considering that the integral on the right-hand side of Eq. (6) is equal to $\sigma_{k,i}\Delta S_k$, from expression (6) we get the following SLAE with a rectangular matrix:

$$\sigma_{k,i} - \frac{1}{2\pi\Delta S_k} \sum_{\substack{j=1 \\ j\neq k}}^{N_S} \sigma_{j,i} \int\limits_{\Delta S_k}\int\limits_{\Delta S_j} \frac{(\mathbf{r}_{PQ}, \mathbf{n}_Q)}{r_{PQ}^3} dS_P dS_Q = -\frac{\varepsilon_0\mu_0}{2\pi\Delta S_k} \sum_{m=1}^{N_V} \frac{\partial \boldsymbol{\delta}_{m,i}}{\partial t} \mathbf{n}_Q \int\limits_{\Delta S_k}\int\limits_{\Delta V_m} \frac{dV_N dS_Q}{r_Q N}$$
$$-\frac{2\varepsilon_0}{\Delta S_k} \int\limits_{\Delta S_k} \frac{\partial}{\partial t} \left(\mathbf{A}_0(Q,t_i), \mathbf{n}_Q\right) dS_Q, \quad k = 1,2,\ldots,N_S; \quad i = 0,1,\ldots,N_T - 1. \tag{11}$$

In the derivation of Eq. (11), it is taken into account that since the surface ΔS_k^- is close to the surface ΔS_k, integration over area ΔS_k^- can be replaced by integration over area ΔS_k.

The system of Equation (11) replaces the system of Equation (1). Its distinctive feature is that it is obtained from the boundary condition written in integral form in Eq. (4) and not from the boundary condition in Eq. (3) written at the point, and therefore the problem of collocation points is missing for it.

3.2. The study of the Properties of the Matrix of the System of Equation (11)

The matrix of system in Eq. (11) is degenerate: its rows are linearly dependent. Let us cover an arbitrary conducting non-ferromagnetic body, in the volume of which the eddy currents of density $\boldsymbol{\delta}(Q,t)$ are closed by the closed surface $\tilde{S}$ infinitely close to the surface of the body S (Figure 3).

From the surface $\tilde{S}$ we select the element, which is infinitely close to the element ΔS_{N_S} of the partitioning of the body surface S (Figure 3). For this element, you can write:

$$\varepsilon_0 \int\limits_{\Delta\tilde{S}_{N_S}} E_n^- dS = \varepsilon_0 \oint\limits_{\tilde{S}} E_n^- dS - \varepsilon_0 \sum_{k=1}^{N_S-1} \int\limits_{\Delta\tilde{S}_k} E_n^- dS, \tag{12}$$

where $\Delta\tilde{S}_k$ is the elements of the surface $\tilde{S}$, infinitely close to the elements of the partition ΔS_k.

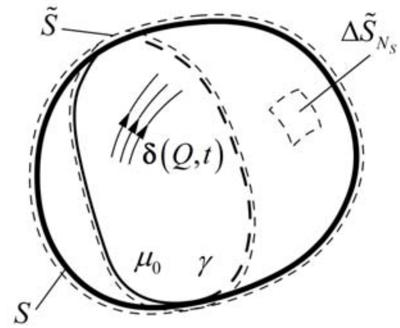

Figure 3. Arbitrary conductive non-ferromagnetic body with eddy currents, covered by a closed surface.

For the integral over a closed contour included in Eq. (12), in accordance with Maxwell's postulate, we can write:

$$\varepsilon_0 \oint_{\tilde{S}} E_n^- dS = \sum_{k=1}^{N_S} \int_{\Delta S_k} \sigma_{k,i}\, dS + q\left(t_i\right). \tag{13}$$

In the right-hand side of Eq. (13) is the total charge covered by the closed surface $\tilde{S}$, where $q(t_i)$ is the charge of the solitary conductor at time t_i $(i = 0, 1, \dots, N_T - 1)$.

