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SCATTERING OF A PLANE WAVE BY ROUGH SURFACES: A NEW CURVILINEAR COORDINATE SYSTEM BASED APPROACH

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Abstract—We present a curvilinear coordinate based method giving the field scattered by a plane surface with a cylindrical local perturbation illuminated by a plane wave. The boundary-value problem turns on the same scalar eigen equation that is solved in the spectral domain. Numerical results are successfully compared with those obtained by other rigorous methods

1. INTRODUCTION

Scattering of waves by rough surfaces has aroused the interest of physicists and engineers for many years because of its large domain of application in optics, radio wave propagation and radar techniques. When the geometrical parameters are close to the incident light wavelength, rigorous vectorial methods have to be used. For periodic surfaces, i.e., gratings, the curvilinear coordinate method (C.C.M) is now well established and is known to provide both simplicity and versatility [1]. We believe that it should also be very powerful in the field of rough surfaces. The essence of the C.C.M method is to choose a coordinate system in such a way that the boundaries of the physical problem coincide with coordinate surfaces. The simplest one is the so called translation coordinate system. The TE and TM fields are derived from the same scalar eigenequation in which the geometry appears in terms of the derivative of the profile function. This system is solved in the spectral domain by using the method of moments with sinc as basis functions and Dirac delta function as weighting functions. The C.C.M has already been extended to the case of non periodic surfaces that are invariant along one direction [2, 3]. However, the present work is different in two aspects: first, the expansion basis sinc has not been used before, second the scalar operator that takes into account the new coordinate system is deduced without using tensorial calculus. Hence, both the method itself and its numerical counterpart remain very simple.

The comparison of our results with those obtained by other authors and the confirmation of the energy balance criterion allow the verification of the validity of our computer code.

2. DEFINITION OF THE PROBLEM

2.1. Notation

We consider the electromagnetic problem in which vacuum and a linear, homogeneous, non magnetic, media are separated by a cylindrical surface (S), that is invariant along the z axis in the Cartesian coordinate system $(Oxyz)$ see Fig. 1. Such a surface, described by an equation: $y = a(x)$ is illuminated from above by a linear polarized monochromatic plane wave with a vacuum wavelength λ, an angular frequency ω, and a wave number $k = 2\pi/\lambda$. The incident medium is supposed to be vacuum and the wave vector, inclined at θ to the y axis has the following Cartesian components: $k_x = k \sin\theta$, $k_y = -k \cos\theta$. Time dependence is expressed by the factor $\exp(i\omega t)$. Such a problem is reduced to the study of the two fundamental cases of polarization,

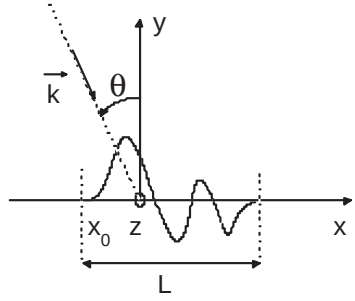

Figure 1. Geometry of the problem.

and the unknown function is the z component of the electric or the magnetic field for TE and TM polarization, respectively. We shall denote by Ψ the complex amplitude of this component. For the sake of simplicity, we shall assume that the incident field has a unit amplitude:

$$\Psi^i = \exp(-i\alpha_0 x)\exp(i\beta_0 y) \tag{1}$$

with $\alpha_0 = k\sin\theta$ and $\beta_0 = \sqrt{k^2 - \alpha_0^2}$. We denote by Ψ^s the scattered field defined by: $\Psi^s = \Psi - \Psi^i - \Psi^r$, Ψ^r being the field which would be reflected if the surface were a plane.

$$\Psi^r = \begin{cases} r_E \exp(-i\alpha_0 x)\exp(-i\beta_0 y) & \text{for TE polarization} \\ r_M \exp(-i\alpha_0 x)\exp(-i\beta_0 y) & \text{for TM polarization} \end{cases} \tag{2}$$

for perfect conductors, $r_E = -1$ and $r_M = 1$, in the general case they are the Fresnel coefficients.

2.2. Plane Wave Decomposition

Let y_M be the maximum value of $a(x)$. For $y > y_M$, Ψ^s can be represented by a continuous spectrum of outgoing plane waves:

$$\Psi^s(x,y) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} R(\alpha)\exp(-i\beta(\alpha))y\exp(-i\alpha x)d\alpha \tag{3}$$

For the propagating waves $|\alpha| < k$ and $\alpha = k\sin\theta$ $\beta(\alpha) = \sqrt{k^2 - \alpha^2} = k\cos\theta$, and for the evanescent waves $|\alpha| > k$ and $\beta(\alpha) = -i\sqrt{\alpha^2 - k^2}$.

