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# SIMULTANEITY, CAUSALITY, AND SPECTRAL REPRESENTATIONS

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**Abstract**–Recently Zangari and Censor discussed the non-uniqueness of the spatiotemporal world-view, and proposed a representative alternative based on the Fourier transform as a mathematical model. It was argued that this so called spectral representation, by virtue of the invertibility of the Fourier transform, is fully equivalent to our conventional spatiotemporal world-view, although in the two systems the information is ordered in a radically different manner.

Criticism of the new conception can be traced back to the fundamental principles of simultaneity and causality, whose role in the spectral domain has not been sufficiently demonstrated. These questions are carefully investigated in the present study.

Simple but concise examples are used to verbally and graphically clarify the mathematics involved in integral transforms, like the Fourier transform under consideration.

The transition from the spatiotemporal domain to the spectral domain entails not only a different patterning of data points. What is involved here is that *every* point in one domain is affecting *all* points in the other domain, and to follow what happens to simultaneity and causality under such circumstances is not a trivial feat. Even for the general reader, the discussion based on the simple examples should suffice to critically follow the arguments as they unfold. For completeness, the general mathematical formulations are given too.

In order to follow the footprints of the spatiotemporal simultaneity and causality concepts into the spectral domain, a special strategy is implemented here: Certain spatiotemporal situations are stated, and then their outcome in the spectral domain is examined. For example, it is shown that if a causal sequence of events is flipped over in time, thus reversing the order of cause and effect, in the spectral domain the

associated spectrum will become a mirror image of the original one. The claim that the spectral transforms are invertible, consequently no information is lost in the spectral world-view, is thus substantiated.

These ideas are extended to situations involving both space and time. Of particular interest are cases where relatively moving observers are involved, each at rest with respect to an appropriate spatial frame of reference, measuring proper time in this frame. In such cases, time and space are intertwined, hence simultaneity and causality must be appropriately redefined. Both the Galilean, and the Special Relativistic Lorentzian transformations in the spatiotemporal domain, and their corresponding spectral domain Doppler transformations, fit into our argument. Special situations are assumed in the spatiotemporal domain, and their consequent footprints in the spectral domain are investigated. Although a great effort is made to keep the presentation and notation as simple as possible, in some places more sophisticated mathematical concepts, such as the Jacobian associated with the change of integration variables, must be incorporated. Here the general reader will have to accept the (mathematical) facts without proof.



## 1. INTRODUCTION AND STATEMENT OF THE PROBLEM

In a recent paper, Zangari and Censor [1] discussed the non-uniqueness of the conventional spatiotemporal world-view, and as an example proposed an alternative spectral representation, based on the Fourier transform as a mathematical tool for the metaphysical world-view model. The choice was motivated by the fact that the Fourier transform is extensively used in science and engineering, and many biological systems involved in human and animal perception submit, at least approximately, to modeling of spectral rather than spatiotemporal nature. Inasmuch as the Fourier transform is invertible, such that coding spatiotemporal data into the spectral domain, and decoding it back, yields the original data, it was argued that the spectral world-view, in spite of apparently being totally different, is equivalent to our conventional spatiotemporal world-view: The two alternatives contain the same information, although it is ordered, or coded, in a different manner.

The germs of Zangari and Censor's approach [1] can be found in other places in the literature. For example, it is reminiscent of Bohm's ideas regarding explicate and implicate order [2], roughly dealing with coding of "explicate" data which originate (empirically) in some physical reality, into an "implicate" code, which still contains the original information, but orders data in a new manner. The "explicate" data can be retrieved by applying the inverse transform, i.e., the associated code which rearranges the data into its original pattern (provided the coding was done in such a manner that the inversion is feasible). Bohm [2] provides two interesting examples, which although merely illustrative to his argument, are important to place the spatiotemporal versus spectral world-views in a proper perspective. Thusly Bohm mentions the method of holography, which codes three-dimensional scenes onto a photographic plate. Looking through the photographic plate, all one can see is some kind of Moiré-like patterns of wavy shades. In his example, this constitutes the implicate representation. Using an appropriate setup will display to the observer a simulacrum of the original scene. Hence it is argued that the coded, or transformed, "object", the hologram in this case, contains all the data, or information, of the original explicate "object", i.e., the scene which was recorded. The Fourier transform used in [1] is closely related to the mathematical description of holography [1]. The second example concerns the coding taking place in a television (or video) camera. The image of the scene recorded by the camera is transformed into a temporal sequence of

[1] In holography the fact that the photographic plate emulsion is sensitive to intensity only is taken into account, while the Fourier transform involves both intensity and phase.

electrical pulses (which can be further coded according to some algorithm on a magnetic tape, and in that case we obtain a spatial coding once more, although different from the original object). The implicate object comprised of the temporal sequence of electrical pulses can be later decoded by an appropriate television receiver, where the coded sequence will be ordered on the screen in a way that creates a simulacrum of the original scene. Due to the fact that the original image is scanned along horizontal lines, and then line by line from top to the bottom, there exists an interesting feature here, whereby space data are represented in time. Moreover, two spatially adjacent points on different sweep lines will usually be non-adjacent in the time sequence, and the flow of time, in this coding example, does not imply time flow or causality in the original scene (and vice-versa).

In [1] this idea was taken one step further: Starting with the conventional spatiotemporal world-view, it was asked if a completely spectral world-view is self-consistent, and could thus provide an alternative (e.g., for some imaginary creature). In the wake of this work [1], many questions have been raised by students and colleagues, including a commentary by Gaudiano, whose very title "Being in the right place at the right time" [3] points to the problematics: It appears that the doubts and uneasiness with the new conception can be traced back to the fundamental principles of simultaneity and causality, and that we failed in clearly demonstrating how these are displayed in the spectral domain. One way of pinning us down was to ask how the spectral creature perceives simultaneity, and causality, or sometimes birth and death, in a world-view where time and space do not explicitly exist.

Essentially, dealing with this issue of investigating the footprints of spatiotemporal processes as they are perceived in the spectral domain, is the main theme of the present study. In doing so, one must be aware of the targeted gamut of readers. There is no point in regurgitating material that can be found in various mathematics or physics tracts. Nor is it desired to oversimplify matters to the point where the verbiage ends up in losing any vestiges of rigorousness. Simple, but concise examples are exploited below to verbally and graphically clarify the mathematical ideas involved. Even for a reader not familiar with the mathematical tools, this should suffice for critically following the arguments as they unfold. For completeness, the general mathematical formulations follow, using language which is not too succinct. Another strategy used below is to avoid the trap of quoting and interpreting early philosophers as was attempted in [1]. Getting into a squabble regarding the correct understanding of earlier metaphysicists could only degrade the clarity of the present discussion. This is left for a later time. As in many other cases, it turns out that the key to the answer is the adequate formulation of the question. The meaningful strategy implemented here is to assume certain spatiotemporal situations, and then consider the outcome in the spectral domain. For example, it is shown that if a temporal sequence of events, which can be assumed to be causal, is flipped over in time, reversing the order of cause and effect, the spectrum will become a mirror image of the original in the spectral domain. Thus the claim that the spectral transformations are invertible, and no information loss is incurred in the spectral world-view, is vindicated.

A few words about causality: Physicists are usually not interested in causality, simultaneity and other metaphysical concepts *per se*, but only to the extent that they can relate such concepts to their mathematical models by which they “describe”, and “measure” empirical reality. Because of the complexity of the mathematics involved in such models, let alone the experiments involving increasingly sophisticated apparatus, the garden variety natural scientist is deep entrenched in his professional aspects. It comes as a rude shock when he realizes that almost all his mathematical models, usually in the form of systems of partial differential equations, are non-causal, or in other words, indifferent to the direction of time flow. However, causality cannot be ignored, it is deeply ingrained in us. Even as secondary education students, solving problems which involve second order algebraic equations, we came across answers like negative time intervals, and the textbook typically comments at such a point that “since a negative time interval has no meaning for the time it takes to fill the pool with water, this answer is to be ignored because it is non-physical”. Thus the synthetic principle of causality is already used for very practical everyday reasons, without delving into the question as to why a “good” mathematical model yields “bad” answers. On a higher level of understanding, it is realized that most mathematical models used by physicists are in themselves indifferent to causality, which must be introduced into them *a posteriori* and heuristically. It is interesting to quote Morse and Feshbach (see p. 206, they even venture broader remarks, see pp. 843–844) [4], an authority on mathematical physics, considering the time symmetry of solutions of the wave equation, with causal and anti-causal solutions referred to in the lingua franca of this subject area as "retarded potential" and "advanced potential", respectively: "It should not be thought that this cause-and-effect relation, employed here, is obvious. The unidirectionality of the flow of time is apparent for macroscopic events, but it is not clear that one can extrapolate this experience to microscopic phenomena. Indeed the equations of motion in mechanics and the Maxwell equations, both of which may lead to a wave equation, do not have any asymmetry in time. It may thus be possible, for microscopic events, for "effects" to propagate backwards in time..."

