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HOMOGENIZATION OF AN ARRAY OF S-SHAPED PARTICLES LOCATED ON A DIELECTRIC INTERFACE

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Abstract—An analytical model of a grid composed of small S-shaped conducting particles located on the surface of a dielectric slab is presented. This approach replaces the original one-layer structure with metallic particles printed on the interface by a multilayered structure with homogenized permittivities for each layers. This way one can homogenize the arrays of small resonant particles. The analytical model is verified by numerical simulations for the case of normal incidence of the plane wave. The homogenization is possible due to the small sizes of S-particles compared to the resonant wavelength in the substrate and due to the small thickness of the whole structure.

5 Conclusion

References

1. INTRODUCTION

In [1, 2], the authors presented the study of coplanar waveguide isolators (CPWI) containing a magnetized ferrite slab. In order to improve the performance of the CPWI, it was proposed to add to the structure a thin layer of dielectric with high permittivity. Without this layer, the spatial distribution of the field of the guided mode gives a weak interaction between the field and the ferrite. The addition of a dielectric layer with $\epsilon_{eff} \gg 1$ under the ferrite layer leads to the re-distribution of the field, so that the influence of the ferrite becomes dramatically higher. Finally the performance of the CPWI is improved. The use of a very thin piezoelectric layer for this purpose (as it was suggested in [1]) is possible but very expensive. On account of this problem, we come to the idea of a small artificial dielectric layer (composite) instead of a piezoelectric one.

Artificial dielectrics have been known for a long time (see e.g., [3] and [4]). Conventional artificial dielectric materials are obtained with simple inclusions like disks, spheres, ellipsoids or needles in a dielectric matrix. These inclusions (that we will often name "particles") have small sizes compared to the wavelength. The distance between them is also small enough with respect to λ. Such materials can be represented by an effective permittivity ϵ_{eff}. Generally this effective permittivity can reach values of 4–10 within a wide frequency band. In some applications, however, one needs to obtain higher values of ϵ_{eff}, at least within a narrow frequency band. A solution to this problem is to prepare *resonant* artificial dielectrics.

At ultrashort waves resonant dielectric composites can be prepared with interrupted wires [5]. For microwave applications planar technology is preferable. One way to design artificial dielectrics at microwave frequencies is to put conducting strips with complex shape on a dielectric interface. In many works the problem of *self-resonant grids* (meshes) was studied. For example, in [6] a grid composed of Jerusalem crosses has been presented. However, such grids used as artificial dielectrics have a serious disadvantage. For self-resonant grids the cell size is close to $\lambda/4$ [6]. This makes the size of the whole structure be rather large and as a consequence it is not applicable in microwave devices like CPWI.

In [7], an artificial dielectric is realized using a set of electronically controlled self-resonant grids (wires containing PIN-diodes inserted

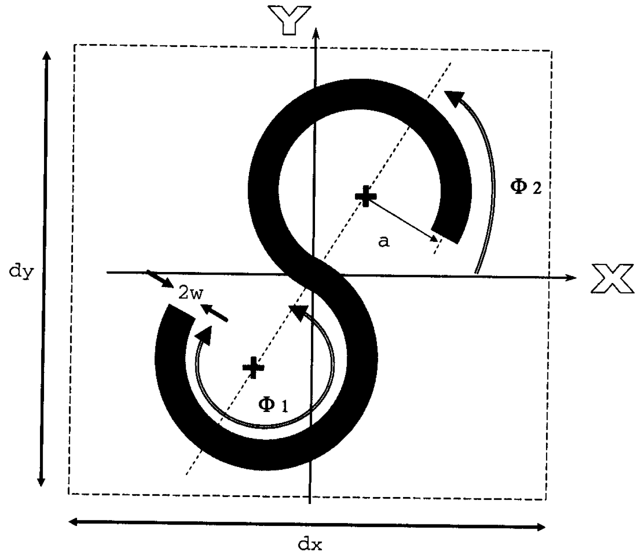

Figure 1. S-shaped particle geometry and grid unit cell parameters.

periodically). This leads to an *active artificial dielectric* but this is a very complicated device. Finally its size is also larger than λ).

