Energy Efficiency Optimization for Wireless Powered Relay Networks

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Abstract—In this paper, we focus on the energy efficiency (EE) optimization for an amplify-and-forward (AF) relay network, where the energy-constrained relay harvests energy from a transmitter using power splitting (PS) scheme. We aim to maximize the EE of the network via jointly optimizing the transmit precoding, relay beamforming, and PS ratio, under the constraints of transmit power and spectral efficiency. To solve the formulated fractional programming, we approximate the problem via two layer optimization, where the outer problem is handled by the Dinkelbach method, and the inner problem is solved by penalized difference-of-convex (DC) and constrained concave-convex procedure (CCCP). Finally, an iterative method is proposed. Simulation results demonstrate the performance of the proposed design.

1. INTRODUCTION

Traditionally, energy is directly harvested from external sources without exploiting the resources of the communication network itself. However, when the natural environment is not able to provide stable energy, wireless devices have to find an alternative energy source, which can be the information-carrying radio frequency (RF) signal radiated by the fixed transmitters such as base station, and hot spots [1]. EH-enabled wireless devices can harvest energy from either RF signals or ambient energy sources which enable them to operate continuously, thus, energy harvesting (EH) is considered as one of the effective approaches for improving the energy efficiency of a wireless network [2].

On the other hand, the recently development in chemical materials has promoted the implement of EH [3–6]. Specifically, in [3], the authors investigated the determination of frying sunflower oil usage time for local potato samples by using microwave transmission line based sensors measurement. In [4], a new design of a microstrip power limiter which is based on microstrip technology and zero bias Schottky diode is introduced. In [5], the authors investigated the detection of chemical materials with a metamaterial-based sensor incorporating oval wing resonators. In [6], an electromagnetic band gap integrated monopole antenna in a deformed cavity resonator was investigated.

Recently, energy efficiency (EE), which is measured in bits per Joule, has become an emerging important metric in energy-constrained wireless networks due to the energy shortage [7]. Specifically, in [8], the authors proposed an EE optimization method for simultaneous wireless information and power transfer (SWIPT) system. In [9], the authors proposed a global EE design in multiuser multiple-input single-output (MISO) networks. Recently, the EE design has been investigated in a more complex network such as the cognitive radio networks in [10], the multiple-input multiple-output (MIMO) channel in [11], the wireless sensors network in [12], and the heterogeneous networks in [13], respectively. Among these references, the Dinkelbach method in [14] is a main tool for handling the fractional EE programming.

The above works mainly focus on the direct link scenario. In wireless cooperative networks, the relay nodes often have limited battery storage and require external charging mechanisms to remain active.
Thus, the EE maximization for relay networks is particularly important [15]. Specifically, in [16], the authors proposed a destination-aided beamforming (BF) method for multi-antenna self-sustainable relay networks to maximize the EE. In [17] and [18], the authors proposed an EE design in MIMO one-way and two-way MIMO amplify-and-forward (AF) relay networks with EH, respectively. Furthermore, in [19], the authors proposed a robust EE optimization for traditional MIMO AF relay networks, which was extend in [20] with EH at the relay. In [21], the authors proposed an energy efficient BF and power splitting (PS) design in an EH-enabled relay network, which was extended in [22] with consideration of the EE fairness for multi-pair systems. However, when all nodes in the relay network are equipped with multi-antennas, the EE optimization for a wireless powered relay network has not been investigated yet.

Motivated by these observations, in this paper, we focus on the EE optimization in a wireless powered AF relay network. Specifically, we investigate the joint transmit precoding, relay BF, and PS design to maximize the EE, subject to the transmit power threshold and information rate constraints. The formulated fractional programming is highly non-convex, due to non-convex objective and coupled variables. To handle this obstacle, we transform it into a Dinkelbach-type optimization problem via penalized difference-of-convex (DC) and constrained concave-convex procedure (CCCP). Then, a two layer iterative method is proposed to solve the approximated problem. Finally, numerical results indicate the effectiveness of the proposed scheme.

