

DIFFRACTION OF A TRANSVERSE ELECTRIC (TE) X-WAVE BY CONDUCTING OBJECTS

A. M. Attiya and E. El-Diwany

Department of Microwave Engineering
Electronic Research Institute
Dokki, Giza, 12211, Egypt

A. M. Shaarawi [†]

The Physics Department
The American University in Cairo
P.O. Box 2511, Cairo 11511, Egypt

I. M. Besieris

The Bradley Department of Electrical and Computer Engineering
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061, USA

Abstract—A study of the diffraction and scattering of a transverse electric X -wave by conducting bodies is presented based on the time-domain, uniform theory of diffraction method and the pulsed plane wave representation of an X -wave. The latter allows the calculation of the diffraction and scattering of each pulsed plane wave component of the incident X -wave at the observation point. The superposition of the individual diffracted and scattered pulsed plane wave components yields the diffracted and scattered field due to an incident X -wave. First, the scattering from a perfectly conducting infinite wedge is studied. Then, the case of a circular conducting disk is considered as an example of a finite scatterer. Numerical results illustrating the effectiveness of the approach, as well as an estimate of the limits of its applicability, are provided.

[†] On leave from the Department of Engineering Physics and Mathematics, Faculty of Engineering, Cairo University, Giza 12211, Egypt.

1 Introduction**2 Formulation****3 X -Wave Scattering by a Perfectly Conducting Wedge**

3.1 The Direct Field

3.2 The Reflected Field

3.3 The Diffracted Field

4 Numerical Examples4.1 Scattering of a TE X -wave Incident on a Perfectly Conducting Wedge4.2 Scattering of a TE X -wave Incident on a Perfectly Conducting Circular Disk**5 Concluding Remarks****Appendix A.****References****1. INTRODUCTION**

Localized waves (LWs) are ultra-wideband, slowly dispersing fields that have extended focused depths [1–4]. Such properties make them worthy candidates for applications involving detection and identifications of buried objects. In previous work, the transmission of one type of LW, the X -wave, through a planar interface separating two different media has been investigated [5–7]. This has been undertaken for acoustic as well as electromagnetic waves. Furthermore, the case of a dispersive lossy half space has been studied using a new technique based on Prony’s method [8, 9]. These investigations have been facilitated by a useful representation of the X -wave solution as a superposition over pulsed plane waves the propagation vectors of which form a conic surface. This pulsed plane wave representation has been formally introduced in Refs. [10] and [11] and has been applied to the reflection and transmission of X -waves at a planar interface separating two different media [5, 6]. Earlier attempts to investigate the planar interface problem include the graphical approach by Donnelly et al. [12] applied to the transmission and reflection of 2-D Focus Wave Modes (FWMs). In their work, Donnelly et al. criticize conclusions reached by Hillion [13] who argued that FWMs are transmitted across a discontinuity surface only if they are normally incident on that surface, while for oblique incidence only reflection takes place.

To study the application of LWs to the detection and identification of buried objects, one needs to carry out a systematic investigation of the scattering of acoustic and electromagnetic LW pulses from various objects. Comprehensive investigations of the scattering of LW pulses are very few. One such study deals with the scattering of acoustic Modified Power Spectrum (MPS) pulses from spheres [14]. In that study, Power et al. used a spectral approach to demonstrate that backscattered acoustic MPS pulses could be used to identify the sizes as well as the material properties of the scattering spheres. A few other scattering problems of LW pulses have been considered. These include the diffraction of a 2-D FWM by a perfectly conducting half plane [15, 16]. Hillion showed that the reflected FWM pulses preserved their shape in the illuminated region, i.e., the reflected pulses were basically FWMs. However, outside that region, Hillion pointed out that it becomes very difficult to analyze the behavior of the diffracted field.

Numerical approaches used for solving the time domain differential Maxwell's equations are usually based on the discretization of space and the stepping in time. Thus, these approaches are mainly limited by the computational resources. The size of the spatial grid is limited by the minimum wavelength of the field and the features of the scattering structure. The time step that assures stability of the solution is limited by the minimum spatial discretization. Large computational resources are required to simulate three-dimensional problems involving the scattering of pulsed fields using the finite-difference, time-domain (FDTD) technique. This view applies to conventional situations involving either pulsed plane or spherical waves. For the case of localized waves, the problem becomes more severe. To make full use of the advantages of localization, the lateral waist of the localization region should be comparable to or smaller than the dimension of the scattering structure. Furthermore, the scanning region around the scattering structure should also be of the same order. What makes the problem more complicated is the indirect calculation of the scattered fields in the far-field region, which is based on calculating the scattered field due to each frequency component of the pulse spectrum and then inverting these spectral components to the time domain via a discrete Fourier transform. For an electrically large problem, this scheme requires a huge amount of computational time, in addition to significant storage capabilities. Such requirements pose restrictions on studying the problem of scattering and diffraction of electromagnetic X -wave using one of the marching-in-time techniques dedicated to solving Maxwell's equations.

In this work, we investigate the effectiveness of using high

frequency techniques in studying the scattering and diffraction of transverse electric (TE) X -waves from conducting bodies, such as wedges and disks. Our approach is based on the pulsed plane wave representation. In earlier publications, we have shown that X -waves can be represented as an azimuthal angular superposition over *pulsed plane waves* [5, 6]. Such pulsed plane wave components are propagating along wave vectors restricted to a circular conic surface characterized by the apex angle θ_0 [17]. The advantage of the pulsed plane wave representation is that it allows the direct application of known results for the diffraction of individual pulsed plane waves. This is done by calculating the diffracted field for each pulsed plane wave component and then superimposing them by integrating over the azimuthal angle. In the present investigation, we make use of the fact that the spectra of the higher order TE X -waves are concentrated at higher frequencies. For this reason, the scattering and diffraction of the first order X -wave is studied in our work using the time domain versions of high frequency techniques. In particular, the uniform theory of diffraction [18, 19] is used for determining the diffracted field of an X -wave incident on an infinite conducting wedge. In addition, results are presented for the diffraction of a first-order X -wave by a circular metallic disk. It is demonstrated that the main difficulty with our approach arises from the infinite field amplitudes appearing at caustics, and that the diffracted field cannot be calculated outside Keller's diffraction cone [20, 21].

The analysis used to study the scattering of a TE X -wave from a perfectly conducting infinite wedge is presented in Sec. 2. Numerical results illustrating the effectiveness of our method are provided in Sec. 3. The method introduced in Sec. 2 is then extended to the case of a finite circular disk, and numerical examples illustrating the behavior of the scattered X -wave are presented. Although the method advocated in this paper seems to be quite effective in the near-field range, it is expected that the evaluation of the fields at farther distances from the scattering disk may be inaccurate. An estimate of the limits of applicability of our method is deduced in Sec. 3. Finally, discussions and concluding remarks are provided in Sec. 4.

2. FORMULATION

The geometry of a perfectly conducting wedge is shown in Fig. 1. The edge is located along the y -axis. The two faces of the wedge are at angles α_0 and $-\alpha_1$ from the x -axis on the x - z plane. The wedge is located in the positive x half space. The normal and tangential

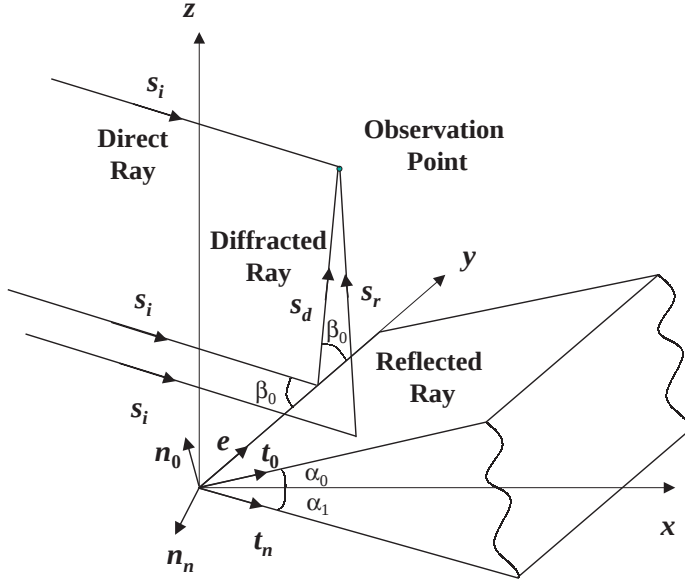


Figure 1. Geometry of the perfectly conducting wedge.

directions to the faces and the direction of the edge are given by

$$\vec{n}_0 = -\vec{a}_x \sin \alpha_0 + \vec{a}_z \cos \alpha_0, \quad (1a)$$

$$\vec{n}_n = -\vec{a}_x \sin \alpha_1 - \vec{a}_z \cos \alpha_1, \quad (1b)$$

$$\vec{t}_0 = \vec{a}_x \cos \alpha_0 + \vec{a}_z \sin \alpha_0, \quad (1c)$$

$$\vec{t}_n = \vec{a}_x \cos \alpha_1 - \vec{a}_z \sin \alpha_1, \quad (1d)$$

and

$$\vec{e} = \vec{a}_y. \quad (1e)$$

The total field at any observation point consists generally of three parts, as shown in Fig. 1; namely, the direct field, the reflected field and the diffracted field. The incident field is chosen to be the first-order X-wave. It has been selected because most its spectral components are in the higher frequency range. This allows one to use the Uniform Theory of Diffraction (UTD) method for the asymptotic evaluation of the diffracted field. The incident TE field component of the first order X-wave can be expressed as

$$E_\phi(\rho, z, t) = \operatorname{Re} \left(\frac{Z_0 \sin \theta_0}{4\pi^2 c^2} \int_0^{2\pi} d\phi_{inc} \vec{a}_\phi \cdot \vec{e}_i(\phi_{inc}) \int_0^\infty d\omega \omega^3 \right)$$

$$e^{-\omega a_0/c} e^{j\omega((t-t_0)-(\vec{R}-\vec{R}_0)\cdot\vec{s}_i(\phi_{inc})/c)} \Big), \quad (2a)$$

or

$$E_\phi(\rho, z, t) = \text{Re} \left(\frac{6Z_0 \sin \theta_0}{4\pi^2 c^2} \int_0^{2\pi} d\phi_{inc} \frac{\cos(\phi_{inc} - \phi_0)}{\left[(a_0/c) - j \left((t - t_0) - (\vec{R} - \vec{R}_0) \cdot \vec{s}_i(\phi_{inc})/c \right) \right]^4} \right), \quad (2b)$$

where Z_0 denotes the intrinsic impedance of free space; furthermore, $\rho = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, $\vec{\rho}_0 = x_0 \vec{a}_x + y_0 \vec{a}_y$ and $\phi_0 = \tan^{-1}((y - y_0)/(x - x_0))$. The vectors $\vec{R} = x \vec{a}_x + y \vec{a}_y + z \vec{a}_z$ and $\vec{R}_0 = x_0 \vec{a}_x + y_0 \vec{a}_y + z_0 \vec{a}_z$ correspond to the observation point and a reference point on the wavefront of the incident pulsed plane wave component at the initial time $t = t_0$, respectively.

