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PHASE-ONLY CONTROL OF ANTENNA SUM PATTERNS

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Abstract–We describe how antenna sum patterns can be controlled by modifying just the phase distribution of the excitation. As examples, we calculate linear and circular aperture distributions affording symmetric sum patterns with low side lobe levels, and linear aperture distributions affording patterns with low sidelobe levels on one side of the beam.

1. INTRODUCTION

For many kinds of antenna, it is much easier to control the excitation phase distribution than the amplitude distribution. Methods for the synthesis of desired radiation power patterns by phase-only control have previously been described in [1] and [2] for line source sum patterns with asymmetric side lobes, in [3] for minimization of the side lobe levels of linear and planar arrays, and in [4] for adaptive introduction of nulls in the patterns of linear arrays.

In this paper we extend our earlier work on linear apertures [2] to obtain a very general strategy for phase-only control that is applicable to linear and circular continuous apertures and to linear and planar arrays. As examples of its application we calculate linear and circular aperture distributions affording symmetric sum patterns with low side lobe levels, and linear aperture distributions affording patterns with low sidelobe levels on one side of a central main beam.

2. METHOD

The new method optimizes the excitation phase distribution of the antenna by perturbing the zeros of a starting power pattern S_0 such as the pattern corresponding to uniform excitation of the antenna. In each iteration of the optimization procedure, the zeros of S_0 are perturbed, the excitation distribution h affording the new pattern is calculated, a new excitation distribution h' is constructed by combining the phases of h with the amplitudes of the excitation distribution h_0 of the starting pattern, and the pattern S' afforded by h' is calculated and compared with the desired pattern. The optimization method used to obtain all the numerical results presented was the simulated annealing algorithm described in [5].

3. EXAMPLES

The implementation of the general strategy described above is best shown by examples rather than through universal equations. The examples presented in this section show how well phase-only control can achieve some standard design objectives for linear or circular apertures: lowering the maximum side lobe level as much as possible or, for a linear aperture, lowering the maximum side lobe level on one side of the main beam as much as possible. All the numerical results were obtained using, for the simulated annealing algorithm, initial simplexes defined by λ values of 0.6–0.8, an initial temperature of 100, and a cooling factor of 0.8 applied every 25 iterations.

3.1 Line Sources: Real Perturbations

The distribution-dependent factor of the power pattern of a line antenna of length $2a$ with the constant excitation distribution $h_0(\zeta) = K$, K being a positive real number and ζ distance to the right of the center of the aperture, is

$$S_0(u) = 2Ka\frac{\sin(\pi u)}{\pi u} \tag{1}$$

where $u = (2a/\lambda)\cos\theta$ if the pointing angle θ is measured from endfire. The zeros u_{n0} of this array factor are the non-zero integers. The levels of its side lobes are modified without significantly broadening the main beam if, following Taylor [6], we move some of these zeros along the real axis; if $\Omega = \{-(n_L - 1), \ldots, -1, 1, \ldots, n_R - 1\}$ $(n_L, n_R > 0)$ is the set of indices n of the zeros that are perturbed from their original value $u_{n0} = n$ to a new value

$$u_n = u_{n0} + \delta_n \tag{2}$$

then the distribution-dependent factor becomes [7]

$$S(u) = S_0(u) \frac{\prod_{n\in\Omega} (1 - u/u_n)}{\prod_{n\in\Omega} (1 - u/u_{n0})} \tag{3}$$