Another way to design resonant artificial dielectric is to use isolated resonant particles. For this kind of materials, the resonance is given by the resonance of individual particles. Classically, these inclusions are called *bianisotropic* particles and their size is of the order $\lambda/10$ (see for example papers on Ω-shaped conducting particles [8, 9]). A huge body of literature is devoted to these microwave composites (named *bianisotropic composites*) which possess resonant material parameters (permittivity, permeability and magnetoelectric coupling resonances). An overview is presented in book [10].

Considering the problem of CPWI [1], it is obvious that the self resonant grids and the active artificial dielectrics give too big thicknesses for the required layer and that the solution is to turn toward composites with resonant particles. In the present paper we suggest using S-shaped conductive particles in order to compose an artificial dielectric layer. These particles have been first proposed to create photonic crystals with unusual frequency properties in the microwave frequency range [19]. Particles are printed on one side of a dielectric film and arranged in a grid (Fig. 2). The grid periods d_x, d_y are much smaller than the wavelength of the particle resonance in the medium of the substrate.

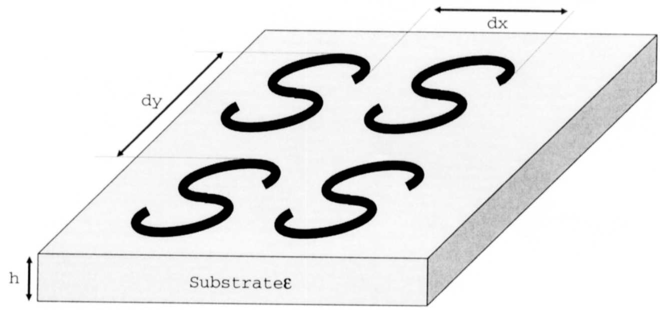

Figure 2. Structure under consideration: a square grid of S-particles.

For our purpose, we do not need a structure with bianisotropic properties. The scatterer we suggest in order to create an artificial dielectric is equivalent to a resonant electric dipole. By the way it has also bianisotropic and paramagnetic properties but at frequencies higher than those of our interest. The goal of our present work is to show that an array of S-shaped particles printed on a dielectric interface can be successfully interpreted as a bulk structure with an effective anisotropic permittivity. For this purpose we develop the homogenization model for a such structure and validate it by comparison with the numerical study of the normal reflection and transmission of a plane wave in the needed frequency range.

Quasi-static model of homogenization of dipole lattice of finite thickness (with a few dipoles over the thickness) has been developed [12]. The theory originates in very old classical works by Ewald and Oseen. The frequency bounds of validity for this quasi-static approximation have been studied analytically [13] and numerically [14]. However, the case of resonant dipoles is still weakly studied. Meanwhile, it can be noted that these kinds of structures are of increasing interest. Thus, in [16], grids of resonant dipoles have been suggested in order to obtain, in microwave band, an effective magnetic wall at distances much smaller than $\lambda/4$ over a metal ground planes. However, in [16], the question of the effective dielectric properties of such grids was not discussed.

In the current paper we present a model for evaluating the effective constitutive parameters of a structure that is equivalent to a grid of resonant electric dipoles. Following [17] and [14], a planar grid of dipoles embedded in a dielectric host medium is presented as a layer of effective bulk medium. This can be done under two conditions. Firstly, the grid of particles must be square. Secondly, the period of the grid must be much smaller than λ. In [18] the theory has been modified for the special case when the particles are located on the dielectric interface.

The main question that appears when one tries to homogenize an array of resonant particles is the following. At the resonance the effective permittivity ϵ_{eff} becomes high. So, the wavelength in the effective medium $\lambda/\sqrt{\epsilon_{eff}}$ is getting smaller than the array period. So, the homogenization model should become internally inconsistent.

