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THE ELECTROMAGNETIC FIELD PRODUCED BY A HORIZONTAL ELECTRIC DIPOLE OVER A DIELECTRIC COATED PERFECT CONDUCTOR

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Abstract—The analytical expressions for the electromagnetic field generated by a horizontal electric dipole over a dielectric coated perfect conductor are derived by transformation of integral path. From the expressions, it can be clearly observed that the excited field consists of the direct wave, reflected wave, trapped surface wave and lateral wave. The propagation wave number of trapped surface wave, which depends on electric parameters and thickness of the dielectric layer, is between the wave number k_0 and k_1.

1. INTRODUCTION

The investigation for electromagnetic field of a horizontal dipole over a dielectric coated a conducting or dielectric medium is a very old but important topic, which have been intensively studied in the past [2–14], after the pioneering work of Sommerfeld in 1909 [1]. These contributions include the theoretical representation of the problem and their numerical implementations. In the theoretical representations, a typical solution was for Hertz potential with an oscillating horizontal electric dipole on the boundary as the source. The associated components of the electric and magnetic field are determined by differentiation.

One of the shortcomings of a solution in terms of the Hertz potential represented by complex integral transforms are the difficulties of interpreting them physically. Attempts to eliminate this important obstacle have been numerous and include a lot of work, especially the work of Norton in representing the surface-wave term and that of Banos and Wait and Campbell in developing explicit approximate formulas for the components of the electromagnetic field. These formulas are quite simple but are limited to restricted. The restriction was removed by King [11] who derived six components of the electromagnetic field at all points in the air, subject only to the condition

$$k_0^2 \ll k_1^2 \leq \left|k_2^2\right|, \tag{1a}$$

$$k_1^2 l^2 \ll 1. \tag{1b}$$

where

$$k_0 = \omega/c = \omega\sqrt{\mu_0\varepsilon_0}, \tag{2a}$$

$$k_1 = \sqrt{\varepsilon_{1r}}k_0; \qquad k_2 = k_0\sqrt{\varepsilon_{2r} + i\sigma_2/\omega\varepsilon_0}, \tag{2b}$$

are the wave numbers of half-space of air (region 0, $z \geq 0$), dielectric layer with uniform thickness l (region 1, $-l \leq z \leq 0$), and a conducting or dielectric medium (region 2, $z \leq -l$), as illustrated in Figure 1. ε_r is the relative permittivity, σ is the conductivity, and use is made of the time dependence $e^{-i\omega t}$. Because $k_1 l$ is too small, the authors didn't consider the trapped surface wave along the surface of the dielectric layer. In some case, however, the necessary restrictions are no longer satisfied.

It is the purpose of this paper to determine the analytical expressions of electric field generated by a horizontal electric dipole in the air over a dielectric coated a perfect conductor without restriction (1). The extension show that the electromagnetic field produced by a horizontal electric dipole in the air or on the boundary consists of the direct wave, reflected wave, trapped surface wave and lateral wave.

2. INTEGRAL EXPRESSIONS OF ELECTRIC FIELD IN THE AIR

The expressions of three components $E_{0\rho}(\rho,\varphi,z)$, $E_{0\varphi}(\rho,\varphi,z)$, and $E_{0z}(\rho,\varphi,z)$ of the electric field at (ρ,φ,z) in the air when a horizontal

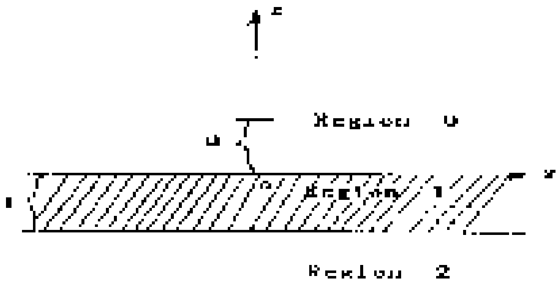

Figure 1. Unit electric dipole at height d over plane boundary ($z = 0$) between air and a layer of dielectric over a conducting or dielectric half-space.