The excitation distribution affording this factor is

$$h(\zeta) = \frac{1}{2a} \sum_{n\in\Omega} S(n) e^{-jn\pi\zeta/a} \tag{4}$$

In general, $|h(\zeta)| \neq |h_0(\zeta)|$. If we wish to control the distribution-dependent factor by altering the phase but not the amplitude of the excitation, then we can optimize the perturbations δ_n so as to minimize the difference (in some appropriate sense) between the desired pattern and the distribution-dependent factor $S'(u)$ generated by the excitation distribution

$$h'(\zeta) = |h_0(\zeta)| \arg[h(\zeta)] \tag{5}$$

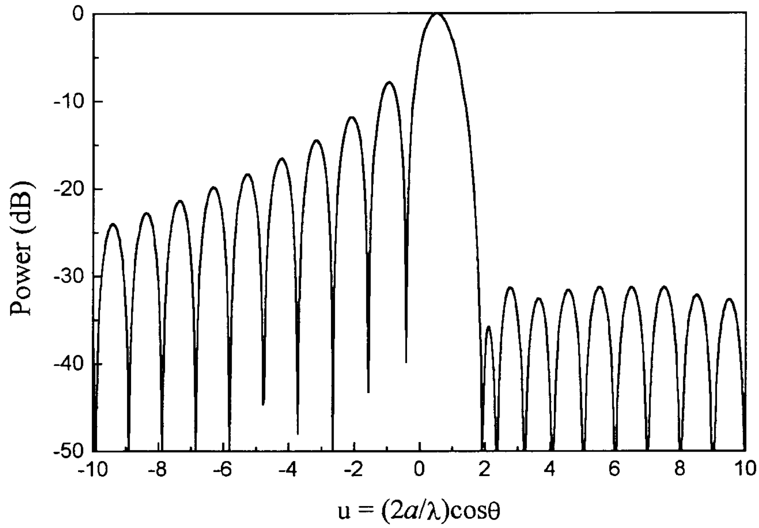
which is given by

$$S'(u) = \int_{-a}^{a} h'(\zeta) e^{j\pi u\zeta/a} d\zeta \tag{6}$$

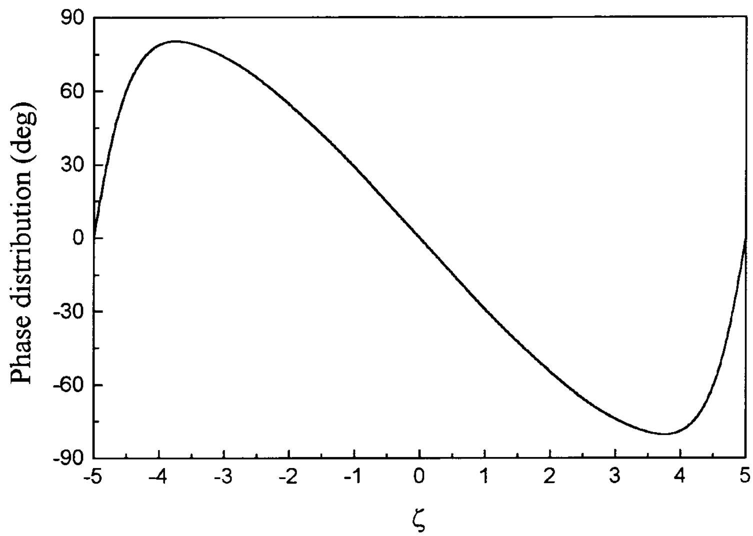
If, for example, we wish, like [1], to lower the maximum side lobe level on one side of the main beam to some particularly low level SLL_d (even at the expense of the SLL on the other side), then a suitable cost function quantifying the difference between the desired and achieved array factors is $C = |\mathrm{SLL}_d - \mathrm{SLL}_a|^2$, where SLL_a is the level of the highest side lobe on the relevant side of the achieved pattern. When the strategy described above was applied with this cost function to the case $a = 5\lambda$ with $\Omega = \{-6, \ldots, 1, 1, \ldots, 6\}$ starting from a uniformly excited aperture, it allowed synthesis of a distribution-dependent factor with SLLs that had risen 5.4 dB to − 7.84 dB on the left and had fallen by 17.8 dB to − 31.27 dB on the right (Fig. la), a drop that compares favourably with the drop of about 6 dB achieved by Frey and Elliott for the same example. As in Frey and Elliott's pattern, the main beam has undergone a slight shift to the right that could be redressed by introducing a progressive phase shift in the excitation distribution. The set Ω used was found to be optimal among sets of the form $\{-(n_L - 1), \ldots, -1, 1, \ldots, n_R - 1\}$ for positive integers n_L, n_R; also, the left-right SLL difference was found to be more sensitive to n_L than to n_R.

