

# **ELECTROMAGNETIC IMAGING FOR AN IMPERFECTLY CONDUCTING CYLINDER BURIED IN A THREE-LAYER STRUCTURE BY THE GENETIC ALGORITHM**

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**Abstract**—The imaging of an imperfectly conducting cylinder buried in a three-layer structure by the genetic algorithm is investigated. An imperfectly conducting cylinder of unknown shape and conductivity buried in the second layer scatters the incident wave from the first layer or the third layer. We measure the scattered field in the first and third layers. Based on the boundary condition and the recorded scattered field, a set of nonlinear integral equations is derived and the imaging problem is reformulated into an optimization problem. The genetic algorithm is then employed to find out the global extreme solution of the cost function. Numerical results demonstrated that, even when the initial guess is far away from the exact one, good reconstruction can be obtained. In such a case, the gradient-based methods often get trapped in a local extreme. In addition, the effect of uniform noise on the reconstruction is investigated.

## **1 Introduction**

## **2 Theoretical Formulation**

### **2.1 Imaging Problem**

### **2.2 Steady-State Genetic Algorithm**

## **3 Numerical Results**

## **4 Conclusions**

## **Appendix A.**

## **References**

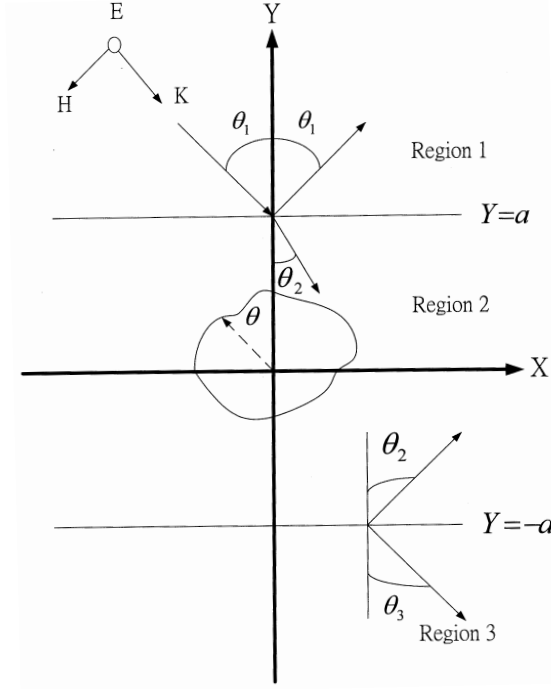
## 1. INTRODUCTION

The inverse scattering techniques for imaging the shape of imperfectly conducting objects have attracted considerable attention in recent years. They can apply in noninvasive measurement, medical imaging, and biological application. In the past 20 years, many rigorous methods have been developed to solve the exact equation. However, inverse problems of this type are difficult to solve because they are ill-posed and nonlinear. As a result, many inverse problems are reformulated as optimization problems. General speaking, two main kinds of approaches have been developed. The first is based on gradient searching schemes such as the Newton-Kantorovitch method [1–3], the Levenberg-Marguart algorithm [4–6] and the successive-overrelaxation method [7]. These methods are highly dependent on the initial guess and tend to get trapped in a local extreme. In contrast, the second approach is based on the evolutionary searching schemes [8, 9]. They tend to converge to the global extreme of the problem, no matter what the initial estimate is [10, 11]. Owing to the difficulties in computing the Green's function by numerical method, the problem of inverse scattering in a three-layer structure has seldom been attacked. In our knowledge, there are still no numerical results by the genetic algorithm for imperfectly conducting scatterers buried in a three-layer structure. In this paper, the electromagnetic imaging of an imperfectly conducting cylinder buried in a three-layer structure is investigated. The steady state genetic algorithm is used to recover the shape and the conductivity of the scatterer. It is found the steady-state genetic algorithm [12, 13] can reduce the calculation time of the image problem compared with the generational genetic algorithm. In Section 2, the theoretical formulation for the electromagnetic imaging is presented. The general principle of the genetic algorithm and the way we applied them to the imaging problem are described. Numerical results for various objects of different shapes are given in Section 3. Section 4 is the conclusion.

## 2. THEORETICAL FORMULATION

### 2.1. Imaging Problem

Let us consider a two-dimensional three-layer structure as shown in Fig. 1, where  $(\varepsilon_i, \sigma_i)$   $i = 1, 2, 3$ , denote the permittivities and conductivities in each layer and an imperfectly conducting cylinder is buried in second layer. The metallic cylinder with cross section described by the equation  $\rho = F(\theta)$  is illuminated by an incident plane wave whose electric field vector is parallel to the  $Z$  axis (i.e.,



**Figure 1.** Geometry of the problem in  $(x, y)$  plane of three-layer structure.

TM polarization). We assume that the time dependence of the field is harmonic with the factor  $\exp(j\omega t)$ . Let  $E^{inc}$  denote the incident field from region 1 with incident angle  $\theta_1$  as follow:

$$E^{inc} = E_1^+ e^{+jk_1 \cos \theta_1 y} e^{-jk_1 \sin \theta_1 x} \hat{z} \quad (1)$$

