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CYLINDRICAL VECTOR WAVE FUNCTION REPRESENTATION OF GREEN'S DYADIC IN GYROTROPIC BIANISOTROPIC MEDIA

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Abstract—This paper presents an eigenfunction expansion of the electric-type dyadic Green's function (DGF) for unbounded gyrotropic bianisotropic media in terms of cylindrical vector wave functions. The DGF is obtained based on the well-known Ohm-Rayleigh method together with dyadic identities formed by the differential, curl and dot product of the constitutive tensors and the cylindrical vector wave functions. Utilization of the dyadic identities greatly simplifies the process of finding the vector expansion coefficients of the DGF for gyrotropic bianisotropic media. The DGF derived is expressed in terms of the contribution from the irrotational vector wave functions and another contribution from the solenoidal vector wave functions, with the λ-domain integrals removed using the residue theorem. This result can be used to characterise electromagnetic waves in gyrotropic bianisotropic media and the idea can be extended to the development of DGF for some other media.

1. INTRODUCTION

The *d*yadic *G*reen's *f*unctions (DGFs) technique [1–4] has been widely used to characterize the electromagnetic waves in various boundary value problems for more than 20 years. It has long been proven to be a valuable tool in the representation of electromagnetic fields; and especially the DGFs necessitate as necessary kernels in the integral equation techniques including the Method of Moments and the Boundary Element Method. However, with the growing property complexity of the material associated with the electromagnetic process, the representation of DGF for the medium also becomes complicated. The dyadic Green's functions of canonical problems may be formulated in several ways. One of the common approaches is to express the Green's functions for defining electromagnetic vector potentials or fields in terms of the Fourier transform, whereas another approach is to represent the Green's functions for defining electromagnetic fields in terms of coordinates vectors and from a set of appropriate electric and magnetic vector potentials. Among all the available approaches, the vector wave function expansion approach is most widely employed to derive the dyadic Green's functions [1–3].

In recent years, due to the advances in material science and technology which have manifested manufacturing of various composite materials, increasing interest has been devoted to the study of electromagnetic wave interactions with complex media. These complex media include chiral [5]; Faraday chiral [6]; uniaxial chiral [7] and [8]; gyroelectric chiral [9–11]; uniaxial bianisotropic [12] and [13]; transversely bianisotropic uniaxial [14]; biisotropic [15]; and gyrotropic bianisotropic [16] and [17] materials. A generalization of these materials mentioned so far would be the gyrotropic bianisotropic media which have all the constitutive dyadics expressed in coaxially gyrotropic carrying a total of twelve independent scalars/pseudoscalars not necessarily constrained by losslessness, reciprocity [2], or uniformity conditions. This class of materials may be fabricated by immersing simultaneously various elements as described in [18] into a host dielectric/ferrite/gyroelectric medium.

Although the DGFs in isotropic media have been well-studied in the last three decades, complete formulation of the DGFs in various media using the eigenfunction expansion technique has not been achieved so far, especially for the case discussed here. Although the field representations in terms of cylindrical vector wave functions have been obtained [13, 19–21], however, the procedure requires formulation in the Cartesian Fourier domain followed by cumbersome rectangular-cylindrical coordinates transformation and grouping of terms. Most importantly, the irrotational Green's dyadic was missing in many of the previous representations. Hence, a more rigorous and direct method is proposed in this paper whereby the DGF for gyrotropic bianisotropic media (consisting of irrotational and solenoidal contributions) is obtained. Comparison of the DGF obtained here gyrotropic bianisotropic media with those available ones for some special types of media are made after the reduction from the general form in the present paper shows a good agreement and it is also revealed that all the other special Green's dyadics can be easily obtained by simply reducing the parameter tensors in the constitutive relations.

2. DGFS FOR UNBOUNDED GYROTROPIC BIANISOTROPIC MEDIA

A homogeneous gyrotropic bianisotropic medium is characterized by the following constitutive equations:

$$\boldsymbol{D} = \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} + \overline{\boldsymbol{\xi}} \cdot \boldsymbol{H}, \tag{1a}$$

$$\boldsymbol{B} = \overline{\boldsymbol{\zeta}} \cdot \boldsymbol{E} + \overline{\boldsymbol{\mu}} \cdot \boldsymbol{H}, \tag{1b}$$

where the constitutive tensors/pseudodyadics $\overline{\boldsymbol{\epsilon}}$, $\overline{\boldsymbol{\mu}}$, $\overline{\boldsymbol{\xi}}$ and $\overline{\boldsymbol{\zeta}}$ take, respectively, the form

$$\overline{\boldsymbol{g}} = \begin{bmatrix} g_t & -g_a & 0 \\ g_a & g_t & 0 \\ 0 & 0 & g_z \end{bmatrix}. \tag{2}$$

It should be mentioned here that g_a is in fact an imaginary quantity; hence it appears as g_a and $-g_a$ to denote itself and its complex conjugate, respectively.

Substituting (1) into the source-incorporated Maxwell's equations for an electric source $\boldsymbol{J}(\boldsymbol{r}')$ leads to

$$\begin{aligned} \nabla \times [\overline{\boldsymbol{\alpha}} \cdot \nabla \times \boldsymbol{E}(\boldsymbol{r})] - i\,\omega \nabla \times \left[\overline{\boldsymbol{\beta}} \cdot \boldsymbol{E}(\boldsymbol{r})\right] \\ + i\,\omega \overline{\boldsymbol{\gamma}} \cdot \nabla \times \boldsymbol{E}(\boldsymbol{r}) - \omega^2 \overline{\boldsymbol{\delta}} \cdot \boldsymbol{E}(\boldsymbol{r}) = i\,\omega \boldsymbol{J}(\boldsymbol{r}') \end{aligned} \tag{3}$$

where and subsequently the following notations are assumed to shorten the lengthy expressions:

$$\overline{\boldsymbol{\alpha}} = \overline{\boldsymbol{\mu}}^{-1}, \tag{4a}$$

$$\overline{\boldsymbol{\beta}} = \overline{\boldsymbol{\mu}}^{-1} \cdot \overline{\boldsymbol{\zeta}}, \tag{4b}$$

$$\overline{\boldsymbol{\gamma}} = \overline{\boldsymbol{\xi}} \cdot \overline{\boldsymbol{\mu}}^{-1}, \tag{4c}$$

$$\overline{\boldsymbol{\delta}} = \overline{\boldsymbol{\epsilon}} - \overline{\boldsymbol{\xi}} \cdot \overline{\boldsymbol{\mu}}^{-1} \cdot \overline{\boldsymbol{\zeta}}. \tag{4d}$$

