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PLANE WAVE DIFFRACTION BY DIELECTRIC LOADED THICK-WALLED PARALLEL-PLATE IMPEDANCE WAVEGUIDE

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Abstract—The high frequency diffraction of E_z-polarized plane waves by a dielectric loaded thick-walled parallel-plate impedance waveguide is investigated rigorously by using Fourier transform technique in conjunction with the mode-matching method. Relying upon the image bisection principle, the original problem is splitted up into two simpler ones and each individual boundary-value problem is formulated with this mixed method which gives rise to a scalar Wiener-Hopf equation of the second kind. The solution of each Wiener-Hopf equation contains infinitely many constants satisfying an infinite system of linear algebraic equations. A numeric solution of this system is obtained for various values of the dielectric constant, plate impedances plate thickness, and the distance between the plates through which the effect of these parameters on the diffraction phenomenon is studied.

1. INTRODUCTION

A good deal of investigations has been devoted to the diffraction of electromagnetic waves by systems of parallel half planes since it constitutes an important subject in diffraction theory and it is relevant to several engineering applications. In this context the diffraction of plane waves by three parallel infinitely thin soft half-planes has been considered first by D. S. Jones who formulated the problem as a three dimensional matrix Wiener-Hopf equation [1]. These equations are converted into a pair involving a two dimensional matrix and scalar Wiener-Hopf equation. The three parallel half-planes problem has been also considered by Abrahams [2] who presented a more simpler approach to achieve the Wiener-Hopf factorization of the kernel matrix. The diffraction of plane waves by a thick-walled parallel-plate impedance waveguide is investigated by Büyükaksoy and Polat [3] who used the Wiener-Hopf technique in conjunction with the mode matching method. Besides, the diffraction of plane waves by three parallel thick impedance half-planes is treated by Alkumru [4] who used the same method described in [3].

In the present work the E_z-polarized plane wave diffraction by a thick-walled parallel plate impedance waveguide filled with two part dielectric material will be analyzed rigorously by using the Wiener-Hopf technique in conjunction with the mode-matching method. Using the classical Fourier transform technique leads to a modified matrix Wiener-Hopf equation which can not be solved by considering the known techniques. In this work an alternative method of formulation which is introduced in [3] will be used. Because of the symmetry of the diffracting structure, the image bisection principle is used and the original problem is splitted-up into even and odd excitation cases. The scattered field in the waveguide region is expanded into a series of normal modes and the Fourier integral representation is used elsewhere. This yields a scalar modified Wiener-Hopf equation of the second kind for each excitation, which can be solved by using the standard techniques. The solution of each equation contains a set of infinitely many unknown constants satisfying an infinite system of linear algebraic equations. Numerical solution of these system is obtained for various values of the dielectric constants, plate impedances, plate thickness, and the distance between the plates through which the effect of these parameters on the diffraction phenomenon is studied.

A time factor $e^{-i\omega t}$ with ω being the angular frequency is assumed and suppressed throughout the paper.

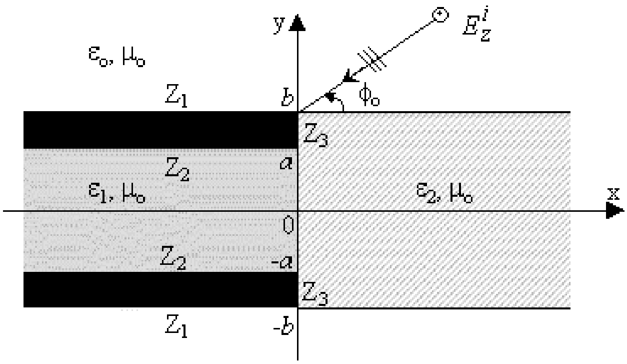

Figure 1. Geometry of diffraction problem.

2. ANALYSIS

We consider the diffraction of an E_z-polarized plane wave by a waveguide formed by two semiinfinite impedance plates defined by $S_1 = \{(x, y, z);\ x \in (-\infty, 0),\ y \in (a, b),\ z \in (-\infty, \infty)\}$, and $S_2 = \{(x, y, z);\ x \in (-\infty, 0),\ y \in (-b, -a),\ z \in (-\infty, \infty)\}$, respectively, as depicted in Fig. 1. The surface impedances of the horizontal walls $y = \pm b,\ x < 0$, and $y = \pm a,\ x < 0$ are denoted by $Z_1 = \eta_1 Z_0$ and $Z_2 = \eta_2 Z_0$ respectively, while the impedance of the vertical walls $x = 0,\ y \in (a, b)$ and $x = 0,\ y \in (-b, -a)$ is $Z_3 = \eta_3 Z_0$, with Z_0 being the characteristic impedance of the free-space.

In order to determine the scattered field, one can proceed by decomposing the incident wave into even and odd excitations as indicated in Fig. 2a and Fig. 2b, respectively. Relying upon the image bisection principle, it can be shown that the configurations shown in Fig. 2a and Fig. 2b are equivalent to those depicted in Fig. 2c and Fig. 2d, respectively. In what follows, the even and odd excitations will be treated separately.

2.1. Even Excitation

Let us consider first the configuration depicted in Fig. 2c, which is equivalent to the even excitation case. Since in this case the field is symmetrical about the plane $y = 0,\ x \in (-\infty, \infty)$ (magnetic wall).

For analysis purposes, it is convenient to express the total field as

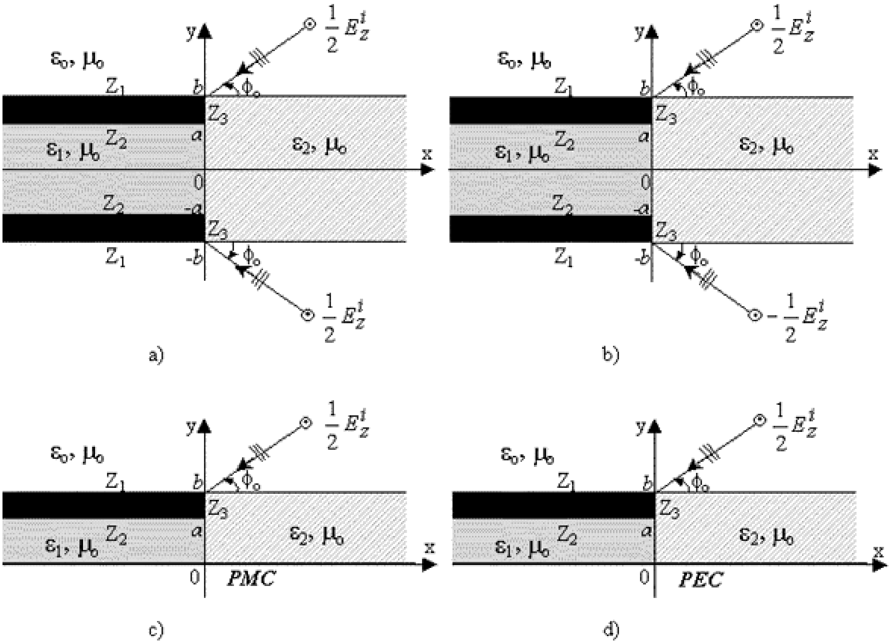

Figure 2. Equivalent problems. a) Symmetric (even) excitation. b) Asymmetric (odd) excitation. c) Equivalence to symmetric (even) excitation. d) Equivalence to asymmetric (odd) excitation.

follows:

$$u_T^{(e)}(x,y) = \begin{cases} u^i + u^r + u_1^{(e)}, & y > b \\ u_2^{(e)}, & 0 < y < a,\ x < 0 \\ u_3^{(e)}, & 0 < y < b,\ x > 0 \end{cases} \tag{1a}$$

Here, u^i is the incident field given by

$$E_z^i = u^i(x,y) = \exp\{-ik_0(x\cos\phi_0 + y\sin\phi_0)\} \tag{1b}$$

while u^r denotes the field reflected from the plane $y = b$, namely

$$u^r(x,y) = \frac{\eta_1\sin\phi_0 - 1}{\eta_1\sin\phi_0 + 1}\exp\{-ik_0[x\cos\phi_0 - (y-2b)\sin\phi_0]\} \tag{1c}$$

with k_0 is the free space wave number which is assumed to have a small positive imaginary part and denoted by $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ with ε_0 and μ_0 being the dielectric permittivity and the magnetic permeability of the free space. The lossless case can then be obtained by making $\Im m(k_0) \to 0$ at the end of the analysis.