Owing to the interfaces, the incident plane wave generates three waves that would exist in the absence of the conducting object. Thus, the unperturbed field is given by

$$E = \begin{cases} E_1 = E_1^+ e^{+jk_1 \cos \theta_1 y} e^{-jk_1 \sin \theta_1 x} \hat{z} + E_1^- e^{-jk_1 \cos \theta_1 y} e^{-jk_1 \sin \theta_1 x} \hat{z}, & y \geq a \\ E_2 = E_2^+ e^{+jk_2 \cos \theta_2 y} e^{-jk_2 \sin \theta_2 x} \hat{z} + E_2^- e^{-jk_2 \cos \theta_2 y} e^{-jk_2 \sin \theta_2 x} \hat{z}, & a \geq y \geq -a \\ E_3 = E_3^+ e^{+jk_3 \cos \theta_3 y} e^{-jk_3 \sin \theta_3 x} \hat{z}, & y \leq -a \end{cases} \quad (2)$$

where  $E_1^+$  is set to be 1 and

$$\begin{aligned}
 E_1^- &= \frac{e^{+j2k_1 \cos \theta_1 a} \left[ \frac{(Z_1 + Z_2)(Z_3 - Z_2)e^{-j2k_2 \cos \theta_2 a}}{-(Z_1 - Z_2)(Z_3 + Z_2)e^{+j2k_2 \cos \theta_2 a}} \right]}{(Z_1 + Z_2)(Z_3 + Z_2)e^{+j2k_2 \cos \theta_2 a} - (Z_1 - Z_2)(Z_3 - Z_2)e^{-j2k_2 \cos \theta_2 a}} \\
 E_2^+ &= \frac{1}{2} e^{jk_2(\sin \theta_2 x - \cos \theta_2 a)} \left[ \frac{Z_1 + Z_2}{Z_1} e^{-jk_1(\sin \theta_1 x - \cos \theta_1 a)} \right. \\
 &\quad \left. + \frac{Z_1 - Z_2}{Z_1} E_1^- e^{-jk_1(\sin \theta_1 x + \cos \theta_1 a)} \right] \\
 E_2^- &= \frac{1}{2} e^{jk_2(\sin \theta_2 x + \cos \theta_2 a)} \left[ \frac{Z_1 - Z_2}{Z_1} e^{-jk_1(\sin \theta_1 x - \cos \theta_1 a)} \right. \\
 &\quad \left. + \frac{Z_1 + Z_2}{Z_1} E_1^- e^{-jk_1(\sin \theta_1 x + \cos \theta_1 a)} \right] \\
 E_3^+ &= \frac{2Z_3}{Z_2 + Z_3} E_2^+ e^{-jk_2(\sin \theta_2 x + \cos \theta_2 a)} e^{jk_3(\sin \theta_3 x + \cos \theta_3 a)}
 \end{aligned}$$

$$k_1 \sin \theta_1 = k_2 \sin \theta_2 = k_3 \sin \theta_3$$

$$k_i^2 = \omega^2 \varepsilon_i \mu_0 - j\omega \mu_0 \sigma_i \quad i = 1, 2, 3 \quad \text{Im}(k_i) \leq 0$$

$$\begin{aligned}
 Z_1 &= \frac{\eta_1}{\cos \theta_1}, \quad Z_2 = \frac{\eta_2}{\cos \theta_2}, \quad Z_3 = \frac{\eta_3}{\cos \theta_3}, \\
 \eta_1 &= \sqrt{\frac{\mu_0}{\varepsilon_1}}, \quad \eta_2 = \sqrt{\frac{\mu_0}{\varepsilon_2}}, \quad \eta_3 = \sqrt{\frac{\mu_0}{\varepsilon_3}}
 \end{aligned}$$

At an arbitrary point  $(x, y)$  (or  $(r, \theta)$  in polar coordinates) in regions 1 and 3 the scattered field,  $\vec{E}_s = \vec{E} - \vec{E}_i$ , can be expressed as

$$E_s(\vec{r}) = - \int_0^{2\pi} G(\vec{r}, F(\theta'), \theta') J(\theta') d\theta' \quad (3)$$

where

$$J(\theta) = -j\omega \mu_0 \sqrt{F^2(\theta) + F'^2(\theta)} J_s(\theta)$$

and

$$G(x, y; x', y') = \begin{cases} G_1(x, y; x', y'), & y > a \\ G_2(x, y; x', y'), & a > y > -a \\ G_3(x, y; x', y'), & y < -a \end{cases} \quad (4)$$

$$\begin{aligned}
G_1 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} j e^{-j\gamma_1(y-a)} \\
&\quad \frac{(\gamma_2 + \gamma_3)e^{j\gamma_2(y'+a)} + (\gamma_2 - \gamma_3)e^{-j\gamma_2(y'+a)}}{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} e^{-j\alpha(x-x')} d\alpha \\
G_2 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{j}{2\gamma_2} \\
&\quad \left\{ \left[ \frac{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)e^{-j\gamma_2[|y-y'|-2a]} + (\gamma_2 - \gamma_1)(\gamma_2 - \gamma_3)e^{j\gamma_2[|y-y'|-2a]}}{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} \right] \right. \\
&\quad \left. + \left[ \frac{(\gamma_2 - \gamma_1)(\gamma_2 + \gamma_3)e^{j\gamma_2[y+y']}] + (\gamma_2 - \gamma_3)(\gamma_1 + \gamma_2)e^{-j\gamma_2[y+y']}}{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} \right] \right\} \\
&\quad \cdot e^{-j\alpha(x-x')} d\alpha \\
G_3 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} j e^{j\gamma_3(y+a)} \\
&\quad \frac{(\gamma_1 + \gamma_2)e^{-j\gamma_2(y'-a)} + (\gamma_2 - \gamma_1)e^{j\gamma_2(y'-a)}}{(\gamma_1 + \gamma_2)(\gamma_2 + \gamma_3)e^{j\gamma_2(2a)} + (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)e^{-j\gamma_2(2a)}} e^{-j\alpha(x-x')} d\alpha
\end{aligned}$$

with  $\gamma_i^2 = k_i^2 - \alpha^2$ ,  $i = 1, 2, 3$  and  $\text{Im}(\gamma_i) \leq 0$ .