The azimuthal superposition given in Eq. (2b) yields the following closed-form expression for the electric field of the incident pulse

$$\begin{aligned} & E_\phi(\rho, z, t) \\ &= \frac{Z_0 \sin^2 \theta_0}{2\pi c^3} \text{Re} \left(\frac{3j\rho}{\left((\rho \sin \theta_0/c)^2 + ((a_0/c) - j((t - t_0) + (z - z_0) \cos \theta_0/c))^2 \right)^{5/2}} \right. \\ & \quad \left. - \frac{15j\rho((a_0/c) - j((t - t_0) + (z - z_0) \cos \theta_0/c))^2}{\left((\rho \sin \theta_0/c)^2 + ((a_0/c) - j((t - t_0) + (z - z_0) \cos \theta_0/c))^2 \right)^{7/2}} \right). \end{aligned} \quad (2c)$$

The above closed form expression represents a pulse moving in the negative z -direction and is polarized in the ϕ -direction. In Eq. (2a), the polarization vector $\vec{e}_i(\phi_{inc})$ and the propagation direction vector $\vec{s}_i(\phi_{inc})$ of each azimuthal component have been chosen to be equal to

$$\vec{e}_i(\phi_{inc}) = -\sin \phi_{inc} \vec{a}_x + \cos \phi_{inc} \vec{a}_y, \quad (3a)$$

$$\vec{s}_i(\phi_{inc}) = -\cos \phi_{inc} \sin \theta_0 \vec{a}_x - \sin \phi_{inc} \sin \theta_0 \vec{a}_y - \cos \theta_0 \vec{a}_z, \quad (3b)$$

$$\vec{a}_\phi = -\sin \phi_0 \vec{a}_x + \cos \phi_0 \vec{a}_y. \quad (3c)$$

It should be noted that the incident X -wave field given in Eq. (2a) is constructed as an azimuthal angular superposition over pulsed plane waves having the following form:

$$\begin{aligned} & \vec{E}_{inc}(\vec{R}, t; \phi_{inc}) \\ &= \frac{1}{\pi} \text{Re} \left[\int_0^\infty d\omega E_0(\omega) \vec{e}_i(\phi_{inc}) \exp \left(j\omega \left\{ (t - t_0) - (\vec{R} - \vec{R}_0) \cdot \vec{s}_i / c \right\} \right) \right]. \end{aligned} \quad (4)$$

The spectral amplitude of each azimuthally dependent pulsed component has the form $E_0(\omega) \propto \omega^3 \exp(-\omega a_0/c)$. This spectral dependence has the advantage of allowing the derivation of closed form TD-UTD expressions for the diffracted pulsed wave components [cf. Eq. (A2)].

3. X -WAVE SCATTERING BY A PERFECTLY CONDUCTING WEDGE

A well-established technique for solving the problem of diffraction from a perfectly conducting wedge is the UTD. The aim of this section is to use the UTD to study the scattering of the first-order TE X -wave from a perfectly conducting wedge. As indicated earlier, the X -wave is a superposition of pulsed plane waves propagating at a tilted angle θ_0 with respect to the direction of propagation of the peak of the pulse. The results of the UTD method derived for each pulsed plane wave component is superimposed to yield the scattered field for the first order X -wave.

3.1. The Direct Field

If the observation point (x, y, z) and the reference point (x_0, y_0, z_0) of an incident pulsed plane wave component lie on the same side of the plane $z = 0$, the corresponding direct field component is given by Eq. (4). However, if they lie on two different sides of the plane $z = 0$, the intersection point x_t through this plane for each ray of the incident pulsed plane waves is given by

$$x_t = x_0 - \frac{s_{ix}}{s_{iz}} z_0. \quad (5)$$

If the transmission point lies on the positive x -axis, the observation point lies in the shadow region. Consequently, the directly transmitted

part of the total field vanishes. The above conditions can be formulated in a systematic form as follows:

$$\vec{E}_{dir}(\vec{R}, t; \phi_{inc}) = [\mathbf{U}(-x_t)\mathbf{U}(-zz_0) + \mathbf{U}(zz_0)] \vec{E}_{inc}(\vec{R}, t; \phi_{inc}). \quad (6)$$

Here, $\mathbf{U}(x)$ is the unit step function.

Applying the result expressed in Eq. (6) to the azimuthal superposition, given in (2b), the directly transmitted part of the X -wave is given by

$$\begin{aligned} \vec{E}_{\phi dir}(\vec{R}, t) = & \operatorname{Re} \left(\frac{6Z_0 \sin \theta_0}{4\pi^2 c^2} \int_0^{2\pi} d\phi_{inc} \right. \\ & \left. \frac{\cos(\phi_{inc} - \phi_0) [\mathbf{U}(-x_t(\phi_{inc}))\mathbf{U}(-zz_0) + \mathbf{U}(zz_0)]}{\left[(a_0/c) - j \left((t - t_0) - (\vec{R} - \vec{R}_0) \cdot \vec{s}_i(\phi_{inc})/c \right) \right]^4} \right) \end{aligned} \quad (7)$$

In this case, the transmission point x_t is a function of ϕ_{inc} and depends on the propagation direction of the incident ray as given by Eq. (5).

3.2. The Reflected Field

The second part of the total field is the reflected part. The reflection may occur due to the face 0 or face n . Reflection occurs only if the angle between the direction of the incident ray and the normal direction to the face of the wedge is greater than $\pi/2$, i.e.,

$$\vec{n}_v \cdot \vec{s}_i(\phi_{inc}) < 0, \quad (8)$$

where $\vec{n}_v$ is either $\vec{n}_0$ or $\vec{n}_n$. The location of the reflection point is determined by the reflection propagation direction and the location of the observation point. The direction of the reflected ray is given by [19]

$$\vec{s}_{rv}(\phi_{inc}) = \vec{s}_i(\phi_{inc}) - 2(\vec{n}_v \cdot \vec{s}_i(\phi_{inc})) \vec{n}_v. \quad (9)$$

The locations of the reflection points on the two faces, in terms of the location of the observation point (x, y, z) and the direction of the reflected ray, are given by

$$\begin{aligned} & (x_{ref0}, y_{ref0}, z_{ref0}) \\ = & \left(\frac{s_{rz0}x - s_{rx0}z}{s_{rz0} - s_{rx0} \tan \alpha_0}, y - \frac{s_{ry0}}{s_{rx0}} \left(x - \frac{s_{rz0}x - s_{rx0}z}{s_{rz0} - s_{rx0} \tan \alpha_0} \right), \right. \\ & \left. \frac{s_{rz0}x - s_{rx0}z}{s_{rz0} - s_{rx0} \tan \alpha_0} \tan \alpha_0 \right) \end{aligned} \quad (10a)$$

$$\begin{aligned}
 & (x_{refn}, y_{refn}, z_{refn}) \\
 = & \left(\frac{s_{rzn}x - s_{rxn}z}{s_{rzn} - s_{rxn} \tan \alpha_1}, y - \frac{s_{ryn}}{s_{rxn}} \left(x - \frac{s_{rzn}x - s_{rxn}z}{s_{rzn} + s_{rxn} \tan \alpha_1} \right), \right. \\
 & \left. \frac{-(s_{rzn}x - s_{rxn}z)}{s_{rzn} + s_{rxn} \tan \alpha_1} \tan \alpha_1 \right) \quad (10b)
 \end{aligned}$$

For the configuration shown in Fig. 1, a sufficient condition ensuring that a reflection point exists is that $x_{refv} > 0$.

The polarization of the incident field should be decomposed into TE and TM components with respect to the normal direction on the surface. Such polarization directions are determined by the propagation direction of the incident field and the normal direction at the reflection point; specifically [19],

$$\vec{a}_{iTMv}(\phi_{inc}) = \frac{\vec{s}_i(\phi_{inc}) \times (\vec{n}_v \times \vec{s}_i(\phi_{inc}))}{|\vec{s}_i(\phi_{inc}) \times (\vec{n}_v \times \vec{s}_i(\phi_{inc}))|}, \quad (11a)$$

$$\vec{a}_{iTEv}(\phi_{inc}) = \vec{s}_i(\phi_{inc}) \times \vec{a}_{iTMv}(\phi_{inc}) = -\frac{(\vec{n}_v \times \vec{s}_i(\phi_{inc}))}{|(\vec{n}_v \times \vec{s}_i(\phi_{inc}))|}. \quad (11b)$$

Each polarization component is reflected by the corresponding reflection coefficient evaluated in the appropriate reflection direction. The unit vectors for the polarization directions of the reflected TE and TM components are given by [19]

$$\vec{a}_{rTMv}(\phi_{inc}) = \frac{\vec{s}_{rv}(\phi_{inc}) \times (\vec{n}_v \times \vec{s}_{rv}(\phi_{inc}))}{|\vec{s}_{rv}(\phi_{inc}) \times (\vec{n}_v \times \vec{s}_{rv}(\phi_{inc}))|}, \quad (12a)$$

$$\vec{a}_{rTEv}(\phi_{inc}) = \vec{s}_{rv}(\phi_{inc}) \times \vec{a}_{rTMv}(\phi_{inc}) = -\frac{(\vec{n}_v \times \vec{s}_{rv}(\phi_{inc}))}{|(\vec{n}_v \times \vec{s}_{rv}(\phi_{inc}))|}. \quad (12b)$$