Actually, in our case, it is not so and the homogenization is possible for arrays of resonant dipoles even if their sizes are larger than $\lambda/\sqrt{\epsilon_{eff}}$. The reason is that the value $\lambda/\sqrt{\epsilon_{eff}}$ has no physical meaning for a single 2D array of resonant particles illuminated by a plane wave (we do not consider the grazing incidence). Our goal is to interpret the structure as an anisotropic dielectric in order to predict properly the reflection and the transmission of plane waves. For this purpose a small number of layers of particles across the structure is favorable for the bulk homogenization procedure. In order to represent a grid of particles as a layer of a bulk medium one needs only to define the procedure of averaging the microscopic field and the polarization in a self-consistent way so as to obtain the correct boundary conditions at the two effective interfaces (continuous normal component of $\mathbf{D}$ and continuous tangential component of $\mathbf{E}$). Then we can nicely introduce the effective permittivity of a single layer (or of a few layers) of particles. Both averaged field and local field acting on each inclusion will not differ significantly from the incident wave field. So, the crucial requirement for the homogenization in this case is the smallness of the array period d compared to λ and not with $\lambda/\sqrt{\epsilon_{eff}}$. Of course, the situation will change dramatically in the case with many layers of particles. When there are many layers and when ϵ_{eff} is high, the effects of spatial dispersion appear even in the case $d \ll \lambda$ since both averaged and local field will vary from layer to layer with respect to the small wavelength. At a given frequency within the particle resonance band the disagreement with the numerical data will increase proportionally to the number of layers N. So, our case when $N = 1$ is the best in order to neglect the spatial dispersion.

2. DESCRIPTION OF THE ARRAY

The S-shaped particle geometry is shown in Fig. 1. This particle can be considered at low frequencies, i.e., in the case when its size is small compared to λ, as a resonant electric dipole. Hence, the magnetic dipole moments of the two broken loops cancel out.

The polarizability of a particle is the tensor that relates the dipole moment to the local electric field as

$$\mathbf{p} = \overline{\overline{a}} \cdot \mathbf{E}^{\text{loc}} \tag{1}$$

The main difficulty with the S-particle is due to the existence of a cross component in the electric polarizability tensor of the particle: a y-polarized electric field produces an x-component of the dipole moment and vice versa. This effect also turns out to be resonant. To model an artificial dielectric layer we must take into account all the non-zero components of the polarizability tensor: a^{xx}, a^{yy} and $a^{xy} = a^{yx}$. However, an appropriate choice of the particle parameters can give a much higher resonant frequency for a^{yy} and a^{xy} than the resonant frequency of a^{xx}.

As an example, we have chosen the following parameters (see Fig. 1): $a = 0.62\,\text{mm}$, $2w = 0.03\,\text{mm}$, $\phi_1 = 345°$, $\phi_2 = -10°$. The particle is assumed to be perfectly conducting and it is put on a dielectric substrate with permittivity $\epsilon = 10.2$ and thickness $h = 0.635\,\text{mm}$ (this corresponds to the fluoropolymer composite film R01030 delivered by Rogers Corporation).

With these particular choices, in the frequency range of our interest $8 < f < 12\,\text{GHz}$ we can neglect the contribution of both a^{yy} and a^{xy} into the effective permittivity of the composite layer. This significantly simplifies our analytical model.

3. HOMOGENIZATION OF THE STRUCTURE

In this section we briefly explain the theory of homogenization using the model of [17] which has been developed also in [14, 15, 18]. This model replaces the square grid of parallel identical dipoles with polarizability a and period d by an effective layer of thickness d_p and bulk dielectric susceptibility κ_p. This susceptibility, which has in our case only an xx-component, must be calculated using the laws of electromagnetic interaction between dipoles in a 2D grid. These equations give the relation between the local field and the bulk averaged field; both are calculated at dipole centers. This relation, as the usual Clausius-Mossotti relation, allows to find the constitutive parameters of the bulk structure which is equivalent to the original 2D grid (in the meaning explained above). This equivalence has been confirmed and has been used to calculate the reflection and transmission coefficients for various frequencies [14] and angles of incidence [17, 15] for the case of non-resonant particles.