$u_j^{(e)}(x,y),\ j = 1,2,3,$ which satisfy the Helmholtz equation as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_{0,1,2}^2\right) u_j^{(e)}(x,y) = 0. \tag{1d}$$

Here, k_1 and k_2 are the wave numbers denoted by $k_1 = \omega\sqrt{\varepsilon_1\mu_0}$ and $k_2 = \omega\sqrt{\varepsilon_2\mu_0}$ with ε_1 and ε_2 being the dielectric permittivity in the regions $x < 0,\ y \in (-a,a)$ and $x > 0,\ y \in (-b,b)$, respectively. $u_j^{(e)}(x,y),\ j = 1,2,3,$ are to be determined with the aid of the the following boundary and continuity conditions:

$$\left(1 + \frac{\eta_1}{ik_0}\frac{\partial}{\partial y}\right) u_1^{(e)}(x,b) = 0,\ x < 0, \tag{2a}$$

$$\left(1 - \frac{\eta_2}{ik_1}\frac{\partial}{\partial y}\right) u_2^{(e)}(x,a) = 0,\ x < 0, \tag{2b}$$

$$\frac{\partial}{\partial y}u_2^{(e)}(x,0) = 0,\ x < 0, \tag{2c}$$

$$\left(1 + \frac{\eta_3}{ik_2}\frac{\partial}{\partial x}\right) u_3^{(e)}(0,y) = 0,\ y \in (a,b), \tag{2d}$$

$$\frac{\partial}{\partial y}u_3^{(e)}(x,0) = 0,\ x > 0, \tag{2e}$$

$$u_2^{(e)}(0,y) = u_3^{(e)}(0,y),\ 0 < y < a, \tag{2f}$$

$$\frac{\partial}{\partial x}u_2^{(e)}(0,y) = \frac{\partial}{\partial x}u_3^{(e)}(0,y),\ 0 < y < a, \tag{2g}$$

$$u_1^{(e)}(x,b) + u^i(x,b) + u^r(x,b) = u_3^{(e)}(x,b),\ x > 0, \tag{2h}$$

$$\frac{\partial}{\partial y}u_1^{(e)}(x,b) + \frac{\partial}{\partial y}u^i(x,b) + \frac{\partial}{\partial y}u^r(x,b) = \frac{\partial}{\partial y}u_3^{(e)}(x,b),\ x > 0, \tag{2i}$$

$$u_1^{(e)}(x,b) - u_3^{(e)}(x,b) = -\frac{2\eta_1\sin\phi_0}{1+\eta_1\sin\phi_0}e^{-ik_0 b\sin\phi_0}e^{-ik_0 x\cos\phi_0},\ x > 0 \tag{2j}$$

and

$$\frac{\partial}{\partial y}u_1^{(e)}(x,b) - \frac{\partial}{\partial y}u_3^{(e)}(x,b) = \frac{2ik_0\sin\phi_0}{1+\eta_1\sin\phi_0}e^{-ik_0 b\sin\phi_0}e^{-ik_0 x\cos\phi_0},\ x > 0. \tag{2k}$$

Since $u_1^{(e)}(x,y)$ satisfies the Helmholtz equation in the range $x \in (-\infty,\infty)$, its Fourier transform with respect to x gives

$$\left[\frac{\partial^2}{\partial y^2}+(k_0^2-\alpha^2)\right]F^{(e)}(\alpha,y)=0 \tag{3a}$$

with

$$F^{(e)}(\alpha,y)=F_+^{(e)}(\alpha,y)+F_-^{(e)}(\alpha,y) \tag{3b}$$

where

$$F_\pm^{(e)}(\alpha,y)=\pm\int_o^{\pm\infty}u_1^{(e)}(x,y)e^{i\alpha x}dx. \tag{3c}$$

By taking into account the following asymptotic behaviors of $u_1^{(e)}$ for $x\to\pm\infty$

$$u_1^{(e)}(x,y)=\begin{cases}O\left(\dfrac{e^{-ik_0x}}{\sqrt{-x}}\right), & x\to-\infty\\ O(e^{-ik_0x\cos\phi_0}), & x\to\infty\end{cases} \tag{4}$$

one can see that $F_+^{(e)}(\alpha,y)$ and $F_-^{(e)}(\alpha,y)$ are regular functions of α in the half planes $\Im m(\alpha)>\Im m(k_0\cos\phi_0)$ and $\Im m(\alpha)<\Im m(k_0)$, respectively. The general solution of (3a) satisfying the radiation condition for $y\to\infty$ reads

$$F_+^{(e)}(\alpha,y)+F_-^{(e)}(\alpha,y)=A^{(e)}(\alpha)e^{iK_0(\alpha)(y-b)} \tag{5a}$$

with

$$K_0(\alpha)=\sqrt{k_0^2-\alpha^2}. \tag{5b}$$

The square-root function is defined in the complex α-plane cut along $\alpha=k_0$ to $\alpha=k_0+i\infty$ and $\alpha=-k_0$ to $\alpha=-k_0-i\infty$ such that $K_0(0)=k_0$.

In the Fourier transform domain, the boundary condition (2a) takes the form

$$F_-^{(e)}(\alpha,b)+\frac{\eta_1}{ik_0}\dot{F}_-^{(e)}(\alpha,b)=0. \tag{6}$$

Where the dot $(\cdot)$ specifies the derivative with respect to y. By using the derivative of (5a) with respect to y and (6), one gets

$$R_+^{(e)}(\alpha) = \frac{K_0(\alpha)}{k_0\chi(\alpha)} A^{(e)}(\alpha) \tag{7a}$$

where

$$R_+^{(e)}(\alpha) = F_+^{(e)}(\alpha, b) + \frac{\eta_1}{ik_0}\dot{F}_+^{(e)}(\alpha, b) \tag{7b}$$

and

$$\chi(\alpha) = \left[\eta_1 + \frac{k_0}{K_0(\alpha)}\right]^{-1} \tag{7c}$$

On the other hand, the field $u_3^{(e)}(x, y)$ also satisfies the Helmholtz equation in the region $x \in (0, \infty),\ y \in (0, b)$ with the wave number k_2:

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} + k_2^2\right] u_3^{(e)}(x, y) = 0. \tag{8}$$

The half-range Fourier transform of (8) yields

$$\left[\frac{\partial^2}{\partial y^2} + K_2^2(\alpha)\right] G_+^{(e)}(\alpha, y) = f^{(e)}(y) + \alpha g^{(e)}(y) \tag{9a}$$

with

$$f^{(e)}(y) = \frac{\partial}{\partial x} u_3^{(e)}(0, y), \tag{9b}$$

$$g^{(e)}(y) = -iu_3^{(e)}(0, y) \tag{9c}$$

and

$$K_2(\alpha) = \sqrt{k_2^2 - \alpha^2}. \tag{9d}$$

$G_+^{(e)}(\alpha, y)$, which is defined by

$$G_+^{(e)}(\alpha, y) = \int_0^\infty u_3^{(e)}(x, y) e^{i\alpha x} dx \tag{10}$$

is a function regular in the half-plane $\Im m(\alpha) > \Im m(-k_2)$. The general solution of (9a) satisfying the Neumann boundary condition at $y = 0$

reads

$$G_+^{(e)}(\alpha, y) = B^{(e)}(\alpha)\cos[K_2(\alpha)y] + \frac{1}{K_2(\alpha)} \cdot \int_0^y [f^{(e)}(t) + \alpha g^{(e)}(t)] \sin[K_2(\alpha)(y-t)]dt. \tag{11}$$

Combining (2j) and (2k) one obtains

$$R_+^{(e)}(\alpha) = G_+^{(e)}(\alpha, b) + \frac{\eta_1}{ik_0}\dot{G}_+^{(e)}(\alpha, b) \tag{12}$$

and $B^{(e)}(\alpha)$ can be solved uniquely to give

$$M^{(e)}(\alpha)B^{(e)}(\alpha) = R_+^{(e)}(\alpha) - \int_0^b [f^{(e)}(t) + \alpha g^{(e)}(t)] \times \left[\frac{\sin[K_2(\alpha)(b-t)]}{K_2(\alpha)} + \frac{\eta_1}{ik_0}\cos[K_0(\alpha)(b-t)]\right] dt \tag{13a}$$

with

$$M^{(e)}(\alpha) = \cos[K_2(\alpha)b] - \frac{\eta_1}{ik_0}K_2(\alpha)\sin[K_2(\alpha)b]. \tag{13b}$$