electric dipole is at the height d are readily obtained from formulas in King [11]. With $k_2 \to \infty$, they are

$$E_{0\rho}(\rho,\varphi,z) = -\frac{\omega\mu_0}{4\pi k_0^2}\cos\varphi[F_{\rho 0}(\rho,z-d) - F_{\rho 0}(\rho,z+d) + F_{\rho 1}(\rho,z+d)] \tag{3}$$

$$E_{0\varphi}(\rho,\varphi,z) = \frac{\omega\mu_0}{4\pi k_0^2}\sin\varphi[F_{\varphi 0}(\rho,z-d) - F_{\varphi 0}(\rho,z+d) + F_{\varphi 1}(\rho,z+d)] \tag{4}$$

$$E_{0z}(\rho,\varphi,z) = \frac{i\omega\mu_0}{4\pi k_0^2}\cos\varphi[F_{z0}(\rho,z-d) - F_{z0}(\rho,z+d) + F_{z1}(\rho,z+d)] \tag{5}$$

Where

$$\left.\begin{matrix} F_{\rho 0}(\rho,z-d) \\ F_{\varphi 0}(\rho,z-d) \end{matrix}\right\} = \int_0^\infty \left\{\frac{\gamma_0}{2}[J_0(\lambda\rho) \mp J_2(\lambda\rho)] + \frac{k_0^2}{2\gamma_0}[J_0(\lambda\rho) \pm J_2(\lambda\rho)]\right\} e^{i\gamma_0|z-d|}\lambda d\lambda,$$

$$\left.\begin{matrix} F_{\rho 0}(\rho,z+d) \\ F_{\varphi 0}(\rho,z+d) \end{matrix}\right\} = \int_0^\infty \left\{\frac{\gamma_0}{2}[J_0(\lambda\rho) \mp J_2(\lambda\rho)] + \frac{k_0^2}{2\gamma_0}[J_0(\lambda\rho) \pm J_2(\lambda\rho)]\right\} e^{i\gamma_0|z+d|}\lambda d\lambda,$$

$$F_{z0}(\rho,z-d) = \pm\int_0^\infty J_1(\lambda\rho)e^{i\gamma_0|z-d|}\lambda^2 d\lambda; \qquad \left\{\begin{matrix} z > d \\ 0 \le z \le d \end{matrix}\right. ,$$

$$F_{z0}(\rho, z+d) = \int_0^\infty J_1(\lambda\rho) e^{i\gamma_0(z+d)} \lambda^2 d\lambda,$$

$$F_{\rho 1}(\rho, z+d) = F_{\rho 2}(\rho, z+d) + F_{\rho 3}(\rho, z+d) \tag{6}$$

$$F_{\varphi 1}(\rho, z+d) = F_{\varphi 2}(\rho, z+d) + F_{\varphi 3}(\rho, z+d) \tag{7}$$

$$F_{z1}(\rho, z+d) = \int_0^\infty (Q_3+1) J_1(\lambda\rho) e^{i\gamma_0(z+d)} \lambda^2 d\lambda \tag{8}$$

$$\left.\begin{matrix} F_{\rho 2}(\rho, z+d) \\ F_{\varphi 2}(\rho, z+d) \end{matrix}\right\} = \frac{1}{2}\int_0^\infty \gamma_0 (Q_3+1)[J_0(\lambda\rho) \mp J_2(\lambda\rho)] e^{i\gamma_0(z+d)} \lambda d\lambda \tag{9}$$

$$\left.\begin{matrix} F_{\rho 3}(\rho, z+d) \\ F_{\varphi 3}(\rho, z+d) \end{matrix}\right\} = -\frac{k_0^2}{2}\int_0^\infty \gamma_0^{-1} (P_3-1)[J_0(\lambda\rho) \pm J_2(\lambda\rho)] e^{i\gamma_0(z+d)} \lambda d\lambda \tag{10}$$

With

$$\gamma_j = \sqrt{k_j^2 - \lambda^2} \qquad j = 0, 1$$

$$-Q_3 = \frac{k_1^2\gamma_0 + ik_0^2\gamma_1 \tan\gamma_1 l}{k_1^2\gamma_0 - ik_0^2\gamma_1 \tan\gamma_1 l}, \quad -P_3 = \frac{-\gamma_1 - i\gamma_0 \tan\gamma_1 l}{\gamma_1 - i\gamma_0 \tan\gamma_1 l}$$

Obviously, the first two terms in (3)–(5) are the direct field and the perfect-image field with a negative image, respectively. They can be evaluated without approximation with the help of standard formulas. For convenience, the formula (3) can be represented as follows:

$$E_{0\rho}(\rho, \varphi, z) = E_{0\rho}^{(1)}(\rho, \varphi, z) + E_{0\rho}^{(2)}(\rho, \varphi, z) + E_{0\rho}^{(3)}(\rho, \varphi, z) \tag{11}$$