Then, in view of Equation (13), Equation (12) takes the form:

$$\varepsilon_0 \int_{\Delta\tilde{S}_{N_S}} E_n^- dS = q\left(t_i\right) + \sum_{k=1}^{N} \int_{\Delta S_k} \sigma_{k,i}\, dS - \varepsilon_0 \sum_{k=1}^{N_S-1} \int_{\Delta\tilde{S}_k} E_n^-\, dS. \tag{14}$$

Since the elements of the surface $\Delta\tilde{S}_k$ are geometrically infinitely close to the elements of the partition ΔS_k, the integration in Eq. (14) with respect to $\Delta\tilde{S}_k$ can be replaced by the integration with respect to ΔS_k $(k = 1, 2, \dots, N_S)$. Then Eq. (14) will take the following form:

$$\varepsilon_0 \int_{\Delta S_{N_S}} E_n^- dS - \int_{\Delta S_{N_S}} \sigma_{N_S,i}\, dS = -\varepsilon_0 \sum_{k=1}^{N_S-1} \int_{\Delta S_k} E_n^-\, dS + \sum_{k=1}^{N_S-1} \int_{\Delta S_k} \sigma_{k,i}\, dS + q\left(t_i\right). \tag{15}$$

Equation (11) is an equivalent record of Equation (6). If we compare Eqs. (6) and (15), we can see that the left side of Eq. (15) is equivalent to Equation (11) for $k = N_S$, and the right side of Eq. (15) is equivalent to the sum of the first $N_S - 1$ of Equation (11) and the value of the charge of the solitary conductor. From this, it follows that the equations of system in Eq. (11) are linearly dependent.

In connection with the above, it is proposed to replace one of Equation (11) with an equation that is linearly independent of the other Equation (11). As such an equation the following is proposed:

$$\sum_{k=1}^{N_S} \sigma_{k,i} \Delta S_k = q\left(t_i\right), \quad i = 0, 1, \dots, N_T - 1. \tag{16}$$

In the particular case $q(t) = 0$, if the conductive body is not charged.

Thus, the SLAE proposed in this work consists of Equations (11), (16) and (2).

4. COMPARISON OF A CONSTRUCTED NUMERICAL MODEL WITH EXISTING APPROACHES

The disadvantages of the collocation method are known and are noted by many authors who are investigating the method of boundary integral equations. In some works, it is proposed to solve the problem of collocation points using the Galerkin method to obtain a SLAE from the corresponding boundary integral equation [17–19]. Mathematically, this method leads to SLAE, the coefficients of which are expressed in terms of double integrals. One of these integrals is taken over the observation point Q, and the second integral is taken over the integration point P. That is, these coefficients will coincide with those obtained in SLAE of Eq. (11) of this article. This means that there is some similarity between the method proposed here and the Galerkin method. However, we believe that the method described here is independent and more general than the Galerkin method. Here are the arguments:

– The arguments given in this article provide a clear explanation of the reason for the unsatisfactory accuracy of the collocation method in the case of a complex boundary of the computational domain. This reason is the lack of accuracy in satisfying the boundary conditions. Therefore, in order to increase accuracy, a boundary condition written at a point was proposed to be replaced with a boundary condition written in integral form in Eq. (4). However, the Galerkin method does not give such clarity but is merely a mathematical technique.
– In this article, it is proved that SLAE in Eq. (11) has a degenerate matrix, since its rows are linearly dependent. This evidence is repelled by the interpretation that underlies the conclusion of this SLAE. At the same time, a simple solution was immediately found that allows one to escape from degeneracy. Obviously, SLAE obtained in [17] by the Galerkin method is also degenerate, but the authors do not write about this.
– The approach proposed in this article can be generalized to other types of boundary secondary sources, for example, on sources such as a simple layer of magnetization currents [20–22]. However, when deriving a SLAE for this type of sources, the law of total current should be applied. This law is written in the form for closed circuits, covering the elements of discretization. Obviously, the SLAE obtained in this way will not coincide with the SLAE obtained by the Galerkin method.

5. EXAMPLE OF CALCULATION AND ANALYSIS OF THE CALCULATING EFFICIENCY OF THE METHOD

As an example, we consider the problem of calculating eddy currents in a conducting non-ferromagnetic plate, having the shape of a parallelepiped and containing a thin section (Figure 4).

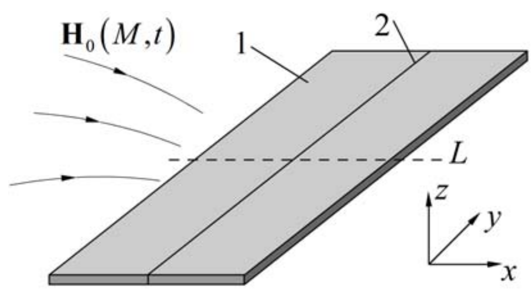

Figure 4. Conductive parallelepiped in a variable field: 1 — conductor; 2 — cut.