Einstein's Special Relativity theory [5–7] established a new cornerstone in our understanding of simultaneity and causality: In a world where relatively moving observers use different spatial frames of reference and clocks are measuring different time rates for various observers, the naive meanings of simultaneity and causality as temporal processes must be redefined. In a nutshell — space and time are intertwined and must be considered together. In this more general context the terms coincidency (as extending simultaneity), and compellency (as extending causality), are dubbed. For relatively moving observers we need in the spatiotemporal domain space-time transformations, be it the "classical" Galilean-, or the "modern" Special Relativity Lorentzian-, transformation. We need also the associated Galilean and Relativistic Doppler transformations, respectively, in the spectral domain.

Some related mathematical concepts, such as the Jacobian associated with the change of integration variables cannot be avoided, and are too technical to explain in simple language — the general reader will have to "take my word for it" when some mathematical consequences are heralded, although a great effort is made to keep the presentation as simple and self-contained as possible.

## 2. SYNOPSIS

The rest of the paper is structured as follows: We start with a section *Diffraction at Your Fingertips* which is intended for the general reader or even the initiated but too theoretical mathematician, who were not exposed to the physics of optics and diffraction phenomena, wondering how all this might relate to the question of spatiotemporal and spectral world-views. On the level of a simple parlor trick, we show how diffraction can be created absolutely without any sophisticated optical bench equipment. What would be the effect if our ocular apertures (i.e., the eye pupil) were so small that diffraction would be part of our biological makeup? How would this affect our world-view?

Fun and games over, the following section is *On Transformations, a Simple Example, and the Temporal Fourier Transform.* This serves to introduce the concept of transforms that map all points in one domain onto all points in another one, which is characteristic of integral transforms in general, and the Fourier transform in particular. It also serves to introduce terms and notation needed later to discuss simultaneity and causality and their footprints as perceived in the spectral domain. The feasibility of such mappings is discussed for a simple example, verbally and graphically, without going into elaborate mathematical proofs. The one-dimensional temporal Fourier transform pair is introduced, and linked to the simplified example.

In the next section, *Positive and Negative Time Flow, Temporal Causality*, a situation is devised in which the time axis is inverted, and it is shown that this causes the spectral axis to be inverted too. Consequently, if for a certain process the direction of time flow signifies causality, it can be stated that the footprint of temporal causality, and its opposite, i.e., the interchange of cause and effect, is manifested in the spectral domain by the spectrum becoming a mirror image of the original one. These findings are proved more abstractly, in the mathematical context of the one-dimensional (temporal) Fourier transform. This part of the section may be skipped on a first run through the paper.

Next considered is the subject of *Simultaneity Effects in the Spectral Domain.* Once again we cannot look for a direct equivalent of the time domain simultaneity in the spectral domain, where time was scrambled and distributed among all data points. But we can ask what is the effect in the spectral domain, due to a perturbation of simultaneity in the spatiotemporal domain. It is shown that such an effect exists and can be mathematically traced.

In the section *Simultaneity in the Spatiotemporal Domain*, dubbed *Coincidency*, we consider the problem of simultaneity in the general case, involving time and space, including the case of observers in relative motion. The concept relies on definitions of coordinate transformations, whether Galilean or Lorentzian. Upon relating phenomena in two frames of reference by an appropriate formula, coincidency is defined as the substitution of the coordinate transformation that renders this formula an identity.

This facilitates the discussion of *Coincidency Effects in the Spectral Domain.* The novel feature here are the effects of spatiotemporal coincidency in the spectral domain. It is shown that the generalized definition of coincidency in the spatiotemporal domain implies coincidency in the spectral domain as well, in the same sense that spatiotemporal coincidency is defined.

Extending the above simple example of causality, *Compellency in the Spatiotemporal and Spectral Domains* are now considered. Here once again we use the simple graphic-verbal argument on a simple example, as above. Once again a situation is devised in which the role of cause and effect are interchanged, and it is shown that the same conclusion applies, namely, the footprint in the spectral domain is a symmetrical inversion of the original spectrum.

For completeness, the mathematical statement of *Compellency and the Four-Dimensional Fourier Transform* is formulated. The general mathematical calculation leads to the same conclusion: The footprint in the spectral domain is an inversion of the original spectrum.

The *Concluding Remarks* complete the discussion.

## 3. DIFFRACTION AT YOUR FINGERTIPS

Unlike a physicist, versed in diffraction theory and its associated mathematics, where the (far-field Fraunhofer) diffraction pattern is readily identified as a Fourier transform, theoretically inclined mathematicians and philosophers might find it beneficial to see at least one simple example, before delving into the more abstract details.

This is easily achieved by holding your index finger and thumb close to your eye, and squinting at the gap between them. Fig. 1 is a sketch of your fingertips and the gap between the fingercushions. Holding your fingers against a lighted background and varying the width of the gap, you will readily see a series of lines appearing in the gap. You are looking at the diffraction pattern, which is, loosely speaking, a Fourier transform of the gap image[2]. This is very close to what one would see looking at a hologram of a single stripe. The argument whether such blurred (for the human observer) images are inferior or superior

[2] This will be recognized as the Fourier transform of a “rectangular” or “boxcar” shaped slit, the so called sinc function, i.e., $\mathrm{sinc}(x) = \frac{\sin(x)}{x}$. Of course, due to the fact that the retina is sensitive to intensity and not phase, we actually see the absolute value $|\mathrm{sinc}(x)|$ and not the exact Fourier transform of the slit.

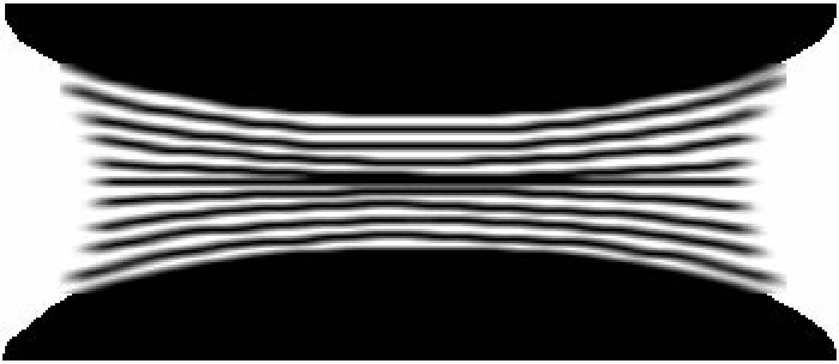

**Figure 1.** A sketch of the diffraction pattern, i.e., bright and dark regions, seen in the narrow slit created by holding close together two fingers near the eye. Closely related to the Fourier transform of a slit (see text).

compared to the geometrical, or ray optics, images that we usually see, is irrelevant. What is important is that the two representations within the Fourier transform pair are equivalent and invertible. Actually there are situations where the diffraction pattern looks simpler, i.e., seems to be less intricate, than the original scene. An example is shown in Fig. 2, depicting a pattern of gradually changing, alternating bright and dark stripes. This quite elaborate structure, because of its repetitive nature, possesses a simple Fourier transform consisting of a pair of lines [3].

From here on, we will develop the abstract aspects needed for the comparison of simultaneity and causality in the spatiotemporal and spectral domains.

## 4. ON TRANSFORMATIONS, A SIMPLE EXAMPLE, AND THE TEMPORAL FOURIER TRANSFORM

A paramount feature of the Fourier transform (and other integral transforms) is that it is intrinsically unlike simplistic mappings where the order of data points is changed in a one-to-one fashion. An example to such simplistic mappings is provided by shuffling around the labels on

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[3] This example will be readily recognized as the transform of a sinusoid, resulting in a pair of equal weight Dirac $\delta$-impulses, being of equal sign for $\cos(x)$ and opposite sign for $\sin(x)$.