Replacing (13a) into (11) one gets

$$G_+^{(e)}(\alpha, y) = \frac{\cos[K_2(\alpha)y]}{M^{(e)}(\alpha)} \times \left\{R_+^{(e)}(\alpha) - \int_0^b [f^{(e)}(t) + \alpha g^{(e)}(t)] \cdot \left[\frac{\sin[K_2(\alpha)(b-t)]}{K_2(\alpha)} + \frac{\eta_1}{ik_0}\cos[K_2(\alpha)(b-t)]\right]\right\} dt + \frac{1}{K_2(\alpha)}\int_0^y [f^{(e)}(t) + \alpha g^{(e)}(t)] \sin[K_2(\alpha)(y-t)]dt. \tag{14}$$

Although the left-hand side of (14) is regular in the upper half-plane $\Im m(\alpha) > \Im m(-k_2)$, the regularity of the right-hand side is violated by the presence of simple poles occurring at the zeros of $M^{(e)}(\alpha)$, namely at $\alpha = \alpha_m^e$ satisfying

$$M^{(e)}(\alpha_m^e) = 0, \quad \Im m(\alpha_m^e) > \Im m(k_2), \quad m = 1, 2, 3, \ldots \tag{15}$$

These poles can be eliminated by imposing that their residues are zero. This gives

$$R_+^{(e)}(\alpha_m^e) = \frac{\sin[K_{2m}^e b]}{2K_{2m}^e}\left[1 - \frac{\eta_1^2}{k_0^2}(K_{2m}^e)^2\right] v_m^e[f_m^e + \alpha_m^e g_m^e] \tag{16a}$$

where K_{2m}^e, v_m^e, f_m^e and g_m^e specify

$$K_{2m}^e = K_2(\alpha_m^e), \tag{16b}$$

$$v_m^e = b + \frac{\eta_1}{ik_0}\sin^2[K_{2m}^e b] \tag{16c}$$

and

$$\begin{bmatrix} f_m^e \\ g_m^e \end{bmatrix} = \frac{2}{v_m^e}\int_0^b \begin{bmatrix} f^{(e)}(t) \\ g^{(e)}(t) \end{bmatrix}\cos[K_{2m}^e t]dt. \tag{16d}$$

Since $\frac{2}{v_m^e}$ is the square of the norm related to the eigenfunctions $\cos[K_{2m}^{(e)} y]$, it should be positive. For $\Im m(k_0) \to 0$, this requirement is satisfied if $\Im m(\eta_1) > -\frac{k_0 b}{\sin^2[K_{2m}^e b]}$ and $\Re e\,(\eta_1) = 0$.

In the region $x \in (-\infty, 0)$, $y \in (0, a)$ the total field can be expressed in terms of Fourier cosine series as

$$u_2^{(e)}(x, y) = \sum_{n=1}^{\infty} c_n^e \cos[\xi_n^e y]e^{-i\beta_n^e x} \tag{17a}$$

with

$$\cos[\xi_n^e a] + \frac{\eta_2}{ik_1}\xi_n^e \sin[\xi_n^e a] = 0, \quad n = 1, 2 \ldots \tag{17b}$$

and

$$\beta_n^e = \sqrt{k_1^2 - (\xi_n^e)^2}. \tag{17c}$$

From the continuity relations (2f), (2g), and (9b), (9c) one obtains

$$\frac{\partial}{\partial x}u_2^{(e)}(0, y) = f^{(e)}(y), \quad 0 < y < a \tag{18a}$$

and

$$u_2^{(e)}(0, y) = ig^{(e)}(y), \quad 0 < y < a \tag{18b}$$

Using (2f), (2g) and (2d) one can write

$$\left(1 + \frac{\eta_3}{ik_2}\frac{\partial}{\partial x}\right)u_3^{(e)}(0, y) = \left\{\begin{matrix} \left(1 + \frac{\eta_3}{ik_2}\frac{\partial}{\partial x}\right)u_2^{(e)}(0, y), & 0 < y < a \\ 0, & a < y < b \end{matrix}\right\}. \tag{19}$$

Hence one obtains

$$\frac{\eta_3}{ik_2} f^{(e)}(y) + i g^{(e)}(y) = \left\{ \begin{matrix} \left(1 + \frac{\eta_3}{ik_2}\frac{\partial}{\partial x}\right) u_2^{(e)}(0, y), & 0 < y < a \\ 0, & a < y < b \end{matrix} \right\}. \tag{20}$$

By taking into account (16d), $f^{(e)}(y)$ and $g^{(e)}(y)$ can be expressed in terms of the following complete set of orthogonal functions:

$$\begin{bmatrix} f^{(e)}(y) \\ g^{(e)}(y) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} f_m^e \\ g_m^e \end{bmatrix} \cos[K_{2m}^e y]. \tag{21}$$

Substituting (17a) and (21) into (18a, 18b) and (20) one gets

$$\sum_{m=1}^{\infty} [f_m^e] \cos[K_{2m}^e y] = -i \sum_{n=1}^{\infty} c_n^e \beta_n^e \cos[\xi_n^e y], \quad 0 < y < a \tag{22}$$

and

$$i \sum_{m=1}^{\infty} [g_m^e - \frac{\eta_3}{k_2} f_m^e] \cos[K_{2m}^e y] = \left\{ \begin{matrix} \sum_{n=1}^{\infty} c_n^e \left(1 - \frac{\eta_3}{k_2}\beta_n^e\right) \cos[\xi_n^e y], 0 < y < a \\ 0, \quad a < y < b \end{matrix} \right\}. \tag{23}$$

Let us multiply both sides of (22) by $\cos[\xi_l^e y]$ and integrate from $y = 0$ to $y = a$ to obtain

$$c_l^{(e)} = -\frac{2i\xi_l^e}{\beta_l^e \mu_l^e} \sin[\xi_l^e a] \sum_{m=1}^{\infty} \frac{\Omega_m^e}{\vartheta_{ml}^e} f_m^e, \quad l = 1, 2, \ldots \tag{24a}$$

with μ_l^e, Ω_m^e and ϑ_{ml}^e being defined by

$$\mu_l^e = a - \frac{\eta_2}{ik_1} \sin^2[\xi_n^e a], \tag{24b}$$

$$\Omega_m^e = \cos[K_{2m}^e a] + \frac{\eta_2}{ik_1} K_{2m}^e \sin[K_{2m}^e a] \tag{24c}$$

and

$$\vartheta_{ml}^e = (K_{2m}^e)^2 - (\xi_l^e)^2. \tag{24d}$$

Similarly, the multiplication of both side of (23) by $\cos[K_{2m}^e y]$ and its integration from $y = 0$ to $y = b$ yields

$$g_l^e - \frac{\eta_3}{k_2} f_l^e = \frac{2i\Omega_l^e}{v_l^e} \sum_{n=1}^{\infty} \frac{\xi_n^e \sin[\xi_n^e a] \left(1 - \frac{\eta_3}{k_2}\beta_n^e\right)}{\vartheta_{ln}^e} c_n, \quad l = 1, 2, \ldots \tag{25}$$

with v_l^e being defined by (16c).
Consider the continuity relation (2i) which reads, in the Fourier transform domain

$$\dot{F}_+^{(e)}(\alpha, b) - \dot{G}_+^{(e)}(\alpha, b) = -\frac{2k_0 \sin\phi_0}{1 + \eta_1 \sin\phi_0} \frac{e^{-ik_0 b \sin\phi_0}}{\alpha - k_0 \cos\phi_0}. \tag{26}$$

Taking into account (5a),(7a), and (14) one gets

$$ik_0 \frac{\chi(\alpha)}{M^{(e)}(\alpha)N^{(e)}(\alpha)} R_+^{(e)}(\alpha) - \dot{F}_-^{(e)}(\alpha, b) = -\frac{2k_0 \sin\phi_0}{1+\eta_1 \sin\phi_0} \frac{e^{-ik_0 b \sin\phi_0}}{\alpha - k_0 \cos\phi_0} + \frac{1}{M^{(e)}(\alpha)} \int_0^b [f^e(t) + \alpha g^e(t)] \cos[K_2^e(\alpha)t] dt \tag{27a}$$

with

$$N^{(e)}(\alpha) = \frac{K_0^e(\alpha)}{[K_0^e(\alpha)\cos(K_2^e b) - iK_2^e(\alpha)\sin(K_2^e b)]}. \tag{27b}$$