The exact direct field and perfect-image field component are

$$E_{0\rho}^{(1)}(\rho, \varphi, z) = \frac{\omega\mu_0}{4\pi k_0}\cos\varphi\left\{e^{ik_0 r_1}\left[\frac{2}{r_1^2} + \frac{2i}{k_0 r_1^3} + \left(\frac{z-d}{r_1}\right)^2\left(\frac{ik_0}{r_1} - \frac{3}{r_1^2} - \frac{3i}{k_0 r_1^3}\right)\right]\right\} \tag{12a}$$

$$E_{0\rho}^{(2)}(\rho, \varphi, z) = \frac{\omega\mu_0}{4\pi k_0}\cos\varphi\left\{-e^{ik_0 r_2}\left[\frac{2}{r_2^2} + \frac{2i}{k_0 r_2^3} + \left(\frac{z+d}{r_2}\right)^2\left(\frac{ik_0}{r_2} - \frac{3}{r_2^2} - \frac{3i}{k_0 r_2^3}\right)\right]\right\} \tag{12b}$$

Where

$$r_1 = \sqrt{\rho^2 + (z-d)^2} \tag{13a}$$

$$r_2 = \sqrt{\rho^2 + (z+d)^2} \tag{13b}$$

with (3), (6), (9) and (10), we obtain

$$
\begin{aligned}
&E_{0\rho}^{(3)}(\rho,\varphi,z)\\
&\quad=\frac{\omega\mu_0}{4\pi}\cos\varphi\left\{\int_0^\infty \frac{i\gamma_0\gamma_1\tan\gamma_1 l}{k_1^2\gamma_0-ik_0^2\gamma_1\tan\gamma_1 l}[J_0(\lambda\rho)-J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda\right.\\
&\qquad\left.+\int_0^\infty \frac{i\tan\gamma_1 l}{\gamma_1-i\gamma_0\tan\gamma_1 l}[J_0(\lambda\rho)+J_2(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda\right\} \qquad (14)
\end{aligned}
$$

Because

$$
\begin{aligned}
J_n(\lambda\rho) &= \frac{1}{2}\left[H_n^{(1)}(\lambda\rho)+H_n^{(2)}(\lambda\rho)\right]\\
H_n^{(1)}(-\lambda\rho) &= (-1)^{n+1}H_n^{(2)}(\lambda\rho)
\end{aligned}
$$

and γ_0, γ_1 are even functions with respect to λ. (14) becomes

$$
\begin{aligned}
&E_{0\rho}^{(3)}(\rho,\varphi,z)\\
&\quad=\frac{\omega\mu_0}{8\pi}\cos\varphi\left\{\int_{-\infty}^\infty \frac{i\gamma_0\gamma_1\tan\gamma_1 l}{k_1^2\gamma_0-ik_0^2\gamma_1\tan\gamma_1 l}[H_0^{(1)}(\lambda\rho)-H_2^{(1)}(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda\right.\\
&\qquad\left.+\int_{-\infty}^\infty \frac{i\tan\gamma_1 l}{\gamma_1-i\gamma_0\tan\gamma_1 l}[H_0^{(1)}(\lambda\rho)+H_2^{(1)}(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda\right\} \qquad (15)
\end{aligned}
$$

Of course, the same results can be obtained from (4)–(5)

$$
\begin{aligned}
&E_{0\varphi}(\rho,\varphi,z)\\
&\quad=-\frac{\omega\mu_0}{4\pi k_0}\sin\varphi\left[e^{ik_0r_1}\left(\frac{ik_0}{r_1}-\frac{1}{r_1^2}-\frac{i}{k_0r_1^3}\right)-e^{ik_0r_2}\left(\frac{ik_0}{r_2}-\frac{1}{r_2^2}-\frac{i}{k_0r_2^3}\right)\right]\\
&\qquad+E_{0\varphi}^{(3)}(\rho,\varphi,z) \qquad (16)
\end{aligned}
$$

$$
\begin{aligned}
&E_{0\varphi}^{(3)}(\rho,\varphi,z)\\
&=-\frac{\omega\mu_0}{8\pi}\sin\varphi\left\{\int_{-\infty}^\infty \frac{i\gamma_0\gamma_1\tan\gamma_1 l}{k_1^2\gamma_0-ik_0^2\gamma_1\tan\gamma_1 l}[H_0^{(1)}(\lambda\rho)+H_2^{(1)}(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda\right.\\
&\qquad\left.+\int_{-\infty}^\infty \frac{i\tan\gamma_1 l}{r_1-i\gamma_0\tan\gamma_1 l}[H_0^{(1)}(\lambda\rho)-H_2^{(1)}(\lambda\rho)]e^{i\gamma_0(z+d)}\lambda d\lambda\right\} \qquad (17)
\end{aligned}
$$