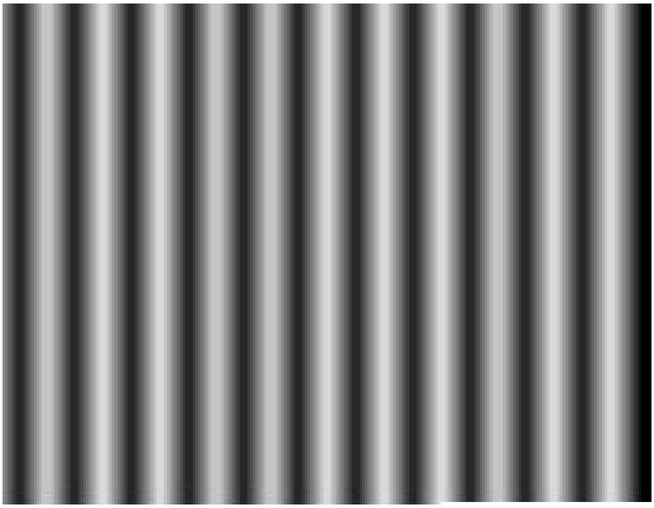

**Figure 2.** A sketch of alternating gradually varying dark and bright strips, whose Fourier transform is a pair of lines (see text).

your keyboard, so that pressing the "A" labeled key, you see a "P" on the screen, etc. Here each point in one domain is individually mapped onto a point in the other domain. Another example is the mapping of a scene in front of us on the eye's retina: A topsy-turvy simulacrum of the scene is imprinted on the retina, but still — regions of the scene are mapped onto regions of the retina in a one-to-one fashion. This one-to-one scheme is lost if the wrong corrective lenses are employed by a person, then the vision becomes blurred. This is exactly what happens in the above example of producing the line pattern between the finger-tips. But blurred images can also be unscrambled, if the scrambling algorithm is known. (and if it is an algorithm that in principle admits to inversion; this is sometimes referred to as deconvolution.)

In the spectral, or Fourier transform scheme, *all* the points in one domain are mapped onto *all* the points in the other domain, and *vice-versa.* This "all for one and one for all" (*a-la* Alexandre Dumas père's The Three Musketeers' motto) happens in the holography mapping mentioned above. Therefore a totally different new space is constructed, with new properties, and this is what makes the possibility of a spectral world-view so exciting. That feasibility of such mappings is discussed for a simple example, verbally and graphically,

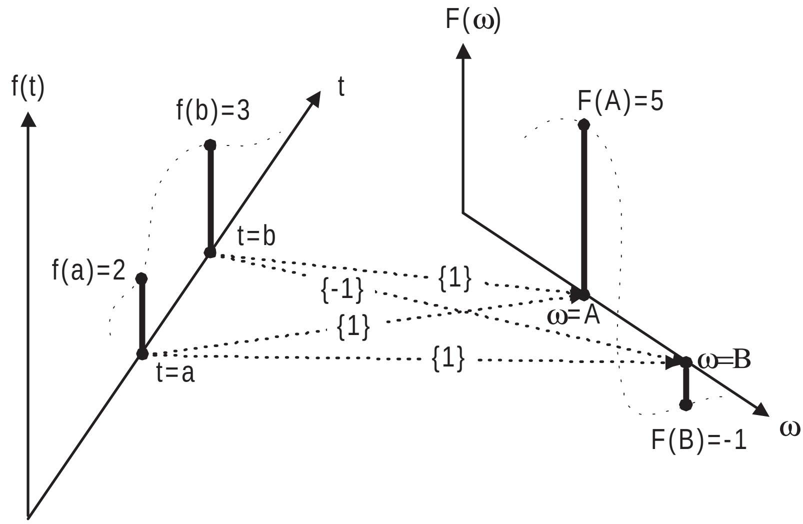


**Figure 3.** A simple transformation from the $t$-domain to the $\omega$-domain. The dotted-line connections and the weights (in braces) describe the transformation rule. The values of $f(t)$ at $t = a$, and $t = b$, and the transformed $F(\omega)$ at $\omega = A$, $\omega = B$ are discussed in the text. Sketches of a general time-signal $f(t)$, and the corresponding $F(\omega)$ are shown in dashed lines.

without going into mathematical proofs.

Mathematically, the Fourier transform, which is the basis for the spectral world-view [1], is a representative of a class of invertible integral transforms. For brevity, henceforth the spatiotemporal and the spectral domains, are denoted as $X$ and $K$, respectively. The following simple example serves to explain, in a verbal and graphical manner, how such transforms work. In Fig. 3, two spaces are juxtaposed: On the left is shown the $t$-axis, representing the $X$-domain, on the right is the $K$-domain, represented by the $\omega$-axis [4]. The two spaces are related by the transformation rule, so the fact that in Fig. 3 the $t$-axis points into the page, while the $\omega$-axis is directed towards the reader, is arbitrary and irrelevant. It is not sufficient to define the two domains, one must also consider phenomena, or events, occurring at labeled $X$

[4] With $t$, $\omega$, denoting time and (angular) frequency, correspondingly, this is the conventional notation.

and $K$ coordinates. Let us denote the events in the $X$-domain, for any specific point like $t = a,\ t = b$ (Fig. 3), by $f(a),\ f(b)$, respectively, or generically $f(t)$. I.e., $f(t)$ is a function, or a rule , expressed by some mathematical relation, table, or verbal explanation, such that for any arbitrary value $t$ substituted into $f(t)$, a corresponding value of the function $f$, is obtained. The observer in $X$ records the coordinates (say the time $t$) and measures the corresponding events $f(t)$, or in general $f(X)$. For sake of example, $t = a,\ t = b$ (Fig. 3) are singled out, and the corresponding values $f(a),\ f(b)$, are depicted on the left hand vertical plane. The events $f(t)$ can be thought of as any arbitrary empirical data that can be measured by an observer. Now let us set the rules for the mapping (which are chosen arbitrarily for this example): Points $\omega = A,\ \omega = B$ are chosen on the $\omega$-axis. The dotted lines connect $X$ and $K$ coordinates, and numbers in braces express the weights associated with these connections [5]. According to the following rule, $K$-domain events are determined by $X$-domain ones: $F(A)$ is obtained as the sum of products $f(a)$ times 1, and $f(b)$ times 1; $F(B)$ is obtained by the sum of $f(a)$ times 1 and $f(b)$ times $(-1)$. This yields for the $K$-domain events $F(\omega)$ the values $F(A) = 5,\ F(B) = -1$ as shown (Fig. 3). Written as equations, we have:

$$\begin{aligned} F(A) &= f(a)\cdot 1 + f(b)\cdot 1 = 2 + 3 = 5 \\ F(B) &= f(a)\cdot 1 + f(b)\cdot(-1) = 2 - 3 = -1 \end{aligned} \tag{1}$$

Recognizing (1) as two algebraic equations with two unknowns, we might solve for $f(a),\ f(b)$, getting,

$$\begin{aligned} f(a) &= [F(A)\cdot 1 + F(B)\cdot 1]/2 = [5 - 1]/2 = 2 \\ f(b) &= [F(A)\cdot + F(B)\cdot(-1)]/2 = [5 + 1]/2 = 3 \end{aligned} \tag{2}$$

In (2) we recognize another set of weights, by which $X$ events are determined by $K$ ones (not shown in Fig. 3). Evidently, in (2) we have defined the associated code, which facilitates the reconstruction of the original $X$-domain data. A simple example shows that not all codes are adequate: If in Fig. 3 all the weights are made $+1$, in (1) we would get two identical equations, amounting to just one, and not enough for solving for two unknowns.

In general, one would assume continuous sequences of events $f(t)$, labeled by points $t$ on the $t$-axis, and following some transformation

[5] The graph-theorist would call them directed weighted branches.

rule, $F(\omega)$ for points $\omega$ exist in the other domain. In Fig. 3 $f(t)$, $F(\omega)$ are sketched as dashed lines. Instead of (1), (2), we can denote a general transform pair by:

$$F(\omega) = \{\alpha; \omega, t\} \cdot f(t) \tag{3}$$

$$f(t) = \{\beta; t, \omega\} \cdot F(\omega) \tag{4}$$

respectively. The way to read (3) is: Given a general prescription for the weight $\alpha$ for a certain coordinate $t$ in the $X$-domain and $\omega$ in the $K$-domain. Compute the factor $\alpha$ and create the product $\alpha \cdot f(t)$. The braces indicate that for a given $\omega$, to get $F(\omega)$ we sum the individual contributions $\alpha \cdot f(t)$ over the whole range of coordinates $\omega$ involved. Similarly, with the obvious changes, for (4).