The rest of this paper is organized as follows. The system model and problem statement are given in Section 2. Section 3 investigates the EE optimization, wherein a penalized DC and CCCP based reformulation is proposed, which is beneficial for the Dinkelbach-type optimization. Simulation results are illustrated in Section 4. Section 5 concludes this paper.

Notations: Throughout the paper, we use upper case boldface letters for matrices and lower case boldface letters for vectors. The superscripts ($\cdot^T$, $\cdot^\dagger$, $\cdot^H$) represent the transpose, conjugate, and conjugate transpose, respectively. $|\cdot|$ and $\|\cdot\|$ denote the absolute value and Frobenius norm, respectively. The trace of matrix $A$ is denoted as $\text{Tr}(A)$. $a = \text{vec}(A)$ stands for stacking the columns of matrix $A$ into a vector $a$. $\text{vec}^{-1}(\cdot)$ is the inverse operation of $\text{vec}(\cdot)$. $\otimes$ denotes the Kronecker product. $I$ denotes an identity matrix with appropriate size. $\mathcal{CN}(0, I)$ denotes a circularly symmetric complex Gaussian random vector with mean $0$ and covariance matrix $I$.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. System Model

We consider a relay network with EH as shown in Fig. 1, in which a transmitter (T) sends information to the destination (D), with the aid of an AF relay (R). We assume that the T, R, and D are equipped with $N_t$, $N_r$, and $N_d$ antennas, respectively. In addition, the direct link between the T and D is negligible. Let $H \in \mathbb{C}^{N_r \times N_t}$ and $G \in \mathbb{C}^{N_d \times N_r}$ denote the channel vectors from the T to R, and from the R to D, respectively. Since the R operates in a half-duplex mode, one transmission round is composed of two phases.

![Figure 1](image-url)  
Figure 1. The wireless powered AF relay system.

In the 1st phase, the received signal at the R is [17]

$$y_r = H^H F s + n_r,$$

where $s$ is the transmitted signal vector satisfying $E\{ss^H\} = I$, and $F \in \mathbb{C}^{N_r \times N_t}$ is the transmitter precoding matrix. In addition, $n_r \in \mathbb{C}^{N_r \times 1}$ is the relay noise with $n_r \sim \mathcal{CN}(0, \sigma_r^2 I)$. 


The R is assumed energy-constrained, and there is no other energy source available at the R, thus it is very difficult to always maintain the active state. In fact, we assume that the R not only processes the received signal, but also harvests and stores the energy from the RF signals transmitted by the T for forwarding the received signals to the D.

Since the PS scheme is utilized by the relay, the signal used for ID is

\[ y_{ID}^r = \sqrt{\rho} (H^H F s + n_r) + n_p, \]

(2)

where 0 ≤ ρ ≤ 1 is the PS ratio, and \( n_p \in \mathbb{C}^{N_R \times 1} \) is the additional processing noise introduced by the EH circuit with \( n_p \sim \mathcal{CN} (0, \sigma^2_p) \).

On the other hand, the signal used for EH is

\[ y_{EH}^r = \sqrt{1 - \rho} (H^H F s + n_r). \]

(3)

Thus, the information rate in the 1st phase is

\[ R_1 = \frac{1}{2} \ln \left| I + \frac{\rho H^H F F^H H}{\sigma^2 I} \right|. \]

(4)

It should be mentioned that here we assume that \( \sigma^2 = \sigma^2_r + \rho \sigma^2_p \). In fact, this term should be \( \sigma^2 = \sigma^2_r + \rho \sigma^2_p \). We make this approximation to simplify the problem. In fact, since 0 ≤ ρ ≤ 1, Eq. (4) can be seen as a lower bound of the practical information rate in the 1st phase.

