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AXIALLY SLOTTED ANTENNA ON A CIRCULAR OR ELLIPTIC CYLINDER COATED WITH METAMATERIALS

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Abstract—The radiation properties of an axially slotted circular or elliptical antenna coated with metamaterials are investigated. The fields inside and outside the dielectric coating are expressed in terms of Mathieu functions. The boundary conditions at various surfaces are enforced to obtain the unknown field expansion coefficients. Numerical results are presented graphically for the radiation pattern, aperture conductance and antenna gain for the TM case. It was found that slotted antenna coated with metamaterials has more directive beam with lower sidelobes compared to coated with conventional dielectric material.

1. INTRODUCTION

Radiation properties of an axially slotted antenna are very important in communications and airplane industries. Numerous authors in the literature have investigated the radiation by dielectric coated slotted circular and elliptical cylinders. For example, Hurd [1] studied the radiation pattern of a dielectric axially slotted cylinder. The external admittance of an axial slot on a dielectric coated metal cylinder was investigated by Knop [2]. Shafai [3] obtained the radiation properties of an axial slotted antenna coated with a homogenous material. Wong [4, 5] investigated the radiation properties of slotted cylinder of elliptical cross section while Richmond [6] studied the radiation from an axial slot antenna on a dielectric coated elliptic cylinder. The analysis was later extended to the radiation by axial slots on a dielectric coated nonconfocal conducting elliptic cylinder [7]. Recently, Hussein and Hamid [8] studied the radiation by N axially slotted cylinder of elliptical cross section coated with a lossy dielectric material.

To the best of our knowledge, the radiation produced by an axially slotted circular or elliptical cylinder coated with metamaterials has not been investigated. Recently, materials posses both negative permittivity and permeability have gained considerable attention by many researches [9–12].

This paper presents an analytical solution of the radiation by an axially slotted antenna on a conducting elliptic cylinder coated with metamaterials based on the boundary value method. The presented numerical results will show the effect of the metamaterials coating on the radiation pattern, aperture conductance and gain of slotted antenna.

2. FORMULATION OF THE RADIATION PROBLEM

The geometry of the perfectly conducting elliptic cylinder with an axially slotted antenna covered by metamaterial is shown in Fig. 1. The structure is assumed to be infinite along the z-axis. The symbols a_c and b_c correspond to the conducting core semi-major and semi-minor axes, respectively, while a and b are the semi-major and semi-minor axes of the dielectric coating material. The axial slot coordinates on the conducting elliptic cylinder are denoted by v_1 and v_2. The elliptical coordinate system (u, v, z) is assumed, and it can be represented in terms of the Cartesian coordinate system (x, y, z) as,

$$x = F \cosh(u) \cos(v) \quad (1)$$

$$y = F \sinh(u) \sin(v) \quad (2)$$

where F is the semifocal length of the elliptical cross section. The radiated electric field outside the dielectric coating (region I and $\xi > \xi_1$) can be expressed in terms of Mathieu functions as follows

$$E_z^I = \sum_{m=0}^{\infty} C_{em} R_{em}^{(4)}(c_0, \xi) S_{em}(c_0, \eta) + \sum_{m=1}^{\infty} C_{om} R_{om}^{(4)}(c_0, \xi) S_{om}(c_0, \eta) \quad (3)$$

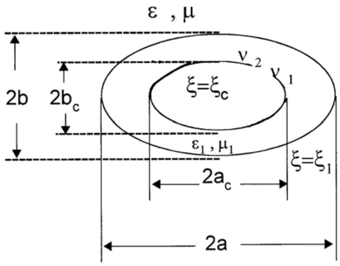

Figure 1. Geometry of an axially slotted antenna on elliptic cylinder coated with metamaterials.