We are now ready to display the temporal Fourier transform pair:

$$f(t) = \int_{\omega=-\infty}^{\omega=+\infty} d\omega q \cdot e^{-i\cdot\omega\cdot t} \cdot F(\omega) \tag{5}$$

$$F(\omega) = \int_{t=-\infty}^{t=+\infty} dt q \cdot e^{+i\cdot\omega\cdot t} \cdot f(t) \tag{6}$$

$$q = \sqrt{\frac{1}{2 \cdot \pi}} \tag{7}$$

where $q$ is simply a numerical factor which must be included. Instead of the braces in (3), (4) the summations and their range are indicated by the integration signs (originally introduced by Newton) $\int_{\omega=-\infty}^{\omega=+\infty} d\omega$, $\int_{t=-\infty}^{t=+\infty} dt$, and the weights are $q \cdot e^{-i\cdot\omega\cdot t}$, $q \cdot e^{+i\cdot\omega\cdot t}$, respectively. In (5), (6), $i$ is the unit imaginary number such that $i^2 = -1$. The functions $f(t)$, $F(\omega)$, are referred to as the time signal, the spectrum, respectively.

Except for mathematical intricacies, which are beyond the scope of the present discussion, the transform pairs (3), (4), and (5), (6), are essentially identical. In the following, mostly simple operations, involving change of sign, will be encountered, which can be followed by the general reader even if the details of the Fourier transform are not familiar.

## 5. POSITIVE AND NEGATIVE TIME FLOW, TEMPORAL CAUSALITY

We are now ready to discuss the flow of time, and the associated temporal causality, as expressed in the time, $t$-domain, and the ensuing effects in the spectral, $\omega$-domain. This is a special case of the full-fledged $X$ and $K$ spaces compellency to be discussed below.

To that end, we now wish to create a function which is the mirror image of $f(t)$, (Fig. 3), with respect to a reflection in the vertical axis, as depicted in and Fig. 4. This new function $f^*(t)$ is obtained from the original $f(t)$ upon replacing $t$ by $-t$. I.e.,

$$f^*(t) = f(-t) \tag{8a}$$

obviously (8a) allows us to substitute $t = -t$ in (8a), yielding $f^*(-t) = f(t)$. For convenience, define a new time variable pointing in the opposite direction,

$$t^* = -t \tag{9}$$

thus (8a) can also be recast as,

$$f^*(t^*) = f(t) \tag{8b}$$

with (8b) possessing a more symmetrical structure that will aid the subsequent discussion.

In Fig. 4, the aspect has been changed, with the $t$, $t^*$, axes now pointing towards, away from, the observer, respectively. See also Fig. 5, giving a broadside aspect of the two axes. Originally $t = a$ precedes $t = b$, i.e., for positive time $a < b$ (Fig. 3).

Causality usually presupposes the flow of time, i.e., even if the delay is infinitesimal, it is always assumed that the cause precedes the effect. To introduce causality, we now ordain the two events $f(a)$, $f(b)$, as cause and effect, whatever their physical nature is. Inspecting Fig. 4, we see that here the time sequence of the events is interchanged: The image of $f(a)$ is now $f(-a)$, or, by our new definitions, $f^*(a^*)$, similarly the image of $f(b)$ is now $f(-b)$, or $f^*(b^*)$. Obviously, on the $t$-scale, $-b$ precedes $-a$, or $-a > -b$, indicating the inversion of the time order. But in terms of the new $t^*$ variable, we have formally $a^* < b^*$. Hence, *formally*, the t and $t^*$ spaces are identical, i.e., they posses the same form, or structure, although *intrinsically*, or as ascribed to the physical reality, they refer to the time t and its inverse $t^*$. In view of the formal identity, it is expected that the formal structure in $\omega$-space will again be preserved if we duly introduced similar new definitions. We define a new variable which effectively flips around the $\omega$-axis (Fig. 4), as in (9):

$$\omega^* = -\omega \tag{10}$$

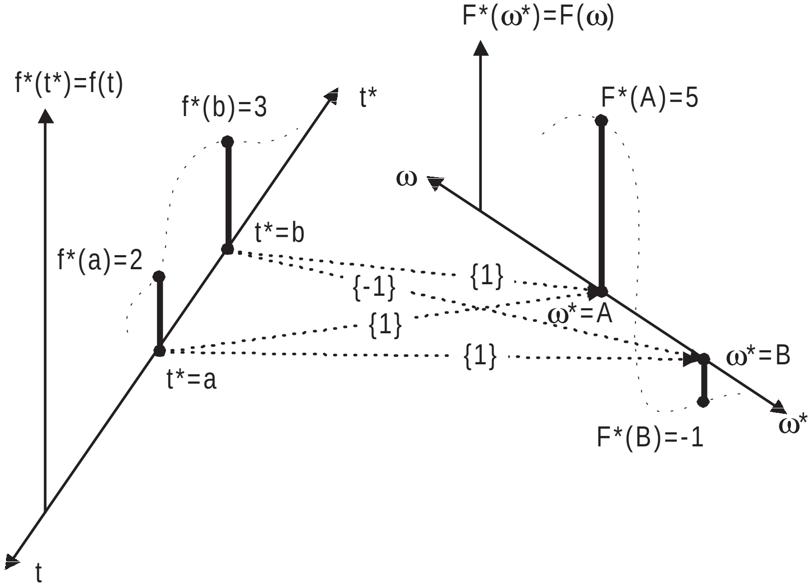


**Figure 4.** Inversion of the $t$-axis, and the associated inversion of the $\omega$-axis. Cf. Fig. 3.

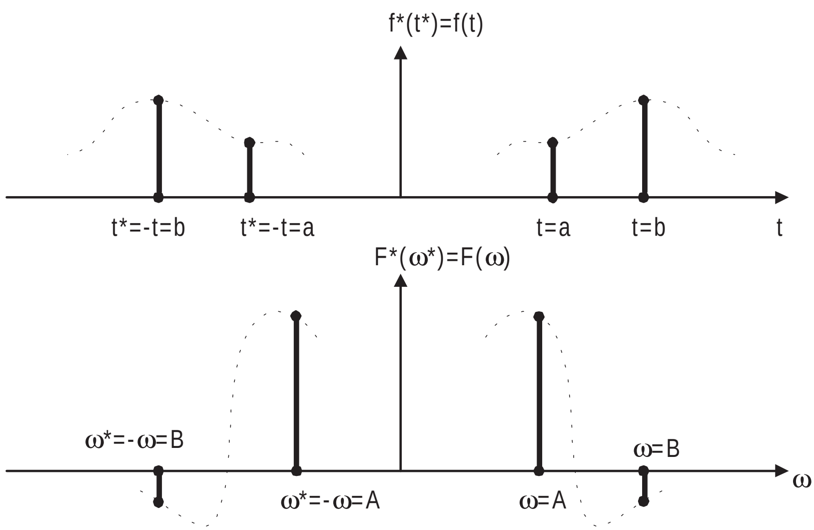


**Figure 5.** A juxtaposing and broadside aspect of Figs. 3, 4.

and similarly to (8a), (8b), we now recast the $\omega$-space events in terms of the new definitions:

$$F^*(\omega) = F(-\omega) \tag{11a}$$

$$F^*(\omega^*) = F(\omega) \tag{11b}$$

respectively. What we now see in Fig. 4, is $F^*(\omega^*)$, a mirror image of $F(\omega)$, with the singled out points $\omega = -A$, or equivalently $\omega^* = A$, and the corresponding $\omega^* = B$.

Consequently, the *formal* structure of Fig. 4 for $\omega$-space and $\omega^*$-space is preserved, and the same transform rules apply, therefore the weights attached to the dotted lines are the same in Figs. 3, 4.

The conceptual outcome of this geometrical constructions is what we were looking for — the footprints of causality in the spectral domain: One cannot look for causality in the $\omega$-space in terms of its raw, $t$-space, time sequence structure. This is a result of the fact that a single time instance is splashed (so to speak), over the entirety of the spectral domain. The question in this form would simply not be valid. But by violating causality, interchanging the cause and effect events, we can see the effect in the spectral domain. What we have found is that the spectrum is flipped over in the spectral domain!