The harvested power at the relay is [18]

\[ P_{EH} = \varepsilon (1 - \rho) \text{Tr} (H^H F F^H H), \]

(5)

where 0 ≤ ε ≤ 1 is the energy harvesting efficiency. Without loss of generality (W.l.o.g.), we assume that \( \varepsilon = 1 \) in the following part.

In the 2nd phase, the received signal at the destination is [19]

\[ y_d = G^H W y_{ID}^r + n_d, \]

(6)

where \( W \in \mathbb{C}^{N_r \times N_r} \) is the relay BF matrix. \( n_d \in \mathbb{C}^{N_d \times 1} \) is the received noise at the D with \( n_d \sim \mathcal{CN} (0, \sigma^2_d I) \).

By substituting \( y_{ID}^r \) into Eq. (6), \( y_d \) can be rewritten as [20]

\[ y_d = \sqrt{\rho} G^H W H^H F s + N, \]

(7)

where \( N \) is the equivalently noise at the relay, which is given by

\[ N = G^H W (\sqrt{\rho} n_r + n_p) + n_d. \]

(8)

Thus, the information rate at the 2nd phase is

\[ R_2 (\rho, F, W) = \frac{1}{2} \ln \left| I + \frac{\rho G^H W H F F^H H W^H G}{\sigma^2 G^H W W^H G + \sigma^2_d I} \right|. \]

(9)

Here, we utilize the assumption \( \sigma^2 = \sigma^2_r + \sigma^2_p \) again to simplify the problem, and Eq. (9) can be seen as a lower bound of the actual information rate.

In addition, the transmit power at the relay is

\[ P_r = \text{Tr} (W (\rho H^H F F^H H + \sigma^2 I) W^H). \]

(10)

2.2. Problem Statement

Assuming that the circuit power consumptions at the T, R, and D are irrelevant with the information rate, which are denoted as \( P_{ct}, P_{cr}, \) and \( P_{cd}, \) respectively. Thus, the total power consumption of the relay network can be modeled as

\[ P_{tot} (F) = \frac{\text{Tr} (F F^H)}{\eta} + P_c, \]

(11)
where \(0 \leq \eta \leq 1\) is the transmitter’s power amplifier efficiencies. In addition, \(P_c = P_{ct} + P_{cr} + P_{cd}\) is the total circuit power consumption for the network [19].

EE is commonly defined as the number of information bits delivered per unit energy (bits per Joule) [17–20], which can be expressed as

\[
EE(\rho, F, W) = \frac{R_2(\rho, F, W)}{P_{tot}(F)} \text{ [bits/Joule].}
\]  

(12)

In this paper, we investigate the joint precoding, BF, and PS ratio design to maximize the EE. Mathematically, the problem is given as

\[
P_1: \max_{\rho, F, W} EE(\rho, F, W) \tag{13a}
\]

s.t. \(P_T + P_{cr} \leq P_{EH}\), \(R_1 \geq R_{th}\), \(\text{Tr}(FF^H) \leq P_s\), \(0 \leq \rho \leq 1\), \(\tag{13b} \tag{13c} \tag{13d} \tag{13e}\)

where Eq. (13b) denotes the total power threshold for the R, and Eq. (13c) denotes the spectral efficiency constraint in the 1st phase with \(R_{th}\) being the minimal spectral efficiency threshold. Besides, \(P_s\) denotes the total transmit power threshold for the T. The EE problem is hard to handle due to non-convex objective (13a) and the constraints (13b), (13c). In the following, we will propose a Dinkelbach method based iterative algorithm to solve Eq. (13).

3. THE JOINT PRECODING, BF, AND PS DESIGN

Firstly, we simplify the coupled relationship in variables \(F, W, \text{ and } \rho\) for Eqs. (13a)–(13c), by the following variable substitutions: \(A = \sqrt{\rho WH^H F}, B = WW^H, N = \rho H^H FF^H H, Y = \rho^2 I, \text{ and } M = (G^H A)^H (\sigma^2 G^H B G + \sigma_d^2 I)^{-1} (G^H A), \) respectively.