where C_{em} and C_{om} are the unknown field expansion coefficients, S_{em} and S_{om} are the even and odd angular Mathieu functions of order m, and $R_{em}^{(4)}$ and $R_{om}^{(4)}$ are the even and odd modified Mathieu functions of the fourth kind. It should be noted that $\xi = \cosh u$, $\eta = \cos v$, $c = kF$, and $k = \omega\sqrt{\mu\varepsilon}$. Similarly, the electric field inside the dielectric coating (region II) for $\xi_c < \xi < \xi_1$ can be expressed in terms of Mathieu functions as

$$\begin{aligned} E_z^{II} &= \sum_{m=0}^{\infty} \left[A_{em} R_{em}^{(1)}(c_1,\xi) + B_{em} R_{em}^{(2)}(c_1,\xi)\right] S_{em}(c_1,\eta) \\ &\quad + \sum_{m=1}^{\infty} \left[A_{om} R_{om}^{(1)}(c_1,\xi) + B_{om} R_{om}^{(2)}(c_1,\xi)\right] S_{om}(c_1,\eta) \end{aligned} \tag{4}$$

where $c_1 = k_1 F$, $k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$, $A_{\substack{em\\om}}$ and $B_{\substack{em\\om}}$ are the unknown field expansion coefficients, $R_{\substack{em\\om}}^{(1)}$ and $R_{\substack{em\\om}}^{(2)}$ are the radial Mathieu functions of the first and second kind, respectively. The angular and the radial Mathieu functions are defined in [13]. The magnetic field component in regions (I) and (II) are obtained using Maxwells equations and written as

$$H_v^I = \frac{-j}{\omega\mu h}\left\{\sum_{m=0}^{\infty} C_{em} R_{em}^{\prime(4)}(c_0,\xi) S_{em}(c_0,\eta) + \sum_{m=1}^{\infty} C_{om} R_{om}^{\prime(4)}(c_0,\xi) S_{om}(c_0,\eta)\right\} \tag{5}$$

$$\begin{aligned} H_v^{II} &= \frac{-j}{\omega\mu_1 h}\left\{\sum_{m=0}^{\infty} \left[A_{em} R_{em}^{\prime(1)}(c_1,\xi) + B_{em} R_{em}^{\prime(2)}(c_1,\xi)\right] S_{em}(c_1,\eta)\right. \\ &\quad \left. + \sum_{m=1}^{\infty} \left[A_{om} R_{om}^{\prime(1)}(c_1,\xi) + B_{om} R_{om}^{\prime(2)}(c_1,\xi)\right] S_{om}(c_1,\eta)\right\} \end{aligned} \tag{6}$$

where $h = F\sqrt{\cosh^2 u - \cos^2 v}$. The prime in equations (5) and (6) denotes derivative with respect to u.

We require E_z to be continuous across the interface at $\xi = \xi_1$, this leads to

$$\left[A_{en}R_{en}^{(1)}(c_1,\xi_1) + B_{en}R_{en}^{(2)}(c_1,\xi_1)\right] N_{en}(c_1) = \sum_{m=0}^{\infty} C_{em}R_{em}^{(4)}(c_0,\xi_1)M_{enm}(c_1,c_0) \tag{7}$$

where

$$N_{en}(c) = \int_0^{2\pi} [S_{en}(c,\eta)]^2 dv \tag{8}$$

$$M_{enm}(c_1,c_0) = \int_0^{2\pi} S_{en}(c_1,\eta)S_{em}(c_0,\eta)dv \tag{9}$$

Continuity of the tangential magnetic field components at $\xi = \xi_1$ require that

$$\left[A_{en}R_{en}^{\prime(1)}(c_1,\xi_1) + B_{en}R_{en}^{\prime(2)}(c_1,\xi_1)\right] N_{en}(c_1) = \frac{\mu_1}{\mu} \sum_{m=0}^{\infty} C_{em}R_{em}^{\prime(4)}(c_0,\xi_1)M_{enm}(c_1,c_0) \tag{10}$$

Similar equations for the odd solution are needed and may be obtained by replacing 'e' with 'o' in equations (7) and (10).