This conductor is placed in an alternating electromagnetic field created by a coil with round turns. The axis of the coil is perpendicular to the base of the parallelepiped.

Conductor material — aluminum with conductivity $\gamma = 3,77 \cdot 10^7$ S/m. Parallelepiped dimensions: width 12 mm, length 14 mm, height 0.3 mm. The thickness of the incision is 40 µm. A non-sinusoidal current flows through the coil: $i(t) = 4 \cdot t \cdot I_m/T + 2I_m$, if $-T/2 < t \leq -T/4$; $i(t) = -4 \cdot t \cdot I_m/T$, if $-T/4 < t \leq T/4$; $i(t) = 4 \cdot t \cdot I_m/T - 2I_m$, if $T/4 < t \leq -T/2$ ($T = 1/f$ — current period). The amplitude of the current is $I_m = 5$ A.

Figure 5 shows the distribution of the normal component of the magnetic field generated by the eddy currents and currents of the coil, along the line L, which is shown in Figure 4 (this line is perpendicular to the section and divides the conductor into two equal parts) obtained for different values of the frequency of the current f_1 and f_2. The solid lines in the graph show the total field; the dotted lines indicate the field of eddy currents; and the dotted line with a dot shows the field of the inductor. Graphics of Figure 5 correspond to time point $t = -T/4$.

As can be seen, there is a significant field gradient in the region above the cut and above the edges of the plate. This effect is confirmed by numerous experiments in the field of magneto-optic eddy current introscopy [23].

The presented result was obtained on the basis of the solution of the system of Eqs. (11), (16), (2).

When calculating the field at a frequency of 20 kHz, the method based on solving the system of Eqs. (11), (16), (2) converges when the number of surface splitting elements is $N_S = 10560$. This partition is obtained by numerous calculations with a gradual increase in the number of partitions. The number of partitions increased as long as the resulting solution was changed by more than 1%.

Figure 6 shows the density distribution of eddy currents in the cross section of the plate (the cross section is drawn along line L) for the time point $t = -T/4$ at a current frequency of $f = 20$ КГП.

As can be seen from Figure 6, there is a discontinuity in the current in the region of the cut plate. This gap provides an extremum of the field in the area above the cut, which is visible in Figure 5.

However, as shown by numerous calculations, when solving a problem using the collocation method, it is not possible to achieve a characteristic current distribution. Figure 7 shows the current distribution,

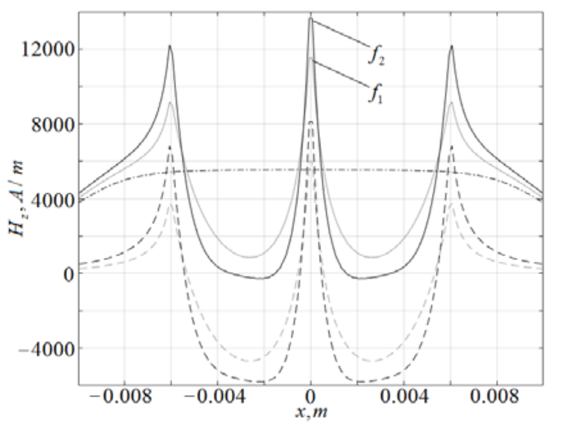

Figure 5. The distribution of the magnetic field above the plate: f_1 — 20 kHz; f_2 — 60 kHz.

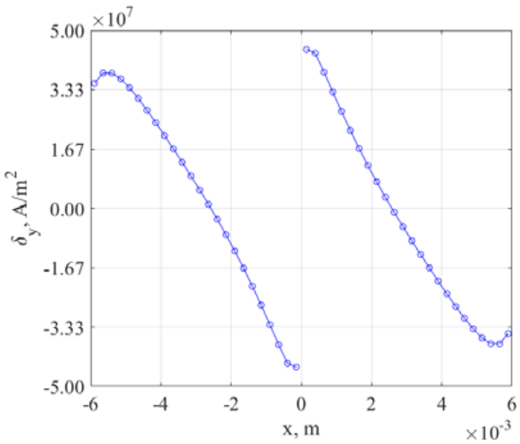

Figure 6. The distribution of y-component of the eddy current density along the axis x at $y = 0$ at time $t = -T/4$.