In terms of the mathematics of the temporal Fourier transform pair (5)–(7), the following happens: Substituting in $f(t)$ (5), $-t$ for $t$, yields the new function $f(-t) = f^*(t)$, which is the flipped over mirror image of $f(t)$ (as in Figs. 4, 5). But $f(-t) = f^*(t)$ implies $f(t) = f^*(-t)$, i.e., (8b). Therefore (5) already represents $f^*(-t)$. By taking (5) and implementing (9), i.e., substituting $t^*$ for $-t$, we obtain $f^*(t^*)$,

$$f^*(t^*) = \int_{\omega=-\infty}^{\omega=+\infty} d\omega q \cdot e^{+i\cdot\omega\cdot t^*} \cdot F(\omega) \tag{12a}$$

Now use (10) and (11b) in (12a), this yields,

$$f^*(t^*) = -\int_{\omega^*=+\infty}^{\omega^*=-\infty} d\omega^* q \cdot e^{-i\cdot\omega^*\cdot t^*} \cdot F^*(\omega^*) \tag{12b}$$

which recovers the original sign of the exponent. It is a well known property of integrals, that the sign is inverted if the lower and upper limits are interchanged, hence we finally obtain,

$$f^*(t^*) = \int_{\omega^*=-\infty}^{\omega^*=+\infty} d\omega^* q \cdot e^{-i\cdot\omega^*\cdot t^*} \cdot F^*(\omega^*) \tag{12c}$$

Obviously, the only formal change of (12c) from (5) is that in (12c) the asterisks are added. The same argument, with the relevant formulas above, yields the inverse transform,

$$F^*(\omega^*) = \int_{t^*=-\infty}^{t^*=+\infty} dt^* q \cdot e^{+i\cdot\omega^*\cdot t^*} \cdot f^*(t^*) \tag{13}$$

which has the same formal structure as (6).

Thus the mathematical expressions lead us to the same conclusions reached in the verbal-graphic example discussed above.

## 6. SIMULTANEITY EFFECTS IN THE SPECTRAL DOMAIN

Suppose we have an arbitrary process for which values $f(t)$ are observed at time instances $t$. We also observe other processes, e.g., call another such process $g(t)$, and are thus able to observe simultaneous events. Say at some specific time instance $t = t_1$, the events $f(t_1),\ g(t_1)$ are observed, which makes them simultaneous, by definition. In the next section we show that such a simplistic premise applies only to co-locational processes, otherwise space consideration must enter the argument, and the concept of simultaneity in the present simple form does not apply.

For the time being, stipulating simultaneity as done here, the present problem is to demonstrate that simultaneity possesses its footprint in the spectral domain associated with the temporal processes. Once again, we are confronted by the main difficulty stemming from the transformation method, where time is scrambled and distributed among all data points, rendering the original spatiotemporal point of view meaningless. What is left of simultaneity after the scrambling effect took place? The specific time $t = t_1$ considered above is lost! Once again, the strategy to solve this paradox is to devise a situation in the temporal domain, which violates simultaneity, and look for the effect in the spectral domain. Only by such constructs can spatiotemporal effects be examined in the spectral domain, and statements be made, exclusively employing spectral domain language.

The mathematical implementation is actually trivial, and amply used by scientists and engineers, although not in the context of investigating metaphysical concepts, as done here.

Given $f(t)$, we create a discrepancy (or delay) with respect to other processes, by shifting $f(t)$ to an earlier time $t - \tau$, thus having a process $f(t-\tau)$. The effect on the transform pair (5)–(7) is straight forward: All we have to do is substitute $t-\tau$ for $t$. This yields in (5):

$$f(t-\tau) = \int_{\omega=-\infty}^{\omega=+\infty} d\omega q \cdot e^{-i\cdot\omega\cdot t} \cdot \left[e^{+i\cdot\omega\cdot\tau} \cdot F(\omega)\right] \tag{14}$$

and thus, instead of the previous spectrum $F(\omega)$, we now have a different expression for the spectrum, dependent on the time delay $\tau$:

$$E(\omega,\tau) = e^{+i\omega\tau} \cdot F(\omega) \tag{15}$$

differing by the exponential factor [6] The consistency with (6) i.e.,

$$E(\omega,\tau) = \int_{t=-\infty}^{t=+\infty} dtq \cdot e^{+i\cdot\omega\cdot t} \cdot f(t-\tau) \tag{16}$$

is easily demonstrable, but the detailed calculations will not be carried out here.

## 7. SIMULTANEITY IN THE SPATIOTEMPORAL DOMAIN — COINCIDENCY

At a first glance, simultaneity in the spatiotemporal domain, as was dealt with in the previous section, seems to be a straightforward concept, simply describing events occurring "at the same time". To realize that we are actually dealing with a much more complicated concept, consider the following imaginary conversation, where **A** defends the simplicity view, and **B** keeps raising questions:

**B**: ... But how do you know the two events you describe occur "at the same time?"

**A**: You look at the events and see them happen. Elementary, my dear Watson.

**B**: But if the events occur at spatially remote locations?

**A**: Well, so what, you still look at them and see them happen.

**B**: Yes, but the further the location from the observer, the greater will be the time delay for the light you see, to reach your eye, i.e., if you watch an event from $L$ [kilometer] away, and the velocity of light is $c$ [kilometer per second], there will be a delay of $L/c$

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[6] This phase shift factor, due to time delay, is well-known property of the Fourier transform.

[seconds] before you actually see the event. How do we account for this fact?

**A**: I agree, this is a valid comment. So obviously you have to know the distance between the observer and location where the event takes place.

**B**: Aha! But surely this undermines the assumption that simultaneity is simply a matter of time only.

**A**: O.k., let me revise my answer: If the events are at different locations in space, then we put clocks at those locations, each clock automatically recording the event as it occurs at its site. We compare the clock records and determine whether the events occurred simultaneously. Piece of cake.

**B**: But how do you know that the clocks, even if they supposedly run perfect time, are synchronized?

**A**: What a question! Obviously we first synchronize them at one location, say the observer's location, then transport them to the locations where the events are to occur.

**B**: There is still a problem. Transportation means motion over a period of time, and according to Special Relativity, even disregarding the instances of starting the motion and ending it, introduces discrepancies (the so called "twin paradox", discussed in many books, see for example Bohm [7], p. 165ff.). How is this obviated, or taken into account, thus leaving simultaneity a simple barebones temporal concept?

**A**: So let us construct telephone lines, or wireless links between the locations where the events occur and the clocks are deposited in their respective locations, and then the observer synchronizes the clocks by some remote device.

**B**: You are back to the time delay problem, this time delay now occurring over the telephone lines or the wireless wave propagation paths, if you use radio or light signaling links. You do not know the delay that is involved in the act of synchronizing the clocks, and this delay depends on distances.

And so on.... Einstein already considered this clock synchronization problem in his original paper ([5], see Section 1 there). In the context of physics, as we know it to date, there is no way Mr. **A** above can obviate the complexities mentioned, i.e., in general, it is impossible to maintain simultaneity as a purely temporal issue. Of course, once the synchronization issue is settled, the rest of the simultaneity issue indeed becomes simple. But what was to pre-relativistic thinking an *a-priori* stipulation (that synchronization of clocks can be presupposed), must be considered an *a-posteriori* issue now.

But this is only the first revision we have to make to our concept of simultaneity: Suppose there are two relatively moving observers, say one is in a moving train (trains, elevators, and flashlights seem to be favorites with people explaining Special Relativity...), the other on the station's platform. As the train passes the station, they hold out their hands, and manage to touch each other's fingertips. Obviously they will call it an event of simultaneity. They make a record of time and location, each in his/her appropriate frame of reference (coordinate system), each using his/her appropriate timepiece to note the time. What is the meaning of simultaneity in the present context? How are they going to attest as to the location and time of events, in order to unequivocally state that the occurrence was "simultaneous"? The concept of simultaneity, referring to time only, must therefore be generalized to include space information too. Inasmuch as the word coincidence refers to both location and time, we will coin the generic term coincidency to refer to spatiotemporal coincidences like in the above example.

Coincidency is defined on a spatiotemporal coordinate transformation, which must be introduced first. Given two frames of reference denoted as $\Gamma$ and $\Gamma'$, whose proper space-time coordinates are denoted as $X,\ X'$, quadruplets of parameters,

$$X = (x, y, z, i \cdot c \cdot t) \tag{17a}$$

$$X' = (x', y', z', i \cdot c \cdot t') \tag{17b}$$

respectively [7]. $\Gamma$ and $\Gamma'$ are in relative uniform motion, such that to an observer co-moving (i.e., at rest) with respect to $\Gamma$, the system $\Gamma'$ appears to move with a constant velocity $v$ in the direction of the $x$-axis, according to the formula $x = v \cdot t$. The two (so-called "inertial") systems of reference $\Gamma$ and $\Gamma'$ are aligned such that they coincide, i.e., $x' = x,\ y' = y,\ z' = z$ at some time reference, designated as zero time, $t = t' = 0$. This also implies that observed from $\Gamma'$, the origin of frame

[7] Augmenting the time by a factor $ic$ ($c$ being the universal constant of the speed of light in free space. $i^2 = -1$ as above) is arbitrary at this stage. It serves to define $X,\ X'$ as four-vectors in the Minkowski space. See [6, Ch. 17] and [7] for more detail and earlier references.