$$
\begin{aligned}
E_{0z}(\rho,\varphi,z) &= -\frac{\omega\mu_0}{4\pi k_0}\cos\varphi\left[e^{ik_0r_1}\left(\frac{\rho}{r_1}\right)\left(\frac{z-d}{r_1}\right)\left(\frac{ik_0}{r_1}-\frac{3}{r_1^2}-\frac{3i}{k_0r_1^3}\right)\right.\\
&\qquad\left.-e^{ik_0r_2}\left(\frac{\rho}{r_2}\right)\left(\frac{z+d}{r_2}\right)\left(\frac{ik_0}{r_2}-\frac{3}{r_2^2}-\frac{3i}{k_0r_2^3}\right)\right]+E_{0z}^{(3)}(\rho,\varphi,z) \qquad (18)
\end{aligned}
$$

$$
E_{0z}^{(3)}(\rho,\varphi,z)=\frac{\omega\mu_0}{4\pi}\cos\varphi\int_{-\infty}^\infty \frac{\gamma_1\tan\gamma_1 l}{k_1^2\gamma_0-ik_0^2\gamma_1\tan\gamma_1 l}H_1^{(1)}(\lambda\rho)e^{i\gamma_0(z+d)}\lambda^2 d\lambda \qquad (19)
$$

3. TRAPPED WAVE AND LATERAL WAVE OF ELECTRIC FIELD

From (3), (4), (5), (11), (12), (15), (16) and (18), we find that the key to analyze components of electric field is evaluation of integrals in (15), (17) and (19). These integrals converge very slowly and consume quite time when the integrals are calculated along the real axis, because they are Sommerfeld-type integrals (SI) which involves a highly oscillatory and slowly decaying kernel, the Bessel function of the first kind. Therefore, we consider singularity of the integrand then revised original path in λ-plane.

In the following discussion, we suppose $k_0\rho \gg 1$ due to far distance between field-point and source-point practically, and $k_0(z+d)$ is a small real number for considering the propagation characteristic of wave along interface between air and layer of dielectric.

Considering (13), the poles of integrand satisfy the equations

$$q(\lambda) = k_1^2\gamma_0 - ik_0^2\gamma_1 \tan\gamma_1 l = 0 \tag{20}$$

and

$$s(\lambda) = \gamma_1 - i\gamma_0 \tan\gamma_1 l = 0 \tag{21}$$

Suppose that λ is a real number which satisfies $k_0 \ll \lambda \ll k_1$ (neglecting the loss of k_1), then γ_0, γ_1 are a positive imaginary number and a positive real number, respectively. Let

$$f(\lambda) = \frac{k_1^2\sqrt{k_0^2-\lambda^2}}{ik_0^2\sqrt{k_1^2-\lambda^2}} \qquad g(\lambda) = \tan\sqrt{k_1^2-\lambda^2}\,l$$

$$R(\lambda) = \frac{i\sqrt{k_0^2-\lambda^2}}{\sqrt{k_1^2-\lambda^2}}$$

We find that if $n\pi < \sqrt{k_1^2-k_0^2}\,l < (n+1)\pi$, there are $(n+1)$ roots of equation (20), i.e., there are $(n+1)$ poles for the first integrand in (15). These roots were designated by λ_i^*, $i = 1, 2, \ldots, (n+1)$. At the same time, we find if $n\pi < \sqrt{k_1^2-k_0^2}\,l < (n+\frac{1}{2})\pi$, there are n roots of equation (21), i.e., there are n poles for the second integrand in (15). These roots were designated by v_j^*, $j = 1, 2, \ldots, n$.

When the layer of dielectric coated the perfect conductor is low-loss, k_1 is a complex number with a small positive imaginary part. In this case, we may seek real number roots of equation (20), (21), and then find exact roots by using Newton iterative method.

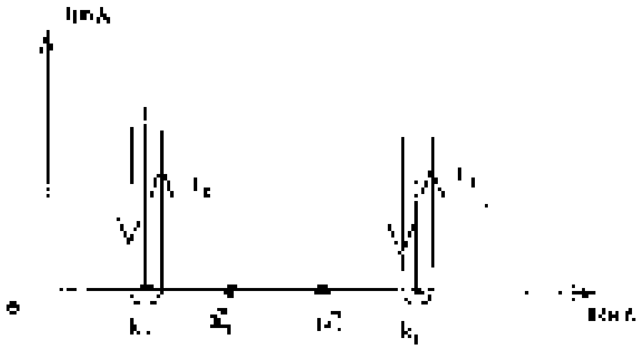

Figure 2. Poles and branch points of integrand.