In region (II), the tangential electric field on the conducting surface ($\xi = \xi_c$) must vanish except at the slot location. This leads to

$$\sum_{m=0}^{\infty} \left[A_{em}R_{em}^{(1)}(c_1,\xi_c) + B_{em}R_{em}^{(2)}(c_1,\xi_c)\right] S_{em}(c_1,\eta) + \sum_{m=1}^{\infty}\left[A_{om}R_{om}^{(1)}(c_1,\xi_c)+B_{om}R_{om}^{(2)}(c_1,\xi_c)\right]S_{om}(c_1,\eta) = \begin{cases} F(v) & v_1 < v < v_2 \\ 0, & \text{elsewhere.} \end{cases} \tag{11}$$

Multiplying both sides of (11) by $S_{en}(c_1,\eta)$ and integrating over $0 < v < 2\pi$, we obtain

$$\left[A_{en}R_{en}^{(1)}(c_1,\xi_c)+B_{en}R_{en}^{(2)}(c_1,\xi_c)\right] N_{en}(c_1) = F_{en} = \int_{v_1}^{v_2} F(v)S_{en}(c_1,\eta)dv \tag{12}$$

A similar equation may be obtained for the odd solution by following the steps in deriving equation (12), this leads to

$$\left[A_{on}R_{on}^{(1)}(c_1,\xi_c)+B_{on}R_{on}^{(2)}(c_1,\xi_c)\right]N_{on}(c_1) = F_{on} = \int_{v_1}^{v_2} F(v)S_{on}(c_1,\eta)dv \tag{13}$$

For the integrals in equations (12) and (13) to be evaluated, we express the field at the slot location as [6, 7]

$$F(v) = E_0 \cos[\pi(v_0 - v)/(2\alpha)] \tag{14}$$

$$v_0 = (v_1 + v_2)/2 \tag{15}$$

$$\alpha = (v_2 - v_1)/2 \tag{16}$$

The Mathieu angular functions are expressed in terms of Fourier series as

$$S_{en}(c,\eta) = \sum_k D_e^k(c,n)\cos(kv) \tag{17}$$

$$S_{on}(c,\eta) = \sum_k D_o^k(c,n)\sin(kv) \tag{18}$$

The summations in (17) and (18) extend over even values of k if n is even, and over odd values of k if n is odd. $D_{\substack{e\\o}}^k$ are the Fourier series coefficients [11]. Substituting equations (14)–(18) into equations (12) and (13), one obtains an expression for F_{en} and F_{on} as

$$F_{en} = E_o \sum_k D_e^k(c_1,n) \int_{v_1}^{v_2} \cos[\pi(v - v_0)/(2\alpha)]\cos(kv)dv \tag{19}$$

$$F_{on} = E_o \sum_k D_o^k(c_1,n) \int_{v_1}^{v_2} \cos[\pi(v - v_0)/(2\alpha)]\sin(kv)dv \tag{20}$$

Solving for B_{en} from equation (12) and using the result in equations (7) and (10) with the elimination of A_{en}, we obtain the following system of linear equations in terms of C_{en}

$$\sum_{n=0} C_{en}Z_{enm} = V_{em} \tag{21}$$

Following the same procedure, we obtain another system of linear equations in terms of C_{on}

$$\sum_{n=1} C_{on}Z_{onm} = V_{om} \tag{22}$$

where

$$Z_{nm} = \left[\frac{R_n^{(4)}(c_0,\xi_1)}{X_m} - \mu_r \frac{R_n^{\prime(4)}(c_0,\xi_1)}{X'_m}\right] R_m^{(2)}(c_1,\xi_c) M_{nm}(c_1,c_0) \quad (23)$$

$$V_m = F_m \left[\frac{R_n^{(2)}(c_1,\xi_1)}{X_m} - \frac{R_n^{\prime(2)}(c_1,\xi_1)}{X'_m}\right] \quad (24)$$

and

$$X_m = \left[R_m^{(1)}(c_1,\xi_0) R_m^{(2)}(c_1,\xi_c) - R_m^{(2)}(c_1,\xi_0) R_m^{(1)}(c_1,\xi_c)\right] N_m(c_1) \quad (25)$$

$$X'_m = \left[R_m^{\prime(1)}(c_1,\xi_0) R_m^{(2)}(c_1,\xi_c) - R_m^{\prime(2)}(c_1,\xi_0) R_m^{(1)}(c_1,\xi_c)\right] N_m(c_1) \quad (26)$$