Note that  $G_1$ ,  $G_2$  and  $G_3$  denote the Green's function which can be obtained by tedious mathematic manipulation for the line source in region 2. Note that we might face some difficulties in calculating the Green's function. The Green's function, given by (4), is in the form of an improper integral which must be evaluated numerically. However, the integral converges very slowly when  $(x, y)$  and  $(x', y')$  approach the interface. Fortunately, we find that the integral in Green's function may be rewritten as a closed-form term plus a rapidly converging integral (see Appendix A). Thus the whole integral in the Green's function can be calculated efficiently.  $J_s(\theta)$  is the induced surface current density, which is proportional to the normal derivative of the electric field on the conductor surface. For an imperfectly conducting scatterer with finite conductivity, the electromagnetic wave is able to penetrate into the interior of a scatterer, so the total tangential electric field at the surface of the scatterer is not equal to zero. As described in [14] and [15], the boundary condition for this case can be approximated by assuming that the total tangential electric field on the scatterer surface is related to surface current density through a

surface impedance  $Z_s(\omega)$ :

$$\hat{n} \times \vec{E} = \hat{n} \times (Z_s \vec{J}_s) \quad (5)$$

where  $\hat{n}$  is the outward unit vector normal to the surface of the scatterer. The scatterer of interest here is a nonmagnetic ( $\mu = \mu_0$ ), imperfectly conducting cylinder with minimum radius of curvature  $a$ . The surface impedance is expressed in [14] and [15] as  $Z_s(\omega) \cong \sqrt{j\omega\mu_0/\sigma}$ . This approximation is valid as long as  $|\text{Im}(N_c)ka| \gg 1$  and  $\sigma \gg \omega\epsilon_0$ , where “Im” means taking the imaginary part, and  $N_c$  is the complex index of refraction of the conductor, given by  $N_c = \sqrt{1 + \frac{\sigma}{j\omega\epsilon_0}}$ .

The boundary condition at the surface of the scatterer given by (5) then yield an integral equation for  $J(\theta)$ :

$$\vec{E}_2^{inc} + \vec{E}_{2s} = Z_s \vec{J}_s$$

$$E_2^{inc}(\vec{r}) = \int_0^{2\pi} G_2(\vec{r}, F(\theta'), \theta') J(\theta') d\theta' + j \sqrt{\frac{j}{\omega\mu_0\sigma}} \frac{J(\theta)}{\sqrt{F^2(\theta) + F'^2(\theta)}} \quad (6)$$

where  $\vec{E}_2^{inc}$  is the incident field and  $\vec{E}_{2s}$  is the scatter field in second layer.

For the direct scattering problem, the scattered field  $E_s$  is calculated by assuming that the shape and conductivity are known. This can be achieved by first solving  $J$  in (6) and then calculating  $E_s$  using (3). For the inverse problem, assume the approximate center of scatterer, which in fact can be any point inside the scatterer, is known. Then the shape function  $F(\theta)$  can be expanded as:

$$F(\theta) = \sum_{n=0}^{N/2} B_n \cos(n\theta) + \sum_{n=1}^{N/2} C_n \sin(n\theta) \quad (7)$$

where  $B_n$  and  $C_n$  are real coefficients to be determined, and  $N + 1$  is the number of unknowns for the shape function. In the inversion procedure, the steady state genetic algorithm is used to minimize the following cost function:

$$CF = \left\{ \frac{1}{M_t} \sum_{m=1}^{M_t} \left| E_s^{\text{exp}}(\vec{r}_m) - E_s^{\text{cal}}(\vec{r}_m) \right|^2 / |E_s^{\text{exp}}(\vec{r}_m)|^2 + \alpha |F'(\theta)|^2 \right\}^{1/2} \quad (8)$$

where  $M_t$  is the total number of measurement points.  $E_s^{\text{exp}}(\vec{r})$  and  $E_s^{\text{cal}}(\vec{r})$  are the measured and calculated scattered fields, respectively.

The factor  $\alpha|F'(\theta)|^2$  can be interpreted as the smoothness requirement for the boundary  $F(\theta)$ . To make sure that our numerical result (by the moment method) is correct, the scattered field of the cylinder of circular cross section is first calculated by the analytic theorem and compare with those obtained by the moment method, it is found good agreement has been achieved. Moreover, the discretization number for the direct problem is two times that for the inverse problem in our simulation, since it is crucial that the synthetic data generated by a direct solver are not like those obtained by the inverse solver.