2.1. General Formulation of Unbounded DGFs

The electric field can thus be expressed in terms of the DGF and the current source distribution as

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega \int_{V'} \overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r'}) \cdot \boldsymbol{J}(\boldsymbol{r'})\, dV', \tag{5}$$

where V' denotes the volume occupied by the exciting current source. Similarly, substituting (5) into (3) leads to

$$\boldsymbol{\nabla} \times \left(\overline{\boldsymbol{\alpha}} \cdot \boldsymbol{\nabla} \times \overline{\boldsymbol{G}}\right) - i\,\omega \boldsymbol{\nabla} \times \left(\overline{\boldsymbol{\beta}} \cdot \overline{\boldsymbol{G}}\right) + i\,\omega \overline{\boldsymbol{\gamma}} \cdot \boldsymbol{\nabla} \times \overline{\boldsymbol{G}} - \omega^2 \overline{\boldsymbol{\delta}} \cdot \overline{\boldsymbol{G}} = \overline{\boldsymbol{I}} \delta(\boldsymbol{r} - \boldsymbol{r'}), \tag{6}$$

where $\overline{\boldsymbol{I}}$ denotes the dyadic identity while $\delta(\boldsymbol{r}-\boldsymbol{r'})$ stands for the Dirac delta function.

According to the well-known Ohm-Rayleigh method, the source term in (6) can be expanded in terms of the solenoidal and irrotational cylindrical vector wave functions in cylindrical coordinate system. Thus, we have

$$\overline{\boldsymbol{I}} \delta(\boldsymbol{r} - \boldsymbol{r'}) = \int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_n(h,\lambda)\boldsymbol{A}_n(h,\lambda) + \boldsymbol{N}_n(h,\lambda)\boldsymbol{B}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda)\boldsymbol{C}_n(h,\lambda)\right], \tag{7}$$

where $\boldsymbol{M}_n(h,\lambda)$ & $\boldsymbol{N}_n(h,\lambda)$ are the solenoidal, and $\boldsymbol{L}_n(h,\lambda)$ is the irrotational, cylindrical vector wave functions while λ and h are the spectral longitudinal and radial wave numbers, respectively. The solenoidal and irrotational cylindrical vector wave functions are defined as [3]

$$\boldsymbol{M}_n(h,\lambda) = \boldsymbol{\nabla} \times \left[\Psi_n(h,\lambda)\widehat{\boldsymbol{z}}\right], \tag{8a}$$

$$\boldsymbol{N}_n(h,\lambda) = \frac{1}{k_\lambda} \boldsymbol{\nabla} \times \boldsymbol{M}_n(h,\lambda), \tag{8b}$$

$$\boldsymbol{L}_n(h,\lambda) = \boldsymbol{\nabla} \left[\Psi_n(h,\lambda)\right], \tag{8c}$$

where $k_\lambda = \sqrt{\lambda^2 + h^2}$, and the generating function is given by

$$\Psi_n(h,\lambda) = J_n(\lambda\rho)e^{i(n\phi+hz)}. \tag{9}$$

The vector expansion coefficients in (7), $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, and $\boldsymbol{C}_n(h,\lambda)$, are to be determined from the orthogonality relationships among the cylindrical vector wave functions [1].

Therefore, by taking the scalar product of (7) with $\boldsymbol{M}_{-n'}(-h',-\lambda')$, $\boldsymbol{N}_{-n'}(-h',-\lambda')$ and $\boldsymbol{L}_{-n'}(-h',-\lambda')$ each at a time, the vector expansion coefficients are given by:

$$\boldsymbol{A}_n(h,\lambda) = \frac{1}{4\pi^2\lambda}\boldsymbol{M'}_{-n}(-h,-\lambda), \tag{10a}$$

$$\boldsymbol{B}_n(h,\lambda) = \frac{1}{4\pi^2\lambda}\boldsymbol{N'}_{-n}(-h,-\lambda), \tag{10b}$$

$$\boldsymbol{C}_n(h,\lambda) = \frac{\lambda}{4\pi^2(\lambda^2+h^2)}\boldsymbol{L'}_{-n}(-h,-\lambda), \tag{10c}$$

where the prime notation of the cylindrical vector wave functions denotes the expressions at the source point $\boldsymbol{r'}$.

The dyadic Green's function can thus be expanded as:

$$\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \left[\boldsymbol{M}_n(h,\lambda)\,\boldsymbol{a}_n(h,\lambda) + \boldsymbol{N}_n(h,\lambda)\,\boldsymbol{b}_n(h,\lambda) + \boldsymbol{L}_n(h,\lambda)\,\boldsymbol{c}_n(h,\lambda)\right], \tag{11}$$

where the vector expansion coefficients $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$ and $\boldsymbol{c}_n(h,\lambda)$ are obtained by substituting (11) and (7) into (6), which the dyadic Green's function must satisfy. Noting the common properties of the vector wave functions [1]

$$\boldsymbol{M}_n(h,\lambda) = \frac{1}{k_\lambda}\boldsymbol{\nabla}\times\boldsymbol{N}_n(h,\lambda), \tag{12a}$$

$$\boldsymbol{N}_n(h,\lambda) = \frac{1}{k_\lambda}\boldsymbol{\nabla}\times\boldsymbol{M}_n(h,\lambda), \tag{12b}$$

$$\boldsymbol{\nabla}\times\boldsymbol{L}_n(h,\lambda) = 0, \tag{12c}$$

and those uncommon properties of the vector wave functions newly obtained here

$$\boldsymbol{\nabla}\times[\overline{\boldsymbol{g}}\cdot\boldsymbol{M}_n(h,\lambda)] = -i\,h\,g_a\boldsymbol{M}_n(h,\lambda) + k_\lambda g_t\boldsymbol{N}_n(h,\lambda), \tag{13a}$$

$$\boldsymbol{\nabla}\times[\overline{\boldsymbol{g}}\cdot\boldsymbol{N}_n(h,\lambda)] = \frac{1}{k_\lambda}\left(h^2\,g_t+\lambda^2\,g_z\right)\boldsymbol{M}_n(h,\lambda) - i\,h\,g_a\boldsymbol{N}_n(h,\lambda), \tag{13b}$$

$$\boldsymbol{\nabla}\times[\overline{\boldsymbol{g}}\cdot\boldsymbol{L}_n(h,\lambda)] = i\,h\,(g_z-g_t)\,\boldsymbol{M}_n(h,\lambda) - k_\lambda g_a\boldsymbol{N}_n(h,\lambda), \tag{13c}$$

