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# STUDY OF GENERALIZED RESONANCE IN MULTI-ANTENNA SYSTEM AND GENERALIZED FOSTER REACTANCE THEOREM

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**Abstract**—This paper begins with a complete description of the complex Poynting theorem, followed by a rigorous study of the generalized resonance in a multi-antenna system. The condition generating the generalized resonance is discussed, which is the balance of the electromagnetic fields energy stored in the antennas open system. The matrix expression of the generalized resonant factor ($GRF$) is derived. On this basis, the generalized Foster reactance theorem for an arbitrary antenna system is presented and radiation $Q$ is used to further describe the generalized resonance behaviors. Some practical examples have shown that the generalized resonance may take on the phenomena of strong and sharp fields in the near zone and super-directivity in the far zone of the antenna system.

## 1. INTRODUCTION

With the development of the electronic devices and wireless communication, the electromagnetic environment becomes more and more numerous and complicated. It is well known that there are scores of antennas, even hundreds of antennas simultaneously located on an airplane, ship or communication device, which compose a complicated multi-antenna system. The interaction and mutual coupling among antenna elements sometimes give rise to the strong electromagnetic oscillation phenomena. Some researches of the electromagnetic compatibility (EMC) for warships have found the phenomenon of strong electric field spots near missile trays when some antennas are simultaneously working, which will result in the free-running of missile trays [1, 2]. Especially for the high-power radiating systems which have found that the power that may be transmitted can be severely limited by the breakdown of the air around the antennas [3]. Fante [3] studied the bounds on the electric field outside a radiating system. Liang [1] and Jin [2] gave an analysis of the special phenomenon and presented the concept of generalized resonance, which is based on an open system and is caused by strong interaction of multi-object. However, the nature of the generalized resonance in an open system can not be revealed completely. Therefore, a rigorous analysis of resonant behaviors in EMC has been an interesting and challenging problem for years [1–10].

In this paper, we desire to give a more rigorous study of the generalized resonance in a multi-antenna open system. In Section 2, we present a complete description of the complex Poynting theorem for electromagnetic fields, in order to derive the electric and magnetic field energy stored in the near zone of the antenna system. In Section 3, based on the complex Poynting theorem, it is shown that the multi-antenna system is essentially equivalent to a complicated lossy multi-port network and the electromagnetic oscillation still exists in the open system. When the stored electric field energy and magnetic field energy are identical, namely the balance of the stored electric and magnetic field energy, the generalized resonance will take place. The matrix expression of the generalized resonant factor ($GRF$) is derived, which can be used to determine the generalized resonant frequency. The conventional Foster reactance theorem is usually stated for a lossless network. For an antenna system, the loss represents the radiated power from the antenna, which prohibits the direct use of the Foster reactance theorem, because the slope of reactance for the antenna can be negative. In Section 4, we further present the generalized Foster reactance theorem for antennas and radiation $Q$ computed in complex frequency domain to describe the generalized resonance behaviors. Finally, by analyzing some practical multi-antenna systems presented in this paper, we have shown that the generalized resonance may take on the phenomena of strong and sharp fields in the near zone and super-directivity in the far zone of the antenna systems.

## 2. POYNTING THEOREM AND ELECTRIC AND MAGNETIC FIELD ENERGY STORED IN OPEN SYSTEM

The differential form of the complex Poynting theorem [8] for time harmonic field in an isotropic medium is given by

$$\nabla \cdot \left( \frac{1}{2} \vec{E} \times \vec{H}^* \right) = -\frac{1}{2} \vec{J}^* \cdot \vec{E} - j2\omega(w_m - w_e) \tag{1}$$

where $w_m = \frac{1}{4}\mu \vec{H} \cdot \vec{H}^*$ and $w_e = \frac{1}{4}\varepsilon \vec{E} \cdot \vec{E}^*$ are the magnetic and electric field energy densities, respectively. We take the integration of (1) over a region $V$, which is enclosed by a sphere surface $S$, as shown in Fig. 1. Let $V_0$ be the volume occupied by the source $\vec{J}$ and $S_0$ be the surface surrounding $V_0$. Using the divergence theorem, we get

$$\int_S \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot d\vec{s} = \int_{V_0} -\frac{1}{2} \vec{J}^* \cdot \vec{E} dv - j2\omega \int_V (w_m - w_e) dv \tag{2}$$