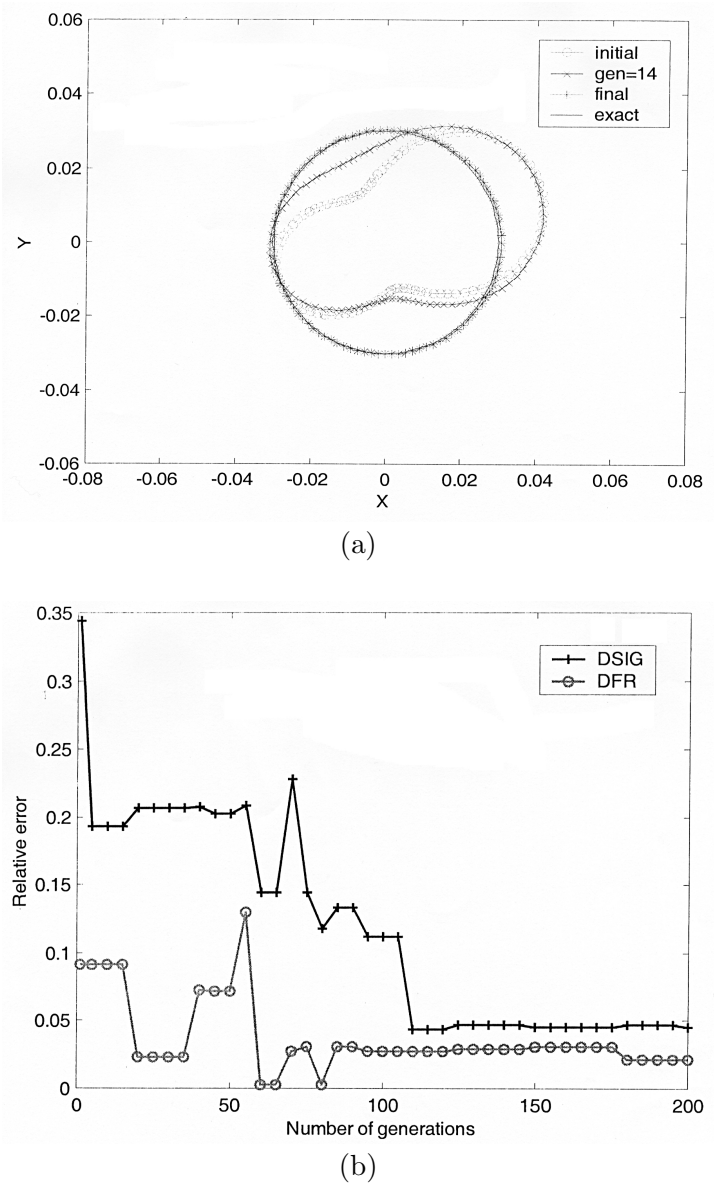
## 2.2. Steady-State Genetic Algorithm

Genetic algorithm is a global numerical optimization method based on genetic recombination and evolution in nature. They use the iterative optimization procedures that start with some randomly selected population of potential solutions, and then gradually evolve toward a better solution through the application of the genetic operators: reproduction, crossover and mutation operators. In our problem, both parameters  $B_n$  and  $C_n$  are encoded using Gray code. We employ steady-state genetic algorithm for the imaging problem investigated. The variance of the steady-state genetic algorithm is to insert the new individuals generated by crossover and mutation into the parent population to form a temporary population. We obtained new offspring by using rank selection scheme. As soon as the cost function ( $CF$ ) changes by  $< 1\%$  in two successive generations, the algorithm will be terminated and the final solution is then obtained.

It should be noted that the calculation of the Green's function is quite computational expensive. Steady-state genetic algorithm has not only the characteristic of faster convergence [12, 13], but also the lower rate of crossover. As a result, it is a suitable scheme to effectively save the calculation time for the inverse problem as compared with the generational GA.

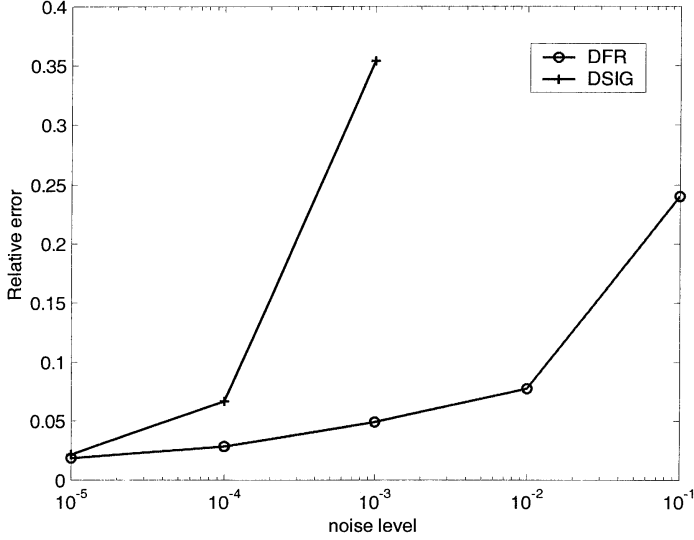
## 3. NUMERICAL RESULTS

We illustrate the performance of the proposed inversion algorithm and its sensitivity to random noise in the scattered field. Consider a lossless three-layer structure ( $\sigma_1 = \sigma_2 = \sigma_3 = 0$ ) and an imperfectly conducting cylinder buried in region 2. The permittivity in each region is characterized by,  $\varepsilon_1 = \varepsilon_0$ ,  $\varepsilon_2 = 2.55\varepsilon_0$  and  $\varepsilon_3 = \varepsilon_0$ , respectively, as shown in Fig. 1. The frequency of the incident wave is chosen to be 3 GHz, with the incident angles equal to  $45^\circ$  and  $315^\circ$ , respectively. The width of the second layer is 0.3 m. Ten measurement points are



**Figure 2.** (a) shape function for example 1. The star curve represents the exact shape, while the solid curves are calculated shape in iteration process. (b) Shape and conductivity function error for example 1 in each generation.