Since the poles of (15) are determined, the trapped surface wave is given by

$$\begin{aligned} E_{0\rho}^{(3)sur}(\rho,\varphi,z) \;=\; & -\frac{\omega\mu_0}{4}\cos\varphi\left\{\sum_i \frac{\gamma_0(\lambda_i^*)\gamma_1(\lambda_i^*)\tan\gamma_1(\lambda_i^*)l\cdot\lambda_i^*}{q'(\lambda_i^*)}\right. \\ & e^{i\gamma_0(\lambda_i^*)(z+d)}\left[H_0^{(1)}(\lambda_i^*\rho)-H_2^{(1)}(\lambda_i^*\rho)\right] \\ & +\sum_j \frac{\tan\gamma_1(v_j^*)l\cdot v_j^*}{s'(v_j^*)} \\ & \left. e^{i\gamma_0(v_j^*)(z+d)}\left[H_0^{(1)}(v_j^*\rho)+H_2^{(1)}(v_j^*\rho)\right]\right\} \end{aligned} \tag{22}$$

Where

$$\begin{aligned} q'(\lambda) &= -\frac{k_1^2\lambda}{\gamma_0}+\frac{ik_0^2\lambda}{\gamma_1}\tan\gamma_1 l+ik_0^2\cdot\lambda\cdot l\cdot\sec^2\gamma_1 l, \\ s'(\lambda) &= -\frac{\lambda}{\gamma_1}+\frac{i\lambda}{\gamma_0}\tan\gamma_1 l+\frac{i\gamma_0}{\gamma_1}\cdot\lambda\cdot l\cdot\sec^2\gamma_1 l. \end{aligned}$$

General speaking, the number of poles in (22) is related to characteristic and thickness of dielectric layer.

There are two branch-points of integrand in (15), which locate at k_0 and k_1. We make use of the branch cuts integration. The contour of the integration in the complex λ-plane is shown in Fig. 2. The contours were denoted by Γ_0 and Γ_1. Because the integrand is an even function with respect to γ_1, the integrals (15) equal to zero along Γ_1. Therefore, the only thing we do calculate the integrals along Γ_0.

In order to evaluate the integrals along Γ_0, let

$$\lambda = k_0(1 + i\tau^2). \tag{23}$$

Where τ is from $-\infty$ to 0 in the left of Γ_0 and from 0 to $+\infty$ in the right of Γ_0, respectively. At the same time, plane angle of γ_0 is $\frac{3}{4}\pi$ in the right of Γ_0 and $-\frac{\pi}{4}$ in the left of Γ_0, respectively. Then

$$H_0^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi k_0\rho}} e^{ik_0\rho - i\frac{\pi}{4}} e^{-k_0\rho\tau^2}, \tag{24a}$$

$$H_2^{(1)}(\lambda\rho) \approx \sqrt{\frac{2}{\pi k_0\rho}} e^{ik_0\rho - i\frac{5\pi}{4}} e^{-k_0\rho\tau^2}. \tag{24b}$$

From (23), (24), we obtain

$$\gamma_0 = \sqrt{k_0^2 - \lambda^2} \approx k_0 e^{i\frac{3}{4}\pi}\sqrt{2}\cdot\tau, \tag{25}$$