Taking the imaginary part of (2), we easily obtain

$$\mathrm{Im} \int_S \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot d\vec{s} = \mathrm{Im} \int_{V_0} -\frac{1}{2} \vec{J}^* \cdot \vec{E} dv - 2\omega \int_V (w_m - w_e) dv \tag{3}$$

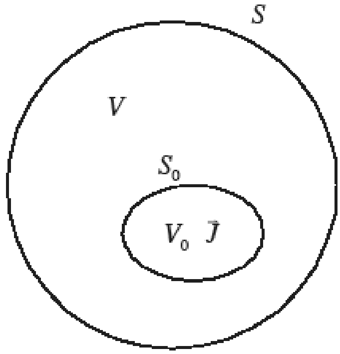



**Figure 1.** A volume $V$ containing source region.

Let us choose $V = V_\infty$, where $V_\infty$ is the region enclosed by a sphere with radius $r_\infty$, where $r_\infty$ is sufficiently large so that it lies in the far field region of the antenna system. Since the complex Poynting vector is a real vector in the far field region, we have

$$\mathrm{Im}\int_{V_0} -\frac{1}{2}\vec{J}^*\cdot\vec{E}dv = 2\omega\int_{V_\infty}(w_m - w_e)dv \tag{4}$$

Substituting (4) into (3), we obtain

$$\mathrm{Im}\int_S \frac{1}{2}(\vec{E}\times\vec{H}^*)\cdot d\vec{s} = 2\omega\int_{V_\infty - V}(w_m - w_e)dv \tag{5}$$

Equation (5) shows that the surface integral of the imaginary part of the Poynting vector depends on the integration surface $S$ in the near field region. Taking the real part of (2), we obtain the radiated power

$$P_{rad} = \mathrm{Re}\int_S \frac{1}{2}(\vec{E}\times\vec{H}^*)\cdot d\vec{s} = \mathrm{Re}\int_{V_0} -\frac{1}{2}\vec{J}^*\cdot\vec{E}dv \tag{6}$$

Equation (6) shows that the surface integral of the real part of the Poynting vector is independent of the surface $S$ as long as it encloses the source region $V_0$, that is, $P_{rad}(S) = P_{rad}(S_0) = P_{rad}(S_\infty) = \mathrm{Re}\int_{V_0} -\frac{1}{2}\vec{J}^*\cdot\vec{E}dv$. Combining (5) and (6), we get

$$\begin{aligned}\int_S \frac{1}{2}(\vec{E}\times\vec{H}^*)\cdot d\vec{s} &= \mathrm{Re}\int_{V_0} -\frac{1}{2}\vec{J}^*\cdot\vec{E}dv + j\mathrm{Im}\int_{V_0} -\frac{1}{2}\vec{J}^*\cdot\vec{E}dv \\ &\quad - j2\omega\int_V (w_m - w_e)dv \\ &= P_{rad} + j2\omega\int_{V_\infty - V}(w_m - w_e)dv \end{aligned} \tag{7}$$

The above expression is the integration form of the complex Poynting theorem for the region shown in Fig. 1, which indicates that the complex power flowing out of $S$ is equal to the radiation power plus the reactive power outside $S$. Some controversies about the applicability of the complex Poynting theorem and the time dependent Poynting theorem in the calculation of antenna $Q$ were proposed [11–16]. A misunderstanding is actually caused by an improper explanation of the power balance relation (7).