$$\overline{\boldsymbol{g}} \cdot \boldsymbol{M}_n(h,\lambda) = g_t \boldsymbol{M}_n(h,\lambda) - \frac{i\,h}{k_\lambda} g_a \boldsymbol{N}_n(h,\lambda) + \frac{\lambda^2}{k_\lambda^2} g_a \boldsymbol{L}_n(h,\lambda), \quad (13\text{d})$$

$$\overline{\boldsymbol{g}} \cdot \boldsymbol{N}_n(h,\lambda) = -\frac{i\,h}{k_\lambda} g_a \boldsymbol{M}_n(h,\lambda) + \frac{h^2\, g_t + \lambda^2\, g_z}{k_\lambda^2} \boldsymbol{N}_n(h,\lambda) + \frac{i\,h\,\lambda^2}{k_\lambda^3}(g_t - g_z)\boldsymbol{L}_n(h,\lambda), \quad (13\text{e})$$

$$\overline{\boldsymbol{g}} \cdot \boldsymbol{L}_n(h,\lambda) = -g_a \boldsymbol{M}_n(h,\lambda) + \frac{i\,h}{k_\lambda}(g_z - g_t)\boldsymbol{N}_n(h,\lambda) + \frac{1}{k_\lambda^2}\left(h^2\, g_z + \lambda^2\, g_t\right)\boldsymbol{L}_n(h,\lambda); \quad (13\text{f})$$

we end up with

$$\begin{aligned}\int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \Big[& k_\lambda \left(\boldsymbol{\nabla}\times\overline{\boldsymbol{\alpha}}\cdot\boldsymbol{N}\boldsymbol{a} + \boldsymbol{\nabla}\times\overline{\boldsymbol{\alpha}}\cdot\boldsymbol{M}\boldsymbol{b}\right) \\ & - i\omega\left(\boldsymbol{\nabla}\times\overline{\boldsymbol{\beta}}\cdot\boldsymbol{M}\boldsymbol{a} + \boldsymbol{\nabla}\times\overline{\boldsymbol{\beta}}\cdot\boldsymbol{N}\boldsymbol{b} + \boldsymbol{\nabla}\times\overline{\boldsymbol{\beta}}\cdot\boldsymbol{L}\boldsymbol{c}\right) \\ & + i\omega k_\lambda\left(\overline{\boldsymbol{\gamma}}\cdot\boldsymbol{N}\boldsymbol{a} + \overline{\boldsymbol{\gamma}}\cdot\boldsymbol{M}\boldsymbol{b}\right) - \omega^2\left(\overline{\boldsymbol{\delta}}\cdot\boldsymbol{M}\boldsymbol{a} + \overline{\boldsymbol{\delta}}\cdot\boldsymbol{N}\boldsymbol{b} + \overline{\boldsymbol{\delta}}\cdot\boldsymbol{L}\boldsymbol{c}\right)\Big] \\ = \int_0^\infty d\lambda \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} & \left(\boldsymbol{M}\boldsymbol{A} + \boldsymbol{N}\boldsymbol{B} + \boldsymbol{L}\boldsymbol{C}\right) \end{aligned} \quad (14)$$

where the notations $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{M}, \boldsymbol{N}$ and $\boldsymbol{L}$ represent $\boldsymbol{a}_n(h,\lambda)$, $\boldsymbol{b}_n(h,\lambda)$, $\boldsymbol{c}_n(h,\lambda)$, $\boldsymbol{A}_n(h,\lambda)$, $\boldsymbol{B}_n(h,\lambda)$, $\boldsymbol{C}_n(h,\lambda)$, $\boldsymbol{M}_n(h,\lambda)$, $\boldsymbol{N}_n(h,\lambda)$ and $\boldsymbol{L}_n(h,\lambda)$ respectively.

Taking the anterior scalar product of (14) with $\boldsymbol{M}_{-n'}(-h',-\lambda')$, $\boldsymbol{N}_{-n'}(-h',-\lambda')$ and $\boldsymbol{L}_{-n'}(-h',-\lambda')$, respectively, making use of the identities shown in (13), and performing the integration over the entire space, we can formulate the equations satisfied by the unknown vectors and the known scalar and vector parameters in a matrix form as given below:

$$[\boldsymbol{\Omega}][\boldsymbol{X}] = [\boldsymbol{\Theta}], \quad (15)$$

where $[\boldsymbol{\Omega}]$ is a 3×3 matrix given by

$$[\boldsymbol{\Omega}] = [\boldsymbol{\Omega}_1 \boldsymbol{\Omega}_2 \boldsymbol{\Omega}_3] \quad (16)$$

with

$$\boldsymbol{\Omega}_1 = \begin{bmatrix} h^2\alpha_t + \lambda^2\alpha_z - \omega\,h\,(\beta_a - \gamma_a) - \omega^2\delta_t \\ -i\,h\,k_\lambda\alpha_a + \dfrac{i\,\omega}{k_\lambda}\left(h^2\gamma_t + \lambda^2\gamma_z - k_\lambda^2\beta_t + \omega\,h\,\delta_a\right) \\ -\dfrac{\omega\,h\,\lambda^2}{k_\lambda^2}(\gamma_t - \gamma_z) - \dfrac{\omega^2\lambda^2}{k_\lambda^2}\delta_a \end{bmatrix}, \quad (17\text{a})$$

$$\boldsymbol{\Omega}_2 = \begin{bmatrix} -i\,h\,k_\lambda \alpha_a - \dfrac{i\,\omega}{k_\lambda}\left(h^2\beta_t + \lambda^2\beta_z - k_\lambda^2\gamma_t - \omega\,h\,\delta_a\right) \\ k_\lambda^2\alpha_t - \omega\,h\,(\beta_a - \gamma_a) - \dfrac{\omega^2}{k_\lambda^2}\left(h^2\delta_t + \lambda^2\delta_z\right) \\ \dfrac{i\,\omega\lambda^2}{k_\lambda}\gamma_a - \dfrac{i\,h\,\omega^2\lambda^2}{k_{\lambda^3}}(\delta_t - \delta_z) \end{bmatrix}, \tag{17b}$$

$$\boldsymbol{\Omega}_3 = \begin{bmatrix} \omega\,h\,(\beta_z - \beta_t) + \omega^2\delta_a \\ i\,\omega\,k_\lambda\beta_a - \dfrac{i\,\omega^2\,h}{k_\lambda}(\delta_z - \delta_t) \\ -\dfrac{\omega^2}{k_\lambda^2}\left(h^2\delta_z + \lambda^2\delta_t\right) \end{bmatrix}; \tag{17c}$$

while $[\boldsymbol{X}]$ and $[\boldsymbol{\Theta}]$ are two column vectors given, respectively, by

$$[\boldsymbol{X}] = \begin{bmatrix} \boldsymbol{a}_n(h,\lambda) \\ \boldsymbol{b}_n(h,\lambda) \\ \boldsymbol{c}_n(h,\lambda) \end{bmatrix}, \tag{18a}$$

$$[\boldsymbol{\Theta}] = \begin{bmatrix} \boldsymbol{A}_n(h,\lambda) \\ \boldsymbol{B}_n(h,\lambda) \\ \boldsymbol{C}_n(h,\lambda) \end{bmatrix}. \tag{18b}$$