3. NUMERICAL RESULTS

Once the unknown field expansion coefficients $C_{\substack{en\\on}}$ are computed, quantities of interest such as far-field radiation pattern, antenna gain, and the aperture conductance can be obtained. The far-zone radiation pattern for the electric field can be calculated using the asymptotic form of the radial Mathieu functions $R^{(4)}_{\substack{em\\om}}$. Thus the far-zone field of the slotted antenna can be written as

$$E_z^I(\rho,\phi) = \sqrt{\frac{j}{k\rho}} e^{-jk\rho} \left[\sum_{n=0}^{\infty} j^n C_{en} S_{en}(c_0,\cos\phi) + \sum_{n=1}^{\infty} j^n C_{on} S_{on}(c_0,\cos\phi)\right] \quad (27)$$

where ρ and ϕ denote the polar coordinates in the circular cylindrical system. The antenna gain is expresses as [6, 7]

$$G(\phi) = \frac{1}{Z_0 k\rho} \left[\left|\sum_{n=0}^{\infty} j^n C_{en} S_{en}(c_0,\cos\phi)\right|^2 + \left|\sum_{n=1}^{\infty} j^n C_{on} S_{on}(c_0,\cos\phi)\right|^2\right] \quad (28)$$

where Z_0 is the free space impedance. The aperture conductance per unit length of the slotted antenna is defined as [6]

$$G_a = 2\pi\rho \frac{S_{av}}{|E_0|^2} \quad (29)$$

where S_{av} is the average power density averaged over an imaginary cylinder of radius ρ and given as

$$S_{av} = \frac{1}{2\pi Z_0 k\rho} \left[\sum_{n=0}^{\infty} |C_{en}|^2 N_{en}(c_0) + \sum_{n=1}^{\infty} |C_{on}|^2 N_{on}(c_0)\right] \quad (30)$$

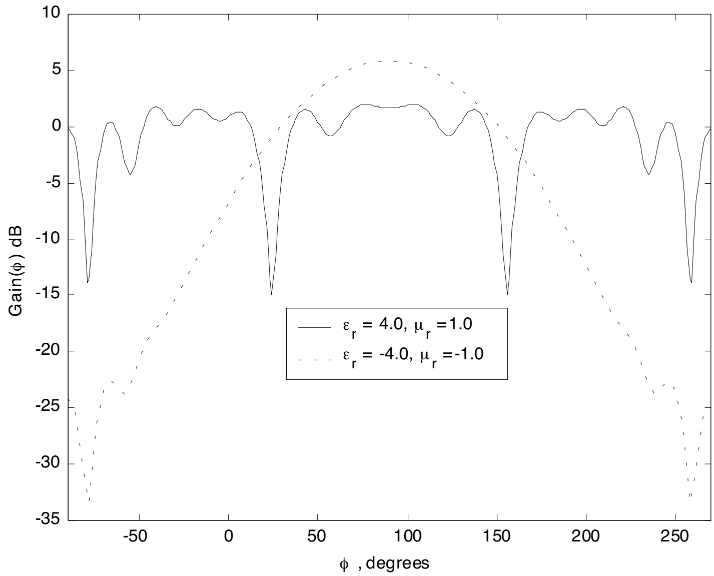

Figure 2. Radiation pattern of an axially slotted elliptic cylinder coated with conventional dielectric and metamaterials.