**Figure 3.** Relative error of shape and conductivity as a function of noise.

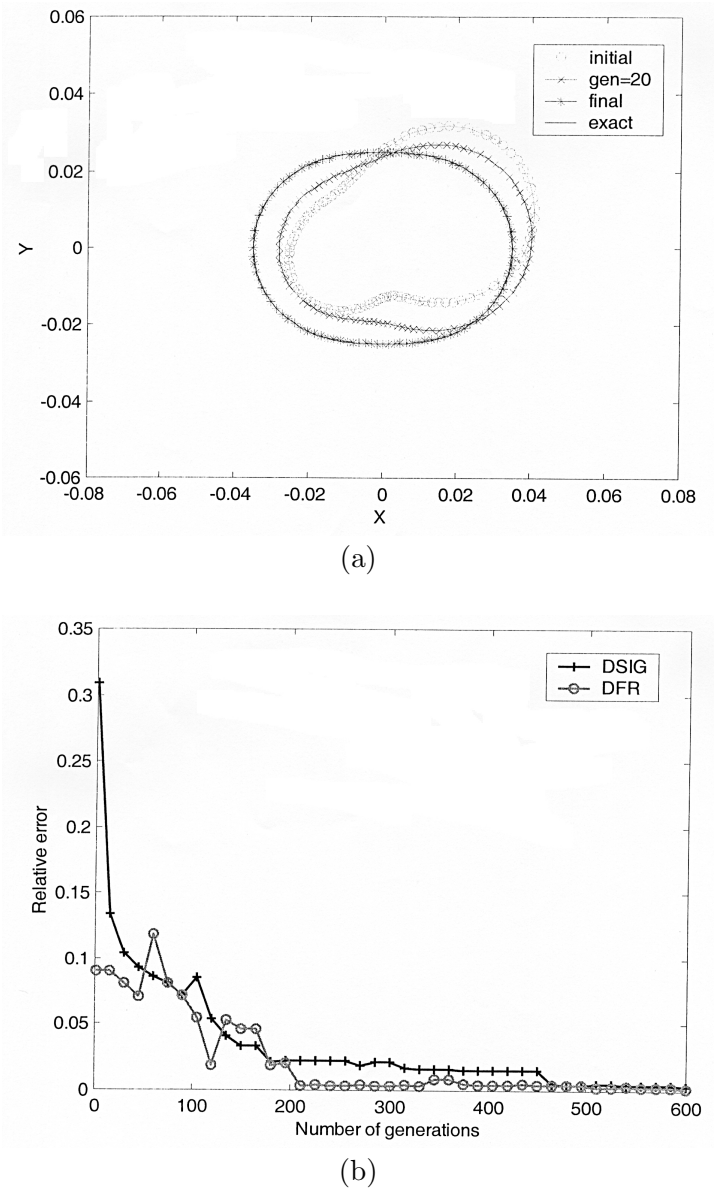
equally separated on two parallel lines at equal spacing in region 1 and region 3. Thus there are totally 20 measurements in each simulation. The population size is chosen as 120.

The coding length of each unknown coefficient,  $B_n$  (or  $C_n$ ), is set to be 20 bits. The search range for the unknown coefficient of the shape function is chosen to be from  $-0.015$  to  $0.015$ ,  $B_0$  is chosen to be  $0.02$  to  $0.05$ . The crossover probability  $p_c$  and mutation probability  $p_m$  are set to be  $0.05$  and  $0.025$ . The range of search of the cylinder conductivity is from  $50$  to  $1000$  (S/m). In the following simulation, the CPU time is about 6 hours per case on a P4 3.0 GHz Computer.