By solving for the inverse of (16), the vector expansion coefficients for the DGF can be shown in an explicit form as follows:

$$\boldsymbol{a}_n(h,\lambda) = \frac{1}{\Gamma}\left[\alpha_1\boldsymbol{A}_n(h,\lambda) - \beta_1\boldsymbol{B}_n(h,\lambda) + \frac{\gamma_1}{\omega}\boldsymbol{C}_n(h,\lambda)\right], \tag{19a}$$

$$\boldsymbol{b}_n(h,\lambda) = \frac{1}{\Gamma}\left[-\alpha_2\boldsymbol{A}_n(h,\lambda) - \beta_2\boldsymbol{B}_n(h,\lambda) + \frac{\gamma_2}{\omega}\boldsymbol{C}_n(h,\lambda)\right], \tag{19b}$$

$$\boldsymbol{c}_n(h,\lambda) = \frac{1}{\Gamma}\left[\frac{\alpha_3}{\omega}\boldsymbol{A}_n(h,\lambda) - \frac{\beta_3}{\omega}\boldsymbol{B}_n(h,\lambda) + \frac{\gamma_3}{\omega^2}\boldsymbol{C}_n(h,\lambda)\right], \tag{19c}$$

where the coefficients α_1, α_2, α_3, β_1, β_2, β_3, γ_1, γ_2, and γ_3 are the numerators, and the coefficient Γ is the denominator, of the elements of the inverse of the matrix Ω respectively. Therefore, the unbounded dyadic Green's function can be written as

$$\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r}') = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^{\infty} \frac{1}{4\,\pi^2\Gamma\lambda} \cdot$$
$$\Big\{\boldsymbol{M}_n(h,\lambda)\Big[\alpha_1\boldsymbol{M'}_{-n}(-h,-\lambda) - \beta_1\boldsymbol{N'}_{-n}(-h,-\lambda)$$
$$+\frac{\gamma_1\lambda^2}{k_\lambda^2\omega}\boldsymbol{L'}_{-n}(-h,-\lambda)\Big] + \boldsymbol{N}_n(h,\lambda)\Big[-\alpha_2\boldsymbol{M'}_{-n}(-h,-\lambda)$$

$$-\beta_2 \boldsymbol{N'}_{-n}(-h,-\lambda) + \frac{\gamma_2\lambda^2}{k_\lambda^2\omega}\boldsymbol{L'}_{-n}(-h,-\lambda)\Big]$$
$$+\boldsymbol{L}_n(h,\lambda)\Big[\frac{\alpha_3}{\omega}\boldsymbol{M'}_{-n}(-h,-\lambda) - \frac{\beta_3}{\omega}\boldsymbol{N'}_{-n}(-h,-\lambda)$$
$$+\frac{\gamma_3\lambda^2}{k_\lambda^2\omega^2}\boldsymbol{L'}_{-n}(-h,-\lambda)\Big]\Big\}. \tag{20}$$

In this way, the dyadic Green's function for an unbounded gyrotropic bianisotropic medium is explicitly represented in the form of the eigenfunction expansion in terms of the cylindrical vector wave functions, as given in (20).

Eq. (20) is represented in the form of a double integral and a single summation. It can be further simplified by eliminating one of the integrals. In order to simplify (20), the residue theorem is needed, and we must first extract the part in (20) which does not satisfy the Jordan lemma as was done in [1]. To do so, we write

$$\boldsymbol{L}_n(h,\lambda) = \boldsymbol{L}_{nt}(h,\lambda) + \boldsymbol{L}_{nz}(h,\lambda), \tag{21a}$$
$$\boldsymbol{L}'_{-n}(-h,-\lambda) = \boldsymbol{L}'_{-nt}(-h,-\lambda) + \boldsymbol{L}'_{-nz}(-h,-\lambda), \tag{21b}$$
$$\boldsymbol{N}_n(h,\lambda) = \boldsymbol{N}_{nt}(h,\lambda) + \boldsymbol{N}_{nz}(h,\lambda), \tag{21c}$$
$$\boldsymbol{N}'_{-n}(-h,-\lambda) = \boldsymbol{N}'_{-nt}(-h,-\lambda) + \boldsymbol{N}'_{-nz}(-h,-\lambda), \tag{21d}$$
$$\boldsymbol{L}_{nt}(h,\lambda) = -\frac{ik_\lambda}{h}\boldsymbol{N}_{nt}(h,\lambda), \tag{21e}$$
$$\boldsymbol{L}'_{-nt}(-h,-\lambda) = \frac{ik_\lambda}{h}\boldsymbol{N}'_{-nt}(-h,-\lambda), \tag{21f}$$
$$\boldsymbol{L}_{nz}(h,\lambda) = \frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda), \tag{21g}$$
$$\boldsymbol{L}'_{-nz}(-h,-\lambda) = -\frac{ihk_\lambda}{\lambda^2}\boldsymbol{N}'_{-nz}(-h,-\lambda). \tag{21h}$$

where the subscripts t and z denote the transverse vector components and the vector z-components of the two functions $\boldsymbol{L}_n(h,\lambda)$ and $\boldsymbol{N}_n(h,\lambda)$, respectively. The coefficient Γ is re-written in the following form in order to perform the λ integration:

$$\Gamma = k_\lambda^4\left(-\epsilon_t\mu_t + \zeta_t\xi_t\right)\left(\lambda^2-\lambda_1^2\right)\left(\lambda^2-\lambda_2^2\right) \tag{22}$$

where

$$\lambda_1 = \sqrt{\frac{1}{2\epsilon_t\mu_t - 2\zeta_t\xi_t}\left[p_\lambda - \sqrt{p_\lambda^2 - q_\lambda}\right]}, \tag{23a}$$

$$\lambda_2 = \sqrt{\frac{1}{2\epsilon_t\mu_t - 2\zeta_t\xi_t}\left[p_\lambda + \sqrt{p_\lambda^2 - q_\lambda}\right]}, \tag{23b}$$

with the coefficients p_λ and q_λ as shown in Appendix A.