In polar coordinates $x = r\sin\theta$, $y = r\cos\theta$ and in the far-field zone $r \gg \lambda$, the above integral can be reduced to [4]:

$$\Psi^s(r,\theta) = k\sqrt{\frac{1}{2\pi}}R(\alpha)\cos\theta\frac{\exp(-ikr)}{\sqrt{kr}}\exp\left(i\frac{\pi}{4}\right) \tag{4}$$

It is seen that the angular dependence is given by the unknown function $R(\alpha)\cos\theta$ from which we deduce the angular scattered power density:

$$P(\theta) = \frac{k\left|R(\alpha)\right|^2\cos^2\theta}{4\pi Z_0} \tag{5}$$

where Z_0 is the vacuum wave impedance.

It is well known that the representation Eq. (3) is not always valid on the surface itself. That is why we have to find another expression to write the boundary conditions.

2.3. Translation Coordinates

For that purpose, we seek a coordinate system in which (S) coincides with a coordinate surface. The simplest one is the so-called translation coordinate system given by:

$$x = u, \quad y = v + a(x), \quad z = w \tag{6}$$

then surface $v = 0$ is the profile surface, thus matching boundary conditions becomes a simple matter. In order to take advantage of the asymptotic expression Eq. (4), we shall express the solutions obtained in the translation coordinate system in terms of plane waves which will permit us to calculate $R\,(\alpha)$.

2.4. Maxwell's Equations in the New Coordinate System

In this section, we derive the operator that takes into account the change of variable Eq. (6). For simplicity, we assume arbitrarily a TE polarization. In a medium with refractive index ν, Maxwell's curl equations and the constitute relations yield:

$$Z_0 H_x = \frac{i}{k}\frac{\partial E_z}{\partial y} \tag{7}$$

$$Z_0 H_y = -\frac{i}{k}\frac{\partial E_z}{\partial x} \tag{8}$$

$$\nu^2 E_z = \frac{i}{k}\left[\frac{\partial Z_0 H_x}{\partial y} - \frac{\partial Z_0 H_y}{\partial x}\right] \tag{9}$$

where E_z, H_x, H_y are the non-null Cartesian components of the electric and the magnetic field, respectively and Z_0 the vacuum impedance. In the new coordinate system $(u,\ v,\ w)$, the covariant components are deduced from the cartesian components by:

$$E_w = E_z \tag{10}$$

$$H_v = H_y \tag{11}$$
$$H_u = H_x + \dot{a}H_y \tag{12}$$

where $\dot{a} = \frac{da}{dx}$. Furthermore, from the chain rule of partial differentiation, the differentiation operators become:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial u} - \dot{a}\frac{\partial}{\partial v} \tag{13}$$
$$\frac{\partial}{\partial y} = \frac{\partial}{\partial v} \tag{14}$$

by substituting Eqs. (10)–(14) into Eqs. (7)–(9), we obtain:

$$Z_0 \left(H_u - \dot{a}H_v\right) = \frac{i}{k}\frac{\partial E_w}{\partial v} \tag{15}$$

$$Z_0 H_v = -\frac{i}{k}\left(\frac{\partial}{\partial u} - \dot{a}\frac{\partial}{\partial v}\right) E_w \tag{16}$$

$$\nu^2 E_w = \frac{i}{k}\left(\frac{\partial}{\partial v} Z_0 \left(H_u - \dot{a}H_v\right) - \left(\frac{\partial}{\partial u} - \dot{a}\frac{\partial}{\partial v}\right) Z_0 H_v\right) \tag{17}$$

At the boundary $y = a(x)$, i.e., $v = 0$, the tangential components of the electric and magnetic fields are continuous. In the present case, these components are proportional to the covariant components E_w and H_u; that is the reason why we eliminate H_v to obtain:

$$Z_0 H_u = \frac{i}{k}\left(\frac{\partial E_w}{\partial v} + \dot{a}\left(\frac{\partial}{\partial u} - \dot{a}\frac{\partial}{\partial v}\right) E_w\right) \tag{18}$$

$$\left[(1 + \dot{a}\dot{a})\frac{\partial^2}{\partial v^2} - \frac{\partial}{\partial v}\left(\dot{a}\frac{\partial}{\partial u}\right) - \frac{\partial}{\partial v}\left(\frac{\partial}{\partial u}\dot{a}\right) + \frac{\partial^2}{\partial u^2} + k^2\nu^2\right] E_w = 0 \tag{19}$$

The above equation is the propagation equation expressed in terms of the new variables u and v. Since the medium is homogeneous it is valid for the TM polarization as well. So we replace E_z with $\Psi(\Psi = E_z$ or $\Psi = Z_0 H_z)$.