$\Gamma$ is observed to move in the opposite direction, i.e., according to the equation $x' = -v \cdot t'$. We are familiar with the "classical" Galilean transformation:

$$\begin{aligned} x' &= x - v \cdot t \\ t' &= t, \quad y' = y, \quad z' = z \end{aligned} \tag{18a}$$

and with the corresponding "modern" relativistic Lorentz transformation [5–7]:

$$\begin{aligned} x' &= \gamma \cdot (x - v \cdot t) \\ t' &= \gamma \cdot (t - \beta \cdot x) \\ y' &= y, \qquad\qquad z' = z \\ \gamma &= 1/\sqrt{1-\beta^2}, \qquad \beta = v/c \end{aligned} \tag{19a}$$

Both (18a) and (19a) will henceforth be abbreviated as

$$X' = \overline{X}'[X] \tag{20a}$$

where like $f(\,)$ etc. used above to describe events, $\overline{X}'[\,]$ in (20a) expresses the rule according to which primed coordinates are expressed in terms of unprimed coordinates. The inverse transformations are calculated by using some algebra, yielding instead of (18a), (19a), (20a) corresponding expressions with primed and unprimed variables interchanged, and $v$ replaced by $-v$. Accordingly we have,

$$\begin{aligned} x &= x' + v \cdot t' \\ t' &= t, \quad y' = y, \quad z' = z \end{aligned} \tag{18b}$$

$$\begin{aligned} x &= \gamma \cdot (x' + v \cdot t') \\ t &= \gamma \cdot (t' - \beta \cdot x') \\ y' &= y, \quad z' = z \end{aligned} \tag{19b}$$

and both relations can be symbolized by,

$$X = \overline{X}[X'] \tag{20b}$$

We are now ready to define coincidency of two phenomena, observed in two inertial systems of reference $\Gamma$ and $\Gamma'$ by properly co-moving observers, each attached to his/her system. The observer in $\Gamma$ measures a phenomenon $f(X)$, the observer in $\Gamma'$ measures a different function $f'(X')$, which might involve different physical entities (e.g., in electromagnetic theory, while charge is measured in $\Gamma$, both charge and current are involved in $\Gamma'$). Only a physical theory, or model, can postulate some connection between the phenomena $f$ and $f'$ measured by the two observers. To make things as simple as possible, let us assume that the two functions are identical[8]. In the language of the example of the person on the station platform and the passenger on the train, we might say that when they touched fingers while they were passing each other, the phenomena were $f$, $f'$, the respective equal pressure they felt on their fingers. Our assumption that the two functions are identical prescribes:

$$f(X) = f'(X') \tag{21}$$

or in words, the $\Gamma$ observer would attest that the pressure was zero for all locations and time instances, except for a specific location and time $X_1 = (x_1, y_1, z_1, i\cdot c\cdot t_1)$ at which the contact occurred and pressure was observed (felt) on his fingertips. Similarly the $\Gamma'$ observer will attest to the function $f'(X')$ and the pressure occurring at a spatiotemporal location $X'_1 = (x'_1, y'_1, z'_1, i\cdot c\cdot t'_1)$. But how are they going to relate the particular coordinate quadruplets? The answer is that the two events are coincident if (20a) or equivalently (20b), is satisfied, including the particular event $X'_1 = \overline{X}'[X_1]$ or $X_1 = \overline{X}[X'_1]$. Mathematically this means that if (20a) is substituted in (21), the equation is satisfied for all $X$, or, as it is called, becomes an identity:

$$f(X) \equiv f'\left(\overline{X}'[X]\right) \tag{22}$$

where "$\equiv$" denotes identity.

It follows that we managed to define spatiotemporal coincidency, as a generalization of the simplistic temporal simultaneity. The new conception includes relative motion of observers, if such is present. In the next section, which is quite mathematical and formalistic, the corresponding effects in the spectral domain are considered.

## 8. COINCIDENCY EFFECTS IN THE SPECTRAL DOMAIN

Our task here is to show that coincidency in the spatiotemporal domain has its counterpart in the spectral domain, and that the information

[8] Thus we have defined $f$ as a scalar invariant.

in one domain finds its expression in the other, in a fully invertible manner.

The first step is to extend the one-dimensional (temporal) Fourier transform pair (5)–(7) to a four-dimensional spatiotemporal Fourier transform. This is readily available. Firstly we will present the extended transformation using certain compacted notation:

$$f(X) = \int_{K=-\infty}^{K=+\infty} d^{(4)}K \; q^4 \cdot e^{i \cdot K \cdot X} \cdot F(K) \tag{23}$$

$$F(K) = \int_{X=-\infty}^{X=+\infty} d^{(4)}X \; q^4 \cdot e^{-i \cdot K \cdot X} \cdot f(X) \tag{24}$$

Similarly to the quadruplets (17a), (17b), we have introduced in (23), (24) a new quadruplet [9] $K$ ,

$$K = (\xi, \eta, \zeta, i \cdot \omega / c) \tag{25a}$$

The integration operators in (23), (24) are shorthand for four-fold integrals, e.g., for (23),

$$\langle K \rangle = \int_{K=-\infty}^{K=+\infty} d^{(4)}K = \int_{\xi=-\infty}^{\xi=+\infty} d\xi \int_{\eta=-\infty}^{\eta=+\infty} d\eta \int_{\zeta=-\infty}^{\zeta=+\infty} d\zeta \int_{\omega=-\infty}^{\omega=+\infty} d\omega \tag{26}$$

and similarly for (24). The symbol $\langle K \rangle$ in (26), and a corresponding $\langle X \rangle$ for (24) further compacts the notation, for subsequent use.

Before discussing the question of coincidency involving observers in relative motion, the simpler question of observers in one and the same frame of reference must be considered. It is assumed that they already synchronized their clocks, taking into account different locations, if such are involved. Essentially, the argument above, (14)–(16), holds here too: Any time advance or delay expressed in $f(X)$ will show in the corresponding spectrum $F(K)$ by introducing exponential phase factor as in (15). Furthermore, this property is valid for the spatial integrations, each constituting a Fourier transform, hence if the space coordinates in $f(X)$ are shifted, this too will show in the spectral domain by introducing additional exponentials analogous to (15). Thus

[9] As in (17a), (17b), the factor $i/c$ is arbitrary and serves to define $K$, $K'$ , as a four-vector.

the footprints of spatiotemporal events in the spectral domain are explained for the present case.

With this out of the way, the much more interesting problem regarding coincidency in the presence of relatively moving observers will be considered. The expressions corresponding to (23), (24), describing the phenomenon recorded by the $\Gamma'$ observer, are:

$$f'(X') = \int_{K'=-\infty}^{K'=+\infty} d^{(4)}K' \; q^4 \cdot e^{i \cdot K' \cdot X'} \cdot F'(K') \tag{27}$$

$$F'(K') = \int_{X'=-\infty}^{X'=+\infty} d^{(4)}X' \; q^4 \cdot e^{-i \cdot K' \cdot X'} \cdot f'(X') \tag{28}$$

and thus we have defined the corresponding quadruplet,

$$K' = (\xi', \eta', \zeta', i \cdot \omega'/c) \tag{25b}$$

Similarly to (20a), (20b), there exist relations, between the spectral components in $\Gamma$ and $\Gamma'$ (see, e.g., [8] for a detailed derivation) extending the celebrated Doppler effect (see [1], [9–12] for historical and general material on the subject, and earlier references), accounting for frequency shifts observed by relatively moving observers.

In terms of the components of $K$, $K'$, we obtain for the Galilean transformation (18a),

$$\begin{aligned} &\omega' = \omega - v \cdot \xi \\ &\xi' = \xi, \quad \eta' = \eta, \quad \zeta' = \zeta \end{aligned} \tag{29a}$$

and subject to the Lorentz transformation (19a) we have,

$$\begin{aligned} &\omega' = \gamma \cdot (\omega - v \cdot \xi) \\ &\xi' = \gamma \cdot \left(\xi - v \cdot \omega/c^2\right) \\ &\eta' = \eta, \qquad \zeta' = \zeta \end{aligned} \tag{30a}$$

which can be summarized, in a notation already familiar from (20a), as,

$$K' = \overline{K}'[K] \tag{31a}$$

The corresponding inverse transformations of (29a), (30a), have the identical structure, except for the interchange of unprimed and primed symbols and replacement of $v$ by $-v$:

$$\begin{aligned} &\omega = \omega' + v \cdot \xi' \\ &\xi = \xi', \quad \eta = \eta', \quad \zeta = \zeta' \end{aligned} \tag{29b}$$

$$\begin{aligned} \omega &= \gamma \cdot (\omega' + v \cdot \xi') \\ \xi &= \gamma \cdot \left(\xi' + v \cdot \omega'/c^2\right) \\ \eta' &= \eta, \qquad \zeta = \zeta' \end{aligned} \tag{30b}$$

summarized by,

$$K = \overline{K}[K'] \tag{31b}$$

Equations (29), (30), and the corresponding (31) are referred to as the Galilean and the relativistic Doppler effects, respectively.