In terms of these functions, (20) can be rewritten in the form

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = &\int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \frac{1}{4\,\pi^2\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)} \\ &[\tau_1 \boldsymbol{M}_n(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda) + \tau_2 \boldsymbol{M}_n(h,\lambda) \\ &\boldsymbol{N'}_{-nt}(-h,-\lambda) + \tau_3 \boldsymbol{M}_n(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda) \\ &+ \tau_4 \boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda) + \tau_5 \boldsymbol{N}_{nt}(h,\lambda) \\ &\boldsymbol{M'}_{-n}(-h,-\lambda) + \tau_6 \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda) \\ &+ \tau_7 \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda) + \tau_8 \boldsymbol{N}_{nt}(h,\lambda) \\ &\boldsymbol{N'}_{-nz}(-h,-\lambda) + \tau_9 \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)]\,,\end{aligned} \tag{24}$$

where the coefficients for τ_1 to τ_9 are as shown in the Appendix A.

2.2. Analytical Evaluation of the λ Integral

Using a similar idea as shown in [1], the irrotational dyadic Green's function can be obtained from (7) as

$$\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\delta(\boldsymbol{r}-\boldsymbol{r}') = \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \frac{1}{4\pi^2\lambda}\frac{k_\lambda^2}{\lambda^2}\boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda). \tag{25}$$

With some algebraic manipulations, we can split (24) into

$$\begin{aligned}\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = &-\int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \frac{1}{4\,\pi^2\lambda}\frac{k_\lambda^2}{\omega^2\epsilon_z\lambda^2} \\ &\cdot \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N}'_{-nz}(-h,-\lambda) \\ &+ \int_0^\infty d\lambda \int_{-\infty}^\infty dh \sum_{n=-\infty}^\infty \frac{1}{4\,\pi^2\lambda(\lambda^2-\lambda_1^2)(\lambda^2-\lambda_2^2)} \\ &[\tau_1 \boldsymbol{M}_n(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda) + \tau_2 \boldsymbol{M}_n(h,\lambda) \\ &\boldsymbol{N'}_{-nt}(-h,-\lambda) + \tau_3 \boldsymbol{M}_n(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda) \\ &+ \tau_4 \boldsymbol{N}_{nt}(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda) + \tau_5 \boldsymbol{N}_{nt}(h,\lambda) \\ &\boldsymbol{M'}_{-n}(-h,-\lambda) + \tau_6 \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nt}(-h,-\lambda) \\ &+ \tau_7 \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{M'}_{-n}(-h,-\lambda) + \tau_8 \boldsymbol{N}_{nt}(h,\lambda) \\ &\boldsymbol{N'}_{-nz}(-h,-\lambda) + \tau_{10} \boldsymbol{N}_{nz}(h,\lambda)\boldsymbol{N'}_{-nz}(-h,-\lambda)]\,.\end{aligned} \tag{26}$$

The first integration term in (26) represents the contribution from the irrotational vector wave functions while the second integration term stands for the contribution from the solenoidal vector wave functions and can be evaluated by making use of the residue theorem in λ-plane. Hence after some mathematical manipulations, we arrive at the final unbounded dyadic Green's function for a gyrotropic medium for $\rho \gtrless \rho'$:

$$\overline{\boldsymbol{G}}_0(\boldsymbol{r},\boldsymbol{r'}) = -\frac{1}{\omega^2\epsilon_z}\,\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r'}) \pm \frac{i}{4\pi}\int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \frac{1}{2(\lambda_1^2-\lambda_2^2)} \sum_{j=1}^{2} \frac{(-1)^{j+1}}{\lambda_j^2}$$
$$\times \begin{Bmatrix} \boldsymbol{M}_n^{(1)}(h,\lambda_j)\boldsymbol{P'}_{-n}(-h,-\lambda_j) + \boldsymbol{Q}_n^{(1)}(h,\lambda_j)\boldsymbol{M'}_{-n}(-h,-\lambda_j) \\ \boldsymbol{M}_n(h,-\lambda_j)\boldsymbol{P'}^{(1)}_{-n}(-h,\lambda_j) + \boldsymbol{Q}_n(h,-\lambda_j)\boldsymbol{M'}^{(1)}_{-n}(-h,\lambda_j) \end{Bmatrix.$$
$$\begin{matrix} +\boldsymbol{U}_n^{(1)}(h,\lambda_j)\boldsymbol{N'}_{-nt}(-h,-\lambda_j) + \boldsymbol{V}_n^{(1)}(h,\lambda_j)\boldsymbol{N'}_{-nz}(-h,-\lambda_j) \\ +\boldsymbol{U}_n(h,-\lambda_j)\boldsymbol{N'}^{(1)}_{-nt}(-h,\lambda_j) + \boldsymbol{V}_n(h,-\lambda_j)\boldsymbol{N'}^{(1)}_{-nz}(-h,\lambda_j) \end{matrix}\Bigg\}, \tag{27}$$

where the superscript (1) of the vector wave functions denotes the first-kind cylindrical Hankel function $H_n^{(1)}(\lambda\rho)$. The vector wave functions $\boldsymbol{P'}_{-n,-h}(-\lambda_j)$, $\boldsymbol{Q}_{n,h}(\lambda_j)$, $\boldsymbol{U}_{n,h}(\lambda_j)$ and $\boldsymbol{V}_{n,h}(\lambda_j)$ are given respectively by

$$\boldsymbol{P'}_{-n,}(-h,-\lambda_j) = \tau_1\boldsymbol{M'}_{-n}(-h,-\lambda_j) + \tau_2\boldsymbol{N'}_{-nt}(-h,-\lambda_j) + \tau_3\boldsymbol{N'}_{-nz}(-h,-\lambda_j), \tag{28a}$$
$$\boldsymbol{Q}_n(h,\lambda_j) = \tau_5\boldsymbol{N}_{nt}(h,\lambda_j) + \tau_7\boldsymbol{N}_{nz}(h,\lambda_j), \tag{28b}$$
$$\boldsymbol{U}_n(h,\lambda_j) = \tau_4\boldsymbol{N}_{nt}(h,\lambda_j) + \tau_6\boldsymbol{N}_{nz}(h,\lambda_j), \tag{28c}$$
$$\boldsymbol{V}_n(h,\lambda_j) = \tau_8\boldsymbol{N}_{nt}(h,\lambda_j) + \tau_{10}\boldsymbol{N}_{nz}(h,\lambda_j). \tag{28d}$$

The DGF obtained above could be used to derive the DGF of other types of media. However, certain conditions have to be satisfied before (27) could be used directly, *i.e.*, (i) the constitutive dyadics of the media must have similar structure to that shown in (2), and (ii) since the residue theorem is applied to the DGF with four pole locations in the complex λ-plane, the media under consideration must also have four poles. This can be checked by ensuring $p_\lambda^2 \neq q_\lambda$.