Equations 2–6, and the synthesis method they describe, are equally valid if instead of starting from the pattern corresponding to uniform excitation we start from any other sum pattern $S_0(u)$ with a main beam at broadside and zeros on the real axis. This allows the power pattern of an antenna with an arbitrary given excitation distribution to be modified by changing only the excitation phase distribution. To take another example used in [1], namely a standard symmetric −25 dB Taylor pattern with $\overline{n} = 7$, application of the present strategy with the same cost function and set Ω as above affords a pattern with a left-hand SLL of −18.6 dB and a right-hand SLL of −37.8 (Fig. 2a), a drop of 12.0 dB (with respect to the Taylor pattern) that is again much larger than the drop of about 6 dB achieved by Frey and Elliott's method for the same case.

As a direct consequence of Eq. (4), the solutions obtained by the present method, like those of Frey and Elliott's method, have anti-symmetric excitation phase distributions (Figs. 1b and 2b). It may be noted that whereas [1] stated that increasing the maximum phase lag beyond about 60° did not significantly improve the performance of their method, the solutions of both the examples considered above involve phase differences greater than 80°. However, if it is desirable to limit the maximum phase lag (for cost reasons, for example), this can be achieved — at the expense, of course, of SLL reduction — by including appropriate constraints (e.g., by adding a suitable term to the cost function).



(a)



(b)

Figure 1. Power pattern (a) and excitation phase distribution (b) obtained by modifying the uniform excitation distribution of a line source of length $2a = 10\lambda$ so as to reduce the side lobe level of the right-hand side as much as possible. ζ and a in units of λ.