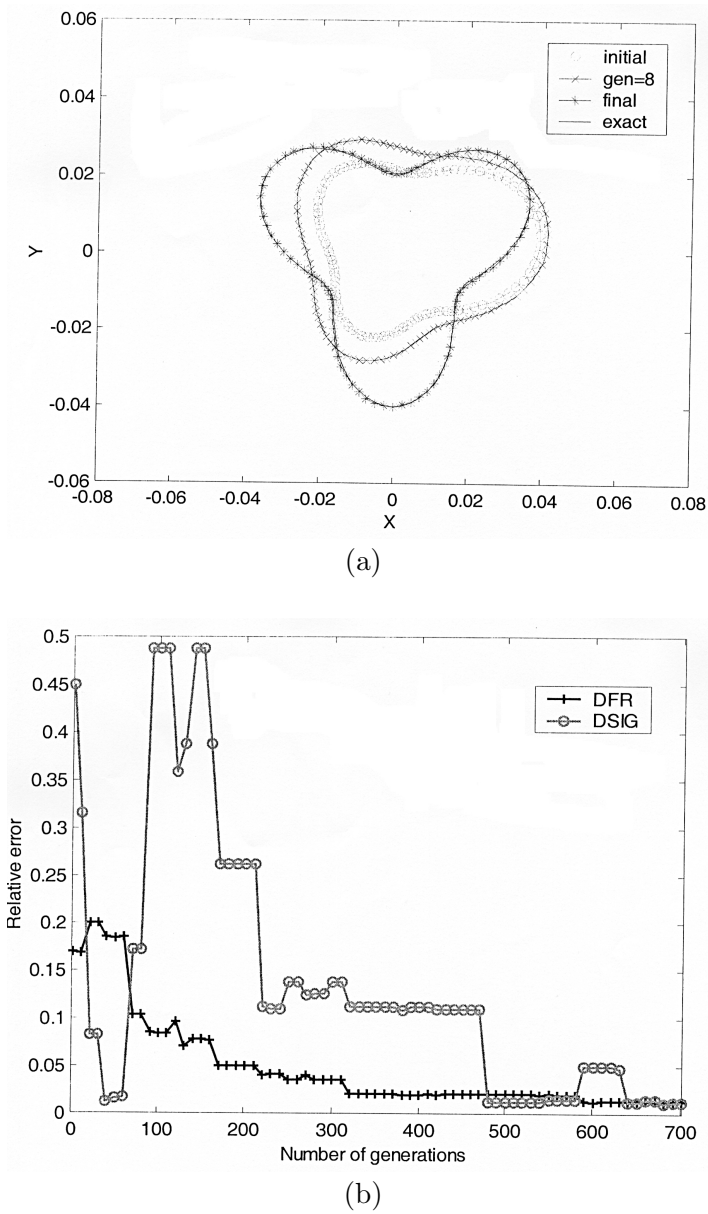
In the first example, the shape function is chosen to be  $F(\theta) = (0.03)\text{m}$ . The chosen conductivity is  $100\text{S/m}$ . The reconstructed shape function for the best population member is plotted in Fig. 2(a) with the shape and the conductivity error shown in Fig. 2(b). The reconstructed result is quite good. Here, the shape function discrepancy is defined as

$$DFR = \left\{ \frac{1}{N'} \sum_{i=1}^{N'} [F^{cal}(\theta_i) - F(\theta_i)]^2 / F^2(\theta_i) \right\}^{1/2} \quad (9)$$

where  $N'$  is set to 1000.



**Figure 4.** (a) shape function for example 2. The star curve represents the exact shape, while the solid curves are calculated shape in iteration process. (b) Shape and conductivity function error for example 1 in each generation.



**Figure 5.** (a) shape function for example 3. The star curve represents the exact shape, while the solid curves are calculated shape in iteration process. (b) Shape and conductivity function error for example 3 in each generation.

The conductivity discrepancy is defined as

$$DSIG = \left| \frac{\sigma^{cal} - \sigma}{\sigma} \right| \quad (10)$$

In the second example, for investigating the sensifivity of the imaging algorithm against random noise, we added the uniform noise to the real and imaginary parts of the simulated scattered fields. We choose the shape function  $F(\theta) = (0.03 + 0.01 \cos 2\theta) \text{ m}$  and the conductivity is 100 S/m. Normalized standard deviations of  $10^{-5}$ ,  $10^{-4}$ ,  $10^{-3}$ ,  $10^{-2}$  and  $10^{-1}$  are used in the simulations. The shape and conductivity error vs. normalized noise level is plotted in Fig. 3. It is found that the effect of noise to the shape reconstruction is negligible for normalized standard deviations below  $10^{-3}$ . But the effect of noise to the conductivity reconstruction is significant for normalized standard deviations over  $10^{-4}$ .

In the third example, the shape function is chosen to be  $F(\theta) = (0.03 + 0.005 \cos 2\theta) \text{ m}$ . The chosen conductivity is 100 S/m. The reconstructed shape function for the best population member is plotted in Fig. 4(a) with the shape and the conductivity error shown in Fig. 4(b). The reconstructed shape error is  $< 5\%$ .

In the fourth example, the shape function is chosen to be  $F(\theta) = (0.03 + 0.01 \sin 3\theta) \text{ m}$ . The chosen conductivity is 100 S/m. The purpose of this example is to show that the proposed scheme is capable to reconstruct the scatterer whose shape has three concavities. The reconstructed shape function for the best population member is plotted in Fig. 5(a) with the shape and the conductivity error shown in Fig. 5(b). The reconstructed shape error is  $< 5\%$ .

## 4. CONCLUSIONS

We have reported a study of applying the genetic algorithm to reconstruct the shapes and the conductivity of an embedded conducting cylinder. Based on the boundary condition and measured scattered field, we have derived a set of nonlinear integral equations and reformulated the imaging problem into an optimization one. The genetic algorithm is then employed to de-embed the microwave image of metallic cylinder. In our experience, the main difficulties in applying the genetic algorithm to the problem are to choose the suitable parameters, such as the population size, coding length of the string ( $L$ ), crossover probability ( $p_c$ ), and mutation probability ( $P_m$ ). Different parameter sets will affect the speed of convergence as well as the computation time. Numerical results illustrate that

the conductivity is more sensitive to noise than the shape function is. Numerical results also show that good shape reconstruction can be achieved as long as the normalized noise level is  $< 10^{-3}$ . But the good conductivity reconstruction can be achieved only the normalized noise level is  $10^{-5}$ .