3. CONCLUSION

This paper presents a complete and straightforward eigenfunction expansion of dyadic Green's function for an unbounded gyrotropic bianisotropic medium. The unbounded dyadic Green's function in the gyrotropic bianisotropic medium is obtained based on the Ohm-Rayleigh method, together with some newly developed tensor relationships as shown in (13) which greatly simplifies the formulation. Especially, the irrotational part of the dyadic Green's function is extracted theoretically and rigorously from the residual integral involving pole points. The results obtained here can be used to obtain the scattering dyadics for a cylindrical layered structure based on the method of scattering superposition. The idea presented here can also be used to obtain more general results for other media in the near future.

APPENDIX A. COEFFICIENTS FOR INTERMEDIATE RESULTS

$$\begin{aligned} p_\lambda =& h^2(-\epsilon_z\mu_t-\epsilon_t\mu_z+\zeta_z\xi_t+\zeta_t\xi_z)+h\Big[\epsilon_t\zeta_z\mu_a+\epsilon_a\zeta_z\mu_t+\epsilon_t\zeta_a\mu_z \\ &-\epsilon_a\zeta_t\mu_z-\epsilon_t\mu_z\xi_a-2\zeta_a\zeta_z\xi_t+\epsilon_a\mu_z\xi_t+\epsilon_z(-\zeta_t\mu_a+\zeta_a\mu_t \\ &-\mu_t\xi_a+\mu_a\xi_t)-(\epsilon_t\mu_a+\epsilon_a\mu_t-2\zeta_t\xi_a)\xi_z\Big]\omega+\Big\{{\epsilon_a}^2\mu_t\mu_z \\ &+{\epsilon_t}^2\mu_t\mu_z-\epsilon_z\zeta_t\mu_a\xi_a+\epsilon_z\zeta_a\mu_t\xi_a+{\zeta_a}^2\zeta_z\xi_t+{\zeta_t}^2\zeta_z\xi_t-\epsilon_z\zeta_a\mu_a\xi_t \\ &-\epsilon_z\zeta_t\mu_t\xi_t+\zeta_t({\xi_a}^2+{\xi_t}^2)\xi_z+\epsilon_a(\zeta_t\zeta_z\mu_a-\zeta_a\zeta_z\mu_t-\zeta_t\mu_z\xi_a \\ &-\zeta_a\mu_z\xi_t-\mu_t\xi_a\xi_z+\mu_a\xi_t\xi_z)-\epsilon_t\Big[\zeta_t\zeta_z\mu_t-\epsilon_z({\mu_a}^2+{\mu_t}^2) \\ &+\zeta_a(\zeta_z\mu_a-\mu_z\xi_a)+\zeta_t\mu_z\xi_t+\mu_a\xi_a\xi_z+\mu_t\xi_t\xi_z\Big]\Big\}\omega^2, \end{aligned} \tag{A1a}$$

$$\begin{aligned} q_\lambda =& 4(\epsilon_t\mu_t-\zeta_t\xi_t)(\epsilon_z\mu_z-\zeta_z\xi_z)\Big\{h^4+2h^3(-\zeta_a+\xi_a)\omega+h^2({\zeta_a}^2 \\ &+{\zeta_t}^2+2\epsilon_a\mu_a-2\epsilon_t\mu_t-4\zeta_a\xi_a+{\xi_a}^2+{\xi_t}^2)\omega^2-2h\Big[-\xi_a({\zeta_a}^2 \\ &+{\zeta_t}^2-\zeta_a\xi_a)+\zeta_a{\xi_t}^2+\epsilon_t(\zeta_t\mu_a-\zeta_a\mu_t+\mu_t\xi_a-\mu_a\xi_t)+\epsilon_a(\zeta_a\mu_a \\ &+\zeta_t\mu_t-\mu_a\xi_a-\mu_t\xi_t)\Big]\omega^3+\Big[{\epsilon_a}^2({\mu_a}^2+{\mu_t}^2)+{\epsilon_t}^2({\mu_a}^2+{\mu_t}^2) \\ &-2\epsilon_a(\zeta_a\mu_a\xi_a+\zeta_t\mu_t\xi_a-\zeta_t\mu_a\xi_t+\zeta_a\mu_t\xi_t)-2\epsilon_t(\zeta_t\mu_a\xi_a-\zeta_a\mu_t\xi_a \\ &+\zeta_a\mu_a\xi_t+\zeta_t\mu_t\xi_t)+({\zeta_a}^2+{\zeta_t}^2)({\xi_a}^2+{\xi_t}^2)\Big]\omega^4\Big\}, \end{aligned} \tag{A1b}$$

$$\tau_1 = \frac{1}{\epsilon_t\mu_t - \zeta_t\xi_t}\Big\{\lambda^2\mu_z(\epsilon_t\mu_t - \zeta_t\xi_t) + h^2\mu_t(\epsilon_z\mu_z - \zeta_z\xi_z) + h(\zeta_t\mu_a - \zeta_a\mu_t + \mu_t\xi_a - \mu_a\xi_t)(\epsilon_z\mu_z - \zeta_z\xi_z)\omega - \Big[\epsilon_t({\mu_a}^2 + {\mu_t}^2) - \zeta_t\mu_a\xi_a + \zeta_a\mu_t\xi_a - \zeta_a\mu_a\xi_t - \zeta_t\mu_t\xi_t\Big](\epsilon_z\mu_z - \zeta_z\xi_z)\omega^2\Big\}, \quad \text{(A1c)}$$

$$\tau_2 = \frac{1}{h(\epsilon_t\mu_t - \zeta_t\xi_t)\omega}\Big\{ik_\lambda\Big[h\lambda^2(\zeta_z\mu_t - \zeta_t\mu_z) + \lambda^2(\zeta_t\zeta_z\mu_a - \zeta_a\zeta_z\mu_t + \epsilon_a\mu_t\mu_z - \zeta_t\mu_z\xi_a)\omega + h^2\mu_a(-\epsilon_z\mu_z + \zeta_z\xi_z)\omega + h(\zeta_a\mu_a + \zeta_t\mu_t - \mu_a\xi_a - \mu_t\xi_t)(\epsilon_z\mu_z - \zeta_z\xi_z)\omega^2 - \Big[\epsilon_a({\mu_a}^2 + {\mu_t}^2) - \zeta_a\mu_a\xi_a - \zeta_t\mu_t\xi_a + \zeta_t\mu_a\xi_t - \zeta_a\mu_t\xi_t\Big](\epsilon_z\mu_z - \zeta_z\xi_z)\omega^3\Big]\Big\}, \quad \text{(A1d)}$$