Combining the above conditions and noting that none of the above factors is frequency dependent, the reflected X -wave reduces to the azimuthal superposition

$$\begin{aligned}
 & \vec{E}_{ref}(\vec{R}, t) \\
 = & \text{Re} \left\{ \frac{6Z_0 \sin \theta_0}{4\pi^2 c^2} \sum_{v=0, n} \int_0^{2\pi} d(\phi_{inc}) \left(\vec{e}_i(\phi_{inc}) \cdot \begin{bmatrix} \vec{a}_{iTMv}(\phi_{inc}) \\ \vec{a}_{iTEv}(\phi_{inc}) \end{bmatrix} \right)^T \right. \\
 & \times \left(\begin{bmatrix} \Gamma^{TMv} & 0 \\ 0 & \Gamma^{TEv} \end{bmatrix} \begin{bmatrix} \vec{a}_{rTMv}(\phi_{inc}) \\ \vec{a}_{rTEv}(\phi_{inc}) \end{bmatrix} \right) \Bigg\}
 \end{aligned}$$

$$\left. \times \frac{U(x_{refv}(\phi_{inc}))U(-n_v \cdot \vec{s}_i(\phi_{inc}))}{\left[(a_0/c) - j \left((t - t_0) - \left((\vec{R} - \vec{R}_{refv}(\phi_{inc})) \cdot \vec{s}_{rv}(\phi_{inc}) + (\vec{R}_{refv}(\phi_{inc}) - \vec{R}_0) \cdot \vec{s}_i(\phi_{inc}) \right) / c \right) \right]^4} \right\}, \quad (13)$$

where $\Gamma^{TMv} = 1$ and $\Gamma^{TEv} = -1$. The TE and the TM polarization vectors of the incident and the reflected fields, the direction of the reflected rays and the location of the reflection point are all functions of ϕ_{inc} . These quantities can be obtained directly as functions of the propagation direction using Eqs. (8) to (12). One should also note that the two sides of the wedge are planar and the incident ray is a plane wave; as a consequence, the spreading factor of the reflected rays equals unity.

3.3. The Diffracted Field

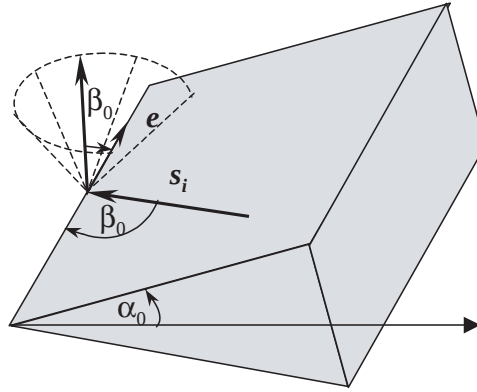
The remaining part of the total field at the observation point is the diffracted field. Fig. 2 shows a schematic diagram for the diffraction due to a wedge. For an incident plane wave component, the diffraction field at a certain observation point originates from a point on the edge of the wedge satisfying the Keller's cone condition. Specifically, the incident and diffracted rays make equal angles with the edge. Since the diffraction point lies on the edge, two of its coordinates are predetermined; namely, $x_d = 0$ and $z_d = 0$. The third coordinate of the diffraction point can be obtained in terms of the observation point and Keller's cone angle β_0 [19]; it is given explicitly as

$$y_d = y - \cot \beta_0 \sqrt{(x - x_d)^2 + (z - z_d)^2} = y - \cot \beta_0 \sqrt{x^2 + z^2}, \quad (14a)$$

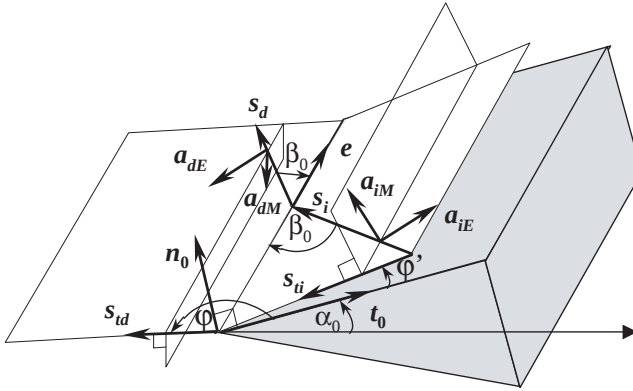
The cosine of Keller's cone angle is determined by the scalar product of the unit vectors representing the direction of the edge $\vec{e}$ and the direction $\vec{s}_i(\phi_{inc})$ of the incident ray; one has, then,

$$\beta_0 = \cos^{-1}(\vec{s}_i(\phi_{inc}) \cdot \vec{e}). \quad (14b)$$

The incident electric field is decomposed into E (soft) and M (hard) polarizations. The E and M components are the electric field components lying on the plane of incidence and on the plane normal to the plane of incidence, respectively, as shown in Fig. 2. The plane of incidence here is determined by the incident ray and the edge direction.



(a)



(b)

Figure 2. The diffraction due to a wedge. (a) Keller diffraction cone, (b) the polarization directions used in the diffraction problem.

The E and M polarization directions are given by

$$\vec{a}_{iM}(\phi_{inc}) = \frac{\vec{e} \times \vec{s}_i(\phi_{inc})}{|\vec{e} \times \vec{s}_i(\phi_{inc})|}, \quad (15a)$$

$$\vec{a}_{iE}(\phi_{inc}) = \vec{s}_i(\phi_{inc}) \times \vec{a}_{iM}(\phi_{inc}). \quad (15b)$$

Each component is diffracted, with a corresponding diffraction coefficient, and is polarized in the appropriate diffraction direction. The E and M polarization directions of the diffracted ray are similarly determined from the direction of the diffracted ray and the direction

of the edge, as illustrated in Fig. 2

$$\vec{a}_{dM}(\phi_{inc}) = \frac{-\vec{e} \times \vec{s}_d(\phi_{inc})}{|\vec{e} \times \vec{s}_d(\phi_{inc})|}, \quad (16a)$$

$$\vec{a}_{dE}(\phi_{inc}) = \vec{s}_d(\phi_{inc}) \times \vec{a}_{dM}(\phi_{inc}). \quad (16b)$$

The diffraction direction $\vec{s}_d$ entering into these expressions is given as

$$\vec{s}_d(\phi_{inc}) = \frac{(\vec{R} - \vec{R}_d(\phi_{inc}))}{|\vec{R} - \vec{R}_d(\phi_{inc})|}. \quad (16c)$$

In the general case of a curved wedge illuminated by a curved wavefront, the diffracted field decays with the distance from the diffraction point in accordance to the spreading factor [18, 19]

$$A(|\vec{R} - \vec{R}_d(\phi_{inc})|) = \sqrt{\frac{\rho_1(\phi_{inc})}{|\vec{R} - \vec{R}_d(\phi_{inc})|(\rho_1(\phi_{inc}) + |\vec{R} - \vec{R}_d(\phi_{inc})|)}}, \quad (17a)$$

where the radius of curvature of the diffracted wavefront ρ_1 can be calculated from the relationship

$$\frac{1}{\rho_1(\phi_{inc})} = \frac{1}{\rho_i} - \frac{\vec{n}_e \cdot (\vec{s}_i(\phi_{inc}) - \vec{s}_d(\phi_{inc}))}{|a_e| \sin^2 \beta_0(\phi_{inc})}. \quad (17b)$$

Here, ρ_i is the radius of curvature of the incident wavefront, a_e is the radius of curvature of the edge at the diffraction point and $\vec{n}_e$ is the direction normal to the edge away from the center of curvature of the edge at the diffraction point [18]. For the case of a plane wave, where $\rho_i = \infty$, and a linear edge, for which $a_e = \infty$, the radius of curvature of the diffracted wavefront is also infinite, i.e., $\rho_i = \infty$. Consequently, the spreading factor reduces to

$$A(|\vec{R} - \vec{R}_d(\phi_{inc})|) = \sqrt{\frac{1}{|\vec{R} - \vec{R}_d(\phi_{inc})|}}. \quad (17c)$$

This is simply the spreading factor of cylindrical waves, i.e., the edge acts as a line source.

Combining the analysis outlined in the above paragraphs, the diffracted field of an incident first-order X -wave can be expressed as

$$\vec{E}_{diff}(\vec{R}, t) = \text{Re} \left\{ \frac{Z_0 \sin \theta_0}{4\pi c^2} \int_0^{2\pi} d\phi_{inc} \frac{1}{\sqrt{|\vec{R} - \vec{R}_d(\phi_{inc})|}} \frac{j\partial^3}{\pi \partial t^3} \right\}$$

$$\begin{aligned}
& \int_0^\infty d\omega \exp(-(\omega/c)a_0) \times \left(\vec{e}_i(\phi_{inc}) \cdot \begin{bmatrix} \vec{a}_{iM}(\phi_{inc}) \\ \vec{a}_{iE}(\phi_{inc}) \end{bmatrix} \right)^T \\
& \times \begin{bmatrix} D^M(\omega, \phi_{inc}) & 0 \\ 0 & D^E(\omega, \phi_{inc}) \end{bmatrix} \begin{bmatrix} \vec{a}_{dM}(\phi_{inc}) \\ \vec{a}_{dE}(\phi_{inc}) \end{bmatrix} \\
& \times \exp \left(j\omega \left((t - t_0) - \left((\vec{R}_d(\phi_{inc}) - \vec{R}_0) \cdot \vec{s}_i(\phi_{inc}) \right. \right. \right. \\
& \left. \left. \left. + (\vec{R} - \vec{R}_d(\phi_{inc})) \cdot \vec{s}_d(\phi_{inc}) \right) / c \right) \right) \} \quad (18)
\end{aligned}$$