$$\gamma_1 = \sqrt{k_1^2 - \lambda^2} \approx \sqrt{k_1^2 - k_0^2}. \tag{26}$$

Thus the integrals in (15) along Γ_0 is

$$\begin{aligned}
&\frac{\omega\mu_0}{8\pi}\cos\varphi\left\{\int_{\Gamma_0}\frac{i\gamma_0\gamma_1\tan\gamma_1 l}{k_1^2\gamma_0 - ik_0^2\gamma_1\tan\gamma_1 l}\left[H_0^{(1)}(\lambda\rho) - H_2^{(1)}(\lambda\rho)\right]e^{i\gamma_0(z+d)}\lambda d\lambda\right.\\
&\qquad\left.+\int_{\Gamma_0}\frac{i\tan\gamma_1 l}{\gamma_1 - i\gamma_0\tan\gamma_1 l}\left[H_0^{(1)}(\lambda\rho) + H_2^{(1)}(\lambda\rho)\right]e^{i\gamma_0(z+d)}\lambda d\lambda\right\}\\
&=\frac{\omega\mu_0}{8\pi}\cos\varphi\left\{-\frac{4k_0^2\sqrt{k_1^2-k_0^2}\tan\sqrt{k_1^2-k_0^2}\,l}{k_1^2}\sqrt{\frac{2}{\pi k_0\rho}}e^{i(k_0\rho-\frac{\pi}{4})+i\frac{k_0\rho}{2}(\frac{z+d}{\rho})^2}\right.\\
&\qquad\left.\times\int_{-\infty}^{\infty}\frac{\tau^2}{\tau - e^{-i\frac{\pi}{4}}\frac{k_0}{k_1^2}\sqrt{\frac{k_1^2-k_0^2}{2}}\tan\sqrt{k_1^2-k_0^2}\,l}e^{-k_0\rho\left(\tau+\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}\right)^2}d\tau\right\}
\end{aligned} \tag{27}$$

In view of

$$r_2 = \sqrt{\rho^2 + (z+d)^2} \approx \rho\left(1 + \frac{1}{2}\left(\frac{z+d}{\rho}\right)^2\right), \tag{28}$$

We define

$$\Delta_0 = \frac{1}{\sqrt{2}}\left(\frac{z+d}{\rho} - i\frac{k_0\sqrt{k_1^2-k_0^2}}{k_1^2}\tan\sqrt{k_1^2-k_0^2}\,l\right). \tag{29}$$

Formula (27) can be rewritten as follows

$$\int_{\Gamma_0} d\lambda = \frac{\omega\mu_0}{8\pi}\cos\varphi\Bigg\{-\frac{4k_0^2\sqrt{k_1^2-k_0^2}\tan\sqrt{k_1^2-k_0^2}l}{k_1^2}$$

$$\cdot\sqrt{\frac{2}{\pi k_0\rho}}\cdot e^{i(k_0\rho-\frac{\pi}{4})+i\frac{k_0\rho}{2}(\frac{z+d}{\rho})^2}$$

$$\times\int_{-\infty}^{\infty}\left(\tau+e^{-i\frac{\pi}{4}}\frac{k_0}{k_1^2}\sqrt{\frac{k_1^2-k_0^2}{2}}\tan\sqrt{k_1^2-k_0^2}l\right)e^{-k_0\rho\left(\tau+\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}\right)^2}d\tau$$

$$-\frac{2k_0^4(k_1^2-k_0^2)\sqrt{k_1^2-k_0^2}\tan^3\sqrt{k_1^2-k_0^2}l}{k_1^6}$$

$$\cdot e^{-i\frac{\pi}{2}}\cdot\sqrt{\frac{2}{\pi k_0\rho}}\cdot e^{i(k_0\rho-\frac{\pi}{4})+i\frac{k_0\rho}{2}(\frac{z+d}{\rho})^2}$$

$$\times\int_{-\infty}^{\infty}\frac{e^{-k_0\rho\left(\tau+\frac{e^{i\frac{\pi}{4}}}{\sqrt{2}}\frac{z+d}{\rho}\right)^2}}{\tau-e^{-i\frac{\pi}{4}}\cdot\frac{k_0}{k_1^2}\sqrt{\frac{k_1^2-k_0^2}{2}}\tan\sqrt{k_1^2-k_0^2}l}\cdot d\tau\Bigg\}$$

$$=\frac{\omega\mu_0}{8\pi}\cos\varphi\Bigg\{\frac{4k_0\sqrt{k_1^2-k_0^2}\tan\sqrt{k_1^2-k_0^2}l}{k_1^2\rho}e^{ik_0r_2}$$

$$\cdot\left(i\frac{k_0}{k_1^2}\sqrt{k_1^2-k_0^2}\tan\sqrt{k_1^2-k_0^2}l+\frac{z+d}{\rho}\right)$$

$$-\frac{2k_0^4(k_1^2-k_0^2)\sqrt{k_1^2-k_0^2}\cdot\tan^3\sqrt{k_1^2-k_0^2}l}{k_1^6}e^{-i\frac{\pi}{2}}$$

$$\cdot\sqrt{\frac{2}{\pi k_0\rho}}\cdot e^{i(k_0r_2-\frac{\pi}{4}}\cdot\int_{-infty}^{\infty}\frac{e^{-k_0\rho t^2}}{t-e^{i\frac{\pi}{4}}\cdot\Delta_0}\cdot dt\Bigg\} \tag{30}$$