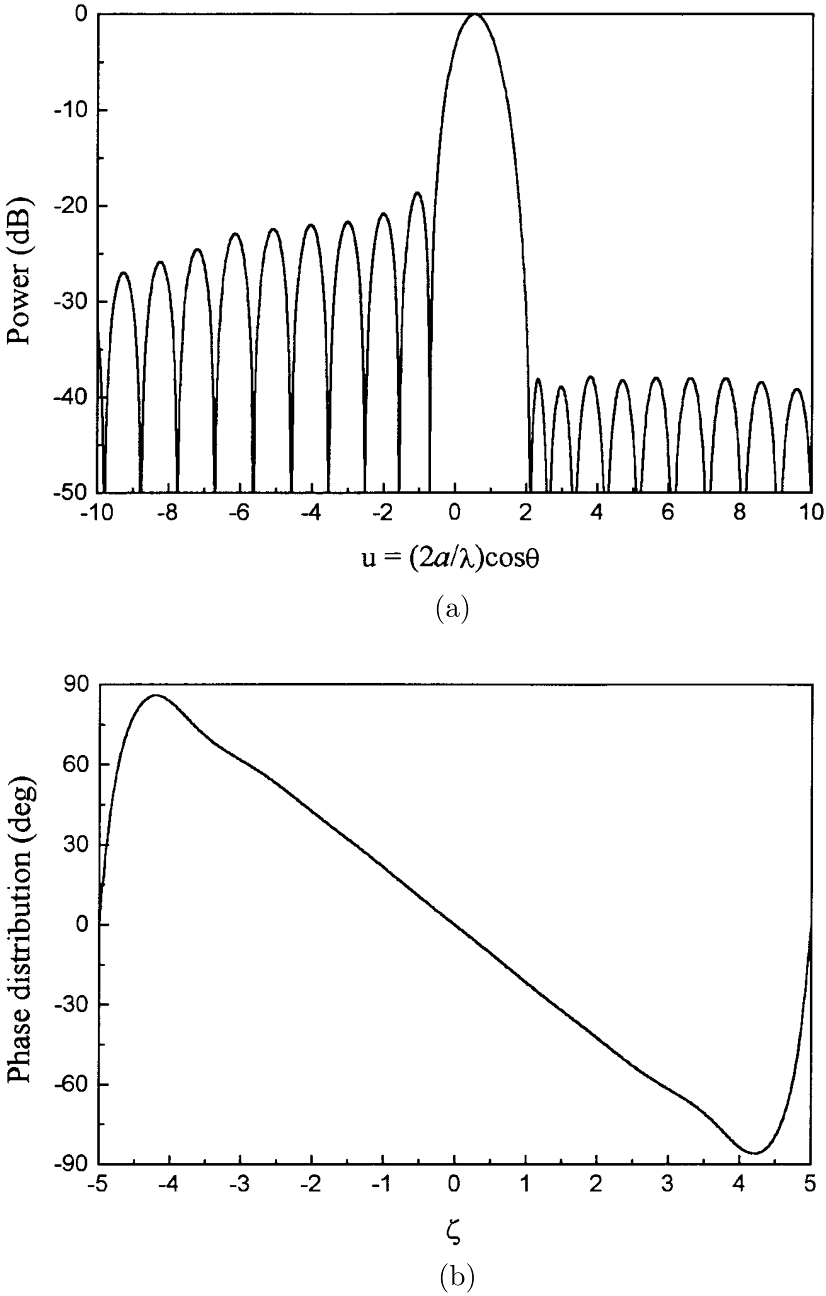


Figure 2. As for Fig. 1, but starting from a $-25\,\text{dB}$ Taylor pattern with $\overline{n} = 7$.

3.2 Line Sources: Complex Perturbations

If we are willing to allow some broadening of the main beam in order to obtain the features desired elsewhere in the power pattern, then we can use complex perturbations δ_n in Eq. (2). For example, the side lobe level of the distribution-dependent factor of the uniformly excited line source considered above can be reduced to -17.3 dB by complex perturbation of the nine innermost zeros on each side (Fig. 3). The cost function that achieves this is the same as before except that SLL_a and SLL_d are now the maxima among all the side lobes, regardless of which side they are on; and symmetry is ensured by making $\delta_{-n} = -\delta_n$. In this case, Eqs. (3) and (4) are equivalent to the generalized Taylor formulae presented in [8]. If instead of starting from a uniform excitation we start from a Taylor pattern, the same procedure lowers the Taylor side lobe level by about 2 dB.

3.3 Circular Apertures

When the general strategy described above is applied to a circular aperture of radius a with axially symmetric excitation [9], Eqs. (4)–(6) are replaced by

$$h(p) = \frac{2}{\pi^2} \sum_{n=0}^{\overline{n}-1} \frac{S(\gamma_{1n})}{J_0^2(\gamma_{1n}\pi)} J_0(p\gamma_{1n}) \tag{7}$$

$$h'(p) = |h_0(p)| \arg[h(p)] \tag{8}$$

and

$$S'(u) = \int_0^\pi p h'(p) J_0(up) dp \tag{9}$$

where ap/π is distance from the center of the aperture, J_0 is the zeroth-order Bessel function of the first kind, $\pi\gamma_{1n}$ is the n-th zero of the first-order Bessel function of the first kind, $u = (2a/\lambda)\sin\theta$ (θ being the polar angle) and, in the interests of the desired geometry δ_{-n} is again set equal to $-\delta_n$ [8]. If the starting pattern is $S_0 = J_1(\pi u)/(\pi u)$ (the distribution-dependent factor of the power pattern generated by a uniform aperture distribution), then for $a = 5\lambda$ use of the same cost function as in Section 3.2 leads to a pencil beam pattern with a side lobe level of -23.1 dB, more than 5 dB below the side lobe level of the starting pattern (Fig. 4). To achieve this result, it is necessary to perturb all the zeros in visible space.