The accuracy of our numerical results is verified against published results for a single slotted circular or elliptic antenna coated with a lossless conventional dielectric material [6, 8]. The geometrical parameters of the slotted antenna used for comparison are $a_c = \lambda$, $b_c = \lambda/2$, $b = b_c + t$, where t is the coating thickness, $v_0 = 90°$ and $\alpha = 2.8657°$. Figure 2 shows the radiation pattern numerical results (gain versus ϕ) obtained for a conventional dielectric coating material represented by solid line, for comparison [6, 8] ($\varepsilon_r = 4$ and $\mu_r = 1$), and metamaterials coating represented by dashed line ($\varepsilon_r = -4$ and $\mu_r = -1$). It can be seen that the metamaterials coating material makes the beam sharper, more directive with higher value especially at $\phi = 90°$, and reduces the side-lobes. The effect of the coating thickness is illustrated in Fig. 3 for the same geometrical parameters as in Fig. 2. One may notice that by increasing the thickness of the metamaterials coating enhances the gain with a decrease in the number of side-lobes. On the other hand, it was earlier shown in [6, 8] that by increasing the thickness of the conventional dielectric coating ($\varepsilon_r = 4$ and $\mu_r = 1$) results in a reduction of the main beam with an increase in the number of sidelobes. Fig. 4 is similar to 3 except for a circular slotted antenna. It can be seen that the circular antenna has a similar behavior as the elliptical antenna.

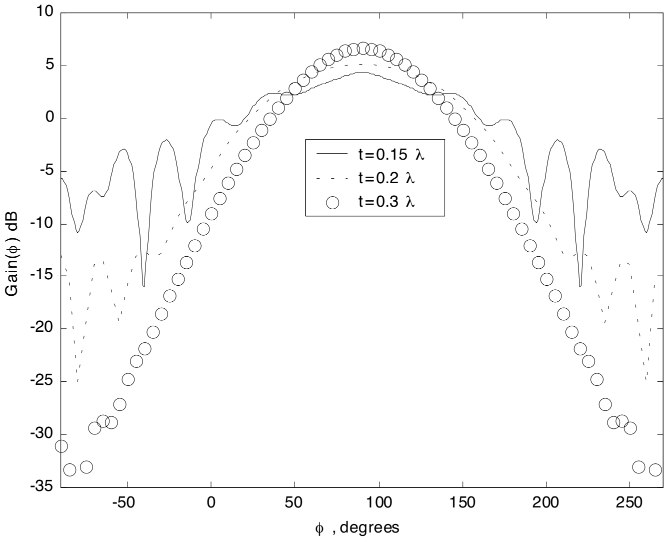

Figure 3. Radiation pattern of an axially slotted elliptic cylinder coated with metamaterials ($\varepsilon_r = -4.0$ and $\mu_r = -1.0$) and various coating thickness.

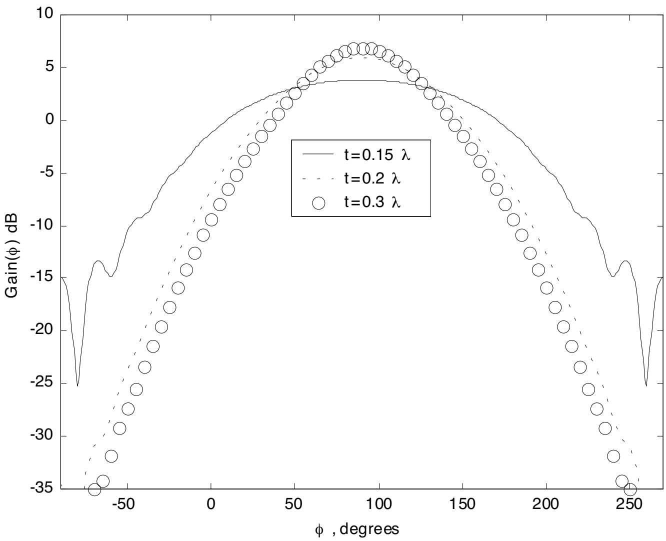

Figure 4. Radiation pattern of an axially slotted circular cylinder coated with metamaterials ($\varepsilon_r = -4.0$ and $\mu_r = -1.0$) and various coating thickness.