## APPENDIX A.

To calculate the Green's function, we can use the following formula.

$$\int_u^\infty x^{r-1} e^{-\beta x} \cos \delta x dx = \frac{1}{2}(\beta + j\alpha)^{-r} \Gamma[r, (\beta + j\delta)u] + \frac{1}{2}(\beta - j\alpha)^{-r} \Gamma[r, (\beta - j\delta)u] \quad (\text{A1})$$

for  $\text{Re}\beta > |\text{Im}\delta|$  where

$$\Gamma(\alpha, Z) = \int_z^\infty e^{-t} t^{\alpha-1} dt$$

$\Gamma$  is the incomplete Gamma function which has the following properties

$$\begin{aligned} \Gamma(-n, z) &= \frac{(-1)^n}{n!} \left[ \Gamma(0, Z) - e^{-z} \sum_{m=0}^{n-1} (-1)^m \frac{m!}{z^{m+1}} \right] \\ \Gamma(0, z) &= -\gamma - \ln z - \sum_{n=1}^{\infty} (-1)^n \frac{z^n}{(n+1)!} \quad [|\arg(z)| < \pi] \quad (\text{A2}) \end{aligned}$$

in which  $\gamma$  is Euler's constant, i.e.,  $\gamma = 0.5772156649$ .

Let us consider the following integral

$$\begin{aligned} G_1 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} j e^{-jr_1(y-a)} \\ &\quad \frac{(r_2 + r_3)e^{jr_2(y'+a)} + (r_2 - r_3)e^{-jr_2(y'+a)}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} e^{-j\alpha(x-x')} d\alpha \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} j e^{-jr_1(y-a)} \\ &\quad \frac{(r_2 + r_3)e^{jr_2(y'+a)} + (r_2 - r_3)e^{-jr_2(y'+a)}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} \cos \alpha(x-x') d\alpha \end{aligned}$$

where  $r_i^2 = k_i^2 - \alpha^2$ ,  $i = 1, 2, 3$ ,  $\text{Im}(\gamma_i) \leq 0$ ,  $y \geq a$ ,  $a \geq y \geq -a$ .

The integral  $G_1$  may be rewritten as follows

$$\begin{aligned}
 G_1 = & \frac{1}{\pi} \int_{-\infty}^{\infty} j e^{-jr_1(y-a)} \\
 & \frac{(r_2 + r_3)e^{jr_2(y'+a)} + (r_2 - r_3)e^{-jr_2(y'+a)}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} \cos \alpha(x - x') d\alpha \\
 & + \frac{1}{2\pi} \int_{\alpha_0}^{\infty} \left[ \frac{e^{-\alpha(y-y')}}{\alpha} + \frac{(k_3^2 - k_2^2) e^{-\alpha(y+y'+2a)}}{4\alpha^3} \right] \cos \alpha(x - x') d\alpha \\
 & - \frac{1}{2\pi} \int_{\alpha_0}^{\infty} \left[ \frac{e^{-\alpha(y-y')}}{\alpha} + \frac{(k_3^2 - k_2^2) e^{-\alpha(y+y'+2a)}}{4\alpha^3} \right] \cos \alpha(x - x') d\alpha
 \end{aligned}$$

in general, we choose  $\alpha_0 \gg |k_i|$ ,  $i = 1, 2, 3$ . By Eq. (A1), we get

$$\begin{aligned}
 & -\frac{1}{2\pi} \int_{\alpha_0}^{\infty} \left[ \frac{e^{-\alpha(y-y')}}{\alpha} + \frac{(k_3^2 - k_2^2) e^{-\alpha(y+y'+2a)}}{4\alpha^3} \right] \cos \alpha(x - x') d\alpha \\
 = & -\frac{1}{4\pi} \{ \Gamma[0, [(y - y') + j(x - x')]\alpha_0] + \Gamma[0, [(y - y') - j(x - x')]\alpha_0] \} \\
 & - \frac{(k_3^2 - k_2^2)}{16\pi} \left\{ \begin{aligned} & [(y + y' + 2a) + j(x - x')]^2 \\ & \cdot \Gamma[-2, [(y + y' + 2a) + j(x - x')]\alpha_0] \\ & + [(y + y' + 2a) - j(x - x')]^2 \\ & \cdot \Gamma[-2, [(y + y' + 2a) - j(x - x')]\alpha_0] \end{aligned} \right\}
 \end{aligned}$$

Using the above relation, we obtain

$$\begin{aligned}
 G_1 = & \frac{1}{\pi} \int_0^{\infty} j e^{-jr_1(y-a)} \\
 & \frac{(r_2 + r_3)e^{jr_2(y'+a)} + (r_2 - r_3)e^{-jr_2(y'+a)}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} \cos \alpha(x - x') d\alpha \\
 & - \frac{1}{2\pi} \int_{\alpha_0}^{\infty} \left[ \frac{e^{-\alpha(y-y')}}{\alpha} + \frac{(k_3^2 - k_2^2) e^{-\alpha(y+y'+2a)}}{4\alpha^3} \right] \cos \alpha(x - x') d\alpha \\
 & - \frac{1}{4\pi} \{ \Gamma[0, [(y - y') + j(x - x')]\alpha_0] + \Gamma[0, [(y - y') - j(x - x')]\alpha_0] \}
 \end{aligned}$$

$$-\frac{(k_3^2 - k_2^2)}{16\pi} \left\{ \begin{aligned} &[(y + y' + 2a) + j(x - x')]^2 \\ &\cdot \Gamma[-2, [(y + y' + 2a) + j(x - x')]\alpha_0] \\ &+ [(y + y' + 2a) - j(x - x')]^2 \\ &\cdot \Gamma[-2, [(y + y' + 2a) - j(x - x')]\alpha_0] \end{aligned} \right\} \quad (\text{A3})$$

Now, the integral  $G_1$  is written as a rapidly converging integral plus a dominate integral. We can use Eq. (A3) to evaluate  $G_1$  by means of Simpson's rule easily.