Within the framework of the uniform theory of diffraction, the coefficients D^E and D^M can be written as [18, 19]

$$\begin{aligned}
& D^E(\varphi, \varphi', \beta_0; \omega, \phi_{inc}) = \\
& \frac{-1}{2n\sqrt{2\pi \sin \beta_0}} \sum_{\ell=1}^4 F\left((\omega/c) \left| \vec{R} - \vec{R}_d(\phi_{inc}) \right| a_\ell(\varphi, \varphi') \sin^2 \beta_0\right) K_\ell^E, \quad (19a)
\end{aligned}$$

$$\begin{aligned}
& D^M(\varphi, \varphi', \beta_0; \omega, \phi_{inc}) = \\
& \frac{-1}{2n\sqrt{2\pi \sin \beta_0}} \sum_{\ell=1}^4 F\left((\omega/c) \left| \vec{R} - \vec{R}_d(\phi_{inc}) \right| a_\ell(\varphi, \varphi') \sin^2 \beta_0\right) K_\ell^M, \quad (19b)
\end{aligned}$$

where

$$K_1^{E,M} = \cot \left[\frac{\pi + (\varphi - \varphi')}{2n} \right], \quad (20a)$$

$$K_2^{E,M} = \cot \left[\frac{\pi - (\varphi - \varphi')}{2n} \right], \quad (20b)$$

$$K_3^{E,M} = \mp \cot \left[\frac{\pi + (\varphi + \varphi')}{2n} \right], \quad (20c)$$

$$K_4^{E,M} = \mp \cot \left[\frac{\pi - (\varphi + \varphi')}{2n} \right], \quad (20d)$$

and

$$n = \frac{2\pi - (\alpha_0 + \alpha_1)}{\pi}. \quad (20e)$$

Here, $n\pi$ is the angle in free space complementing the wedge angle. The case of $n = 2$ represents a semi-infinite half-plane and the case $n = 1$ represents an infinite half-plane. The functions $a_\ell(\varphi, \varphi')$ are given by

$$a_1(\varphi, \varphi') = a^+(\varphi - \varphi'), \quad (21a)$$

$$a_2(\varphi, \varphi') = a^-(\varphi - \varphi'), \quad (21b)$$

$$a_3(\varphi, \varphi') = a^+(\varphi + \varphi'), \quad (21c)$$

$$a_4(\varphi, \varphi') = a^-(\varphi + \varphi'), \quad (21d)$$

where

$$a^{\pm}(X) = 2 \cos^2 \left(\frac{2n\pi N^{\pm} - X}{2} \right). \quad (21e)$$

$N^{\pm}$ is the integer value that nearly satisfies the condition

$$2n\pi N^{\pm} - X \approx \pm\pi. \quad (21f)$$

The transition function $F((\omega/c)l)$ is given explicitly by [22]

$$\begin{aligned} F((\omega/c)l) &= 2\sqrt{j}l \exp(j\omega l/c) \int_{\sqrt{(\omega/c)l}}^{\infty} \exp(-j\tau^2) d\tau \\ &= \sqrt{\pi}l \exp(j\omega l/c) \operatorname{erfc} \left(\sqrt{j\omega l/c} \right), \end{aligned} \quad (22)$$

in terms of the complementary error function [cf. formula (7.1.1) in Ref. 23]. The angles φ' and φ in Eq. (21) are the inclinations of the incident and the diffracted field directions in the $\vec{n}_0 - \vec{t}_0$ plane measured from the $\vec{t}_0$ direction, as shown in Fig. 2. These angles are given by [19]

$$\varphi' = \pi - \left\{ \pi - \cos^{-1}(-\vec{s}_{ti}(\phi_{inc}) \cdot \vec{t}_0) \right\} \operatorname{sgn}(-\vec{s}_{ti}(\phi_{inc}) \cdot \vec{n}_0), \quad (23a)$$

$$\varphi = \pi - \left\{ \pi - \cos^{-1}(\vec{s}_{td}(\phi_{inc}) \cdot \vec{t}_0) \right\} \operatorname{sgn}(\vec{s}_{td}(\phi_{inc}) \cdot \vec{n}_0), \quad (23b)$$

where $\vec{s}_{ti}$ and $\vec{s}_{td}$, the components of the incident and diffracted wave vectors normal to the edge of the wedge, are equal to

$$\vec{s}_{ti}(\phi_{inc}) = \frac{\vec{s}_i(\phi_{inc}) - (\vec{s}_i(\phi_{inc}) \cdot \vec{e})\vec{e}}{|\vec{s}_i(\phi_{inc}) - (\vec{s}_i(\phi_{inc}) \cdot \vec{e})\vec{e}|}, \quad (24a)$$

$$\vec{s}_{td}(\phi_{inc}) = \frac{\vec{s}_d(\phi_{inc}) - (\vec{s}_d(\phi_{inc}) \cdot \vec{e})\vec{e}}{|\vec{s}_d(\phi_{inc}) - (\vec{s}_d(\phi_{inc}) \cdot \vec{e})\vec{e}|}, \quad (24b)$$

To determine the time domain diffracted field one has to evaluate the inverse Fourier transform in Eq. (18). Unlike the reflection coefficients, the diffraction coefficients are frequency dependent. For this reason, the Fourier inversion cannot be carried out exactly in general. For the case of the first-order X -wave, however, the incident field spectrum multiplied by the diffraction coefficients results in expressions whose inverse Fourier transform can be evaluated in a

closed form. Specifically, the diffracted field is given by

$$\begin{aligned} & \vec{E}_{diff}(\vec{R}, t) \\ &= \frac{Z_0 \sin \theta_0}{4\pi c^2} \text{Re} \left\{ \int_0^{2\pi} d\phi_{inc} \frac{1}{\sqrt{|\vec{R} - \vec{R}_d(\phi_{inc})|}} \times \left(\vec{e}_i(\phi_{inc}) \cdot \begin{bmatrix} \vec{a}_{iM}(\phi_{inc}) \\ \vec{a}_{iE}(\phi_{inc}) \end{bmatrix} \right)^T \right. \\ & \quad \times \left. \begin{bmatrix} -G^M(t, \phi_{inc}) & 0 \\ 0 & -G^E(t, \phi_{inc}) \end{bmatrix} \begin{bmatrix} \vec{a}_{dM}(\phi_{inc}) \\ \vec{a}_{dE}(\phi_{inc}) \end{bmatrix} \right\} \end{aligned} \quad (25a)$$

where

$$G^{TE}(t, \phi_{inc}, \vec{R}) = \frac{-1}{2n\sqrt{2\pi} \sin \beta_0(\phi_{inc})} \sum_{\ell=0}^4 \zeta_{\ell}(t, \phi_{inc}, \vec{R}) K_{\ell}^{TE}(\phi_{inc}), \quad (25b)$$

$$G^{TM}(t, \phi_{inc}, \vec{R}) = \frac{-1}{2n\sqrt{2\pi} \sin \beta_0(\phi_{inc})} \sum_{\ell=0}^4 \zeta_{\ell}(t, \phi_{inc}, \vec{R}) K_{\ell}^{TM}(\phi_{inc}). \quad (25c)$$

The quantity $\zeta_{\ell}(t, \phi_{inc}, \vec{R})$ equals

$$\begin{aligned} \zeta_{\ell}(t, \phi_{inc}, \vec{R}) &= \text{Re} \left\{ \frac{j\partial^3}{\pi\partial t^3} \left(\int_0^{\infty} d\omega F\left((\omega/c) \left| \vec{R} - \vec{R}_d \right| a_{\ell}(\varphi, \varphi') \sin^2 \beta_0\right) \right. \right. \\ & \quad \times \exp \left(-\omega \left((a_0/c) - j \left((t - t_0) - \left((\vec{R}_d - \vec{R}_0) \cdot \vec{s}_i \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. + (\vec{R} - \vec{R}_d) \cdot \vec{s}_d \right) / c \right) \right) \right) \right\} \end{aligned} \quad (25d)$$

This quantity has been evaluated explicitly in the Appendix. The substitution of Eqs. (A3) and (A4) into Eq. (25) yields an expression for the diffracted field that can be easily evaluated numerically by integrating over ϕ_{inc} .

The total field resulting from the diffraction of the X -wave by the perfectly conducting wedge can now be obtained by combining the direct, the reflected and the diffracted fields, *viz.*,

$$\vec{E}(\vec{R}, t) = \vec{E}_{dir}(\vec{R}, t) + \vec{E}_{ref}(\vec{R}, t) + \vec{E}_{diff}(\vec{R}, t). \quad (26)$$

4. NUMERICAL EXAMPLES

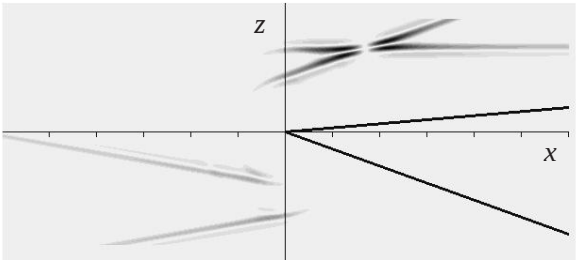
The procedure described above is illustrated numerically in this section, first for the case of a conducting wedge and then for a

conducting disk. The numerical examples are chosen to demonstrate the effectiveness of the azimuthal angular superposition over pulsed plane waves in the evaluation of the total scattered field due to an incident TE X -wave field.