We readily get, from [15]

$$\int_{-\infty}^{\infty}\frac{e^{-k_0\rho t^2}}{t-e^{i\frac{\pi}{4}}\cdot\Delta_0}\cdot dt = i\pi e^{-ip^*}\cdot\sqrt{2}e^{-i\frac{\pi}{4}}F(p^*). \tag{31}$$

Where

$$p^* = k_0\rho\Delta_0^2 = \frac{k_0\rho}{2}\left(\frac{z+d}{\rho}-i\frac{k_0\sqrt{k_1^2-k_0^2}}{k_1^2}\tan\sqrt{k_1^2-k_0^2}l\right)^2, \tag{32}$$

$F(p^*)$ is a Fresnel integral, i.e.,

$$F(p^*) = \frac{1}{2}(1+i) - \int_0^{p^*} \frac{e^{it}}{\sqrt{2\pi t}} dt. \tag{33}$$

Substituting (30)–(33)into (29), we have

$$\begin{aligned} E_{0\rho}^{(3)lat}(\rho,\varphi,z) = \frac{\omega\mu_0}{8\pi}\cos\varphi \Bigg\{ & \frac{4k_0\sqrt{k_1^2-k_0^2}\tan\sqrt{k_1^2-k_0^2}l}{k_1^2\rho} \\ & \cdot e^{ik_0r_2}\left(i\frac{k_0}{k_1^2}\sqrt{k_1^2-k_0^2}\times\tan\sqrt{k_1^2-k_0^2}l + \frac{z+d}{\rho}\right) \\ & + \frac{4ik_0^4(k_1^2-k_0^2)\sqrt{k_1^2-k_0^2}\tan^3\sqrt{k_1^2-k_0^2}l}{k_1^6} \\ & \cdot\sqrt{\frac{\pi}{k_0\rho}}e^{ik_0r_2}e^{-ip^*}F(p^*)\Bigg\}. \end{aligned} \tag{34}$$

It is easy to get the expression of (17) and (19) by using the same procedure.

4. DISCUSSION

1. If we consider the case, i.e., $k_0^2 \ll k_1^2,\ k_1 l \ll 1$, the second term of (34) in this paper is the same as last term of (45) in [11]. As a matter of fact, because

   $$p^* = \frac{k_0\rho}{2}\left(\frac{z+d}{\rho} - ik_0 l\right)^2 \approx \frac{k_0 r_2}{2}\left(\frac{z+d+\varepsilon r_2}{\rho}\right)^2 = p_2,$$

   with the parameter $\varepsilon = -ik_0 l$. The second term of (34) in this paper is

   $$4ik_0^4 l^3\sqrt{\frac{\pi}{k_0\rho}}e^{ik_0r_2}\cdot e^{-ip^*}\cdot F(p^*)$$

   The last term of (45) in [11] is

   $$4k_0\varepsilon^3\left(\frac{r_2}{\rho}\right)\sqrt{\frac{\pi}{k_0r_2}}e^{ik_0r_2}\cdot e^{-ip_2}\cdot F(p_2)$$

   Since r_2 is close to ρ actually, two formulas accord with each other entirely. Similarly, the first term of (34) is the rest of (45) except the first two terms and the last term.

2. From the third part, it is observed that the contribution for the components of electric field is come from branch cut Γ_0 and the poles λ_i^*, v_j^* besides direct wave and reflected wave. Because $e^{i\gamma_0(z+d)}$ is a attenuation factor which is equal to $e^{-\sqrt{\lambda_i^{*2}-k_0^2}(z+d)}$ or $e^{-\sqrt{v_j^{*2}-k_0^2}(z+d)}$, the wave along z direction decay with exponential rule when view-point or field-point is away from interface between air and layer of dielectric. On the other hand, the wave along ρ direction whose propagating factor is $e^{i\lambda_i^*\rho}$ and $e^{iv_j^*\rho}$ varies as $1/\sqrt{\rho}$, that is trapped surface wave. The trapped surface wave can be excited when the dipole is very close to the interface between air and layer of dielectric. Once the dipole is far from the interface, the excited field strengths rapidly decay with exponential rule.
3. The trapped surface wave not only relates with frequency and electric parameter but also relates with the thickness of dielectric layer. Figure 3 shows the curves of moving for λ_1^*/k_0 and v_1^*/k_0 with thickness of dielectric layer when $f = 100\,\text{MHz}$, ε_r is 2.65 and 5, respectively. Thickness of dielectric layer is from 0.15 m to 2 m. It can be seen that the trapped surface wave number obviously differ from k_0. Therefore, the trapped surface wave interferes with the lateral wave on the surface of dielectric layer.
4. When $\varepsilon_{1r} = 1$, which lead to $k_1 = k_0$, the expressions of (20), (34) are equal to zero. This reduces to a half-space problem. The electric field in the air consists of direct wave and reflected wave. Therefore, the formula (15) is actually an amendatory term for perfect-image field.