Remark 1 *It can be easily verified that* $\psi = \exp\left[-i\beta\left(v + -a(u)\right)\right] \exp\left(-i\alpha u\right)$ *is an elementary solution of the new propagation equation provided that* $\beta^2 + \alpha^2 = k^2\nu^2$. *This shows that the elementary solutions of Eq. (19) are just plane waves expressed in the new coordinate system. However, this beautiful property is no longer satisfied as soon as we consider a truncated representation of the elementary waves. From the practical point of view, we have to solve numerically Eq. (19) in order to obtain approximate elementary solutions that take into account at best of the truncation scheme.*

3. NUMERICAL SOLUTION

First, we split Eq. (19) into two first-order differential equations:

$$\begin{bmatrix} \dfrac{\partial^2}{\partial u^2} + k^2\nu^2 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \Psi \\ \dfrac{\partial \Psi}{\partial v} \end{pmatrix} = \begin{bmatrix} \dfrac{\partial}{\partial u}\dot{a} + \dot{a}\dfrac{\partial}{\partial u} & -(1+\dot{a}\dot{a}) \\ 1 & 0 \end{bmatrix} \frac{\partial}{\partial v} \begin{pmatrix} \Psi \\ \dfrac{\partial \Psi}{\partial v} \end{pmatrix} \tag{20}$$

The method of separation of variables suggests to seek solutions of the form:

$$\Psi(u,v) = \exp(-irv)\psi(u) \tag{21}$$

$$\Psi'(u,v) = i\frac{\partial \Psi}{\partial v} = \exp(-irv)\psi'(u) \tag{22}$$

Finally, we have to solve the following eigenequation:

$$\begin{bmatrix} \dfrac{\partial^2}{\partial u^2} + k^2\nu^2 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix} = -ir \begin{bmatrix} \left(\dfrac{\partial}{\partial u}\dot{a} + \dot{a}\dfrac{\partial}{\partial u}\right) & i(1+\dot{a}\dot{a}) \\ i & 0 \end{bmatrix} \begin{pmatrix} \psi \\ \psi' \end{pmatrix} \tag{23}$$

In order to solve numerically the above eigenequation, we have to associate a matrix to each operator coefficient. For that purpose we shall use the method of moments in the spectral domain. Let us designate by $H_u^{(\alpha)}$, $H_v^{(\alpha)}$, $E_z^{(\alpha)}$, and $\dot{a}^{(\alpha)}$ the Fourier transforms of H_u, H_v, E_z, and $\dot{a}$. The Fourier transform is defined by:

$$H_u^{(\alpha)} = \int_{-\infty}^{+\infty} H_u \exp(i\alpha x)\, dx \tag{24}$$

In the spectral domain Eq. (15)–(17) become:

$$Z_0\left(H_u^{(\alpha)} - \dot{a}^{(\alpha)} * H_v^{(\alpha)}\right) = \frac{i}{k}\frac{\partial E_w^{(\alpha)}}{\partial v} \tag{25}$$

$$Z_0 H_v^{(\alpha)} = -\frac{i}{k}\left(i\alpha E_w^{(\alpha)} - \frac{\partial}{\partial v}\dot{a}^{(\alpha)} * E_w^{(\alpha)}\right) \tag{26}$$

$$\begin{aligned} \nu^2 E_w^{(\alpha)} = \frac{i}{k}\Bigg[& Z_0\frac{\partial}{\partial v}\left(H_u^{(\alpha)} - \dot{a}^{(\alpha)} * H_v^{(\alpha)}\right) \\ & - \left(i\alpha Z_0 H_v^{(\alpha)} - \frac{\partial}{\partial v}\dot{a}^{(\alpha)} * Z_0 H_v^{(\alpha)}\right)\Bigg] \end{aligned} \tag{27}$$

where $*$ designates the convolution product. Let us define a window such that $\alpha \in [\alpha_0 - M\Delta\alpha \quad \alpha_0 + M\Delta\alpha]$ and $\alpha_0 = \frac{2\pi}{\lambda}\sin\theta$. We call the integer M the truncation order. Each spectral component of the field is sampled:

$$H_u^{(\alpha)} = \sum_{p=-M}^{p=+M} H_{up} \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_p)\right] \tag{28}$$

$$H_v^{(\alpha)} = \sum_{p=-M}^{p=+M} H_{vp} \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_p)\right] \tag{29}$$

$$E_w^{(\alpha)} = \sum_{p=-M}^{p=+M} E_{wp} \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_p)\right] \tag{30}$$

$$\alpha_p = \alpha_0 + p\Delta\alpha \tag{31}$$

The profile derivative spectrum is also approximated by samples:

$$\dot{a}^{(\alpha)} = \sum_{p=-\infty}^{p=+\infty} \dot{a}_p \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_p)\right] \tag{32}$$

by projecting Eqs. (25)–(27) onto the Dirac pulses $\delta(\alpha - \alpha_m)$, and remarking that $\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_p)\right] * \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_m)\right] = \Delta\alpha \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_{m+p})\right]$, we obtain the following matrix relations

$$Z_0\left(\mathbf{H}_u^{(\alpha)} - \Delta\alpha\dot{\mathbf{a}}^{(\alpha)}\mathbf{H}_v^{(\alpha)}\right) = \frac{i}{k}\frac{\partial \mathbf{E}_w^{(\alpha)}}{\partial v} \tag{33}$$

$$Z_0\mathbf{H}_v^{(\alpha)} = -\frac{i}{k}\left(i\boldsymbol{\alpha}\mathbf{E}_w^{(\alpha)} - \frac{\partial}{\partial v}\Delta\alpha\dot{\mathbf{a}}^{(\alpha)}\mathbf{E}_w^{(\alpha)}\right) \tag{34}$$

$$\begin{aligned}\nu^2\mathbf{E}_w^{(\alpha)} \quad = \quad & \frac{i}{k}\left[\frac{\partial}{\partial v}Z_0\left(\mathbf{H}_u^{(\alpha)} - \Delta\alpha\dot{\mathbf{a}}^{(\alpha)}\mathbf{H}_v^{(\alpha)}\right)\right. \\ & \left. - \left(i\boldsymbol{\alpha}Z_0\mathbf{H}_v^{(\alpha)} - \frac{\partial}{\partial v}\Delta\alpha\dot{\mathbf{a}}^{(\alpha)} * Z_0\mathbf{H}_v^{(\alpha)}\right)\right]\end{aligned} \tag{35}$$

$\mathbf{H}_u$, $\mathbf{H}_v$, $\mathbf{E}_w$ are column vectors whose elements are H_{um}, H_{vm}, E_{wm} respectively; $\boldsymbol{\alpha} = diag(\alpha_m)$; $\dot{\mathbf{a}}^{(\alpha)} = \left[\dot{a}_{mp}^{(\alpha)}\right]$ is a toeplitz matrix such that:

$$\dot{a}_{mp}^{(\alpha)} = \dot{a}_{m-p}^{(\alpha)} = \dot{a}^{(\alpha)}(\alpha_{m-p}) \tag{36}$$

Let us introduce $\Psi^{(\alpha)}$ and $\Phi^{(\alpha)}$:

$$\Psi^{(\alpha)} = \psi^{(\alpha)} \exp(-irv) = \begin{cases} E_w^{(\alpha)} & \text{for TE polarization} \\ Z_0 H_w^{(\alpha)} & \text{for TM polarization} \end{cases} \tag{37}$$

$$\Phi^{(\alpha)} = \phi^{(\alpha)} \exp(-irv) = \begin{cases} Z_0 H_u^{(\alpha)} & \text{for TE polarization} \\ -E_u^{(\alpha)} & \text{for TM polarization} \end{cases} \tag{38}$$

Finally Eq. (23) becomes:

$$\mathbf{L}_A \begin{pmatrix} \boldsymbol{\psi}_q^{(\alpha)} \\ \boldsymbol{\psi}_q'^{(\alpha)} \end{pmatrix} = \frac{1}{r_q} \mathbf{L}_B \begin{pmatrix} \boldsymbol{\psi}_q^{(\alpha)} \\ \boldsymbol{\psi}_q'^{(\alpha)} \end{pmatrix} \tag{39}$$

where

$$\mathbf{L}_A = \begin{bmatrix} -\boldsymbol{\alpha}\Delta\alpha\dot{\mathbf{a}}^{(\alpha)} - \Delta\alpha\dot{\mathbf{a}}^{(\alpha)}\boldsymbol{\alpha} & \mathbf{I} + \Delta\alpha\dot{\mathbf{a}}^{(\alpha)}\Delta\alpha\dot{\mathbf{a}}^{(\alpha)} \\ \mathbf{I} & \mathbf{0} \end{bmatrix} \tag{40}$$

$$\mathbf{L}_B = \begin{bmatrix} k^2\boldsymbol{\nu}^2\mathbf{I} - \boldsymbol{\alpha}\boldsymbol{\alpha} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \tag{41}$$

where $\boldsymbol{\psi}_q^{(\alpha)} = \left(\psi_{mq}^{(\alpha)}\right)$ and $\boldsymbol{\psi}_q'^{(\alpha)} = \left(\psi_{mq}'^{(\alpha)}\right)$are the upper part and the lower part respectively of the eigenvector associated with the qth eigenvalue $1/r_q$ of the matrix $\mathbf{L}_B^{-1}\mathbf{L}_A$; $\phi_q^{(\alpha)}$ is deduced from $\boldsymbol{\psi}_q^{(\alpha)}$ by:

$$\boldsymbol{\phi}_q^{(\alpha)} = \left[\frac{1}{kr_q}\left(k^2\nu^2\mathbf{I} - \boldsymbol{\alpha}\boldsymbol{\alpha}\right) + \frac{1}{k}\boldsymbol{\alpha}\Delta\alpha\dot{\mathbf{a}}^{(\alpha)}\right]\boldsymbol{\psi}_q^{(\alpha)} \tag{42}$$

Finally, in the α space, the general solution is constructed by superposition of elementary solutions:

$$\Psi^{(\alpha)}(\alpha, v) = \sum_q s_q \exp(-ir_q v) \sum_m \psi_{mq} \sin c\left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_m)\right] \tag{43}$$

It is observed numerically that there are two sets of modes, the number of which are equal: those propagating or decaying in the positive y direction and those traveling in the opposite direction.

$$\begin{aligned} \{r_q\} &= \{r_q / \left(r_q \in \mathbb{R}^+\right) \text{ or } (r_q \in \mathbb{C} \text{ and } \Im(r_q) < 0)\} \\ &\cup \{r_q / \left(r_q \in \mathbb{R}^-\right) \text{ or } (r_q \in \mathbb{C} \text{ and } \Im(r_q) > 0)\} \end{aligned} \tag{44}$$

$\Im(z)$ designates the real part of $z \in C$. The numerically computed eigenvalues and eigenvectors depend on the truncation order M. In

the following relation, we use the extra superscript M to indicate this dependence. Numerical experiments show that:

$$\lim_{M \longrightarrow \infty} r_q^M = \beta(\alpha_q) = \beta_q = \sqrt{k^2 - \alpha_q^2} \tag{45}$$

The interesting feature of Eq. (45) is that the limit is independent of the coordinate system. This is not surprising because in a homogeneous space, the eigensolutions of Maxwell's equations are just plane waves. However, the invariance of the eigensolutions is severely destroyed by the matrix truncation that is unavoidable in the numerical implementation. For the C.C.M. method to be successful, M has to be chosen large enough so that the computed real eigenvalues coincide with a great accuracy with the real plane waves eigenvalues.

3.1. Plane Waves in the New Coordinate System

Since the calculated real eigenvalues coincide with $\sqrt{k^2 - \alpha^2}$, the associated eigenvector correspond to the Fourier representation of the corresponding plane wave. We consider the part of the plane wave spectrum that corresponds to the scattered far field. In cartesian coordinate we have:

$$\Psi(x, y) = \int_{-\infty}^{+\infty} R_k(\alpha) \exp\left[-i\beta(\alpha)\, y\right] \exp(-i\alpha x) d\alpha \tag{46}$$

$$\Psi^{(\alpha)}(y) = R_k(\alpha) \exp(-i\beta(\alpha) y) = \int_{-\infty}^{+\infty} \Psi(x, y) \exp(i\alpha x) dx \tag{47}$$

with

$$R_k(\alpha) = \begin{cases} R(\alpha) & \text{if} \quad |\alpha| < k \\ 0 & \text{if} \quad |\alpha| >= k \end{cases} \tag{48}$$

A sampled version of this spectrum can be written as:

$$\Psi^{(\alpha)}(y) = \sum_q \Psi^{(\alpha_q)}(y) \sin c \left[\frac{\pi}{\Delta\alpha}(\alpha - \alpha_q)\right] \tag{49}$$

with

$$\Psi^{(\alpha_q)}(y) = R_k(\alpha_q) \exp(-i\beta(\alpha_q) y) \tag{50}$$

Since Ψ is a field component that remains unchanged under the change of variable we have:

$$\Psi(u, v) = \Psi(x, y = v + a(x)) \tag{51}$$

Taking into account the Fourier representation of $\Psi(u,v)$, we get:

$$\begin{aligned}\Psi(u,v) &= \int_{-\infty}^{+\infty} R_k(\alpha)\exp\left[-i\beta(\alpha)v\right]\left[\exp(-i\beta(\alpha)a(u)\right]\exp(-i\alpha u)d\alpha \\ &= \int_{-\infty}^{+\infty}\sum_q \Psi^{(\alpha_q)}(v)\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_q)\right]\left[\exp(-i\beta(\alpha_q)a(u)\right] \\ &\qquad \exp(-i\alpha u)d\alpha \end{aligned} \tag{52}$$