The field radiated by an antenna consists of a radiation field carrying power to infinity and a localized reactive field. However, for the purpose of evaluating the energy stored in the reactive field, the field can not be separated into a radiation field and a reactive field and treated separately due to nonzero interaction terms. Following the Collin's and Fante's viewpoints [17, 18], the energy density can be physically separated into two parts: one is time reversible and the other is time irreversible. The reversible part should be identified with evanescent stored energy, while the irreversible part should be associated with radiation. Let $w'_e$ and $w'_m$ denote the time-average, nonpropagating, stored electric field and magnetic field energy densities, and $w_e^{rad}$ and $w_m^{rad}$ denote the time-average radiated electric field and magnetic field energy densities, respectively. We can then define

$$\begin{cases} w_m = w'_m + w_m^{rad} \\ w_e = w'_e + w_e^{rad} \end{cases} \tag{8}$$

These calculations are physically appropriate since density is a summable quantity. It is readily seen from (5) that $w_m = w_e$ in the far field zone since the complex Poynting vector becomes real as $V$ approaches $V_\infty$. In addition, according to the far field expression generated by an arbitrary current distribution [15, 19], we can easily obtain $\vec{E}^{rad} = \eta \vec{H}^{rad} \times \hat{n}$, where $\eta$ denotes the wave impedance in free space, i.e., $\eta = 120\pi$. Hence, we have

$$w_m^{rad} = \frac{1}{4}\mu \vec{H}^{rad} \cdot \vec{H}^{rad*} = \frac{1}{4}\varepsilon \vec{E}^{rad} \cdot \vec{E}^{rad*} = w_e^{rad} \tag{9}$$

Mathematically, (9) holds everywhere. It is shown that the electric field energy and magnetic field energy for the radiated field are identical everywhere. Therefore, in the region $V_\infty - V_0$, we obtain

$$\begin{aligned} W'_m - W'_e &= \int_{V_\infty - V_0} (w'_m - w'_e) dv = \int_{V_\infty - V_0} (w_m - w_e) dv \\ &= \frac{1}{2\omega} \mathrm{Im} \int_{S_0} \frac{1}{2} (\vec{E} \times \vec{H}^*) \cdot d\vec{s} \end{aligned} \tag{10}$$

$$\begin{aligned} W'_m + W'_e &= \int_{V_\infty - V_0} (w'_m - w'_e) dv \\ &= \int_{V_\infty - V_0} [(w_m - w_m^{rad}) + (w_e - w_e^{rad})] dv \\ &= \int_{V_\infty - V_0} (w_m + w_e) dv - \frac{r_\infty}{c} \mathrm{Re} \int_{S_\infty} \frac{1}{2} (\vec{E} \times \vec{H}^*) \cdot d\vec{s} \\ &= W_m + W_e - \frac{r_\infty}{c} P_{rad} \end{aligned} \quad (11)$$

where $W_m$ and $W_e$ represent the total time-average magnetic field and electric field energy, and $W'_m$ and $W'_e$ represent the time-average, nonpropagating, stored magnetic field and electric field energy, respectively. $P_{rad}$ is the radiated power, $r_\infty$ is the radius of the sphere surface $S_\infty$ , and $c$ is the speed of light. It is noticed that two integral terms in (11) are divergent as $r_\infty \to \infty$. However, by using a proof similar to that used in Rellich's theorem [20], it can be shown that the net term in (11) is convergent, which implies the reactance energy stored around an antenna system is finite. Thus, $W'_e$ and $W'_m$ can be expressed as follows

$$W'_e = \frac{1}{2} \int_{V_\infty - V_0} (w_m + w_e) dv - \frac{r_\infty}{2c} P_{rad} - \frac{1}{4\omega} \mathrm{Im} \int_{s_0} \frac{1}{2} (\vec{E} \times \vec{H}^*) \cdot d\vec{s} \quad (12)$$

$$W'_m = \frac{1}{2} \int_{V_\infty - V_0} (w_m + w_e) dv - \frac{r_\infty}{2c} P_{rad} + \frac{1}{4\omega} \mathrm{Im} \int_{s_0} \frac{1}{2} (\vec{E} \times \vec{H}^*) \cdot d\vec{s} \quad (13)$$

The clarification of the concept of the electric and magnetic field energy stored in the open system is helpful to gain insight into the nature of the generalized resonance.