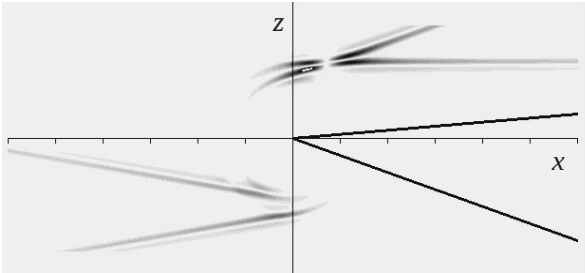
4.1. Scattering of a TE X -wave Incident on a Perfectly Conducting Wedge

In this subsection, we study the diffraction of an incident first-order TE X -wave from a perfectly conducting wedge. The X -wave is characterized by the parameters $a_0 = 10$ cm and $\theta_0 = 10^\circ$. At $t = 0$, the center of the localized X -wave is initially located at $(y_0 = 0, z_0 = 110$ cm) and x_0 varies for the different cases under consideration. Fig. 3 shows the scattered field E_ϕ in the xz plane due to a TE X -wave incident on a perfectly conducting wedge. The incident X -wave is traveling in the negative z direction and the wedge makes angles $\alpha_0 = 5^\circ$ and $\alpha_1 = 20^\circ$ with the xy plane. The different plots, evaluated at $t = 6$ ns correspond to an incident X -wave centered initially ($t = 0$) at $x_0 = 100, 50, 0, -50$, and -100 cm. The plots of the scattered field are based on a sixteen-level gray-scale normalized to the peak value of the incident X -wave. It should be noted that the main localized part is nearly completely reflected in the case that $x_0 = 100$ cm. Also, little distortion is visible near the edge due to the diffracted part of the total field. Two weak arms are transmitted in the left-most part of the figure. The peaks of the diffracted field appear as two light circular wavefronts centered on the edge of the wedge. The diffracted field allows the continuity of the field at the transmission and the shadow boundaries. In Fig. 3b, the initial localization point at $t = 0$ is moved to the point $x_0 = 50$ cm. In this case, the localization region becomes closer to the edge and the diffracted field is more pronounced. Furthermore, the two left arms of the reflected X -wave have changed slightly. In Fig. 3c, the center of the localization region at $t = 0$ is moved to $x_0 = 0$ cm, which is just above the edge. In this case, the X -wave splits into two equal transmitted and reflected parts. The diffracted field is very weak because the null of the field of the TE X -wave, lying at the center of the localization region, passes through the edge. Figs. 3d, 3e display the total field when $x_0 = -50$ cm and $x_0 = -100$ cm, respectively. They show a behavior similar to that illustrated in Figs. 3a and 3b, after interchanging the transmitted and reflected parts of the field.

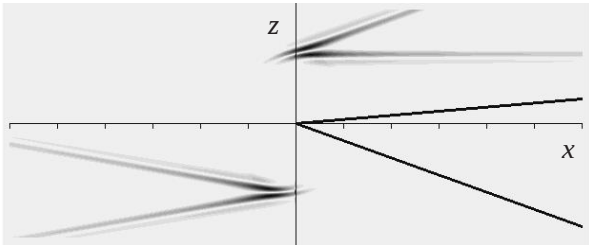
One should note that the reflected X -wave is rotated by an angle of 10° , so that one of the reflected X -wave arms becomes horizontal and the other is reflected at an angle 20° . This is the case because the angle between the two arms of the incident X -wave is equal to



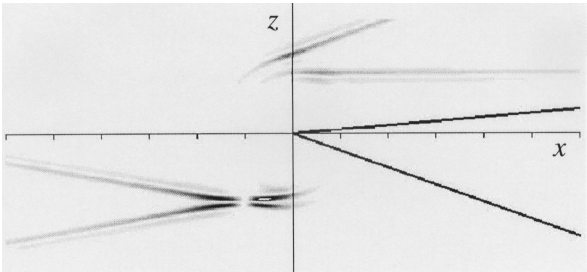
(a) ($x_0 = 100\text{cm}, y_0 = 0, z_0 = 110\text{ cm}$)



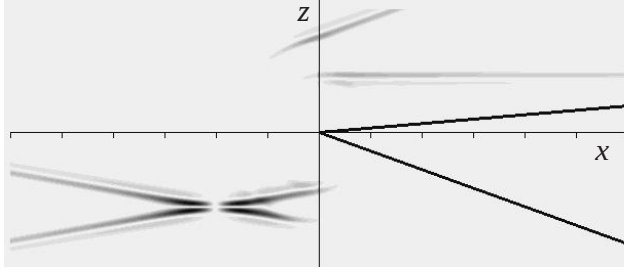
(b) ($x_0 = 50\text{cm}, y_0 = 0, z_0 = 110\text{cm}$)



(c) ($x_0 = 0, y_0 = 0, z_0 = 110\text{ cm}$)



(d) ($x_0 = -50\text{cm}, y_0 = 0, z_0 = 110\text{ cm}$)



(e) ($x_0 = -100$ cm, $y_0 = 0$, $z_0 = 110$ cm)

Figure 3. Total E_ϕ field at $t = 6$ ns due to an X -wave normally incident on a perfectly conducting wedge having $\alpha_0 = 5^\circ$ and $\alpha_1 = 20^\circ$ for different locations of the center of the X -wave at $t = 0$. The parameters of the incident X -wave are $a_0 = 10$ cm and $\theta_0 = 10^\circ$.

10° . The transmitted X -wave is not affected by the angular tilts, α_0 and α_1 , in the surface of the wedge and continues to propagate in the negative z direction. One should also observe that the peaks of the displayed xz section of the diffracted field form two concentric circles. The difference in the time of arrival of these two peaks is a function of the distance x_0 from the edge of the wedge and the angle θ_0 characterizing the incident X -wave. This behavior could allow the use of X -waves for precise determination of the position of the edge of a conducting object. A straightforward scheme is to vary the transverse position of a sequence of incident X -waves while monitoring the arrival time of the peaks of the diffracted fields. In this manner, the initial transverse location of the X -wave yielding the least difference in the measured times of arrival of the diffracted peaks indicates the position of the edge.

4.2. Scattering of a TE X -wave Incident on a Perfectly Conducting Circular Disk

In this subsection, the diffraction of a first-order TE X -wave from a circular perfectly conducting disk is considered. Similarly to the case of diffraction from the wedge, the X -wave is decomposed into a superposition of oblique pulsed plane waves and the diffraction from each pulsed plane wave is calculated. Subsequently, the total diffracted wave is obtained as an azimuthal superposition of the elementary diffracted waves associated with the various pulsed plane-wave components. Since the edge of the disk is curved, the general

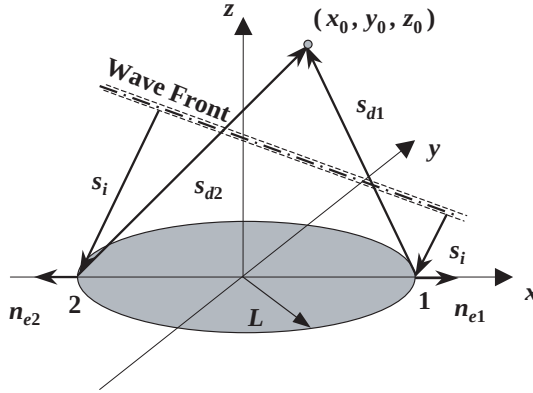


Figure 4. Principal points of diffraction from a circular disk.

spreading factor given in Eq. (17) is used. Specifically [18, 19],

$$A(|\vec{R} - \vec{R}_d|) = \sqrt{\frac{\rho_1}{|\vec{R} - \vec{R}_d|(\rho_1 + |\vec{R} - \vec{R}_d|)}}, \quad (27a)$$

where ρ_1 is given by

$$\frac{1}{\rho_1} = \frac{1}{\rho_i} - \frac{\vec{n}_e \cdot (\vec{s}_i(\phi_{inc}) - \vec{s}_d(\phi_{inc}))}{|a_e| \sin^2 \beta_0}. \quad (27b)$$

Here, ρ_i is the radius of curvature of the incident wavefront, a_e is the radius of curvature of the edge at the diffraction point and $\vec{n}_e$ is the unit normal vector to the edge at the diffraction point directed away from the center of curvature, as shown in Fig. 4. For an incident plane wave component and a circular edge, one obtains

$$\rho_1 = -L \sin^2 \beta_0 / (\vec{n}_e \cdot (\vec{s}_i(\phi_{inc}) - \vec{s}_d(\phi_{inc}))), \quad (27c)$$

where L is the radius of the circular disk and ϕ_{inc} is the azimuthal angle associated with each pulsed plane wave component of the incident X -wave.

In the UTD, the diffraction point illuminating a certain observation point should satisfy the Keller cone condition. Consider a circular disk, centered at the origin, illuminated by a plane wave with a propagation vector $\vec{s}_i$ parallel to the x - z plane. In this case, the main diffracted rays satisfying Keller's diffraction cone lie on the x - z

plane due to the diffraction from only the two edge points on the x -axis as shown in Fig. 4 [24, 25]. The diffracted rays from other points on the circumference of the edge do not satisfy Keller's diffraction cone on the x - z plane, except along the caustic line arising when the denominator of the spreading factor of Eq. (27a) becomes zero, i.e., at $\rho_1 = -|\vec{R} - \vec{R}_d|$. Otherwise, points not lying on the x -axis diffract the field in other directions. At the caustic line, many diffracted rays intersect causing high intensity of the field. For the case of a plane wave normally incident on a circular disc, the caustic line coincides with the axis of the disk [24]. Higher order diffraction occurs through multiple diffractions and creeping waves along the surface of the disk [25–27]. However, such multiple diffractions have appreciable effect in situations involving diffraction in the grazing plane of the disk, a situation outside the scope of the present study. For the diffraction from the principal diffraction points, the angle of incidence with respect to the edge β_0 is equal to 90° . Consequently, ρ_1 as a function of $\vec{s}_i$ and $\vec{s}_d$ is given by

$$\rho_1 = -\frac{L}{\vec{n}_e \cdot (\vec{s}_i(\phi_{inc}) - \vec{s}_d(\phi_{inc}))}. \quad (28)$$

Thus, for each pulsed plane wave component, the caustic is located at $|\vec{R} - \vec{R}_d| = -\rho_1 = L/(\vec{n}_e \cdot (\vec{s}_i(\phi_{inc}) - \vec{s}_d(\phi_{inc})))$.