$\Psi(u,v)$ is a function of the u variable that can be Fourier transformed. Its spectrum is given by:

$$\Psi^{(\alpha)}(v) = \int \Psi(u,v)\exp(i\alpha u)du \tag{53}$$

To be consistent with all previous Fourier representation we seek F_{mq} coefficients such that

$$\Psi^{(\alpha)}(v) = \sum_q \Psi^{(\alpha_q)}(v)\sum_m F_{mq}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right] \tag{54}$$

F_{mq} is given by

$$F_{mq} = \Delta\alpha\int_{-\infty}^{+\infty}\Pi_{\Delta\alpha}(u)\exp\left[i\beta(\alpha_q)a(u)\right]\exp(-i\alpha_q u)\exp(im\Delta\alpha u)du \tag{55}$$

with:

$$\Pi_{\Delta\alpha}(u) = \begin{cases} 1 & \text{if} \quad u\in\left[\dfrac{-1}{2\Delta a} \quad \dfrac{1}{2\Delta a}\right] \\ \text{otherwise} \end{cases}$$

3.2. Application to Scattering from Aperiodic Surfaces

In the preceding section we have made an identification between two representations of the far field. This will enable us to compute very simply the angular power density thanks to Eq. (5). Simultaneously we have solved Maxwell's equations in the so-called translation coordinate system in order to write in a simple manner the boundary conditions and to calculate the coefficients $R_k(\alpha_q)$. We put together the convenient representation of the field by substituting, in the decomposition of the function Ψ on the $\mathbf{B}^{-1}\mathbf{A}$ matrix eigenvectors, the computed eigenvectors associated with real eigen values with the corresponding transformed plane wave spectrum. That is:

$$\sum_{q\in U} s_q\exp(-ir_q v)\sum_{m=-M}^{m=M}\psi_{mq}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right] \tag{56}$$

is replaced by:

$$\sum_{q\in U} R(\alpha_q)\exp(-i\beta(\alpha_q)v)\sum_{m=-M}^{m=M} F_{mq}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right] \tag{57}$$

where

$$U=\left\{q\in[-M,M],k^2-r_q^2>0\right\}=\{q\in[-M,M],|\alpha_q|<k\} \tag{58}$$

Finally, in the Fourier space associated with the translation coordinate system we have the following representations for the scattered field, the incident field and the specular reflected field respectively:

$$\Psi^{s(\alpha)}=\sum_{q\in U} R(\alpha_q)\exp(-i\beta(\alpha_q)v)\sum_{m=-M}^{m=M} F_{mq}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right] \tag{59}$$

$$+\sum_{q\in V} s_q\exp(-ir_q)v)\sum_{m=-M}^{m=M}\Psi_{mq}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right] \tag{60}$$

$$\Psi^{i(\alpha)}=\exp(i\beta(\alpha_0)v)\left[\delta(\alpha-\alpha_0)+\sum_{m=-M}^{m=M}F_{m0}^{i}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right]\right] \tag{61}$$

$$\Psi^{r(\alpha)}=\begin{cases} r_E\exp(-i\beta(\alpha_0)v)\left[\delta(\alpha-\alpha_0)+\sum_{m=-M}^{m=M}F_{m0}^{r}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right]\right] \\ \qquad\qquad \text{for TE polarization} \\ r_M\exp(-i\beta(\alpha_0)v)\left[\delta(\alpha-\alpha_0)+\sum_{m=-M}^{m=M}F_{m0}^{r}\sin c\left[\frac{\pi}{\Delta\alpha}(\alpha-\alpha_m)\right]\right] \\ \qquad\qquad \text{for TM polarization} \end{cases} \tag{62}$$

with

$$V=\{q\in[-M,M],\Im m(r_q)<0\} \tag{63}$$

and

$$F_{m0}^{i}=\int_{x_0}^{x_0+L}\left[\exp\left[i\beta(\alpha_0 a(u)\right]-1\right]\exp(-i\alpha_0 u)]\exp(im\Delta\alpha u)du \tag{64}$$

and

$$F_{m0}^{r}=\int_{x_0}^{x_0+L}\left[\exp\left[-i\beta(\alpha_0 a(u)\right]-1\right]\exp(-i\alpha_0 u)\exp(im\Delta\alpha u)du \tag{65}$$