Substituting the series expansions for $f^{(e)}(y)$ and $g^{(e)}(y)$ in (27a) and evaluating the resulting integral, one obtains the following modified Wiener-Hopf equation of the second kind valid in the strip $\Im m(k_0 \cos\phi_0) < \Im m(\alpha) < \Im m(k_0)$

$$ik_0 \frac{\chi(\alpha)}{M^{(e)}(\alpha)N^{(e)}(\alpha)} R_+^{(e)} - \dot{F}_-^{(e)}(\alpha, b) = -\frac{2k_0 \sin\phi_0}{1+\eta_1 \sin\phi_0} \frac{e^{-ik_0 b \sin\phi_0}}{\alpha - k_0 \cos\phi_0} + \sum_{m=1}^{\infty} \frac{K_{2m}^e \sin[K_{2m}^e b]}{[\alpha^2 - (\alpha_m^e)^2]} [f_m^e + \alpha g_m^e]. \tag{28}$$

By using the classical procedure this Wiener-Hopf equation can easily be solved to give

$$\begin{aligned} &ik_0\frac{\chi_+(\alpha)}{M_+^{(e)}(\alpha)N_+^{(e)}(\alpha)}R_+^{(e)}(\alpha)\\ &\quad=-\frac{2k_0\sin\phi_0}{1+\eta_1\sin\phi_0}\frac{M_-^{(e)}(k_0\cos\phi_0)N_-^{(e)}(k_0\cos\phi_0)}{\chi_-(k_0\cos\phi_0)}\frac{e^{-ik_0b\sin\phi_0}}{\alpha-k_0\cos\phi_0}\\ &\qquad-\sum_{m=1}^{\infty}\frac{K_{2m}^e\sin[K_{2m}^e b]}{2\alpha_m^e}\frac{M_+^{(e)}(\alpha_m^e)N_+^{(e)}(\alpha_m^e)}{\chi_+(\alpha_m^e)}\frac{(f_m^e-\alpha_m^e g_m^e)}{\alpha+\alpha_m^e}. \end{aligned}\tag{29a}$$

Here $M_+^{(e)}(\alpha)N_+^{(e)}(\alpha)$, $\chi_+(\alpha)$, and $M_-^{(e)}(\alpha)N_-^{(e)}(\alpha)$, $\chi_-(\alpha)$ are the split functions, regular and free of zeros in the half-planes $\Im m(\alpha) > \Im m(-k_0)$ and $\Im m(\alpha) < \Im m(k_0)$, respectively, resulting from the Wiener-Hopf factorization of kernel function $\chi(\alpha)/[M^{(e)}(\alpha)N^{(e)}(\alpha)]$ as

$$\frac{\chi(\alpha)}{M^{(e)}(\alpha)N^{(e)}(\alpha)}=\frac{\chi_+(\alpha)}{M_+^{(e)}(\alpha)N_+^{(e)}(\alpha)}\times\frac{\chi_-(\alpha)}{M_-^{(e)}(\alpha)N_-^{(e)}(\alpha)}.\tag{29b}$$

The explicit expression of $M_\pm^{(e)}(\alpha)$ and $N_\pm^{(e)}(\alpha)$ can be obtained by following the procedure outlined in [5]:

$$\begin{aligned} N_+^{(e)}(\alpha)=&\left[\frac{K_0^e(\alpha)}{[K_0^e(\alpha)\cos(K_2^e b)-iK_2^e(\alpha)\sin(K_2^e b)]}\right]^{\frac{1}{2}}\left(\frac{k_0+\alpha}{k_0-\alpha}\right)^{\frac{1}{4}}\prod_{m=1}^{\infty}\left(\frac{\alpha_m^e-\alpha}{\alpha_m^e+\alpha}\right)^{\frac{1}{2}}\\ &\times\exp\frac{1}{2}\left\{\frac{-2K_0^e(\alpha)b}{\pi}\ln\left(\frac{\alpha+iK_0^e(\alpha)}{k_0}\right)-iK_0^e(\alpha)b+q(\alpha)\right\}\\ &\times\exp\left[\frac{i\alpha b}{\pi}\ln\left(\frac{2\alpha}{k_0}\right)\right] \end{aligned}\tag{30a}$$

with

$$\begin{aligned} q^{(e)}(\alpha)=PV\int_o^\infty&\left\{\frac{b}{\pi}-\frac{\varpi^2 b+\left(\frac{\varpi^2}{\varpi'}-\varpi'\right)\sin(\varpi' b)\cos(\varpi' b)}{\pi[(\varpi')^2\sin^2(\varpi' b)+\varpi^2\cos^2(\varpi' b)]}\right\}\\ &\ln\left(\frac{K_0(\varpi)+\alpha}{K_0(\varpi)-\alpha}\right)d\varpi, \end{aligned}\tag{30b}$$

$$\varpi'=\sqrt{\varpi^2+k_2^2-k_0^2},\tag{30c}$$

$$M_+^{(e)}(\alpha) = \left[\cos(k_2 b) + \frac{i\eta_1 k_2}{k_0}\sin(k_2 b)\right]^{1/2} \exp\left\{\frac{i\alpha b}{\pi}\left[1 - C - \ln\left(\frac{\alpha b}{\pi}\right) + i\frac{\pi}{2}\right]\right\}$$
$$\times \prod_{m=1}^{\infty}\left(1 + \frac{\alpha}{\alpha_m^e}\right)\exp\left(\frac{i\alpha b}{m\pi}\right) \tag{30d}$$

and

$$M_-^{(e)}(\alpha) = M_+^{(e)}(-\alpha) \quad \text{and} \quad N_-^{(e)}(\alpha) = N_+^{(e)}(-\alpha) \tag{30e}$$

In (30d) C is the Euler's constant given by $C = 0.57721\ldots$. By following the method described in [6], the split function $\chi_-(\alpha)$ can be expressed explicitly in terms of Maliuzhinets function [6] as follows:

$$\chi_-(k_0\cos\phi_0) = 2^{3/2}\sqrt{\frac{2}{\eta_1}}\sin\frac{\phi}{2} \times \left\{\frac{M_\pi\left(\frac{3\pi}{2} - \phi - \theta\right) M_\pi\left(\frac{\pi}{2} - \phi + \theta\right)}{M_\pi^2\left(\frac{\pi}{2}\right)}\right\}^2$$
$$\times \left\{\left[1 + \sqrt{2}\cos\left(\frac{\frac{\pi}{2} - \phi + \theta}{2}\right)\right]\left[1 + \sqrt{2}\cos\left(\frac{\frac{3\pi}{2} - \phi - \theta}{2}\right)\right]\right\}^{-1} \tag{31a}$$

with

$$\sin\theta = \frac{1}{\eta_1}, \tag{31b}$$

$$\chi_+(k_0\cos\phi_0) = \chi_-(-k_0\cos\phi_0) \tag{31c}$$

and

$$M_\pi(z) = \exp\left\{-\frac{1}{8\pi}\int_0^z \frac{\pi\sin u - 2\sqrt{2}\pi\sin\left(\frac{u}{2}\right) - 2u}{\cos u}du\right\}. \tag{31d}$$

Substituting $\alpha = \alpha_1^e, \alpha_2^e, \ldots$ in (29a) and using (16a) one can get the following equation for f_r^e and g_r^e:

$$\frac{ik_0 v_r^e}{2}\left[1-\frac{\eta_1^2}{k_0^2}(K_{2r}^e)^2\right]\frac{\sin[K_{2r}^e b]}{2K_{2r}^e}\frac{\chi_+(\alpha_r^e)}{M_+^{(e)}(\alpha_r^e)N_+^{(e)}(\alpha_r^e)}[f_r^e+\alpha_r^e g_r^e]=$$
$$-\frac{2k_0\sin\phi_0}{1+\eta_1\sin\phi_0}\frac{M_-^{(e)}(k_0\cos\phi_0)N_-^{(e)}(k_0\cos\phi_0)}{\chi_-(k_0\cos\phi_0)}\frac{e^{-ik_0 b\sin\phi_0}}{\alpha_r^e-k_0\cos\phi_0}$$
$$-\sum_{m=1}^{\infty}\frac{K_{2m}^e\sin[K_{2m}^e b]}{2\alpha_m^e}\frac{M_+^{(e)}(\alpha_m^e)N_+^{(e)}(\alpha_m^e)}{\chi_+(\alpha_m^e)}\frac{(f_m^e-\alpha_m^e g_m^e)}{\alpha_r^e+\alpha_m^e},\quad r=1,2,\ldots \tag{32}$$

Replacing (24a) in (25) g_r^e can be expressed in terms of f_r^e as

$$g_r^e=\frac{\eta_3}{k_2}f_r^e+4\frac{\Omega_l^e}{v_l^e}\sum_{m=1}^{\infty}f_m^e\Omega_m^e\sum_{n=1}^{\infty}\frac{(\xi_n^e)^2\sin^2[\xi_n^e\alpha]\left(1-\frac{\eta_3}{k_2}\beta_n^e\right)}{\beta_n^e\mu_n^e\vartheta_{mn}^e\vartheta_{rn}^e}. \tag{33}$$