$$\tau_3 = -\frac{ik_\lambda}{(\epsilon_t\mu_t - \zeta_t\xi_t)\omega}\Big\{h^2(\zeta_z\mu_t - \zeta_t\mu_z) + h(\zeta_t\zeta_z\mu_a - \zeta_a\zeta_z\mu_t + \epsilon_t\mu_a\mu_z + \epsilon_a\mu_t\mu_z + \zeta_z\mu_t\xi_a - 2\zeta_t\mu_z\xi_a - \zeta_z\mu_a\xi_t)\omega - \Big[(\zeta_a\zeta_z - \epsilon_a\mu_z)(\mu_t\xi_a - \mu_a\xi_t) + \zeta_t(-\zeta_z\mu_a\xi_a + \mu_z{\xi_a}^2 - \zeta_z\mu_t\xi_t + \mu_z{\xi_t}^2) + \epsilon_t\Big(\zeta_z({\mu_a}^2 + {\mu_t}^2) - \mu_z(\mu_a\xi_a + \mu_t\xi_t)\Big)\Big]\omega^2\Big\}, \quad \text{(A1e)}$$

$$\tau_4 = -\frac{k_\lambda^2}{h^2(\epsilon_t\mu_t - \zeta_t\xi_t)\omega^2}\Big\{\lambda^4\mu_t - (\epsilon_z\mu_z - \zeta_z\xi_z)\omega^2\Big[h^2\mu_t + h(\zeta_t\mu_a - \zeta_a\mu_t + \mu_t\xi_a - \mu_a\xi_t)\omega + (-\epsilon_t({\mu_a}^2 + {\mu_t}^2) + \zeta_t\mu_a\xi_a - \zeta_a\mu_t\xi_a + \zeta_a\mu_a\xi_t + \zeta_t\mu_t\xi_t)\omega^2\Big] + \lambda^2\Big[h^2\mu_z + h(-\zeta_z\mu_a - \zeta_a\mu_z + \mu_z\xi_a + \mu_a\xi_z)\omega + (\zeta_t\zeta_z\mu_t - \epsilon_z({\mu_a}^2 + {\mu_t}^2) - \epsilon_t\mu_t\mu_z + \zeta_a(\zeta_z\mu_a - \mu_z\xi_a) + \mu_a\xi_a\xi_z + \mu_t\xi_t\xi_z)\omega^2\Big]\Big\}, \quad \text{(A1f)}$$

$$\tau_5 = -\frac{ik_\lambda}{h(\epsilon_t\mu_t - \zeta_t\xi_t)\omega}\Big\{-h\lambda^2(\mu_z\xi_t - \mu_t\xi_z) + \Big[h^2\mu_a(\epsilon_z\mu_z - \zeta_z\xi_z) + \lambda^2(-\epsilon_a\mu_t\mu_z + \zeta_a\mu_z\xi_t + \mu_t\xi_a\xi_z - \mu_a\xi_t\xi_z)\Big]\omega + h(-\zeta_a\mu_a - \zeta_t\mu_t + \mu_a\xi_a + \mu_t\xi_t)(\epsilon_z\mu_z - \zeta_z\xi_z)\omega^2 + \Big[\epsilon_a({\mu_a}^2 + {\mu_t}^2) - \zeta_a\mu_a\xi_a - \zeta_t\mu_t\xi_a + \zeta_t\mu_a\xi_t - \zeta_a\mu_t\xi_t\Big](\epsilon_z\mu_z - \zeta_z\xi_z)\omega^3\Big\}, \quad \text{(A1g)}$$

$$\tau_6 = \frac{k_\lambda^2}{h(\epsilon_t\mu_t - \zeta_t\xi_t)\omega^2}\Big\{h\lambda^2\mu_t + h^3\mu_z + \lambda^2(\zeta_t\mu_a - \zeta_a\mu_t)\omega + h^2(-2\zeta_a\mu_z + \mu_z\xi_a + \mu_a\xi_z)\omega + h\Big[\mu_z({\zeta_a}^2 + {\zeta_t}^2 + \epsilon_a\mu_a - \epsilon_t\mu_t - 2\zeta_a\xi_a) - (\zeta_a\mu_a + \zeta_t\mu_t - \mu_a\xi_a - \mu_t\xi_t)\xi_z\Big]\omega^2 + \Big[(-\epsilon_a\zeta_a\mu_a - \epsilon_t\zeta_t\mu_a + \epsilon_t\zeta_a\mu_t - \epsilon_a\zeta_t\mu_t)\mu_z + ({\zeta_a}^2 + {\zeta_t}^2)\mu_z\xi_a + \Big(\epsilon_a({\mu_a}^2 + {\mu_t}^2) - \zeta_a\mu_a\xi_a - \zeta_t\mu_t\xi_a + \zeta_t\mu_a\xi_t - \zeta_a\mu_t\xi_t\Big)\xi_z\Big]\omega^3\Big\}, \tag{A1h}$$

$$\tau_7 = -\frac{ik_\lambda}{(\epsilon_t\mu_t - \zeta_t\xi_t)\omega}\Big\{h^2(\mu_z\xi_t - \mu_t\xi_z) + h\Big[\epsilon_t\mu_a\mu_z + \epsilon_a\mu_t\mu_z - 2\zeta_a\mu_z\xi_t + (-\zeta_t\mu_a + \zeta_a\mu_t - \mu_t\xi_a + \mu_a\xi_t)\xi_z\Big]\omega + \Big[\epsilon_a(\zeta_t\mu_a - \zeta_a\mu_t)\mu_z - \epsilon_t(\zeta_a\mu_a + \zeta_t\mu_t)\mu_z + ({\zeta_a}^2 + {\zeta_t}^2)\mu_z\xi_t + \epsilon_t({\mu_a}^2 + {\mu_t}^2)\xi_z - (\zeta_t\mu_a\xi_a - \zeta_a\mu_t\xi_a + \zeta_a\mu_a\xi_t + \zeta_t\mu_t\xi_t)\xi_z\Big]\omega^2\Big\}, \tag{A1i}$$