Then, Eq. (13) can be rewritten as

\[
P_2: \max_{\rho, F, W, A, B, M, N, Y} \frac{\ln |I + M|}{P_{tot}} \tag{14a}
\]

s.t. \(\text{Tr}(AA^H) + \sigma^2 \text{Tr}(B) + P_{cr} \leq \eta(\text{Tr}(H^H FF^H H) - \text{Tr}(N)),\) \(\frac{1}{2} \ln \left| I + \frac{1}{\sigma^2} N \right| \geq \tilde{R}_1,\) \(\text{Tr}(FF^H) \leq P_s,\) \(0 \leq \rho \leq 1,\) \(\tag{14b} \tag{14c} \tag{14d} \tag{14e}\)

where Eq. (14) is still non-convex, mainly due to the fractional objective. In the following, we will turn Eq. (14) into a penalized difference-of-convex (DC) programming problem [23].

Firstly, we need to decouple the variables and introduce the following Lemma.

**Lemma 1:** The equations \(A = \sqrt{\sigma WH^H F}\) and \(B = WW^H\) can be equivalently rewritten as

\[
\begin{bmatrix}
B & A & W \\
A^H & K_1 & F^H H \\
W^H & H^H F & I
\end{bmatrix} \succeq 0,
\]

\(\text{Tr}(B - WW^H) \leq 0,\) \(\tag{15a} \tag{15b}\)

where \(K_1\) is the auxiliary variable.

**Proof:** Please refer to [18] for the proof of Lemma 1.

Similarly, the equations \(N = \rho H^H FF^H H\) and \(Y = \rho^2 I\) can be replaced with

\[
\begin{bmatrix}
Y & N & \rho I \\
N^H & K_2 & H^H FF^H H \\
\rho I & H^H FF^H H & I
\end{bmatrix} \succeq 0,
\]

\(\text{Tr}(Y - \rho^2 I) \leq 0,\) \(\tag{16a} \tag{16b}\)
where $K_2$ is the auxiliary variable.

In addition, $M = (G^H A)^H (\sigma^2 G^H B G + \sigma_d^2 I)^{-1} (G^H A)$ is equivalent to

$$
\begin{bmatrix}
M \\
G^H A \\
\sigma_p^2 G^H B G + \sigma_d^2 I
\end{bmatrix} \succeq 0, \\
\text{Tr}(M) - \text{Tr}((G^H A)^H (\sigma^2 G^H B G + \sigma_d^2 I)^{-1} (G^H A)) \leq 0.
$$

(17a) \\
(17b)

To exploit the convexity of Eq. (17b), we introduce the following Lemma.

**Lemma 2:** Denote $f(A, B) = \text{Tr}((G^H A)^H (\sigma^2 G^H B G + \sigma_d^2 I)^{-1} (G^H A))$, then $f$ is a convex function, where $A \succeq 0$ and $B \succeq 0$.

**Proof:** Please refer to [18] for the proof of Lemma 2.

Then, via using the following linearize technique, we change the objective (14a) as

$$
q = \ln |I + M|/P_{tot},
$$

(18)

then we have the following Lemma.

**Lemma 3:** The sufficient and necessary condition for $q^*$ being the optimal solution is when and only when $q^*$ is the unique solution of the auxiliary function

$$
\max_{\rho, F, W, A, B, M, N, Y} \ln |I + M| - q^* P_{tot} = \ln |I + M^*| - q^* P_{tot} = 0,
$$

(19)

where $\ln |I + M| \geq 0$ and $P_{tot} \geq 0$.

**Proof:** Please refer to [19] for the proof of Lemma 3.

Via Lemma 3, we obtain the following problem

$$
P3 : \min_{\rho, F, W, A, B, M, N, Y} q P_{tot} - \ln |I + M| \\
\text{s.t.} \ (14b) - (14e), (15) - (17).
$$

(20a) \\
(20b)

The objective (20a) is convex; however, all the constraints are non-convex. To overcome this obstacle, in the following we will linearize these constraints via the penalized DC method.