Subject to (21), if we simply equate (23), (27), then we are equating two integrals, which does not imply anything about the integrands [10]. However, these integrals have unique properties: The product $K \cdot X$ in the exponent (23) is a shorthand for,

$$K \cdot X = \xi \cdot x + \eta \cdot y + \zeta \cdot z - \omega \cdot t \tag{32a}$$

and similarly in (27) for,

$$K' \cdot X' = \xi' \cdot x' + \eta' \cdot y' + \zeta' \cdot z' - \omega' \cdot t' \tag{32b}$$

By painstakingly substituting into $K' \cdot X'$ from (18a) and (29a) for the Galilean case, or (19a) and (30a) for the corresponding Lorentzian case, it is verified [11] that,

$$K \cdot X = K' \cdot X' \tag{33}$$

In the jargon of electromagnetics (33) is referred to as: "The phase invariance". The four-fold integration operators in (23), (24), (27), (28), e.g., $\langle K \rangle$ of (23), explicitly defined in (26) have peculiar properties: Both the Galilean and the Lorentz transformations display the property:

$$\left\langle X' \right\rangle = \langle X \rangle \tag{34a}$$

$$\left\langle K' \right\rangle = \langle K \rangle \tag{34b}$$

[10] An integral, as treated here, is a sum, and equality of two sums does not imply equality of the summands, e.g., $6 = 2 + 4 = 1 + 5$ does not imply $2 = 1$ or $2 = 5$.

[11] Actually $K \cdot X$ constitutes an inner product of the two four-vectors, and hence (33) is true without any ado.

in the spatiotemporal and spectral domains, respectively. This is a direct result of the Jacobian determinant [12] for the change of variables and limits of integration, having the value 1. See for example [8, 13]. Thus, subject to (21), if we equate (23) and (27), and subject to (31a) or (31b), the integrands are indeed identical, and therefore we finally obtain in the spectral domain,

$$F(K) = F'(K') \tag{35}$$

which becomes an identity,

$$F(K) \equiv F'\left(\overline{K}'[K]\right) \tag{36}$$

that should be compared with (22).

This means, according to the ideas and definitions stated above, that coincidency in the spatiotemporal domain (22) prescribes (36), which because of the similarity of (22), (36) will be (formally, in the sense of the geometry of the coordinates) referred to as coincidency in the spectral domain. Any departure from coincidency in the $X$-domain will destroy the coincidency (36) in the spectral domain.

## 9. COMPELLENCY IN THE SPATIOTEMPORAL AND SPECTRAL DOMAINS

As long as we accept the concept of an upper bound for the velocity of objects [13], or equivalently, as long as "action at a distance" is negated, compellency must include space as well as time considerations. Here "objects" include any material particle or ponderable body, or any wave pulse whose propagation entails the transport of energy (hence carrying detectable information) through space. Let us firstly consider a case where the whole scenario takes place in one and the same frame of reference, e.g., we think of the cause, or compellency, as associated with a transmitter, sending some signal which is picked up by a receiver, at a later time and at a different location, compelling some effect. For example, think of a viewer sitting at some distance from

[12] The general readers, for whom the mathematical term is unfamiliar, must further pursue this subject elsewhere.

[13] This can be any speed, but in physics, as understood to date, the upper bound is the speed of light in free space.

the TV set and changing channels by pressing the button on the remote control. Transmitter and receiver are at rest with respect to each other in this scenario. The only motion involved is the propagation of the signal from the transmitter to the receiver, be this signal based on ultrasonic waves, infrared light, or any other physical principle. For all we care, the transmitter can be a catapult, hurling stones at the receiver. Thus it becomes apparent that the whole question of compellency reduces to the associated spatiotemporal characteristics of the propagating signal.

As argued above for causality (involving time considerations only), also here it is meaningless to simply ask how spatiotemporal compellency is manifested in the spectral domain. It is not! That is because in the spectral domain's language, time intervals and distances do not exist. Or as noted above, all spatiotemporal coordinates were mapped on all spectral domain coordinate points, thus destroying the identity of specific space and time coordinates. The valid approach is to assume various spatiotemporal situations, and investigate the associated consequences in the spectral domain. Accordingly, we devise two scenarios in the spatiotemporal domain, one demonstrating compellency and the other its opposite, call it anti-compellency, where cause and effect events are interchanged, and study the corresponding outcomes in the spectral domain.

It would take us far away from our subject if we tried to discuss the signal propagation as a full-fledged wave propagation problem. Instead, we think of the signal as a material particle or a very sharp wave pulse traversing the route between the transmitter and the receiver. We are essentially interested in the geometry and time dependent motion of the signal along its trajectory [14]. As such, in general the description of a spatiotemporal signal requires three time-dependent relations,

$$x = \overline{x}(t), \qquad y = \overline{y}(t), \qquad z = \overline{z}(t) \tag{37}$$

which upon substitution of a specific time parameter $t = t_1$ yield the corresponding location coordinates $x_1,\ y_1,\ z_1$. The simplest case of uniform motion along the $x$-axis with a constant velocity $u$ suffices for clarifying our ideas. Thus we adopt the equation of motion,

$$x = u \cdot t \tag{38}$$

[14] Indeed, the physicist would right away recognize the idea as "geometrical optics", although the signal might be mechanical projectile, a sound (or any other mechanical) wave, light, etc.

i.e., the rule $\overline{x}(\,)$ of (37) simply means in (38): "To compute the distance $x$, multiply the time $t$ by the velocity $u$". In the present discussion, the other two coordinates $y,\ z$, do not participate and are suppressed altogether[15]. Consistent with Figs. 3–5, where $f(t),\ f^*(t^*)$, are described by dashed lines, we now have two-dimensional surfaces $f(x,t)$, and $f^*(x^*,t^*)$, sketched in Fig. 6 by dotted line contours: Think of the two dotted line surfaces shown, $f(x,t)$ and its flipped over mirror image $f^*(x^*,t^*) = f(-x,-t)$ as if hovering above the horizontal plane. We are singling out two events, one for the transmitter, the other for the receiver: The transmitter is located at position $x = c$ and acts at time $t = a$, the corresponding receiver parameters are $x = d,\ t = b$. The corresponding values that can be measured for the phenomena are $f(c,a),\ f(d,b)$, see Fig. 6. The signal's velocity $u$ is given by the distance between transmitter and receiver divided by the delay time,

$$u = \frac{+d - c}{+b - a} \tag{39a}$$

and we assign to it a direction pointing from $c$ to $d$ along the $x$-axis, thus indicating that the event $f(c,a)$, at the earlier time $t = a$ is the cause for the event $f(d,b)$ at the later time $t = b$. Now let us perform an interchange of cause and effect, as we did for the purely temporal case in Figs. 4, 5. The corresponding $f(-c,-a),\ f(-d,-b)$ are the flipped over image events, (Fig. 6). The signal's velocity is now given by (39a) with $a,\ldots,d$ replaced by their inverse $-a,\ldots,-d$,

$$u = \frac{-d + c}{-b + a} = \frac{+d^* - c^*}{+b^* - a^*} = u^* \tag{39b}$$

It is significant that replacing the coordinates by their inverse leaves $u$ unchanged, as depicted in Fig. 6, which is tantamount to replacing the cause and effect roles between the events.

By now the effect on the spectral domain should be obvious, without encumbering the discussion with an additional figure: As the case was in the purely temporal case, Figs. 3–5, also in the present case the

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[15] Had we kept all the three parametric equations (37) and still assumed a constant velocity, then (38) would be replaced by three equations $x = u_x \cdot t,\ y = u_y \cdot t,\ z = u_z \cdot t$, where $u_x,\ u_y,\ u_z$ denote the velocity components projected on the corresponding coordinate axes. Or we could express everything vectorially. Again, this is unnecessarily complicating matters for the general reader.