Substituting (33) in (32) one obtains infinitely many equations in infinite number of unknowns that yield the constants f_r^e as follows:

$$\left\{\frac{ik_0 v_r^e}{2}\left(1+\frac{\eta_3}{k_0}\alpha_r^e\right)\left[1-\frac{\eta_1^2}{k_0^2}(K_{2r}^e)^2\right]\frac{\sin[K_{2r}^e b]}{K_{2r}^e}\frac{\chi_+(\alpha_r^e)}{M_+^{(e)}(\alpha_r^e)N_+^{(e)}(\alpha_r^e)}+C_r^e(\alpha_r^e)\right\}f_r^e$$
$$+\sum_{\substack{m=1\\ n\neq r}}^{\infty}C_m^e(\alpha_r^e)f_m^e$$
$$=-\frac{2k_0\sin\phi_0}{1+\eta_1\sin\phi_0}\frac{M_-^{(e)}(k_0\cos\phi_0)N_-^{(e)}(k_0\cos\phi_0)}{\chi_-(k_0\cos\phi_0)}\frac{e^{-ik_0 b\sin\phi_0}}{\alpha_r^e-k_0\cos\phi_0} \tag{34a}$$

with

$$C_m^e(\alpha_r^e)=\left(1+\frac{\eta_3}{k_2}\alpha_m^e\right)\frac{K_{2m}^e\sin[K_{2r}^e b]}{2\alpha_m^e(\alpha_r^e+\alpha_m^e)}\frac{M_+^{(e)}(\alpha_m^e)N_+^{(e)}(\alpha_m^e)}{\chi_+(\alpha_m^e)}$$
$$+\Omega_m^e\sum_{n=1}^{\infty}\frac{(\xi_n^e)^2\sin^2[\xi_n^e a]\left(1-\frac{\eta_3}{k_2}\beta_n^e\right)}{\beta_n^e\mu_n^e\vartheta_{mn}^e}$$
$$\times\left\{ik_0\left[1-\frac{\eta_1^2}{k_0^2}(K_{2r}^e)^2\right]\frac{\alpha_r^e\Omega_r^e}{\vartheta_{rn}^e}\frac{\sin[K_{2r}^e b]}{K_{2r}^e}\frac{\chi_+(\alpha_r^e)}{M_+^{(e)}(\alpha_r^e)N_+^{(e)}(\alpha_r^e)}\right.$$
$$\left.-\sum_{s=1}^{\infty}\frac{2\Omega_s^e}{v_s^e\vartheta_{sn}^e}\frac{K_{2s}^e\sin[K_{2s}^e b]}{(\alpha_r^e+\alpha_s^e)}\frac{M_+^{(e)}(\alpha_s^e)N_+^{(e)}(\alpha_s^e)}{\chi_+(\alpha_s^e)}\right\}. \tag{34b}$$

2.2. Odd Excitation

The solution for odd excitation is similar to that of even excitation. Indeed, by assuming a representation similar to (1a) with the superscript (e) being replaced by (o); it can be seen that all the boundary and continuity relations in (2a–i) remain valid for odd excitation case also, except (2c) and (2e) which are to be changed as

$$u_2^{(o)}(x,0) = 0, \qquad x < 0, \tag{35a}$$

$$u_3^{(o)}(x,0) = 0, \qquad x < 0. \tag{35b}$$

In this case the Wiener-Hopf equation reads

$$k_0 \frac{\chi(\alpha)}{K_0(\alpha) M^{(o)}(\alpha) N^{(o)}(\alpha)} R_+^{(o)}(\alpha) + \dot{F}_-^{(o)}(\alpha, b)$$
$$= \frac{2k_0 \sin\phi_0}{1 + \eta_1 \sin\phi_0} \frac{e^{-ik_0 b \sin\phi_0}}{\alpha - k_0 \cos\phi_0} + \sum_{m=1}^{\infty} \frac{K_{2m}^o \cos[K_{2m}^o b]}{[\alpha^2 - (\alpha_m^o)^2]} [f_m^o + \alpha g_m^o] \tag{36a}$$

with

$$N^{(o)}(\alpha) = \frac{K_2^o(\alpha)}{[K_2^o(\alpha)\cos(K_2^o b) - iK_0^o(\alpha)\sin(K_2^o b)]}, \tag{36b}$$

$$M^{(o)}(\alpha) = \frac{\sin[K_2(\alpha)b]}{K_2(\alpha)} + \frac{\eta_1}{ik_0}\cos[K_2(\alpha)b], \tag{36c}$$

$$K_{2m}^o = \sqrt{k_2^2 - (\alpha_m^o)^2} \tag{36d}$$

and

$$\begin{bmatrix} f_m^o \\ g_m^o \end{bmatrix} = \frac{2}{v_m^o} \int_0^b \begin{bmatrix} f^{(o)}(t) \\ g^{(o)}(t) \end{bmatrix} \sin[K_{2m}^o t] dt, \;\; v_m^o = b + \frac{\eta_1}{ik_0}\cos^2[K_{2m}^o b] \tag{36e}$$

where α_m^o are the roots of

$$\frac{\sin[K_{2m}^o b]}{K_{2m}^o} + \frac{\eta_1}{ik_0}\cos[K_{2m}^o b] = 0, \quad \Im m(\alpha_m^o) > \Im m(k_2). \tag{36f}$$

The application of the Wiener-Hopf procedure to (36a) gives:

$$k_0 \frac{\chi_+(\alpha)}{\sqrt{k_0+\alpha}M_+^{(o)}(\alpha)N_+^{(o)}(\alpha)} R_+^{(o)}(\alpha) = \frac{2k_0 \sin\phi_0}{1+\eta_1 \sin\phi_0}$$
$$\cdot \frac{\sqrt{k_0(1-\cos\phi_0)}M_-^{(o)}(k_0\cos\phi_0)N_-^{(o)}(k_0\cos\phi_0)}{\chi_-(k_0\cos\phi_0)} \frac{e^{-ik_0 b \sin\phi_0}}{\alpha - k_0\cos\phi_0}$$
$$-\sum_{m=1}^{\infty} \frac{K_{2m}^o \cos[K_{2m}^o b]}{2\alpha_m^o} \frac{\sqrt{k_0+\alpha_m^o}M_+^{(o)}(\alpha_m^o)N_+^{(o)}(\alpha_m^o)}{\chi_+(\alpha_m^o)} \frac{(f_m^o - \alpha_m^o g_m^o)}{\alpha+\alpha_m^o}. \tag{37}$$

Here $N_+^{(o)}(\alpha), M_+^{(o)}(\alpha)$ and $N_-^{(o)}(\alpha), M_-^{(o)}(\alpha)$ are the split functions resulting from the Wiener-Hopf factorization of (36b) and (36c) as

$$\frac{\chi(\alpha)}{K_0(\alpha)M^{(o)}(\alpha)N^{(o)}(\alpha)} = \frac{\chi_+(\alpha)}{\sqrt{k_0+\alpha}M_+^{(o)}(\alpha)N_+^{(o)}(\alpha)} \times \frac{\chi_-(\alpha)}{\sqrt{k_0-\alpha}M_-^{(o)}(\alpha)N_-^{(o)}(\alpha)}. \tag{38a}$$

The explicit expressions of $M_\pm^{(o)}(\alpha)$ and $N_\pm^{(o)}(\alpha)$ reads [5]

$$N_+^{(o)}(\alpha) = \left[\frac{K_2^o(\alpha)}{[K_2^o(\alpha)\cos(K_2^o b) - iK_0^o(\alpha)\sin(K_2^o b)]}\right]^{\frac{1}{2}} \prod_{m=1}^{\infty} \left(\frac{\alpha_m^o - \alpha}{\alpha_m^o + \alpha}\right)^{\frac{1}{2}}$$
$$\times \exp\frac{1}{2}\left\{\frac{2K_0^o(\alpha)b}{\pi} \ln\left(\frac{\alpha + iK_0^o(\alpha)}{k_0}\right) - iK_0^o(\alpha)b + q^{(o)}(\alpha)\right\}$$
$$\times \exp\left[\frac{i\alpha b}{\pi}\ln\left(\frac{2\alpha}{k_0}\right)\right], \tag{38b}$$

$$N_-^{(o)}(\alpha) = N_+^{(o)}(-\alpha) \quad \text{and} \quad M_-^{(o)}(\alpha) = M_+^{(o)}(-\alpha) \tag{38c}$$