Let us assume that we know the polarizability a^{xx} of the electric dipoles that compose the square grid with period d. Following [17] and [18], we must calculate the averaged field at any position. To do this, we must average the true electric field (which is the addition of the field of incident wave with the field generated by the grid of dipoles and the dielectric substrate). The averaging is done over a cubic cell of volume $V = d \times d \times d$ which is centered at the observation point. The averaged field E (calculated at the center of an arbitrary dipole) is related with the local field $E^{\rm loc}$ (acting on this dipole) by [18]:

$$E = E^{\rm loc} - 0.359 \frac{p}{\epsilon' \epsilon_0 V} \tag{2}$$

Here the factor 0.359 is called *interaction constant* of a 2D dipole grid [4] and it is denoted $\epsilon' = (\epsilon + 1)/2$ where ϵ is the relative permittivity of the substrate†. Equation (2) is approximate but the accuracy is sufficient if $kd < 1$, where k is the wave number in the host medium. In this section we consider the x-components of the fields and polarization, since the y- and z-component of the array polarization are negligible.

Using the equation $p = a^{xx} E^{\rm loc}$ with relation (2) allows to express the dipole moment of the bulk cell V as a function of the averaged field:

$$p = \frac{a^{xx} E}{1 - \dfrac{0.359}{\epsilon' \epsilon_0 V} a^{xx}} \tag{3}$$

Suppose that the dipole moment is located at the observation point, that is to say at the center of the averaging volume. In this case (see Fig. 3, case (1)) the bulk polarization is equal to $P_p = p/V$ and the effective bulk susceptibility κ_p is equal to $P_p/\epsilon_0 E$. If the observation point is not exactly at the center of the dipole but even so there is a particle in the averaging cell (as it is shown in Fig. 3, case (2)), the dipole moment of this volume is still equal to p and the bulk polarization is still equal to $P_p = p/V$. Hence, κ_p is non-zero within the interval $-d/2 < z < d/2$.

It must be noticed also that the averaging volume includes part of the dielectric substrate. So, in order to obtain the total averaged polarization, we should add to P_p the bulk polarization due to the dielectric substrate which we will denote as P_d. If $d > h$ the substrate occupies a part V_d of the averaging cell. Furthermore V_d depends on the vertical coordinate z. Since the substrate susceptibility is equal to $\kappa_d = (\epsilon - 1)$, we can write that $P_d = \epsilon_0(\epsilon - 1)(V_d/V)E$. To calculate V_d, we suppose that $d > 2h$.

The complete effective susceptibility of our structure is then equal to $\kappa = (P_p + P_d)/\epsilon_0 E$. Deriving from the usual definition:

$$D = \epsilon_0 E + P_p + P_d = \epsilon_0 \epsilon_{eff} E \tag{4}$$

† In [20] this formula has been derived for homogeneous host medium $\epsilon' = \epsilon$.

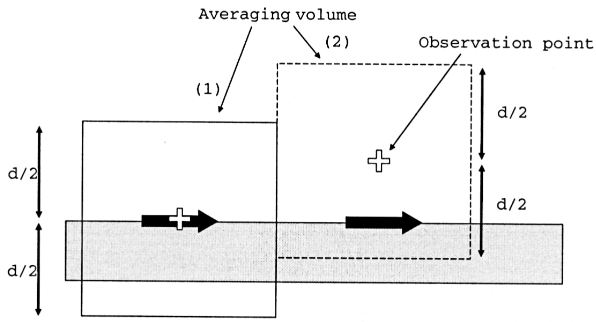

Figure 3. Averaging volumes V. Case (1) observation point coincides with dipole center. Case (2) observation point does not coincide with dipole center.