Similarly,

$$\begin{aligned} G_2 = & \frac{1}{\pi} \int_0^\infty \frac{j}{2r_2} \\ & \left\{ \left[ \frac{(r_1 + r_2)(r_2 + r_3)e^{-jr_2[|y-y'|-2a]} + (r_2 - r_1)(r_2 - r_3)e^{jr_2[|y-y'|-2a]}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} \right] \right. \\ & + \left[ \frac{(r_2 - r_1)(r_2 + r_3)e^{jr_2[y+y']}] + (r_2 - r_3)(r_1 + r_2)e^{-jr_2[y+y']}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} \right] \\ & \left. - \frac{je^{-jr_2|y-y'|}}{2r_2} \right\} \cos \alpha(x - x') d\alpha + \frac{j}{4} H_0^{(2)} \left( k_2 \sqrt{(x - x') + (y - y')} \right) \\ & + \frac{1}{2\pi} \int_{\alpha_0}^\infty \left[ \frac{(k_3^2 - k_2^2)(k_1^2 - k_2^2)e^{-\alpha[4a - |y - y'|]}}{16\alpha^5} \right. \\ & + \frac{(k_1^2 - k_2^2)}{4} \frac{e^{-\alpha(2a - y - y')}}{\alpha^3} + \frac{(k_3^2 - k_2^2)}{4} \frac{e^{-\alpha(y' + y + 2a)}}{\alpha^3} \left. \right] \cos \alpha(x - x') d\alpha \\ & - \frac{(k_1^2 - k_2^2)}{16\pi} \left\{ \begin{aligned} &[(2a - y - y') + j(x - x')]^2 \\ &\cdot \Gamma[-2, [(2a - y - y') + j(x - x')]\alpha_0] \\ &+ [(2a - y - y') - j(x - x')]^2 \\ &\cdot \Gamma[-2, [(2a - y - y') - j(x - x')]\alpha_0] \end{aligned} \right\} \\ & - \frac{(k_3^2 - k_2^2)}{16\pi} \left\{ \begin{aligned} &[(y + y' + 2a) + j(x - x')]^2 \\ &\cdot \Gamma[-2, [(y + y' + 2a) + j(x - x')]\alpha_0] \\ &+ [(y + y' + 2a) - j(x - x')]^2 \\ &\cdot \Gamma[-2, [(y + y' + 2a) - j(x - x')]\alpha_0] \end{aligned} \right\} \\ & - \frac{(k_1^2 - k_2^2)(k_3^2 - k_2^2)}{64\pi} \\ & \cdot \left\{ \begin{aligned} &[4a - |y - y'| + j(x - x')]^4 \Gamma[-4, [4a - |y - y'| + j(x - x')]\alpha_0] \\ &+ [4a - |y - y'| - j(x - x')]^4 \Gamma[-4, [4a - |y - y'| - j(x - x')]\alpha_0] \end{aligned} \right\} \end{aligned}$$

And

$$\begin{aligned}
 G_3 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} j e^{jr_3(y+a)} \\
 &\quad \frac{(r_1 + r_2)e^{-jr_2(y'-a)} + (r_2 - r_1)e^{jr_2(y'-a)}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} e^{-j\alpha(x-x')} d\alpha \\
 &= \frac{1}{\pi} \int_0^{\infty} j e^{jr_3(y+a)} \\
 &\quad \frac{(r_1 + r_2)e^{-jr_2(y'-a)} + (r_2 - r_1)e^{jr_2(y'-a)}}{(r_1 + r_2)(r_2 + r_3)e^{jr_2(2a)} + (r_1 - r_2)(r_2 - r_3)e^{-jr_2(2a)}} e^{-j\alpha(x-x')} d\alpha \\
 &\quad + \frac{1}{2\pi} \int_{\alpha_0}^{\infty} \left[ \frac{e^{-\alpha(y'-y)}}{\alpha} + \frac{(k_1^2 - k_2^2) e^{-\alpha(2a-y-y')}}{4\alpha^3} \right] \cos \alpha(x - x') d\alpha \\
 &\quad - \frac{1}{4\pi} \{ \Gamma[0, [(y' - y) + j(x - x')]\alpha_0] + \Gamma[0, [(y' - y) - j(x - x')]\alpha_0] \} \\
 &\quad - \frac{(k_1^2 - k_2^2)}{16\pi} \\
 &\quad \cdot \left\{ [(2a - y - y') + j(x - x')]^2 \Gamma[-2, [2a - y - y') + j(x - x')]\alpha_0] \right. \\
 &\quad \left. + [(2a - y - y') - j(x - x')]^2 \Gamma[-2, [(2a - y - y') - j(x - x')]\alpha_0] \right\}
 \end{aligned}$$

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