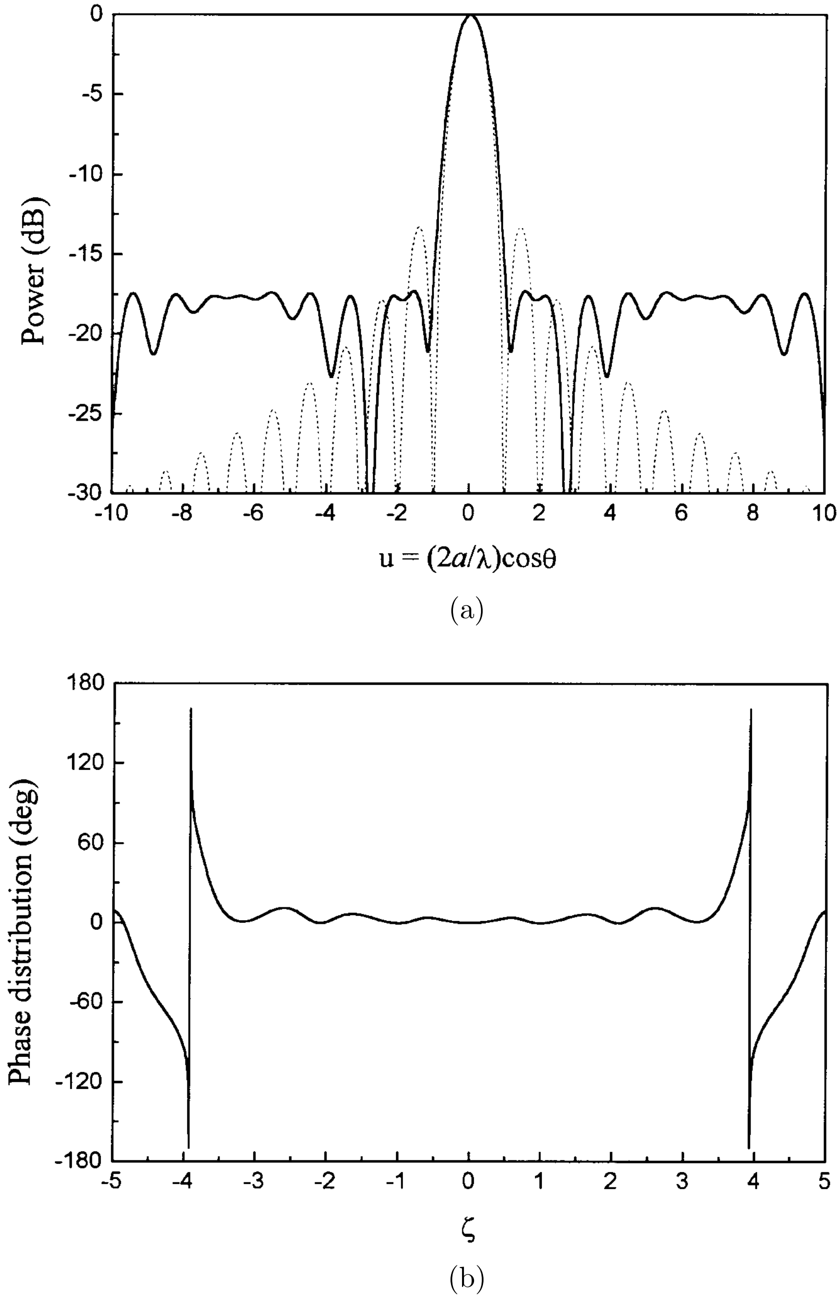
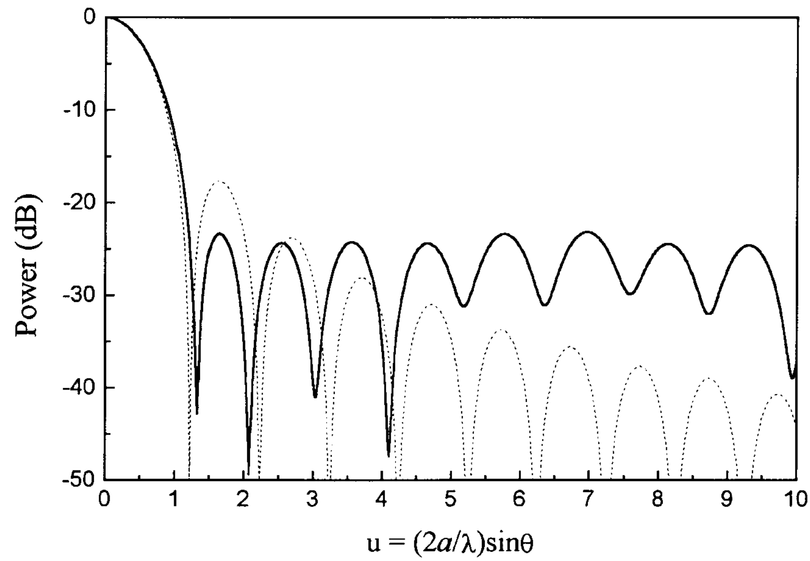
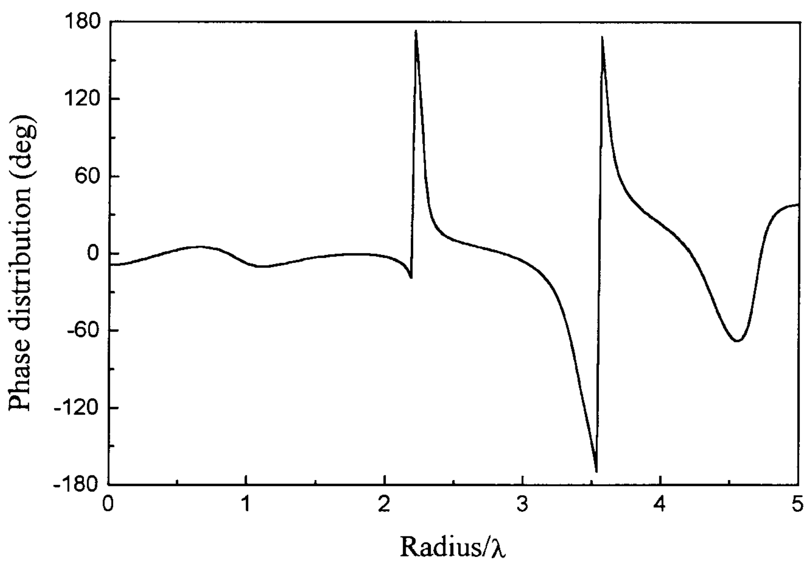


Figure 3. As for Fig. 1, but with the aim of reducing side lobe level as much as possible without altering the symmetry.



(a)



(b)

Figure 4. Power pattern (a) and excitation phase distribution (b) obtained by modifying the uniform excitation distribution of a circular aperture of radius $a = 5\lambda$ so as to reduce the side lobe level as much as possible without altering the symmetry.

4. FINAL REMARKS

This article presents the first general strategy for the application of optimization algorithms to the synthesis of antenna power patterns by modification only of the phase distribution of the aperture excitation. Although only the control of sum pattern side lobe levels has been considered here, the basic manoeuvre, epitomized in Eq. 5, is generalizable to the control of other pattern features and other kinds of pattern.

As in any application of optimization methods, the more complex the problem, the more likely it is that the solution will benefit from some experimentation with the optimization control parameters (e.g., the initial temperature in the case of the simulated annealing technique) or the cost function parameters (e.g., SLL_d). In our experience, best side lobe levels are often achieved by specifying SLL_d's considerably lower than can be hoped for; for the specific examples presented here, trials were made with SLL_d's ranging from -25 to -50 dB. The solutions obtained, which compare well with those afforded by other methods, show how much can be achieved by modifying just the phase distribution of an aperture excitation.

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