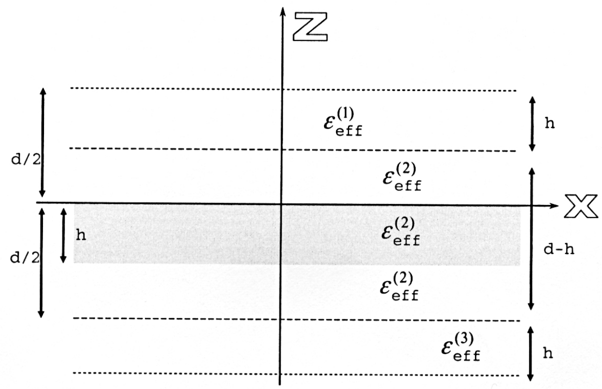

Figure 4. Equivalent 3-layered structure (side view).

where D is the x-component of the electric displacement vector, we can obtain effective relative permittivity of the composite as

$$\epsilon_{eff} = 1 + \kappa \tag{5}$$

Using this method now becomes equivalent to replacing our array on the substrate (see Fig. 2) by a 3-layered dielectric structure. Permittivities of layers are different (see Fig. 4):

1. An effective layer of thickness $d_1 = h$ for $d/2 - h < z < d/2$.

$$\epsilon^{(1)}_{eff} = 1 + \kappa_p + (\epsilon - 1)\left(\frac{1}{2} - \frac{z}{d}\right) \tag{6}$$

2. An effective layer of thickness $d_2 = d - h$ for $-d/2 < z < d/2 - h$.

$$\epsilon_{eff}^{(2)} = 1 + \kappa_p + (\epsilon - 1)\frac{h}{d} \tag{7}$$

3. An effective layer of thickness $d_3 = d_1 = h$ for $-d/2 - h < z < -d/2$.

$$\epsilon_{eff}^{(3)} = 1 + (\epsilon - 1)\left(\frac{z + h}{d} + \frac{1}{2}\right) \tag{8}$$

In these formulae we have

$$\kappa_p = \frac{a^{xx}}{\epsilon_0 V \left(1 - \frac{0.359}{\epsilon' \epsilon_0 V} a^{xx}\right)} \tag{9}$$

To simplify, we average the z-dependence of the susceptibility in the upper and lower layers. Considering that the contribution of the substrate is rather small in these areas, we choose to use the value at the center of the two layers. So, in (6) we substitute $z = d/2 - h/2$ and in (8) $z = -d/2 - h/2$, and we obtain

$$\epsilon_{eff}^{(1)} = 1 + \kappa_p + (\epsilon - 1)\frac{h}{2d} \tag{10}$$

and

$$\epsilon_{eff}^{(3)} = 1 + (\epsilon - 1)\frac{h}{2d} \tag{11}$$

All these formulas are for the xx-components of the permittivity. For other components the particle influence is absent and we have

$$\epsilon_{yy,zz}^{(1),(3)} = 1 + (\epsilon - 1)\frac{h}{2d}, \qquad \epsilon_{yy,zz}^{(2)} = 1 + (\epsilon - 1)\frac{h}{d}$$

The reflection and transmission of the 3-layered structure (Fig. 4) is a special case of a multi-layered dielectric stack which has been considered in [22]. The reflection coefficient of a plane wave under normal incidence is given by:

$$R = \frac{t_1 R_1 e^{-2jkn_{eff}^{(1)} h} - 1}{t_1 - R_1 e^{-2jkn_{eff}^{(1)} h}} \tag{12}$$

R_1 is the reflection coefficient at the plane $z = d/2 - h$:

$$R_1 = \frac{t_2 R_2 e^{-2jkn_{eff}^{(2)}(d-h)} + 1}{t_2 + R_2 e^{-2jkn_{eff}^{(2)}(d-h)}} \tag{13}$$