$$\tau_8 = \frac{k_\lambda^2}{h(\epsilon_t\mu_t - \zeta_t\xi_t)\omega^2}\Big\{h\lambda^2\mu_t + h^3\mu_z - h^2(\zeta_z\mu_a + \mu_z(\zeta_a - 2\xi_a))\omega + \lambda^2(\mu_t\xi_a - \mu_a\xi_t)\omega + h(\zeta_a\zeta_z\mu_a + \zeta_t\zeta_z\mu_t + \epsilon_a\mu_a\mu_z - \epsilon_t\mu_t\mu_z - \zeta_z\mu_a\xi_a - 2\zeta_a\mu_z\xi_a + \mu_z{\xi_a}^2 - \zeta_z\mu_t\xi_t + \mu_z{\xi_t}^2)\omega^2 - \Big[(-\zeta_t\zeta_z + \epsilon_t\mu_z)(\mu_t\xi_a - \mu_a\xi_t) + \zeta_a(-\zeta_z\mu_a\xi_a + \mu_z{\xi_a}^2 - \zeta_z\mu_t\xi_t + \mu_z{\xi_t}^2) + \epsilon_a(\zeta_z({\mu_a}^2 + {\mu_t}^2) - \mu_z(\mu_a\xi_a + \mu_t\xi_t))\Big]\omega^3\Big\}, \tag{A1j}$$

$$\tau_9 = \frac{k_\lambda^2}{\lambda^2(\epsilon_t\mu_t - \zeta_t\xi_t)\omega^2}\Big\{-h^4\mu_z + 2h^3\mu_z(\zeta_a - \xi_a)\omega - h\lambda^2(\zeta_t\mu_a - \zeta_a\mu_t + \mu_t\xi_a - \mu_a\xi_t)\omega + \lambda^2(\epsilon_t({\mu_a}^2 + {\mu_t}^2) - \zeta_t\mu_a\xi_a + \zeta_a\mu_t\xi_a - \zeta_a\mu_a\xi_t - \zeta_t\mu_t\xi_t)\omega^2 + 2h\mu_z\Big[-\xi_a({\zeta_a}^2 + {\zeta_t}^2 - \zeta_a\xi_a) + \zeta_a{\xi_t}^2 + \epsilon_t(\zeta_t\mu_a - \zeta_a\mu_t + \mu_t\xi_a - \mu_a\xi_t) + \epsilon_a(\zeta_a\mu_a + \zeta_t\mu_t - \mu_a\xi_a - \mu_t\xi_t)\Big]\omega^3 - \mu_z\Big[{\epsilon_a}^2({\mu_a}^2 + {\mu_t}^2) + {\epsilon_t}^2({\mu_a}^2 + {\mu_t}^2) - 2\epsilon_a(\zeta_a\mu_a\xi_a + \zeta_t\mu_t\xi_a - \zeta_t\mu_a\xi_t + \zeta_a\mu_t\xi_t) - 2\epsilon_t(\zeta_t\mu_a\xi_a - \zeta_a\mu_t\xi_a + \zeta_a\mu_a\xi_t + \zeta_t\mu_t\xi_t) + ({\zeta_a}^2 + {\zeta_t}^2)({\xi_a}^2 + {\xi_t}^2)\Big]\omega^4 + h^2\Big(-\lambda^2\mu_t - \mu_z({\zeta_a}^2 + {\zeta_t}^2 + 2\epsilon_a\mu_a - 2\epsilon_t\mu_t - 4\zeta_a\xi_a + {\xi_a}^2 + {\xi_t}^2)\omega^2\Big)\Big\}, \tag{A1k}$$

$$\begin{aligned}\tau_{10} = & \frac{k_\lambda^2}{\epsilon_z\lambda^2(\epsilon_t\mu_t-\zeta_t\xi_t)\omega^2}\Big\{\epsilon_t\lambda^4\mu_t+h^2\epsilon_t\lambda^2\mu_z-h^2\zeta_z\lambda^2\xi_t-\zeta_t\lambda^4\xi_t\\
&-h^4\zeta_z\xi_z-h^2\zeta_t\lambda^2\xi_z+h\Big[-\lambda^2[\epsilon_t\Big(\zeta_z\mu_a+\mu_z(\zeta_a-\xi_a)\Big)\\
&-2\zeta_a\zeta_z\xi_t+\epsilon_a(\zeta_z\mu_t-\zeta_t\mu_z+\mu_z\xi_t)]+\Big(2h^2\zeta_z(\zeta_a-\xi_a)\\
&+\lambda^2(\epsilon_t\mu_a+\epsilon_a\mu_t-2\zeta_t\xi_a)\Big)\xi_z\Big]\omega-\lambda^2\Big({\epsilon_a}^2\mu_t\mu_z+{\epsilon_t}^2\mu_t\mu_z\\
&+({\zeta_a}^2+{\zeta_t}^2)\zeta_z\xi_t+\epsilon_a(\zeta_t\zeta_z\mu_a-\zeta_a\zeta_z\mu_t-\zeta_t\mu_z\xi_a-\zeta_a\mu_z\xi_t)\\
&-\epsilon_t(\zeta_a\zeta_z\mu_a+\zeta_t\zeta_z\mu_t-\zeta_a\mu_z\xi_a+\zeta_t\mu_z\xi_t)\Big)+[h^2\zeta_z({\zeta_a}^2+{\zeta_t}^2\\
&+2\epsilon_a\mu_a-2\epsilon_t\mu_t-4\zeta_a\xi_a+{\xi_a}^2+{\xi_t}^2)+\lambda^2\Big(-\epsilon_a\mu_t\xi_a+\zeta_t{\xi_a}^2\\
&+\epsilon_a\mu_a\xi_t+\zeta_t{\xi_t}^2-\epsilon_t(\mu_a\xi_a+\mu_t\xi_t)\Big)]\xi_z\Big]\omega^2+2h\zeta_z\Big(-\xi_a({\zeta_a}^2\\
&+{\zeta_t}^2-\zeta_a\xi_a)+\zeta_a{\xi_t}^2+\epsilon_t(\zeta_t\mu_a-\zeta_a\mu_t+\mu_t\xi_a-\mu_a\xi_t)\\
&+\epsilon_a(\zeta_a\mu_a+\zeta_t\mu_t-\mu_a\xi_a-\mu_t\xi_t)\Big)\xi_z\omega^3-\zeta_z\Big({\epsilon_a}^2({\mu_a}^2+{\mu_t}^2)\\
&+{\epsilon_t}^2({\mu_a}^2+{\mu_t}^2)-2\epsilon_a(\zeta_a\mu_a\xi_a+\zeta_t\mu_t\xi_a-\zeta_t\mu_a\xi_t\\
&+\zeta_a\mu_t\xi_t)-2\epsilon_t(\zeta_t\mu_a\xi_a-\zeta_a\mu_t\xi_a+\zeta_a\mu_a\xi_t\\
&+\zeta_t\mu_t\xi_t)+({\zeta_a}^2+{\zeta_t}^2)({\xi_a}^2+{\xi_t}^2)\Big)\xi_z\omega^4\Big]\Big\}. \qquad \text{(A11)}\end{aligned}$$

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