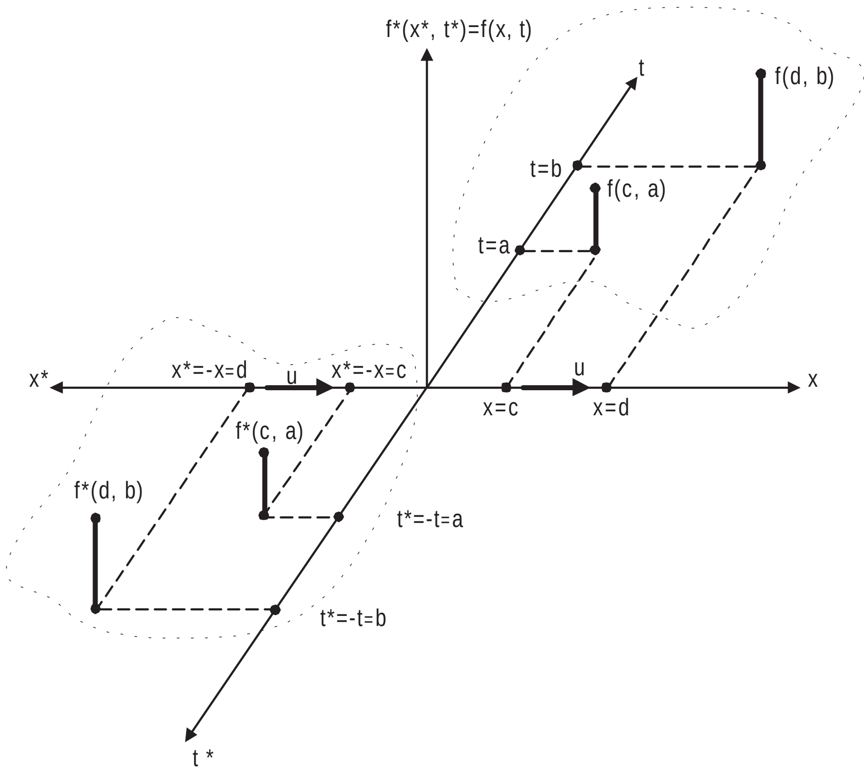


**Figure 6.** A combined spatiotemporal (with only one representative space coordinate) illustration of a function and its inverse. Sketched is the situation depicting compellency (spatiotemporal causality), and the interchange of cause and effect events.

transformation will produce a mirror image configuration in the spectral domain. This is formally verified in the next section. It follows that the violation of compellency in the spatiotemporal domain, i.e., the interchange of cause and effect in our example, results in a clear effect in the spectral domain, namely the appearance of a mirror like symmetrical spectrum. This establishes our claim that although in the spectral domain it is meaningless to directly define compellency, the effect is stored in the data obtained through the transformation from the spatiotemporal domain, is not lost, can be identified by a spectral domain measurement, and can be retrieved through the inverse transform.

Looking back on (39a) one may wonder if the analogous expression in the spectral domain has any significance. According to the pairs $(c, a)$, $(d, b)$, we assume in the spectral domain the corresponding pairs $(C, A)$, $(D, B)$, respectively. Similarly to (39a), consider now the ratio,

$$s = \frac{+D - C}{+B - A} \tag{40}$$

Obviously, like (39b), also (40) retains its value upon inverting the signs of $A, \ldots, D$. Since the expressions (32a), (32b) are dimensionless, it follows that $\xi$, $\omega$, have the dimension of inverse distance, inverse time, respectively, and $s$ in (40) has the dimensions of inverse velocity, i.e., time per distance, hence $s$ is dubbed "slowness". This parameter, its relativistic rules of transformation based on (30a), and its significance are further discussed elsewhere [14]. Under certain conditions, the inverse of (40) $1/s$ not only has the dimensions of velocity, it also describes a physical entity of paramount importance, the so called group velocity [16], describing the motion of wave packets along ray trajectories.

## 10. COMPELLENCY AND THE FOUR-DIMENSIONAL FOURIER TRANSFORM

The somewhat intuitive, verbal and graphic discussion of the previous section is rigorously validated by a procedure similar to the one-dimensional temporal Fourier transform used above. Instead of the transform pair (5), (6), we now start from (23), (24). Forming the mirror image function $f(-X)$, by inverting the sign on all components of the quadruplet (17a), the whole discussion including equations (8) to (11) carries over with the exchange of $X$ for $t$ and $K$ for $\omega$. Thus the manipulation of the temporal Fourier transform (5) which results in (12c) can be repeated, simply by inspection. The analog of (12c) will now be given by:

$$f^*(X^*) = \int_{K^*=-\infty}^{K^*=+\infty} d^{(4)}K^* \; q^4 \cdot e^{i \cdot K^* \cdot X^*} \cdot F^*(K^*) \tag{41}$$

Similarly, the analog of (13) follows, given by:

$$F^*(K^*) = \int_{X^*=-\infty}^{X^*=+\infty} d^{(4)}X^* \; q^4 \cdot e^{i \cdot K^* \cdot X^*} \cdot f^*(X^*) \tag{42}$$

16 For more detail about the subject see [15].

Consequently the same conclusions are reached as for purely temporal causality discussed above: The effect of interchanging roles between cause and effect in the spatiotemporal $X$-domain are definitely detectable in the spectral $K$-domain, to wit, a symmetrical interchange in the $X$-domain (as shown in the example of Fig. 6, and formally in (41), (42)) will create the same symmetrical interchange of spectra in the $K$-domain. From (21) and the corresponding (35), it becomes clear that the present conclusions apply to events for relatively moving observers as well.

This concludes our effort of establishing the footprints of compellency in the spectral domain.

## 11. CONCLUDING REMARKS

A simplistic transformation of a one-to-one mapping of data points cannot, obviously, lead to a new world-view. Although “world-view” was not rigorously defined, the word is used in the context of a unique perception of reality, with some essential difference from other “world-views”. The spatiotemporal and the spectral domains provide an example for such radically different world-views.

If a person is forced to watch TV, say, with the set resting on the table upside down, every pixel of data that usually appears on the normally situated screen is simply appearing in a different place. It is similar to the example mentioned above, of the keyboard on which the labels are shuffled — a person forced to use such devices will learn, after some practicing, to do it perfectly. Like writing with your right hand when you are left-handed. Obviously this does not qualify for the term “different world-view”.

The Fourier transform, as a representative of many other (continuous or discrete) integral transformations, is something completely different: It scrambles all points of data from one domain onto all points of data in the other domain. Nobody will claim that as a matter of course we can look at a Fourier transformed spatial scene, or a spectrum created by transforming a complicated time signal, and directly comprehend the original data, in all its detail. In spite of this, the Fourier transform, like many other integral transforms, is invertible. This means that the original data is retained in the transformed representation, and the coding can be inverted to extract the original by applying the inverse transform.

Specifically in the context of the spatiotemporal "world-view", into which we are born, never mind the evolutional-biological circumstances that caused this "world-view", one might legitimately ask if there are consistent alternatives. This problem has been posed and discussed by Zangari and Censor [1]. Inasmuch as the mathematical Fourier transform is prevalent in science and engineering as a tool for modeling various phenomena, it has been chosen as a candidate for exploring this idea.

Alas, because of the ingrained spatiotemporal world-view, although we are able to logically following the formal discussion, we cannot reconcile the basic principles of simultaneity (coincidency) and causality (compellency) inherent in our world-view, with the lack of them in the spectral domain world-view. Time and space as we know them are at the root of our cognitive processes, and the synthetic principles of simultaneity and causality based on them are indispensable. How can we communicate with an imagined creature possessing a spectral world-view, if we are not allowed to use concepts associated with our world-view basic elements of space and time? It seems impossible. The situation seems to be even worse than communicating with a congenital blind and deaf person (think for a moment about the celebrated Helen Keller, blind and deaf from infancy, who became a celebrated lecturer and author), trying to explain to them the concepts of sight, color, music, assessment of distances by sighting, etc.

The method chosen here to deal with this situation was to strictly use only the language valid for one world-view domain. We don't quiz the spectral domain world-viewers about time and space, simultaneity or causality, because these principles are foreign to them. The only access to them, and the only avenue they can use in communicating with us about their specific concepts, is through the relevant transformations, like the presently discussed Fourier transform. Therefore, in order to probe the effects of spatiotemporal events on the spectral domain, special situations are to be devised in the spatiotemporal domain, and their footprints in the spectral domain are investigated. For example, we first consider spatiotemporally simultaneous events, then repeat the experiment with non-simultaneous events, as using the time delay $\tau$ introduced in (14), (15), and compare the outcome in the spectral domain. The difference in the spectra observed is the only means by which the effect can be communicated to the spectral domain "resident". Similarly for causality, and the extended concepts of compellency, and coincidency in the presence of motion, all discussed above.

Inasmuch as nobody can assert that all intelligent thinking in the universe must be spatiotemporally based, the examination of different world-views sets the stage for new philosophical questions. This, in my view, justifies the original work on the subject [1], and its ramifications discussed presently.

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