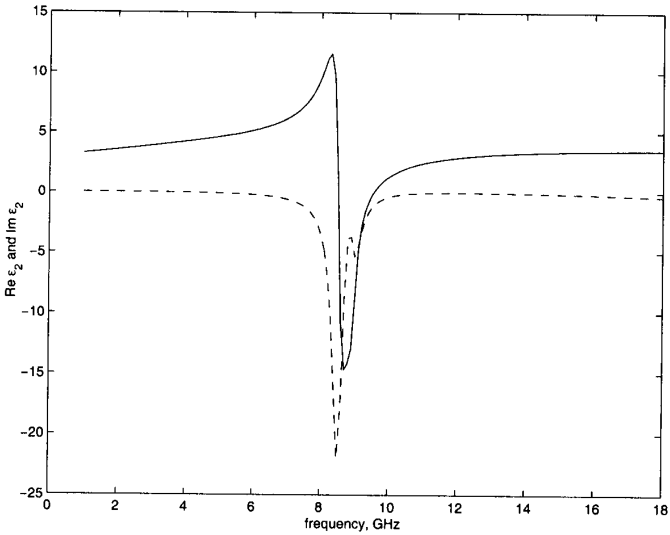

Figure 5. Permittivity $\epsilon_{eff}^{(2)}$ of the central layer versus frequency (GHz). Solid line — real part. Dashed line — imaginary part.

and R_2 is the reflection coefficient at the plane $z = -d/2$:

$$R_2 = \frac{t_2 e^{-2jkn_{eff}^{(3)}h} - t_3}{t_1 t_3 - e^{-2jkn_{eff}^{(3)}h}} \tag{14}$$

where $n_{eff}^{(1,2,3)} = \sqrt{\epsilon_{eff}^{(1,2,3)}}$ are the effective refracted coefficients of the layers and:

$$t_1 = \frac{n_{eff}^{(1)} + 1}{n_{eff}^{(1)} - 1}, \qquad t_2 = \frac{n_{eff}^{(1)} + n_{eff}^{(2)}}{n_{eff}^{(1)} - n_{eff}^{(2)}}, \qquad t_3 = \frac{n_{eff}^{(3)} + 1}{n_{eff}^{(3)} - 1} \tag{15}$$

The transmission coefficient can be found with formula (18.60b) from [22].

The frequency variations of polarizability a^{xx} of a single S-particle printed on the surface of a dielectric layer has been found using the ANSOFT code *ENSEMBLE.* This code gives a resonance frequency of a^{xx} at 8.9 GHz.

4. EFFECTIVE PERMITTIVITY OF AN HOMOGENIZED LAYER AND COMPARISON WITH NUMERICAL SIMULATION

To apply correctly the homogenization model we have presented, the array of dipoles must be square, whereas the structure shown in Fig. 1 is rectangular ($d_x = 3\,\text{mm}$ and $d_y = 1.5$). However, the case $d_x = d_y = 1.5\,\text{mm}$ is physically impossible (particles would overlap) and the case $d_x = d_y = 3\,\text{mm}$ is not very interesting since it leads to values of $|\epsilon_{eff}|$ which do not exceed 14. This value of the permittivity is not high enough for the desired applications. That is why we keep the geometry of Fig. 1 with periods d_x, d_y which are related by $d_x = 2d_y = 3\,\text{mm}$. This relation allows to obtain a square array easily, considering two adjacent S-shaped particles in a cell with dimension $d_x \times d_x$ and containing one dipole with total polarizability $2a^{xx}$. This approach gives for the equivalent permittivity of the central layer $-d/2 < z < d/2-h$ the frequency dependence presented in Fig. 5.

The maximum absolute value of permittivity at the resonance is 29.5, whereas the real part attains values 12 and -15 within the resonance band. This is not a very high value, but the effective thickness of the 3-layered structure in our model is equal to 3.635 mm which is higher than the real physical thickness of slab ($h = 0.635\,\text{mm}$). This big virtual thickness will significantly increase the effect of the presence of our artificial dielectric.

To understand the applicability of our homogenization model we compare its results with those obtained from numerical simulations of the code described in [19] and [21]. This code uses Floquet's expansion for currents and fields and can simulate the reflection and transmission coefficients for a dielectric layer covered by a doubly periodic array of arbitrary planar particles. Numerical simulations give practically the same result for the case where the particles are located on the upper or lower surfaces of the dielectric layer. As for our analytical model, it gives exactly the same results since it does not distinguish these two cases of incidence.