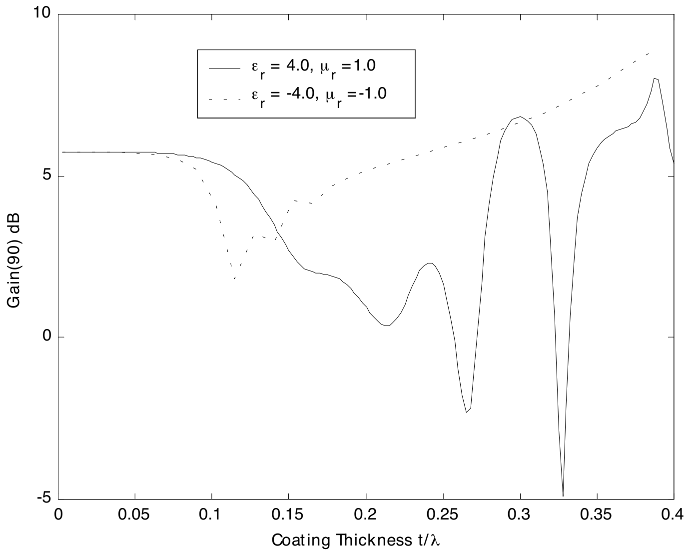

Figure 5. Gain versus coating thickness for an axially slotted elliptic cylinder coated with conventional dielectric and metamaterials.

The gain versus coating thickness for a single slot elliptical antenna with the same geometrical parameters used in Fig. 2 is displayed in Fig. 5. The gain is evaluated at $\phi = 90^\circ$ since the slot is centered at $v = 90^\circ$ where the gain is expected to be maximum. For very small coating thickness, the conventional coating and metamaterials have the same effect on the gain. As t becomes greater than $0.15\,\lambda$, the metmaterials coating increases the antenna gain when compared with the conventional coating. Further, the presence of surface waves in the case of conventional coating starts to disappear in the case of metmaterials coating. Fig. 6 is similar to 5 except for a circular antenna. It can be seen that the circular antenna has similar characteristics as the elliptical antenna.

The gain versus coating layer material permittivity and permeability is plotted in Figs. 7 and 8, respectively, and such plots are not available in earlier publications [6, 8]. In Fig. 7 when both ε_r and μ_r are negative and $t = 0.3\,\lambda$, the gain has higher values with no oscillations when compared with the narrower coating thickness $t = 0.15\,\lambda$.

The aperture conductance for metamaterials and conventional dielectric coating is shown in Fig. 9 for an elliptical antenna with the same geometrical parameters used in Fig. 2. The antenna has

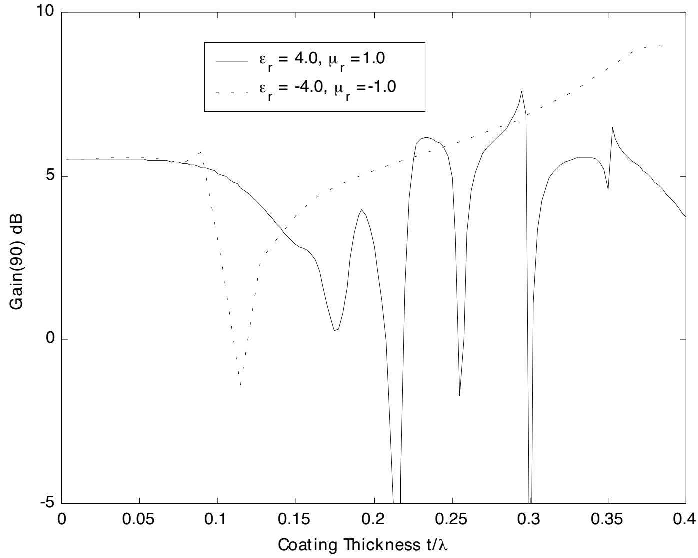

Figure 6. Gain versus coating thickness for an axially slotted circular cylinder coated with conventional dielectric and metamaterials.