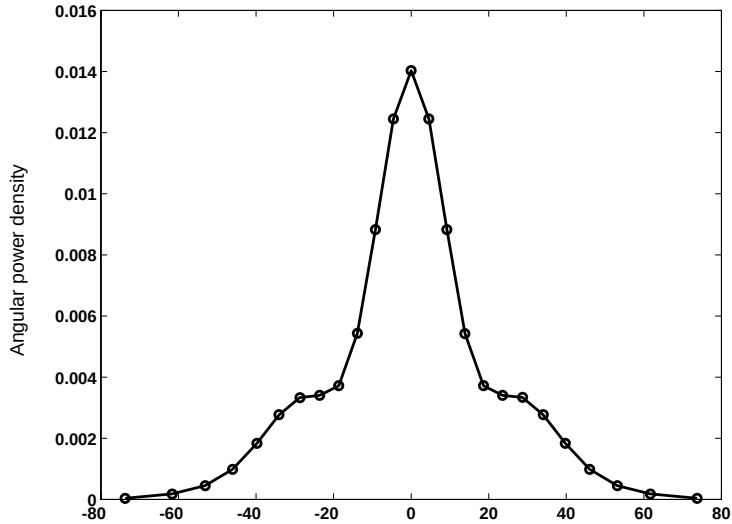

Figure 2. Angular power density for the Shwartz profile.

It is then easy to deduce the other component of interest, E_u or H_u according to the polarization , and to match the field at the surface $v = 0$. The unknown coefficients are obtained by projecting the relations issued from the boundary conditions onto Delta Dirac functions.

4. RESULTS

In order to validate our computer code, we apply the previous theory to a Schwartz profile for which results are available in the literature. It is given by:

$$a(x) = \begin{cases} 0 & \text{if} \quad x \notin \left[-\dfrac{L}{2}, \dfrac{L}{2}\right] \\ h \exp\left(b - \dfrac{bl^2}{l^2 - 4x^2}\right) & \text{if} \quad x \in \left[-\dfrac{L}{2}, \dfrac{L}{2}\right] \end{cases} \tag{66}$$

with the following parameters $h/\lambda = .4$, $l/\lambda = 4$, $b/\lambda = 3$. Fig. 2 shows the angular power density for TE polarization. The comparison with the results obtained by Faure-Geors and Maystre [5] on one hand and by Dusseaux and De Oliveira [3] on the other hand is good. It should be emphasized that the rigorous integral method of Maystre [6]

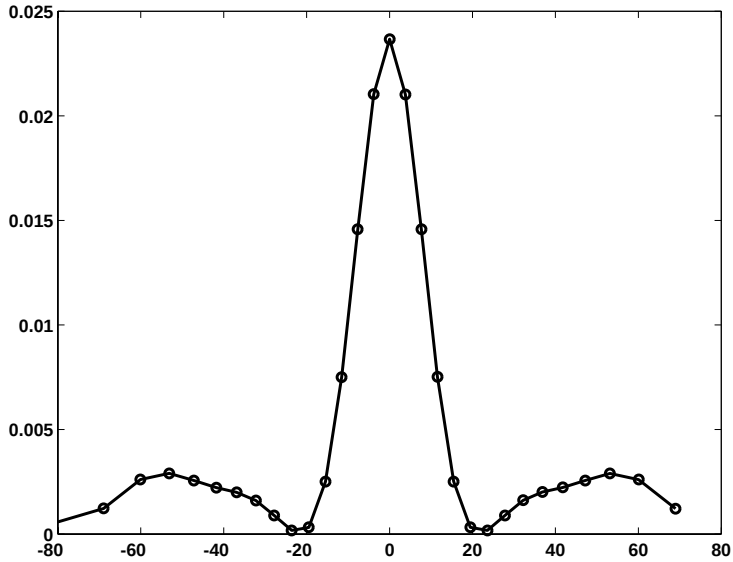

Figure 3. Angular power density of a sinusoidal like profile.

is considered as a reference method. Next, we have investigated the convergence of the method as a function of the numerical parameters M and $\Delta\alpha$. For rough surfaces, the power balance writes:

$$P^s = P_0 \tag{67}$$

with

$$P^s = \int_{-\pi/2}^{+\pi/2} P(\theta) d\theta \tag{68}$$

and

$$P_0 = \frac{1}{Z_0} \Re e \left[R\left(\theta_0\right) \right] \cos\theta_0 \tag{69}$$

where $\Re e(z)$ designates the real part of $z \in \mathbb{C}$. We measure the accuracy on the power balance by introducing the following error function:

$$\Delta P^s(M, \Delta\alpha) = -\log_{10}(1 - \frac{P^s(M, \Delta\alpha)}{P_0(M, \Delta\alpha)} \tag{70}$$

$\Delta\alpha$ gives the spectral resolution and $2M\Delta\alpha$ the width of the spectral window. This bandwidth has to be large enough to include all the evanescent waves that are necessary to represent the near field. It is clear, that for a given bandwidth, the better the resolution, the larger