Fig. 5 shows the ϕ component of the total field on the x - z plane due to a TE X -wave normally incident on a circular disk of radius $L = 50$ cm, with its center lying at the origin ($x = 0$, $y = 0$, $z = 0$). The field is plotted at $t = 6$ ns. The initial localization point at $t = 0$ is $y_0 = 0$ and $z_0 = 110$ cm. The various plots show the diffracted field due to an X -wave the center coordinate x_0 of which is varied from 0 to 100 cm. The parameters of the incident X -wave are $a_0 = 10$ cm and $\theta_0 = 10^\circ$. The total scattered field for $x_0 = 0$ is shown in Fig. 5a. It is seen that the reflected field is a deformed image of the incident X -wave. In particular, the front two arms of the reflected X -wave are shorter than the rear ones. The reflected wave consists of a set of obliquely traveling wavefronts of finite circular extensions determined by the finite size of the circular disk. As these oblique wavefronts of finite extensions move away from the disk, their intersection acquires the asymmetric form of the reflected X -wave.

On the x - z plane, the reflected waves due to incidence at all angles contribute to reflection, whereas the diffracted field is mainly due to plane wave components incident at angles $\phi_{inc} = 0^\circ$ and $\phi_{inc} = 180^\circ$. The remaining plane wave components of the normally incident X -wave do not satisfy Keller's cone condition on the x - z plane, except along the caustic lines. This explains the weak contribution of the diffracted part of the field compared to the total field. The transmitted

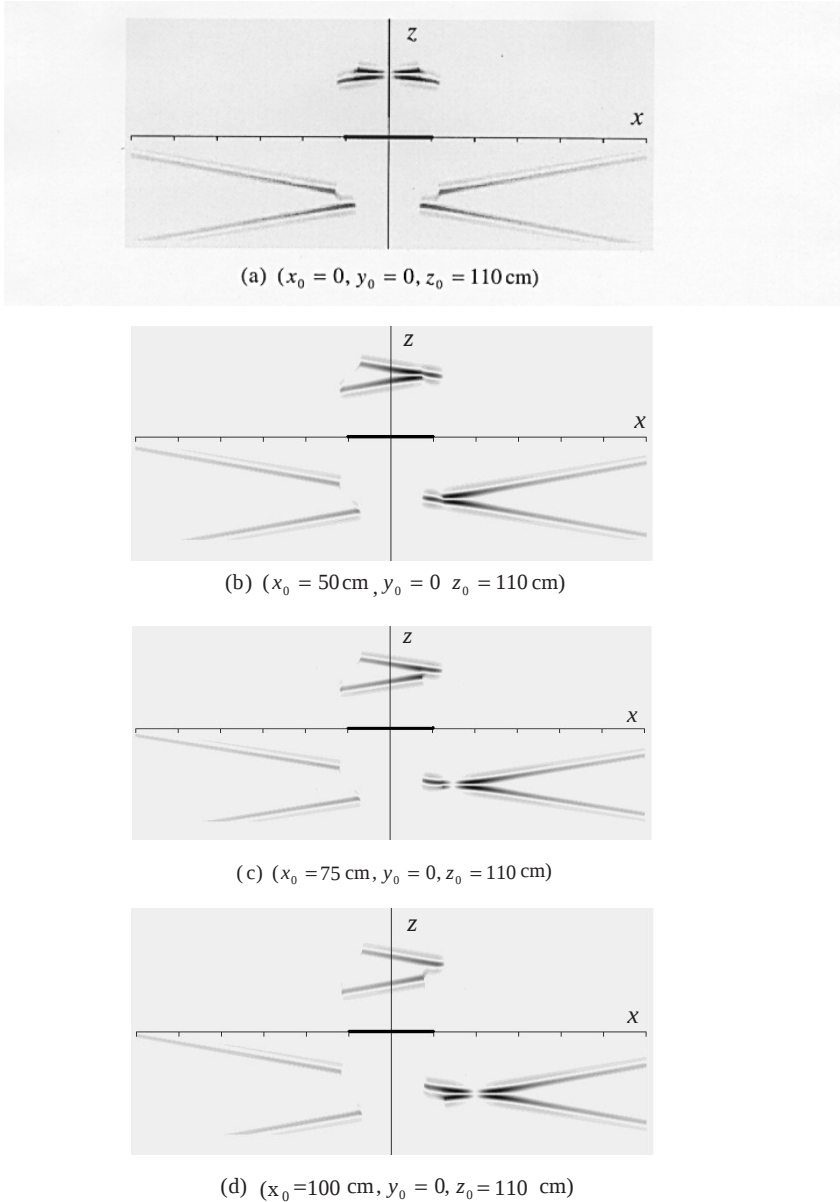


Figure 5. The ϕ component of the total field on the x - z plane for a normally incident TE X -wave on a perfectly conducting circular disk of radius $L = 50 \text{ cm}$ at time $t = 6 \text{ ns}$ for different reference points at $t = 10 \text{ cm}$. The parameters of the incident X -wave are $a_0 = 10 \text{ cm}$ and $\theta_0 = 10^\circ$.

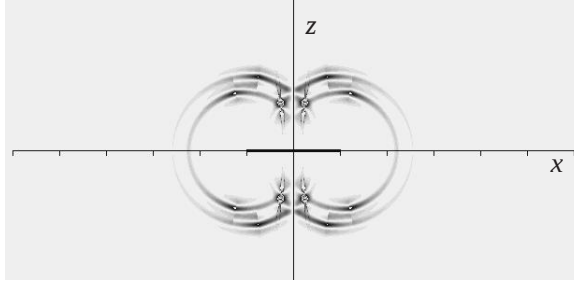


Figure 6. The ϕ component of the diffracted field corresponding to Fig. 5a, normalized to the peak value of the incident X -wave and multiplied by a factor of 80.

part of the field appears to be the complementary part of the reflected field; when fitted together they form the incident X -wave. Fig. 5b shows the total field when $x_0 = 50$ cm at $t = 6$ ns. In this case, the center of the localization region is just above the edge of the disk. One should note that the localization region is not symmetrically split into two equal reflected and transmitted parts as in the case of the infinite wedge [cf. Fig. 3c]. This is due to the curvature of the edge that causes a reduction of the diffracted part of the X -wave. Fig. 5c shows the total field when $x_0 = 75$ cm at $t = 0$. In this case, the center of the localization region is situated out of the edge of the disk. Thus, the arms of the reflected waves do not intersect and the localization region is formed in the transmission region. In Fig. 5d, the center of the localization region is located at $x_0 = 100$ cm. Now, most of the localization region passes undisturbed into the transmission region. However, a slight deformation is observed in the front arm of the transmitted X -wave that lies closer to the edge.

To acquire a better view of the caustics, we provide in Fig. 6 the diffracted field due to the edge of the disk, when the localization point $x_0 = 0$ cm. This case corresponds to the situation considered in Fig. 5a. To improve the visibility of the plots of the diffracted field, the amplitude is multiplied by a factor of eighty and normalized to the peak value of the incident X -wave. The caustics here are not situated on the axis of rotation of the disk because the incident pulsed plane wave components of the X -wave are obliquely incident with respect to the axis of rotation.

The behavior of the reflected and transmitted parts of the X -wave as they move away from the scattering circular disk is illustrated in Fig. 7. For parameters identical to those used for Fig. 5a, the total field is plotted at $t = 8$ and 10 ns. These two cases correspond to

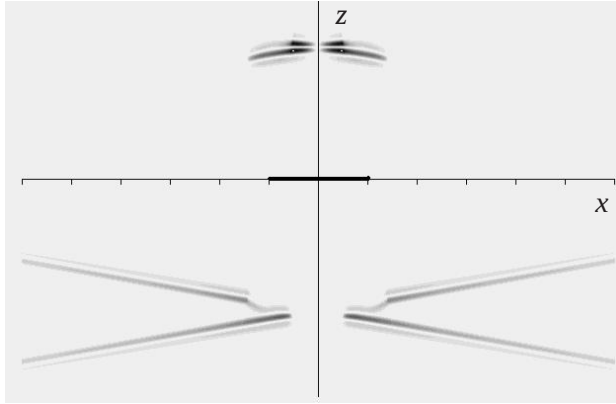
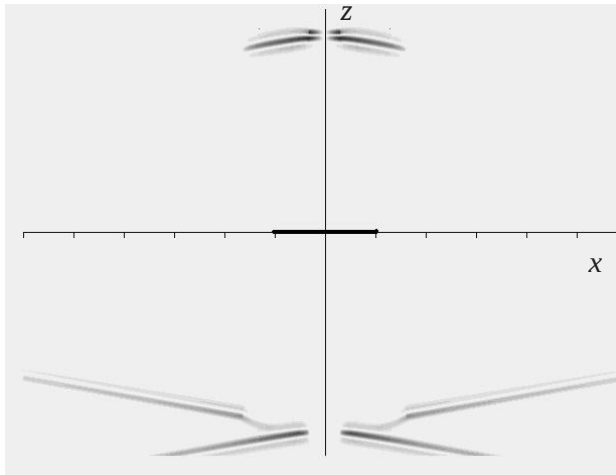
(a) $t = 8$ ns(b) $t = 10$ ns.

Figure 7. Total E_ϕ field of a normally incident TE X -wave on a perfectly conducting circular disk corresponding to Fig. 5a at later times.

the time instants occurring just before the two arms of the scattered X -wave separate from each other. This time is estimated as $t_d = (z_0 + z_d)/(c/\cos\theta_0)$, where $z_d = L/\tan\theta_0$ is the diffraction length of an X -wave associated with a radiator of radius L [3, 28]. For the parameters used in our numerical example, $t_d = 12.92$ ns. A comparison of Figs. 7a and 7b with Fig. 5a, shows that the two arms constituting the localization region of the reflected field separate from

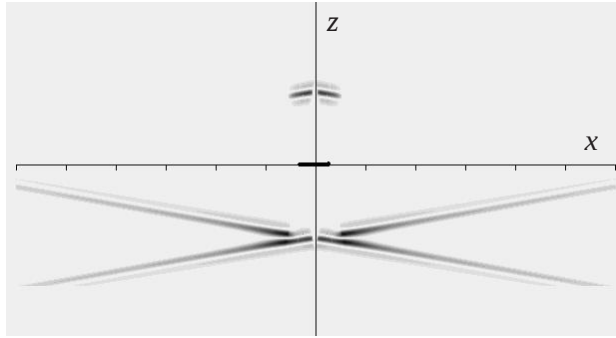
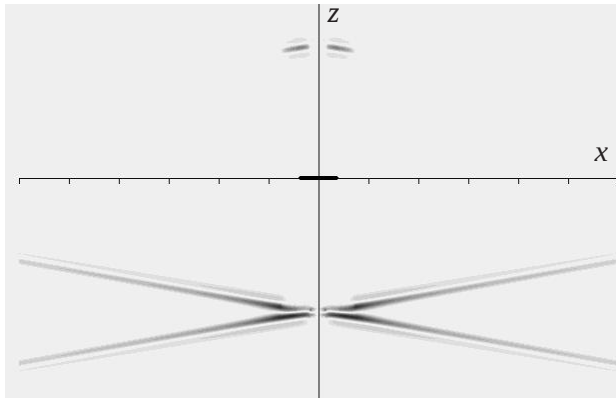
(a) $t = 6$ ns.(b) $t = 8$ ns.