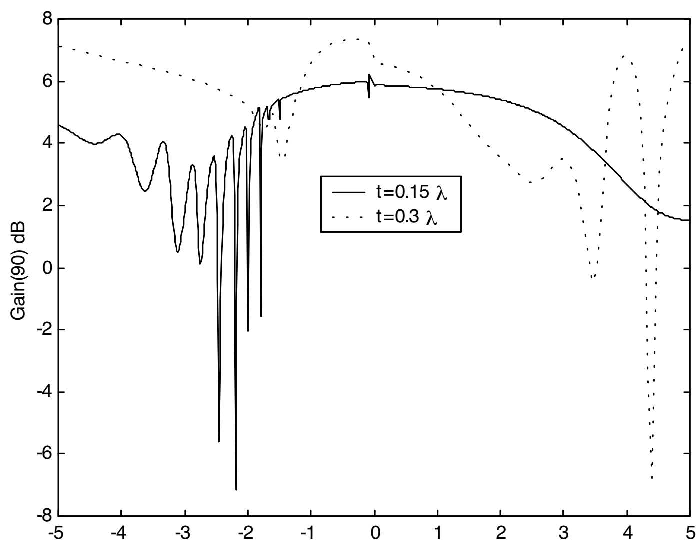

Figure 7. Gain versus ε_r for dielectric coated axially slotted elliptic cylinder and various dielectric coating thickness.

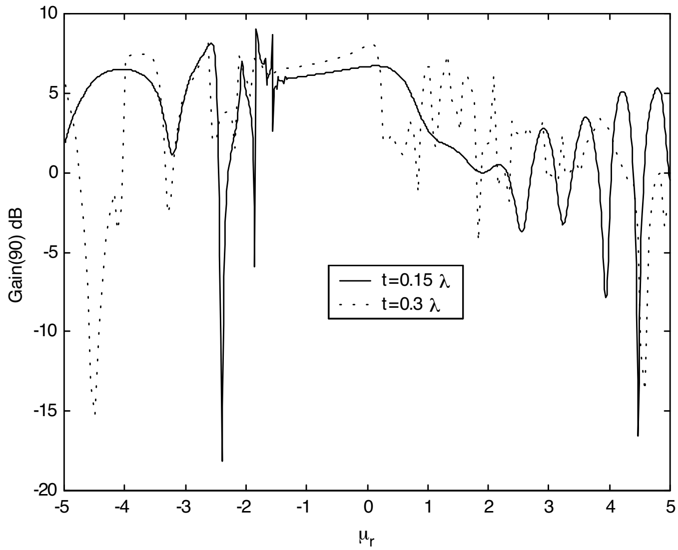

Figure 8. Gain versus μ_r for dielectric coated axially slotted elliptic cylinder and various dielectric coating thickness.

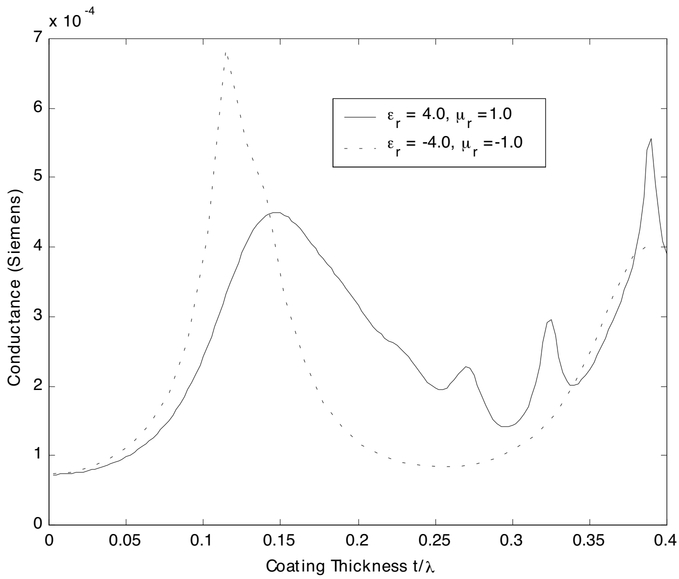

Figure 9. Aperture conductance versus coating thickness for an axially slotted elliptic cylinder coated with conventional dielectric and metamaterials.

higher conductance values for metamaterials coating thickness less than $0.15\,\lambda$, and smaller conductance values for t greater than $0.15\,\lambda$ when compared with the conventional coating material.

4. CONCLUSIONS

The radiation characteristics of an axially slotted circular or elliptical antenna coated with metamaterials were investigated using analytic solution. It was shown that the presence of metamaterials coating has changed significantly the characteristics of the antenna. Finally, the metamaterials can be used to enhance the antenna gain with lower side-lobes over a certain dielectric coating range.

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