The results of the comparison are presented in Figs. 6–8. It can be noted that crosses represent the results for the same dielectric layer without S-particles. We can see that the effect of the particles in both analytical and numerical models is qualitatively the same. Yet, our theory gives for the frequency of resonant reflection and transmission a value which is approximately 18% lower than that obtained with the numerical code. Nevertheless, we can consider that the agreement between the two models is satisfactory. Actually, not only the resonant frequency but also the shapes of all the curves given by the analytical

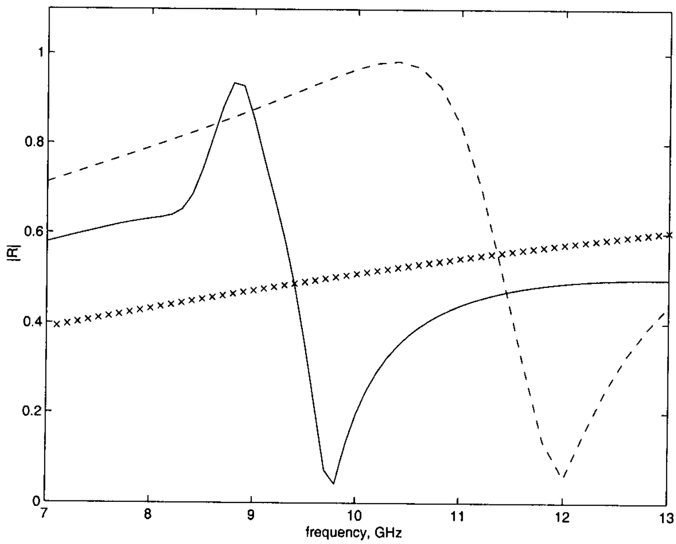

Figure 6. Absolute value of the reflection coefficient of the layer with S-shaped particles on its surface versus frequency (GHz). Solid line — our analytical model (effective 3-layered composite). Dashed line — numerical model [21]. Crosses — the same layer without S-particles.

model are extremely sensitive to input parameters. As an example, a very small change in the particle geometry, in the slab thickness h or in the permittivity ϵ of the substrate, dramatically changes the shape of all analytical curves, so that any agreement between analytical and numerical models disappears.

Another remark can be made. The lower virtual layer $-d/2-h < z < -d/2$, though its permittivity $\epsilon_{eff}^{(3)}$ is not resonant and though its thickness $d_3 = h$ is small, has a significant influence on the results and especially on the phase curves. Indeed, if one removes this effective layer from the analytical model, the agreement also disappears.

In Fig. 7, a second resonance of reflection appears in our frequency band. It is predicted by our analytical model at 12.9 GHz. The numerical model gives for the second resonance the value 16.5 GHz. This great difference can be related with the cross-polarization effect. Actually, at frequencies higher than 13 GHz, the cross-component of particle polarizability a_{ee}^{xy} becomes significant (a_{ee}^{xy} has a resonance at 14 GHz). We do not take into account the influence of a_{ee}^{xy} in order to present a very simple theory. So, our model is not valid at the

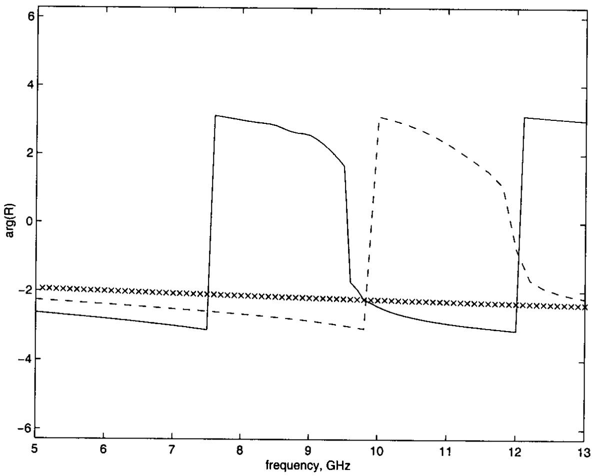

Figure 7. Phase of the reflection coefficient of the layer with S-shaped particles on its surface versus frequency (GHz). Solid line — our analytical model (effective 3-layered composite). Dashed line — numerical model [21]. Crosses — the same layer without S-particles.