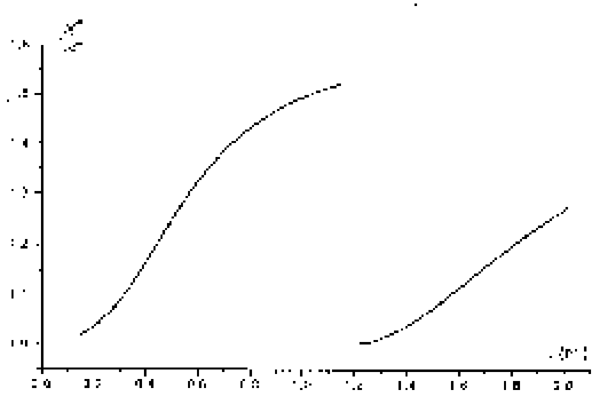

Figure 3a. Variation of λ_1^*/k_0 with thickness of dielectric layer ($\varepsilon_r = 2.65$).

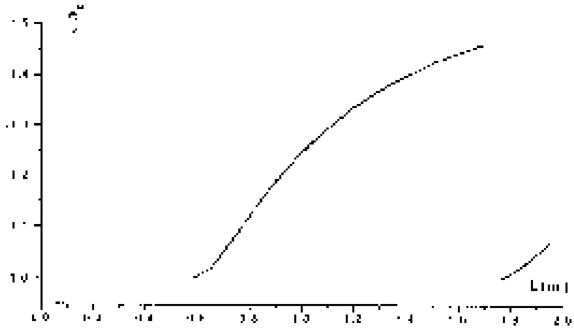

Figure 3b. Variation of v_1^*/k_0 with thickness of dielectric layer ($\varepsilon_r = 2.65$).

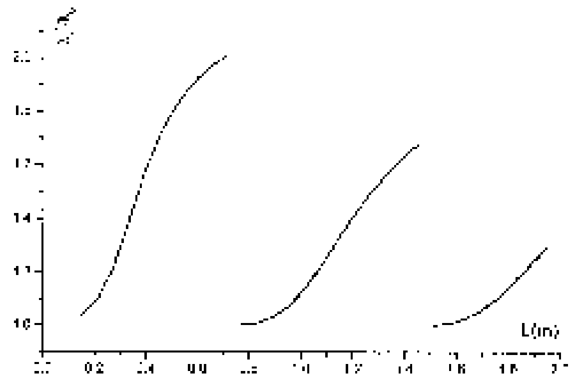

Figure 3c. Variation of λ_1^*/k_0 with thickness of dielectric layer ($\varepsilon_r = 5$).

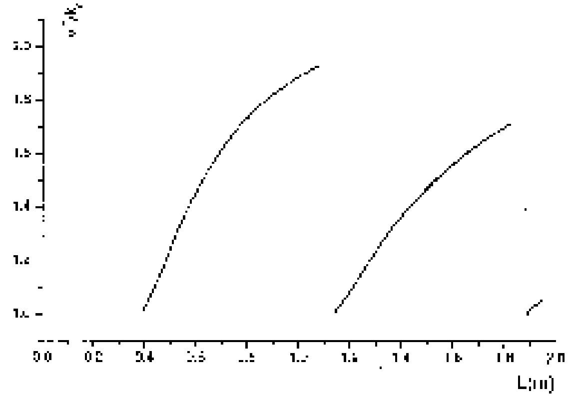

Figure 3d. Variation of v_1^*/k_0 with thickness of dielectric layer ($\varepsilon_r = 5$).

5. If substrate is not an ideal conductor, Q_3 and P_3 is no longer the simple form. However, we anticipate that the conclusion is still true though discussion will be very complicated.

5. CONCLUSION

This paper presents new formulas of electric field generated by a horizontal electric dipole in the air over a dielectric coated a perfect conductor by using residue theorem and revising integral path. The result showed that the excited field in the air consists of the direct wave, reflected wave, trapped surface wave and lateral wave. The propagation wave number of trapped surface wave, which depends on electric parameters and thickness of the dielectric layer, is between the wave number k_0 and k_1.

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