Figure 8. Total E_ϕ field of a normally incident TE X -wave on a perfectly conducting circular disk of radius $L = 15$ cm at different observation times. The reference point at $t = 0$ is $(x_0 = 0, y_0 = 0, z_0 = 110$ cm). The parameters of the incident X -wave are $a_0 = 10$ cm and $\theta_0 = 10^\circ$.

each other as the reflected field moves away from the disk. This separation shortens the intense front arms with respect to the rear ones. As the reflected field moves away from the disk, the shortening increases as shown in Fig. 7b. At farther distances from the disk, the two arms of the reflected X -wave will not intersect with each other. Thus, the localization region starts to disappear from the reflected field. On the other hand, as the transmitted field moves away from the scattering disk, the arms of the transmitted X -wave start to get closer together forming a localization region below the disk. The reason for

this behavior is that the localization in the transmission region far from the disk is generated by the farthest parts of the arms of the incident X -wave that are essentially unaffected by the presence of the disk.

The separation of the reflected X -wave arms can become clearer by considering a scattering disk of a smaller radius, as shown in Fig. 8. In this case, the radius of the disk is decreased to 15 cm and the field is plotted at times $t = 6$ and 8 ns. The diffraction length of the scattered X -wave is approximately equal to 85 cm. The total path length from the initial localization center to the diffraction limit equals $z_0 + z_d = 195$ cm. Consequently, the time needed for the wave to arrive at the diffraction limit is nearly 6.5 ns. Figs. 8a, 8b show the total field for the cases when the wave is nearly at the diffraction length and beyond it. At $t = 6$ ns, the reflected wave fronts have a small radius and the arms of the X -wave are slightly separated, as shown in Fig. 8a. The separation between the two arms increases in Fig. 8b, and there is no localization region because the scattered X -wave in this case exceeds its diffraction length. On the other hand, the localization behavior appears clearly in the transmission region and is less deformed as the transmitted X -wave moves away from the circular disk.

Finally, we address the issue of the range of validity of the approximation used to calculate the X -wave field scattered from a perfectly conducting circular disk. An estimate of such a range can be deduced from the analogy between the reflecting circular disk and a radiator of the same size. It has been argued that a circular aperture of radius L can be used to reconstruct the original shape of a pulsed plane wave within the range [9, 29]

$$R_{0\max} \leq \frac{L^2 - (c\tau)^2}{2(c\tau)}, \quad (29)$$

where $R_{0\max}$ is the maximum normal distance from the center of the aperture and $c\tau$ is the axial width of the initial pulsed plane wave excitation. Within the range $R_{0\max}$, a replica of the wave is generated. The same concept applies to the specularly reflected wave of the UTD solution. At distances larger than $R_{0\max}$, the total wavefront reduces gradually to the first derivative of the original pulse. The spatial width (distance between the front and rear nulls) of the pulsed plane wave components of the first order TE X -wave may be taken to be nearly equal to $0.8a_0$. For such a width and a circular disk illuminated by a first order TE X -wave, it can be shown that the maximum range of validity of the UTD solution, as a specularly reflected replica of the

incident wave, is equal to

$$R_{0\max} = \frac{L^2 - (0.8a_0)^2}{2(0.8a_0)}. \quad (30)$$

For $a_0 = 10$ cm and a circular disk of radius $L = 50$ cm, illustrated in Figs. 5 and 7, the UTD solution may be valid up to 152.25 cm from the disk. At a larger distance, the reflected wave should approach the form of the first derivative of the incident pulse. In Figs. 5 and 7, the initial localization center is located at $z_0 = 110$ cm; therefore, the total path becomes 252.25 cm. This corresponds to a time equal to 8.61 ns. Thus, the result exhibited in Fig. 7b may be less accurate than the ones shown in Figs. 5 and 7a. On the other hand, the UTD solution is valid up to only 10 cm from the disk for the case of the circular disk considered in Fig. 8. For the same X -wave and the same initial localization center, the maximum time for the UTD solution in this case is nearly 3.94 ns. Thus, the approximate method used in arriving at the results of Fig. 8 may need more modification.

5. CONCLUDING REMARKS

In this paper, the diffraction and scattering of a TE X -wave by conducting bodies has been investigated using the time-domain, uniform theory of diffraction (TD-UTD) method. The analysis used in this work has been based on the pulsed plane wave representation of an X -wave introduced in Refs. [10] and [11]. This allows the calculation of the diffraction and scattering of each pulsed plane wave component of the incident X -wave at the observation point. The superposition of the individual diffracted and scattered pulsed plane wave components yields the diffracted and scattered field due to an incident X -wave.

The TD-UTD has first been applied to the case of an infinite perfectly conducting wedge. The scattered field is divided into three parts; namely, the transmitted, the reflected and the diffracted parts. The diffracted part of the field is characterized by its cylindrical wavefront. The total field at the transmission and reflection shadow boundaries satisfies the continuity conditions. After studying the canonical problem of an infinite wedge, the TD-UTD has been applied to a perfectly conducting circular disk, as an example of a finite scattering structure. This problem has been studied for a normally incident X -wave. It has been shown that the diffracted part of the total field is nearly negligible compared to the transmitted and reflected parts of the total field. This behavior is due to the fact that only incrementally small parts of the total X -wave satisfy Keller's cone condition for the circular edge of the disk. It has also been found that

the localization region in the reflected part of the field is formed only when the center of the localization region of the incident X -wave is close to the center of the scattering disk. The localization region in the reflected part vanishes as the reflected part moved away from the circular disk because the two finite arms of the reflected X -wave do not intersect beyond a certain distance. The TD-UTD solution has been shown to be incapable of predicting the correct behavior of the backscattered field beyond a certain distance from a finite scattering structure. Thus, the domain of applicability of the present approach is limited within the distance given in Eq. (30).

In conclusion, it has been shown that the TD-UTD can be used to predict the behavior of the scattered X -wave within a certain range. When combined with the pulsed plane wave representation of X -waves, this approach is a time-efficient technique for the calculation of the scattered field. On the other hand, the limitations of the TD-UTD for the case of a finite scatterer have been pointed out; specifically, the formation of caustics and the limited range of applicability. One way to alleviate these problems is to use other asymptotic techniques, e.g., the physical theory of diffraction or the incremental theory of diffraction [30–36].

We are primarily interested in applying X -waves for the detection and identification of buried objects. Towards this goal, we hope that the method introduced in this paper together with previous work on the transmission and reflection of X -waves from semi-infinite media [5–8] provide the basic tools for studying the scattering of X -waves from buried structures.

APPENDIX A.

The quantity $\zeta_\ell(t, \phi_{inc}, \vec{R})$ in Eq. (25d) can be written as

$$\zeta_\ell(t, \phi_{inc}, \vec{R}) = \text{Re} \left(\frac{j\partial^3}{\pi\partial^3} I_\ell(t, \phi_{inc}, \vec{R}) \right), \quad (\text{A1a})$$

where

$$\begin{aligned} I_\ell(t, \phi_{inc}) = & \int_0^\infty d\omega \quad F \left((\omega/c) |\vec{R} - \vec{R}_d| a_\ell(\varphi, \varphi') \sin^2 \beta_0 \right) \\ & \times \exp \left(-\omega \left((a_0/c) - j \left((t - t_0) - ((\vec{R}_d - \vec{R}_0) \cdot \vec{s}_i \right. \right. \right. \\ & \left. \left. \left. + (\vec{R} - \vec{R}_d) \cdot \vec{s}_d \right) / c \right) \right) \right); \end{aligned} \quad (\text{A1b})$$

$$\begin{aligned}
F((\omega/c)l) &= 2\sqrt{j\bar{l}} \exp(j\omega l/c) \int_{\sqrt{(\omega/c)l}}^{\infty} \exp(-j\tau^2) d\tau \\
&= \sqrt{\pi\bar{l}} \exp(j\omega l/c) \operatorname{erfc}\left(\sqrt{j\omega l/c}\right), \tag{A1c}
\end{aligned}$$

The integration in Eq. (A1b) can be performed using the identity [22]

$$\int_0^{\infty} e^{\omega q} \operatorname{erfc}\left(\sqrt{\omega q}\right) e^{-\omega p} d\omega = \frac{1}{\sqrt{p}\left(\sqrt{p} + \sqrt{q}\right)}, \tag{A2}$$

where $-\pi < \arg(p) \leq \pi$ and $-\pi < \arg(q) \leq \pi$. As a result, one has

$$I_{\ell}(t, \phi_{inc}) = \frac{\sqrt{-j\pi c q_{\ell}(\phi_{inc}, \vec{R})}}{\sqrt{p(t, \phi_{inc}, \vec{R})} \left(\sqrt{p(t, \phi_{inc}, \vec{R})} + \sqrt{q_{\ell}(\phi_{inc}, \vec{R})} \right)}, \tag{A3a}$$

where

$$\begin{aligned}
p(t, \phi_{inc}, \vec{R}) &= (a_0/c) - j \left((t - t_0) - \left((\vec{R}_d(\phi_{inc}) - \vec{R}_0) \cdot \vec{s}_i(\phi_{inc}) \right. \right. \\
&\quad \left. \left. + (\vec{R} - \vec{R}_d(\phi_{inc})) \cdot \vec{s}_d(\phi_{inc}) \right) / c \right), \tag{A3b}
\end{aligned}$$

$$q_{\ell}(\phi_{inc}, \vec{R}) = j \left(\left| \vec{R} - \vec{R}_d(\phi_{inc}) \right| a_{\ell}(\varphi(\phi_{inc}), \varphi'(\phi_{inc})) \sin^2 \beta_0(\phi_{inc}) \right) / c. \tag{A3c}$$