and

$$q^{(o)}(\alpha) = PV\int_o^{\infty} \left\{\frac{b}{\pi} - \frac{\varpi^2 b - \left(\frac{\varpi^2}{\varpi'} - \varpi'\right)\sin(\varpi' b)\cos(\varpi' b)}{\pi[(\varpi')^2\sin^2(\varpi' b) + \varpi^2\cos^2(\varpi' b)]}\right\}$$
$$\ln\left(\frac{K_0(\varpi)+\alpha}{K_0(\varpi)-\alpha}\right) d\varpi \tag{38d}$$

with

$$M_+^{(o)}(\alpha) = \left[\frac{\sin(k_2 b)}{k_2} + \frac{i\eta_1}{k_0}\cos(k_2 b)\right]^{1/2} \exp\left\{\frac{i\alpha b}{\pi}\left[1 - C - \ln\left(\frac{\alpha b}{\pi}\right) + i\frac{\pi}{2}\right]\right\}$$
$$\times \prod_{m=1}^{\infty}\left(1 + \frac{\alpha}{\alpha_m^o}\right)\exp\left(\frac{i\alpha b}{m\pi}\right). \quad (38e)$$

and ϖ' being given by (30c). In (38e) C is the Euler's constant given by $C = 0.57721\ldots$. By using the continuity relations at the aperture $0 < y < b$, $x = 0$ one obtains infinitely many equations in infinite number of unknowns which give the constants f_r^o as follows:

$$\left\{-\frac{ik_0 v_r^o}{2}\left(1+\frac{\eta_3}{k_0}\alpha_r^o\right)\left[1-\frac{\eta_1^2}{k_0^2}(K_{2r}^o)^2\right]\frac{\cos[K_{2r}^o b]}{K_{2r}^o}\frac{\chi_+(\alpha_r^o)}{\sqrt{k_0+\alpha_r^o}M_+^{(o)}(\alpha_r^o)N_+^{(o)}(\alpha_r^o)}\right.$$
$$\left. + C_r^o(\alpha_r^o)\right\} f_r^o + \sum_{\substack{m=1 \\ n\neq r}} C_m^o(\alpha_r^o) f_m^o = \frac{2k_0 \sin\phi_0}{1 + \eta_1 \sin\phi_0}$$
$$\frac{\sqrt{k_0(1-\cos\phi_0)}M_-^{(o)}(k_0\cos\phi_0)N_-^{(o)}(k_0\cos\phi_0)}{\chi_-(k_0\cos\phi_0)}\frac{e^{-ik_0 b\sin\phi_0}}{\alpha_r^o - k_0\cos\phi_0} \quad (39a)$$

with

$$C_m^o(\alpha_r^o) = \left(1 - \frac{\eta_3}{k_2}\alpha_m^o\right)\frac{K_{2m}^o\cos[K_{2r}^o b]}{2\alpha_m^o(\alpha_r^o + \alpha_m^o)}\frac{\sqrt{k_0+\alpha_m^o}M_+^{(o)}(\alpha_m^o)N_+^{(o)}(\alpha_m^o)}{\chi_+(\alpha_m^o)}$$
$$- \Omega_m^o \sum_{n=1}^{\infty}\frac{(\xi_n^o)^2\cos^2[\xi_n^o a]\left(1 - \frac{\eta_3}{k_2}\beta_n\right)}{\beta_n \mu_n^o \vartheta_{mn}^o}$$
$$\times\left\{k_0\left[1-\frac{\eta_1^2}{k_0^2}(K_{2r}^o)^2\right]\frac{\alpha_r^o\Omega_r^o}{\vartheta_{rn}^o}\frac{\cos[K_{2r}^o b]}{K_{2r}^o}\frac{\chi_+(\alpha_r^o)}{\sqrt{k_0+\alpha_r^o}M_+^{(o)}(\alpha_r^o)N_+^{(o)}(\alpha_r^o)}\right.$$
$$\left. + \sum_{s=1}^{\infty}\frac{2\Omega_s^o}{v_s^o\vartheta_{sn}^o}\frac{K_{2s}^o\cos[K_{2s}^o b]}{(\alpha_r^o+\alpha_s^o)}\frac{\sqrt{k_0+\alpha_s^o}M_+^{(o)}(\alpha_s^o)N_+^{(o)}(\alpha_s^o)}{\chi_+(\alpha_s^o)}\right\}. \quad (39b)$$

Here, μ_n^o, Ω_m^o, ϑ_{mn}^o, and β_n^o stand for

$$\mu_n^o = a - \frac{\eta_2}{ik_1}\cos^2[\xi_n^o a], \quad (39c)$$

$$\Omega_m^o = \sin[K_{2m}^o a] - \frac{\eta_2}{ik_1}K_{2m}^o\cos[K_{2m}^o a], \quad (39d)$$

$$\vartheta_{mn}^o = (K_{2m}^o)^2 - (\xi_n^o)^2 \tag{39e}$$

and

$$\beta_n^o = \sqrt{k_1^2 - (\xi_n^o)^2} \tag{39f}$$

with ζ_n^o being the roots of

$$\sin[\xi_n^o a] - \frac{\eta_2}{ik_1}\xi_n^o \cos[\xi_n^o a] = 0. \tag{39g}$$

g_r^o can be expressed in terms of f_n^o as

$$g_r^o = \frac{\eta_3}{k_2} f_r^o + \frac{4\Omega_l^o}{v_l^o} \sum_{m=1}^{\infty} f_m^o \Omega_m^o \sum_{n=1}^{\infty} \frac{(\xi_n^o)^2 \cos^2[\xi_n^o a] \left(1 - \frac{\eta_3}{k_2}\beta_n\right)}{\beta_n \mu_n^o \vartheta_{mn}^o \vartheta_{rn}^o}. \tag{40}$$

3. ANALYSIS OF THE DIFFRACTED FIELD

The diffracted field in theregion $y > b$ for even and odd excitations can be obtained by taking the inverse Fourier transform of $F^{(e)}(\alpha, y)$ and $F^{(e)}(\alpha, y)$ respectively:

$$u_1^{(e)}(x, y) = \frac{1}{2\pi} \int_L A^{(e)}(\alpha) e^{iK_0(\alpha)(y-b)} e^{-i\alpha x} d\alpha, \tag{41a}$$

$$u_1^{(o)}(x, y) = \frac{1}{2\pi} \int_L A^{(o)}(\alpha) e^{iK_0(\alpha)(y-b)} e^{-i\alpha x} d\alpha. \tag{41b}$$

Here L is straight line parallel to the real α-axis lying in the strip $\Im m(k_0 \cos\phi_0) < \Im m(\alpha) < \Im m(k_0)$. The asymptotic evaluation of the integrals in (41a) and (41b) through the saddle-point technique enables us to write for the diffracted field

$$u_1(\rho, \phi) = \frac{u_1^{(e)}(\rho, \phi) + u_1^{(o)}(\rho, \phi)}{2} \tag{42a}$$

with

$$u_1^{(e)}(\rho, \phi) \sim \left\{ u_0 D^{(e)}(\phi, \phi_0) + \frac{e^{i\pi/4}}{\sqrt{2\pi}} \frac{\sin\phi}{1+\eta_1 \sin\phi} \frac{M_-^{(e)}(k_0 \cos\phi) N_-^{(e)}(k_0 \cos\phi)}{\chi_-(k_0 \cos\phi)} \right.$$
$$\left. \times \sum_{m=1}^{\infty} \frac{K_{2m}^e \sin[K_{2m}^e b]}{2\alpha_m^e} \frac{M_+^{(e)}(\alpha_m^e) N_+^{(e)}(\alpha_m^e)}{\chi_+(\alpha_m^e)} \frac{(f_m^e - \alpha_m^e g_m^e)}{\alpha_m^e - k_0 \cos\phi} \right\} \frac{e^{ik_0\rho}}{\sqrt{k_0\rho}}, \tag{42b}$$

$$u_1^{(o)}(\rho,\phi) \sim \left\{ u_0 D^{(o)}(\phi,\phi_0) + \frac{e^{i3\pi/4}}{\sqrt{2\pi}} \frac{\sin\phi}{1+\eta_1 \sin\phi} \right.$$

$$\frac{\sqrt{k_0(1-\cos\phi)}M_-^{(o)}(k_0\cos\phi)N_-^{(o)}(k_0\cos\phi)}{\chi_-(k_0\cos\phi)} \sum_{m=1}^{\infty} \frac{K_{2m}^o \cos[K_{2m}^o b]}{2\alpha_m^o}$$