Specifically, we remove these constraints into the objective and introduce the penalty multiplier $\mu$ to scale the violation of these constraints, then we have the following problem

$$
P4 : \min_{\psi \in \Psi} q P_{tot} - \ln |I + M| + \mu h(\psi),
$$

(21)

where

$$
\Psi = \{\psi = \rho, F, W, A, B, M, N, Y | (14b) - (14e), (15) - (17)\},
$$

(22)

and

$$
h(\psi) = \text{Tr}(A A^H) + \sigma_p^2 \text{Tr}(B) + P_c - \eta \text{Tr}(H^H F F^H H) - \text{Tr}(N)) + \text{Tr}(B - W W^H) \\
+ \text{Tr}(Y - \rho^2 I) + \text{Tr}(M) - \text{Tr}((G^H A)^H (\sigma^2 G^H B G + \sigma_d^2 I)^{-1} (G^H A)).
$$

(23)

Furthermore, Eq. (21) can be reformulated as

$$
\min_{\psi \in \Psi} f_1(\psi) - f_2(\psi),
$$

(24)

where

$$
f_1(\psi) = q P_{tot} - \ln |I + M| + \mu \text{Tr}(A A^H) + \sigma^2 \text{Tr}(B) + P_c + \eta \text{Tr}(N) + \text{Tr}(B) + \text{Tr}(Y) + \text{Tr}(M),
$$

(25)

and

$$
f_2(\psi) = \mu (\eta \text{Tr}(H^H F F^H H) + \text{Tr}(W W^H) + \text{Tr}(\rho^2 I) + \text{Tr}((G^H A)^H (\sigma^2 G^H B G + \sigma_d^2 I)^{-1} (G^H A))).
$$

(26)

Since the difference of two convex functions is non-convex, we need to linearize Eq. (24) via convex approximation. Specifically, we have the following approximated reformulation of $f_2(\psi)$

$$
f_2(\psi; \tilde{\psi}) = f_2(\tilde{\psi}) + \mu (2 \eta \text{Re}(\text{Tr}(H^H F F^H H (F - \tilde{F}))) + 2 \text{Re}(\text{Tr}(\tilde{W}^H (W - \tilde{W})))) \\
+ 2 \text{Re}(\rho (\rho - \tilde{\rho}) I) + 2 \text{Re}(\text{Tr}(\tilde{M} (A - \tilde{A}))) - \sigma^2 \text{Tr}(\tilde{M} (B - \tilde{B}) \Sigma^T),
$$

(27)
where \( \tilde{\psi} \) is a given point and \( \tilde{M} = (G^H \tilde{A})^H(\sigma^2 G^H \tilde{B} G + \sigma_n^2 I)^{-1}(G^H \tilde{A}) \).

Combining the procedures above, we obtain the following approximation of P4

\[
\min_{\psi \in \Psi} f_1(\psi) - f_2(\psi; \tilde{\psi}^{(m)}),
\]

which can be efficiently solved by the convex programming toolbox CVX [24], and \( \tilde{\psi}^{(m)} \) is the optimal solution of Eq. (28) obtained in the \( m \)-th iteration.

To this end, we turn Eq. (13) into a solvable problem in Eq. (28). The entire algorithm is summarized as Algorithm 1.