Substituting in Eq. (A1a), the quantity $\zeta_{\ell}(t, \phi_{inc}, \vec{R})$ acquires the form

$$\begin{aligned}
&\zeta_{\ell}(t, \phi_{inc}, \vec{R}) \\
&= \operatorname{Re} \left(\frac{j\partial^3}{\pi \partial t^3} \frac{\sqrt{-j\pi c q_{\ell}(\phi_{inc}, \vec{R})}}{\sqrt{p(t, \phi_{inc}, \vec{R})} \left[\sqrt{p(t, \phi_{inc}, \vec{R})} + \sqrt{q_{\ell}(\phi_{inc}, \vec{R})} \right]} \right), \tag{A4a}
\end{aligned}$$

or, more explicitly,

$$\begin{aligned}
&\zeta_{\ell}(t, \phi_{inc}, \vec{R}) \\
&= \frac{1}{\pi} \operatorname{Re} \left(\frac{15}{8} \frac{\sqrt{-j\pi c q_{\ell}(\phi_{inc}, \vec{R})}}{\left(p(t, \phi_{inc}, \vec{R}) \right)^{7/2} \left[\sqrt{p(t, \phi_{inc}, \vec{R})} + \sqrt{q_{\ell}(\phi_{inc}, \vec{R})} \right]} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{15}{8} \frac{\sqrt{-j\pi c q \ell(\phi_{inc}, \vec{R})}}{\left(p(t, \phi_{inc}, \vec{R})\right)^3 \left[\sqrt{p(t, \phi_{inc}, \vec{R})} + \sqrt{q \ell(\phi_{inc}, \vec{R})}\right]^2} \\
& + \frac{3}{2} \frac{\sqrt{-j\pi c q \ell(\phi_{inc}, \vec{R})}}{\left(p(t, \phi_{inc}, \vec{R})\right)^{5/2} \left[\sqrt{p(t, \phi_{inc}, \vec{R})} + \sqrt{q \ell(\phi_{inc}, \vec{R})}\right]^3} \\
& + \frac{3}{4} \frac{\sqrt{-j\pi c q \ell(\phi_{inc}, \vec{R})}}{\left(p(t, \phi_{inc}, \vec{R})\right)^2 \left[\sqrt{p(t, \phi_{inc}, \vec{R})} + \sqrt{q \ell(\phi_{inc}, \vec{R})}\right]^4} \Bigg) . \quad (A4b)
\end{aligned}$$

REFERENCES

1. Lu, J. and J. F. Greenleaf, "Nondiffracting X -waves—Exact solutions to free-space scalar wave equation and their finite aperture realization," *IEEE Trans. on Ultrason., Ferroelect. Freq. Contr.*, Vol. 39, 19–31, 1992.
2. Ziolkowski, R. W, "Localized transmission of electromagnetic energy," *Phys. Rev. A*, Vol. 39, 2005–2033, 1989.
3. Shaarawi, A. M., "Comparison of two localized wave fields generated from dynamic apertures," *J. Opt. Soc. Am. A*, Vol. 14, 1804–1816, 1997.
4. Shaarawi, A. M., I. M. Besieris, R. W Ziolkowski, and S. M. Sedky, "Generation of approximate focus-wave-mode pulses from wide-band dynamic Gaussian apertures," *J. Opt. Soc. Am. A*, Vol. 12, 1954–1964, 1995.
5. Shaarawi, A. M., I. M. Besieris, A. M. Attiya, and E. A. El-Diwany, "Acoustic X -wave reflection and transmission at a planar interface: spectral analysis," *J. Acoust. Soc. Am.*, Vol. 107, 70–86, 2000.
6. Attiya, A. M., E. A. El-Diwany, A. M. Shaarawi, and I. M. Besieris, "Reflection and transmission of X -waves in the presence of planarly layered media: the pulsed plane wave representation," *Progress in Electromagnetics Research (PIER)*, Vol. 30, 191–211, 2000.
7. Shaarawi, A. M., I. M. Besieris, A. M. Attiya, and E. A. El-Diwany, "Reflection and transmission of an electromagnetic X -wave incident on a planar air-dielectric interface: spectral

- analysis," *Progress in Electromagnetics Research (PIER)*, Vol. 30, 213–250, 2000.
8. Attiya, A. M., E. A. El-Diwany, and A. M. Shaarawi, "Transmission and reflection of TE electromagnetic X -wave normally incident on a lossy dispersive half-space," *Proceedings of the 17th National Radio Science Conference, NRSC'2000*, B11.1–12, Minufiya University, Egypt, 2000.
 9. Attiya, A. M., "Transverse (TE) electromagnetic X -waves: propagation, scattering, diffraction and generation problems," Ph.D. Thesis, Cairo University, May 2001.
 10. Saari, P. and K. Reivelt, "Evidence of X -shaped propagation-invariant localized light waves," *Phys. Rev. Lett.*, Vol. 79, 4135–4138, 1997.
 11. Saari, P. and H. S. Onajalg, "Pulsed Bessel beams," *Laser Phys.*, Vol. 7, 32–39, 1997.
 12. Donnelly, R. and D. Power, "The behavior of electromagnetic localized waves at a planar interface," *IEEE Trans. Antennas Propagat.*, Vol. 45, 580–591, 1997.
 13. Hillion, P., "How do focus wave modes propagate across a discontinuity in a medium?" *Optik*, 93, 67–72, 1993.
 14. Power, D., R. Donnelly, and MacIsaac, "Spherical scattering of superpositions of localized waves," *Phys. Rev., E*, Vol. 48, 1410–1417, 1993.
 15. Hillion, P., "Diffraction of time-dependent electromagnetic pulse by a conducting plane screen," *Opt. Comm.*, Vol. 102, 199–204, 1993.
 16. Hillion, P., "Diffraction of focus wave modes at a perfectly conducting screen," *Opt. Comm.*, Vol. 123, 215–224, 1996.
 17. J. Fagerholm, A. T. Friberg, J. Huttunen, D. P. Morgan, and M. M. Salomaa, "Angular-spectrum representation of nondiffracting X -waves," *Phys. Rev. E*, Vol. 54, 4347–4352, 1996.
 18. Kouyoumjian, R. G. and P. H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. IEEE*, Vol. 62, 1448–1461, 1974.
 19. McNamara, D. M., C. W. I. Pistorius, and J. A. G. Malherba, "Introduction to the uniform geometrical theory of diffraction," Artech House, Boston, 1990.
 20. Keller, J. B., "Geometrical theory of diffraction," *J. Opt. Soc. Am.*, Vol. 52, 116–130, 1962.
 21. Michaeli, A., "Equivalent edge currents for arbitrary aspects of observation," *IEEE Trans. Antennas and Prop.*, Vol. AP-32, 252–

- 258, 1984.
22. Rousseau, P. R. and P. H. Pathak, "Time-domain uniform geometrical theory of diffraction for a curved wedge," *IEEE Trans. Antennas and Prop.*, Vol. AP-43, 1375–1382, 1995.
 23. Abramowitz, M. and I. A. Stegun, *Handbook of Mathematical Functions*, Dover Publications, New York, 1965.
 24. Knott E. and T. Senior, "Comparison of three high-frequency diffraction techniques," *Proc. IEEE*, Vol. 62, 1468–1474, 1974.
 25. Marsland, D. P., C. A. Balanis, and S. A. Brumley, "Higher order diffractions from a circular disk," *IEEE Trans. Antennas and Prop.*, Vol. AP-35, 1436–1444, 1987.
 26. Shirai, H. and L. B. Felsen, "Modified GTD for generating complex resonances for flat strips and disks," *IEEE Trans. Antennas and Prop.*, Vol. AP-34, 779–790, 1986.
 27. Dominek, A. K., "Transient scattering analysis for a circular disk," *IEEE Trans. Antennas and Prop.*, Vol. AP-39, 815–819, 1991.
 28. Lu J. and J. F. Greenleaf, "Experimental verification of nondiffracting X -waves," *IEEE Trans. on Ultrason., Ferroelect., Freq. Contr.*, Vol. 39, 441–446, 1992.
 29. Attiya, A. M., E. A. El-Diwany, and A. M. Shaarawi, "Generation of an approximate TE electromagnetic X -wave in free space using Huygen's aperture," *Proceedings of the 16th National Radio Science Conference, NRSC'99*, B3.1–11, Ain Shams University, Cairo, Egypt, 1999.
 30. Michaeli, A., "Equivalent edge currents for arbitrary aspects of observation," *IEEE Trans. Antennas and Prop.*, Vol. AP-32, 252–258, 1984.
 31. Michaeli, A., "Elimination of infinities in equivalent edge currents, part I: fringe current components," *IEEE Trans. Antennas and Prop.*, Vol. AP-34, 912–918, 1986.
 32. Knott, E., "The relationship between Mitzner's ILDC and Michaeli's equivalent currents," *IEEE Trans. Antennas and Prop.*, Vol. AP-33, 112–114, 1985.
 33. Johansen, P., "Uniform physical theory of diffraction equivalent edge currents for truncated wedge strips," *IEEE Trans. Antennas and Prop.*, Vol. AP-44, 989–995, 1996.
 34. Johansen, P., "Time-domain version of the physical theory of diffraction," *IEEE Trans. Antennas and Prop.*, Vol. AP-47, 261–270, 1999.
 35. Tiberio, R. and S. Maci, "An incremental theory of diffraction: scalar formulation," *IEEE Trans. Antennas and Prop.*, Vol. AP-

- 42, 600–611, 1994.
36. Tiberio, R., S. Maci, and A. Toccafondi, “An incremental theory of diffraction: electromagnetic formulation,” *IEEE Trans. Antennas and Prop.*, Vol. AP-43, 87–96, 1995.