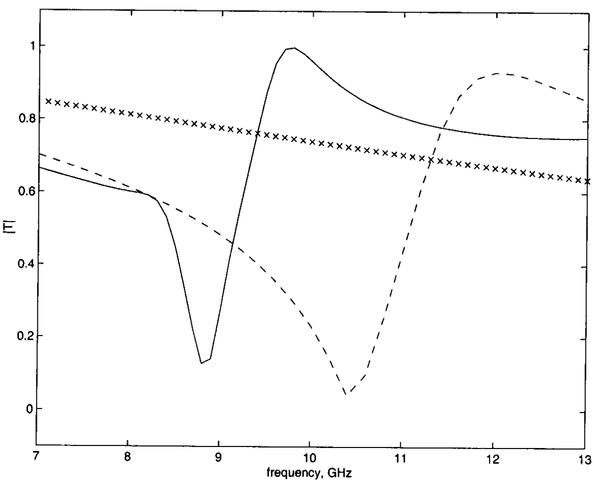

Figure 8. Absolute value of the transmission coefficient of the layer with S-shaped particles on its surface versus frequency (GHz). Solid line — our analytical model (effective 3-layered composite). Dashed line — numerical model [21]. Crosses — the same layer without S-particles.

frequencies higher than 12–12.5 GHz

The next comment is about the shape of the particle. The case where angle ϕ_1 is less than 345° has also been studied. If values of ϕ_1 are getting smaller than 300°, the homogenization model is not applicable any more, even for the first resonance. The reason is quite simple. When ϕ_1 is not high enough each loop of S-particle has a large opened part. So, the particle cannot be modelled as a resonant electric dipole. It is possible to show that the conducting broken loop with a large opened part is the superposition of an electric dipole, a magnetic dipole and an electric quadrupole. Hence, the dipole model of S-particles we have presented, is valid for the cases where an S-particle is close to an 8-particle (as in Fig. 1).

5. CONCLUSION

The starting point of study presented in this paper is the need for materials with rather high permittivity, for applications in guided planar structures like coplanar waveguide isolators. These materials need to be thin and must be feasible with planar technology. To answer this problem we propose to use a resonant artificial dielectric that is able to give a high effective permittivity at resonant frequency. The structure we have decided to study is a dielectric slab with metallic shapes printed on one of its faces. The chosen shapes are called S-particles.

We have proposed an analytical model that homogenizes a square array of S-particles. This model replaces the study of the real structure (dielectric slab and metallic particles) by an electromagnetic equivalent structure. This effective structure is composed of 3 anisotropic dielectric layers with different dyadic permittivities. The analytical model has been validated by comparison with the results of a numerical code and the results are in rather good agreement (though the array under consideration was rectangular and we replaced it by a square array referring the dipole moments of 2 adjacent particles to the square unit cell).

In order to get analytical equations as simple as possible, our modelling has been voluntarily restricted in the frequencies and in the particle geometry. We have indicated its limitations. The model is limited to the upper edge of the first resonance of S-particles and to S-particles with a shape which is close to an 8-particle.

These results can be improved considering magnetic and cross component polarizabilities for example. Nevertheless, our model is very simple to use and the first results we have obtained are good enough to begin the realization of this kind of media.

Concerning the problem of losses in the structure, we can notice that the resonant effective permittivity we have calculated contains high imaginary part (satisfying to Kramers-Kronig relations). There is no physical reason for these losses if the dielectric and metal are perfect as it was in the simulations. However, if we voluntarily assume the imaginary part of permittivity to be zero without changing the real part, the model will become internally inconsistent and the results for R and T will become absurd. So, we should remain these losses if we want to have a consistent bulk homogenization model and should keep in mind that this is significant problem of the model. However, in our case when $h = 0.635\,\text{mm}$ the discrepancy in the condition of the energy conservation[‡] $|R|^2 + |T|^2 = 1$ turns out be close to 1%. This is the consequence of the small thickness of the substrate. For more thick structures our model will lead to big errors in the transmittance.

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[‡] This condition is satisfied perfectly in our numerical simulations.

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