**Algorithm 1** The Dinkelbach based algorithm for solving Eq. (13).

1: Initialize \( n = 1 \), \( \zeta \geq 0 \), \( (F^0, W^0, \rho^0) \in \mathcal{X} \) and \( q_n \) such that \( F(q_n) \geq 0 \).
2: repeat← Level 1
3: Initialize \( i = 0 \), \( R^{n,i} = 0 \), \( \sigma \geq 0 \), and \( (F^{n,i}, W^{n,i}, \rho^{n,i}) = (F^{n-1}, W^{n-1}, \rho^{n-1}) \).
4: repeat← Level 2
5: Solving (13) with \( q = q_i \) and obtain \( (F^{n,i+1}, W^{n,i+1}, \rho^{n,i+1}) \); 6: Compute \( R^{n,i+1} \) via (9); 7: Update \( i = i + 1 \);
8: \( \Delta R = R^{n,i} - R^{n,i-1} \).
9: until \( |\Delta R| \leq \sigma \);
10: \( (F^n, W^n, \rho^n) = (F^{n,i}, W^{n,i}, \rho^{n,i}) \);
11: update \( q_{n+1} = \frac{\rho^n}{\rho^n + \psi_i(F^n, W^n, \rho^n)} \);
12: update \( n = n + 1 \);
13: until \( |q_n - q_{n-1}| \leq \zeta \);
14: Output the EE \( q_n \).

4. SIMULATION RESULTS

In this section, simulation results are presented to evaluate the performance of the proposed design. We set \( N_t = 5 \), \( N_r = 5 \), \( N_d = 5 \), \( P_s = 10 \) dBw, \( P_{ct} = P_{cr} = P_{cd} = -30 \) dBw, \( R_{th} = 2 \) bits/s/Hz, \( \sigma_n^2 = \sigma_s^2 = \sigma_d^2 = 10^{-8} \), \( \eta = 1 \) for the following simulation examples. Besides, each entry of \( H \) and \( G \) is assumed to be independent and identically distributed (i.i.d.) random variables with \( \mathcal{CN}(0, 10^{-4}) \). In addition, we compare the proposed method with the following designs: 1) the fixed PS method, e.g., only optimizing the BF with fixed PS ratio \( \rho = 0.5 \); 2) the proposed design with fixed BF method, e.g., only optimizing the PS ratio \( \rho = 0.5 \) with fixed BF method. These designs are labeled as “the proposed method”, “the fixed PS design”, and “the fixed BF design”, respectively.

Firstly, we investigate the convergency performance of the proposed method by comparing the EE with the iterative numbers. Fig. 2 shows several examples of the convergence behavior with random channel realizations. From this figure, we can see that the proposed method can always converge to the optimal solution within limited iterative numbers.

Secondly, we compare the EE of these schemes to the total power budget for the transmitter \( P_s \), and the result is shown in Fig. 3. From this figure, we can see that the proposed method outperforms the other designs. In addition, the average EE reaches a saturation with the increase of \( P_s \), which is when \( P_s \) is relatively small compared to \( P_t \), and the EE metric is mainly determined by the information rate in the numerator. By contrast, when \( P_t \) becomes large, the EE metric is also determined by the relay transmit power in the denominator. As a result, the EE metric reaches a saturated point.

Lastly, we compare the EE of these schemes versus the relay antenna number \( N_r \), and the result is shown in Fig. 4. From this figure, we can see that for all these methods, the larger the relay antenna number is, the more EE can be obtained. Since higher spatial degree of freedom can be achieved with larger relay antenna number, thus it is beneficial to improve the EE.
Figure 2. Convergence behaviour of the proposed method.

Figure 3. The energy efficiency versus the transmit power.
Figure 4. The energy efficiency versus the number of relay antenna.

5. CONCLUSION

In this paper, the EE optimization in an EH enabled AF relay network has been investigated, where the energy-constrained relay harvests energy from the transmitter. Specifically, we aim to maximize the EE of the network via jointly optimizing the transmit precoding, relay BF, and PS ratio, under the constraints of the transmit power and spectral efficiency. To solve the formulated fractional programming, we approximate the problem via two-layer optimization, where the outer problem is handled by the Dinkelbach method, and the inner problem is solved by the penalized DC and CCCP method. Finally, an iterative method is proposed. Simulation results show that the proposed EE optimization method outperforms some existing schemes in related literatures.

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