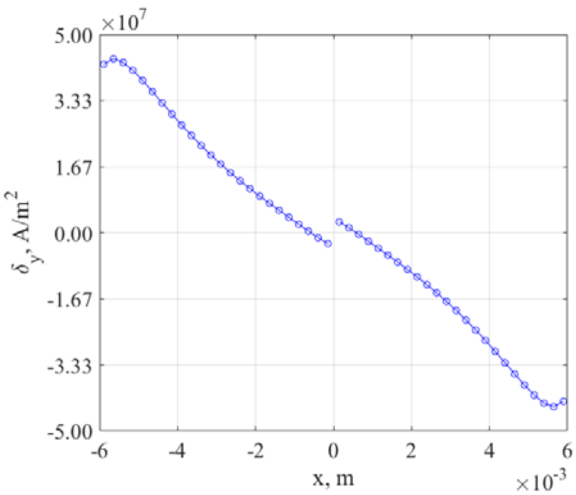

Figure 7. The distribution of y-component of the eddy current density along the axis x at $y = 0$ at time $t = -T/4$. This distribution is obtained by the collocation method.

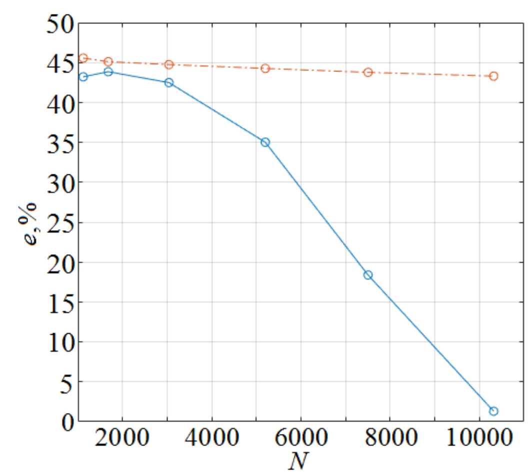

Figure 8. Graph of convergence of the collocation method (dotted line with a dot) and the proposed method (solid line).

which is obtained for a similar point in time but using the collocation method. As can be seen, the current distribution in the conductor volume obtained by the collocation method is not correct. This is evident from the fact that this current is not closed inside each of the halves of the cut conductor. This is a consequence of the violation of boundary conditions. In turn, the proposed method gives the correct current distribution (Figure 6), although the same discretization was set for both methods.

We present a graph showing the process of convergence of a numerical solution with an increase in the number of partitions of the computational domain for the proposed method and the collocation method (Figure 8).

As can be seen from Figure 6, the use of the proposed method obviously gives a substantially fast convergence of the numerical solution with an increase in the number of partitions compared with the collocation method.

When the number of elements of the partitioning of the surfaces of the conductors is equal to 10320, the calculation error of the resulting field by the proposed method is about 40 times lower than the error obtained when solving the problem using the collocation method.

6. CONCLUSION

The proposed method allows to significantly increase the computational efficiency of the method of secondary sources when modeling a three-dimensional quasistationary electromagnetic field in a piecewise-homogeneous medium containing massive conductors.

The proposed method is applicable to the simulation of eddy currents excited in conducting bodies by an external alternating field. The external field varies in time according to an arbitrary law. The law of field change can be periodic or non-periodic. The main limitation is the satisfaction of the field with the conditions of quasistationarity.

Also, the method has no restrictions on the geometry of the calculated area. But its efficiency compared with the collocation method depends on the complexity of the geometry of the computational domain. With the complication of the geometry of the area, the efficiency of the method increases. In particular, if there are angles or very small air gaps in which a large field gradient occurs, the method demonstrates high efficiency. If the computational domain does not contain singularities, then the proposed method gives the same accuracy as the collocation method.

The method was verified experimentally by comparing the simulation results with the observed magneto-optical images of defects in the conductors. The results of this comparison are given in publications from the list (in particular, [23]). The material of this article is devoted exclusively to the development of a theoretical model, so the results of the experiment are not presented here.

In the future, the authors plan to develop the proposed model in the case of piecewise-homogeneous media containing conductors and ferromagnets.

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