$$\left. \frac{\sqrt{k_0+\alpha_m^o}M_+^{(o)}(\alpha_m^o)N_+^{(o)}(\alpha_m^o)}{\chi_+(\alpha_m^o)} \frac{(f_m^o - \alpha_m^o g_m^o)}{\alpha_m^o - k_0\cos\phi} \right\} \frac{e^{ik_0\rho}}{\sqrt{k_0\rho}}, \quad (42c)$$

$$u_0 = e^{-ik_0 b \sin\phi_0}, \quad (42d)$$

$$D^{(e)}(\phi,\phi_0) = e^{i3\pi/4}\sqrt{\frac{2}{\pi}} \frac{\sin\phi_0}{1+\eta_1\sin\phi_0} \frac{\sin\phi}{1+\eta_1\sin\phi}$$
$$\times \frac{M_-^{(e)}(k_0\cos\phi_0)N_-^{(e)}(k_0\cos\phi_0)M_-^{(e)}(k_0\cos\phi)N_-^{(e)}(k_0\cos\phi)}{\chi_-(k_0\cos\phi_0)\chi_-(k_0\cos\phi)}$$
$$\times \frac{1}{\cos\phi_0 + \cos\phi} \quad (42e)$$

and

$$D^{(o)}(\phi,\phi_0) = e^{i3\pi/4}\sqrt{\frac{2}{\pi}} \frac{\sin\phi_0}{1+\eta_1\sin\phi_0} \frac{\sin\phi}{1+\eta_1\sin\phi}$$
$$\times \frac{\sqrt{k_0(1-\cos\phi_0)}M_-^{(o)}(k_0\cos\phi_0)N_-^{(o)}(k_0\cos\phi_0)}{\chi_-^{(o)}(k_0\cos\phi_0)}$$
$$\times \frac{\sqrt{k_0(1-\cos\phi)}M_-^{(o)}(k_0\cos\phi)N_-^{(o)}(k_0\cos\phi)}{\chi_-(k_0\cos\phi)} \frac{1}{\cos\phi_0+\cos\phi} \quad (42f)$$

where (ρ,ϕ) are the cylindrical polar coordinates defined by $x = \rho\cos\phi,\ y-b = \rho\sin\phi$ and u_0 is the expression of the incident field at $y=b,\ x=0$.

In order to observe the influence of the different values of the parameters such as wall thickness, the impedances and the dielectric constants on the diffraction phenomenon, some numerical results concerning the variation of the amplitude of the diffracted field $(20\log|u_1^d \times \sqrt{k_0\rho}|)$ versus the observation angle (ϕ) are presented.

Since the field expressions given in (38) with (26) and (33) include infinite series, one must first check their convergence. From Fig. 3, it is observed that the truncation number can be chosen as $N = 10$. It is required a smaller N for increasing values of a/b as expected.

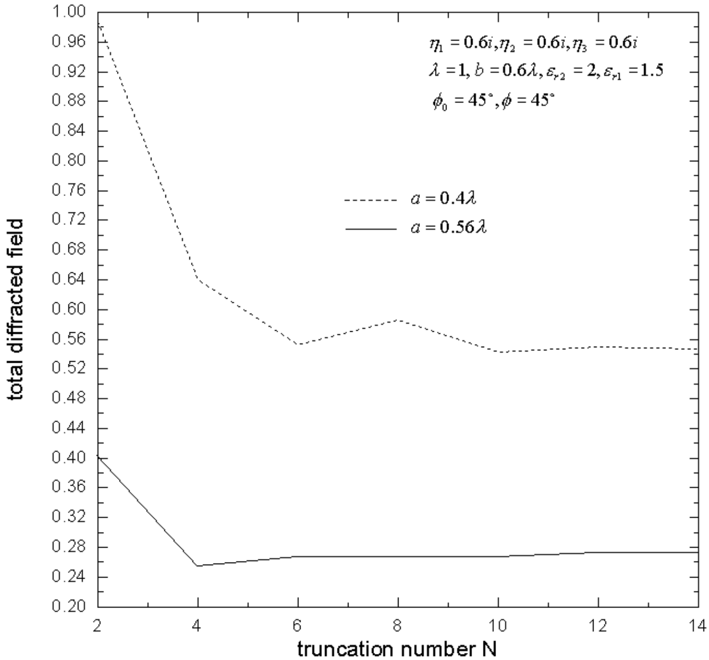

Figure 3. Determination of the truncation number for diffrent values of a.

Fig. 4 shows the dependence of the diffracted field with the observation angle for different values of the wall thickness k_0b.

Fig. 5 and Fig. 6, display the variation of the diffracted field versus the observation angle for different values of the dielectric permittivities. In Figs. 7, 8 and Fig. 9 the effects of the surface reactance of the horizontal and vertical walls of the parallel-plate wave guide on the diffraction phenomenon are presented, respectively.

In all of the figures, it is seen that the amplitude of the diffracted field exhibits a minimum for $\phi \in (-\pi/6, \phi/6)$. This is due to the fact that the diffracted field weakens fast in the dielectric load. It is also observed that the minimum lose its effect when $\varepsilon_{r1} = \varepsilon_{r2} \simeq 1$ (See Fig. 10). Notice that this result is in a good agreement with the one reported in ref. [3].

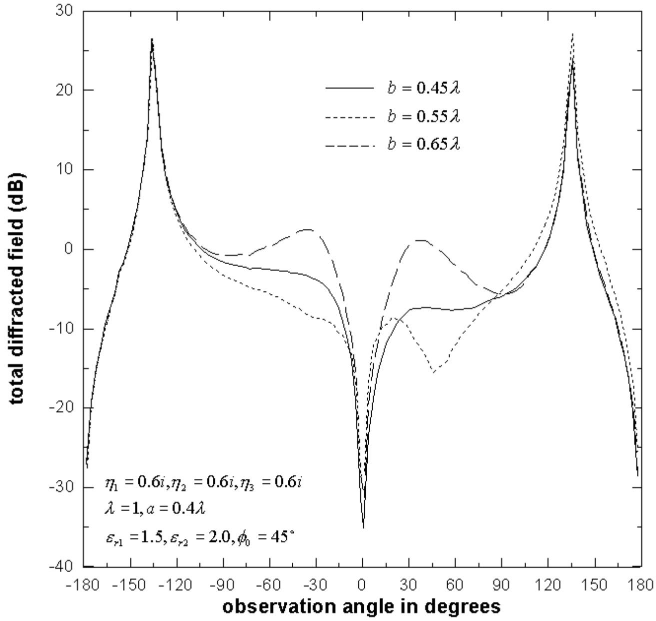

Figure 4. Diffracted field amplitude versus the observation angle for different values of the b.

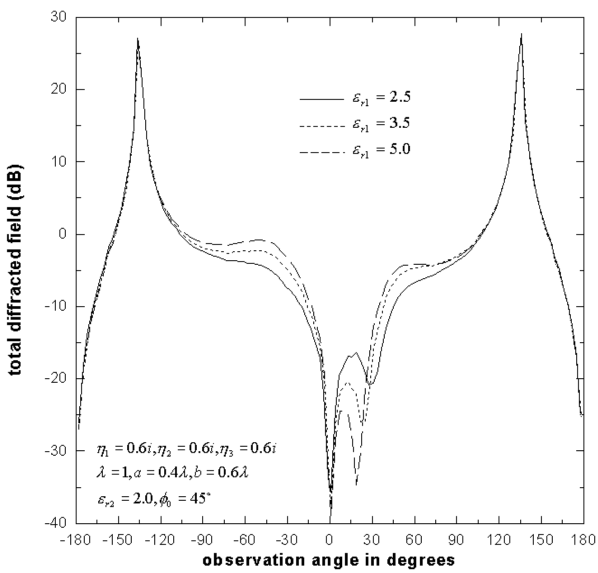

Figure 5. Diffracted field amplitude versus the observation angle for different values of the ε_{r1}.

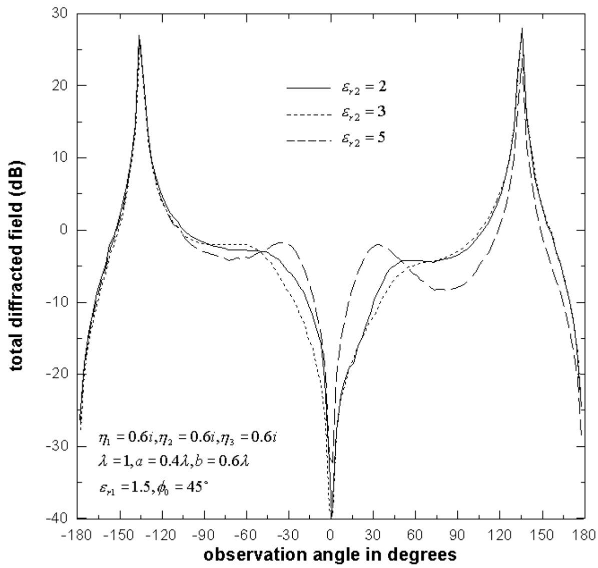

Figure 6. Diffracted field amplitude versus the observation angle for different values of the ε_{r2}.