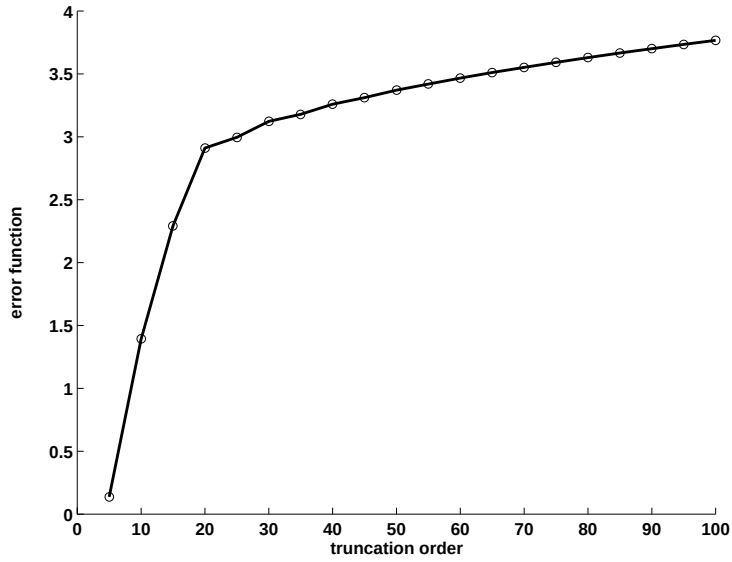

Figure 4. Error function versus the truncation order.

the truncation order is. So, from a practical point of view, there is a compromise to be chosen. For illustrative purpose, let us consider the profile given by:

$$a(x) = \begin{cases} 0 & \text{if} \quad x \notin \left[-\dfrac{L}{2}, \dfrac{L}{2}\right] \\ h\cos\left[\left(\dfrac{2\pi x}{L}\right) + 1\right] & \text{if} \quad x \in \left[-\dfrac{L}{2}, \dfrac{L}{2}\right] \end{cases} \tag{71}$$

with $h/\lambda = 0.4$ and $L/\lambda = 4$. Its angular power density is plotted in Fig. 3. Fig. 4 shows the error function versus the truncation order for a the resolution $\Delta\alpha = \frac{2\pi}{12}$. The error function decreases with the truncation order. In Fig. 5, the error function as a function of the spectral resolution is plotted for two values of the truncation order. The abscissa is graduated in term of $\frac{k}{\Delta\alpha}$: the resolution increases when going from the left to the right. It is seen that the error function increases with the resolution . It is easy to explain : since the truncation order remains constant, the width of the spectral window diminishes and contains les evanescent waves.

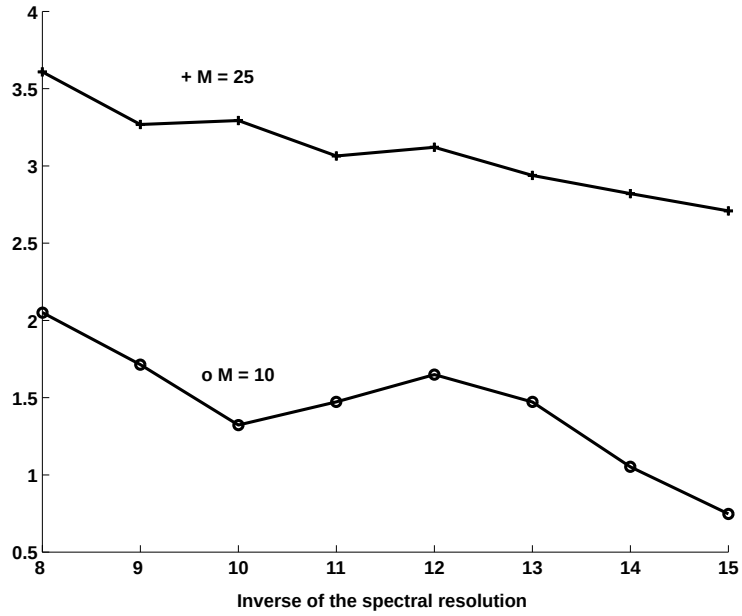

Figure 5. Error function versus the spectral resolution for two values of the truncation order.

5. CONCLUSION

In this paper, we have presented an improved version of the curvilinear coordinate method as applied to rough surfaces. The new operator is derived in a simple manner by considering the new coordinates as a change of variable. The proposed method amounts to solving an eigenvalue system. Its advantages include easy numerical implementation and fast convergence. Although the method is rigorous from the mathematical point of view, it is numerically limited when the local slopes of the profile become too important. However all the refinements developed for the C.C.M and periodic surfaces could be extended when dealing with any surface. That is the reason why the method should be premising for random 1D and 2D rough surfaces where the numerical aspect is of crucial importance.

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