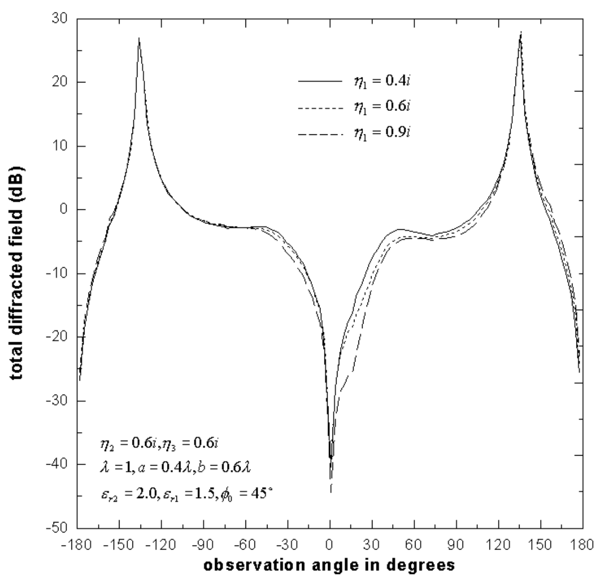

Figure 7. Diffracted field amplitude versus the observation angle for different values of the η_1.

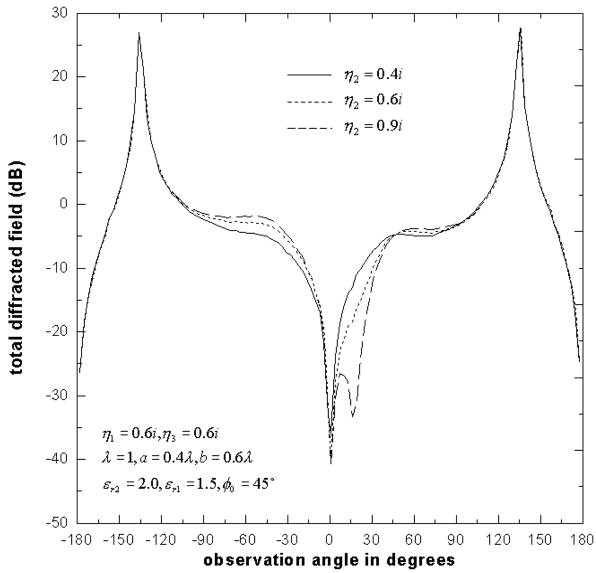

Figure 8. Diffracted field amplitude versus the observation angle for different values of the η_2.

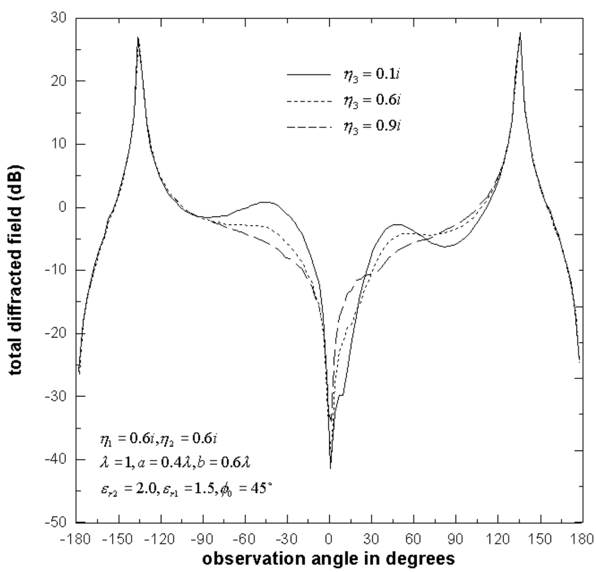

Figure 9. Diffracted field amplitude versus the observation angle for different values of the η_3.

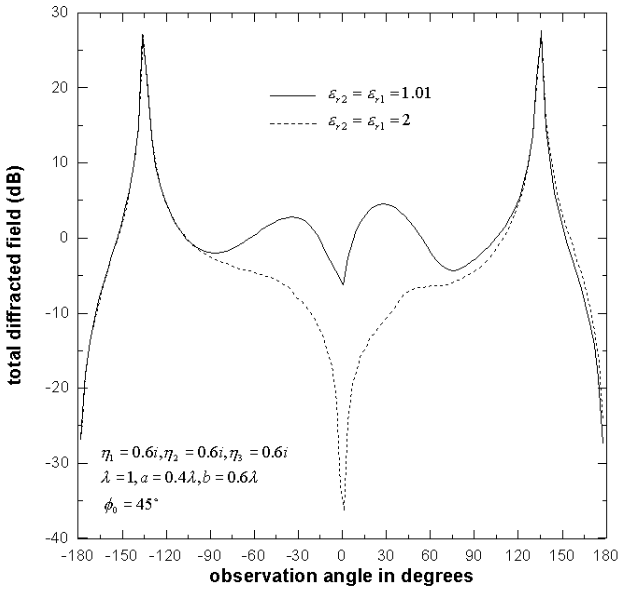

Figure 10. Diffracted field amplitude versus the observation angle for $\varepsilon_{r1} = \varepsilon_{r2}$.

4. CONCLUDING REMARKS

In the present work, the diffraction of the high frequency E_z-Polarized plane waves by a dielectric loaded thick-walled parallel-plate impedance waveguide is considered. In order to obtain the explicit expression of the diffracted field the problem is first reduced to a modified Wiener-Hopf equation of the second kind and then solved rigorously through the Wiener-Hopf technique.

For the special case, where $\varepsilon_{r1} = \varepsilon_{r2} = 1$, we have $K_0(\alpha) = K_1(\alpha) = K_2(\alpha) = K(\alpha) = \sqrt{k^2 - \alpha^2}$ and $M^{(e)}(\alpha)$, $M^{(o)}(\alpha)$, $N^{(e)}(\alpha)$, and $N^{(o)}(\alpha)$ reduce to

$$M^{(e)}(\alpha) \to \cos K(\alpha)b - \frac{\eta_1}{ik} K(\alpha) \sin K(\alpha)b, \tag{43a}$$

$$M^{(o)}(\alpha) \to \frac{\sin K(\alpha)b}{K(\alpha)} + \frac{\eta_1}{ik} \cos K(\alpha)b, \tag{43b}$$

$$N^{(e)}(\alpha) = N^{(o)}(\alpha) \to e^{iK(\alpha)b} \tag{43c}$$

respectively. In this case the diffracted field is reduced to the field diffracted from a thick-walled parallel-plate impedance waveguide [3].

On the other hand, if we consider the case where $a = 0$, then $u_2 = 0$ and the equation (20) reduced to following

$$\frac{\eta_3}{ik_2} f^{(e)}(y) + ig^{(e)}(y) = 0, \qquad 0 < y < b \tag{44a}$$

and

$$g_m = \frac{\eta_3}{k_2} f_m. \tag{44b}$$

In this case the total diffracted field is reduced the field diffracted by the junction of a thick impedance half-plane and a thick dielectric slab [7].

REFERENCES

1. Jones, D. S., "Diffraction by three semi-infinite planes," *Proc. Roy. Soc. Lond.*, Vol. A404, 299–321, 1986.
2. Abrahams, I. D., "Scattering of sound by three semi-infinite planes," *J. Sound Vibr.*, Vol. 112, 396–398, 1987.
3. Büyükaksoy, A. and B. Polat, "Plane wave diffraction by a thick-walled parallel-plate impedance waveguide," *IEEE Trans. Antennas and Propagattion*, Vol. 46, 1692–1699, 1998.
4. Alkumru, A., "Plane wave diffraction by three parallel thick impedance half plane," *JEWA*, Vol. 12, 801–819, 1998.
5. Mittra, R. and S. W. Lee, *Analytical Techniques in The Theory of Guided Waves*, The Macmillan Company, New York, 1971.
6. Senior, T. B. A., "Half-plane edge diffraction," *Radio Sci.*, Vol. 10 645–650, 1975.
7. Tayyar, I. H. and A. Büyükaksoy, "Plane wave diffraction by the junction of a thick impedance half-plane and a thick dielectric slab," *IEE Proc. Sci. Meas. Technol.*, (submitted).