## 3. GENERALIZED RESONANCE IN MULTI-ANTENNA SYSTEM

The generalized resonance is based on an open system and is caused by the strong interaction of multi-object. If there is only one object or antenna in the system, its self-resonance behaviors belong to the generalized resonance presented in this paper. But there are not the special phenomena of the generalized resonance in the mono-object system, such as strong and sharp fields in the near zone around objects. So it is not emphasis of our study. We mainly analyze the generalized resonance in the $N$-antenna system, as shown in Fig. 2. Let $V$, enclosed by $S$, contain $N$ antennas and $V_i$ be the region occupied by $i_{th}$ antenna.

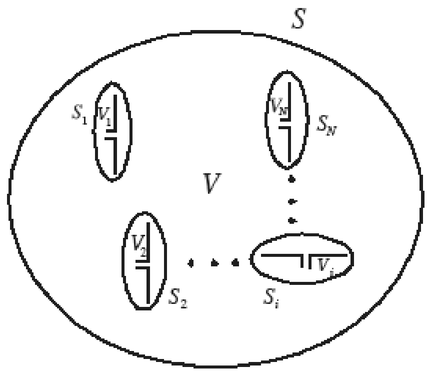



**Figure 2.** A system containing $N$ antennas.

If we choose $S = S_\infty$, using complex Poynting theorem over the region $V_\infty - \sum_{i=1}^{N} V_i$, we obtain

$$\int_{S_\infty} \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot d\vec{s} + \sum_{i=1}^{N} \int_{S_i} \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot d\vec{s} = -j2\omega(W_m - W_e) \quad (14)$$

We choose $S_i$ is coincident with the $i_{th}$ antenna surface and assume the antenna surface is perfectly conducting, and then $(\vec{E} \times \vec{H}^*) \cdot d\vec{s}$ vanishes everywhere on $S_i$ except over the input terminal. For a single-mode transmission line we have

$$\frac{1}{2}\sum_{i=1}^{N} V_i I_i^* = \int_{s_\infty} \frac{1}{2}(\vec{E} \times \vec{H}^*) \cdot d\vec{s} + j2\omega(W_m - W_e) \qquad (15)$$

where $V_i$ and $I_i$ are equivalent voltage and current at the feeding reference plane of $i_{th}$ antenna, respectively. Introduce the equivalent impedance matrix of the multi-antenna system as follows

$$[V] = [Z][I] \qquad (16)$$

Hence, (15) may be expressed concisely in matrix form

$$[I]^+[Z][I] = P_{rad} + j2\omega(W_m - W_e) \qquad (17)$$

where superscript $^+$ indicates conjugation and transposition. The antenna open system shown in Fig. 2 can be viewed as the generalized closed system which enclosed by the surface $S_\infty$ and $\sum_{i=1}^{N} S_i$. Similar to the definition of resonance in the circuit system, we define the generalized resonance as the balance of the stored electric and magnetic fields energy in the antenna system, which is

$$W'_m = W'_e \tag{18}$$

From previously theory, we know $W'_m = W_m - W_m^{rad}$, $W'_e = W_e - W_e^{rad}$, and $W_m^{rad} = W_e^{rad}$. Thus (18) can be expressed as $W_m = W_e$. From (17), we can obtain the matrix description of the generalized resonance as follows

$$\mathrm{Im}([I]^+[Z][I]) = 0 \tag{19}$$

If the antenna system is reciprocal network, (19) can be reduced to

$$[I]^+[X][I] = 0 \tag{20}$$

where $[I]$ is the equivalent current vector, and $[X]$ is the imaginary part matrix of the equivalent impedance matrix of the antenna system. The generalized resonant factor ($GRF$) can be defined as

$$GRF = \mathrm{Im}([I]^+[Z][I]) \tag{21}$$

By analyzing $GRF$, we can accurately determine the generalized resonant frequencies that are the solutions to the equation $GRF = 0$. In other words, the prediction of the generalized resonance occurred in the $N$-antenna system can be made by observing $GRF$. The dual form of $GRF$, using the equivalent admittance matrix of the antenna system, can be expressed as

$$GRF = \mathrm{Im}([V]^+[Y]^+[V]) \tag{22}$$

The above relations indicate that the generalized resonance is related to not only the antenna system itself and modes, but also the complicated excitations and loads. When $W'_m = W'_e$, the generalized resonance will take place. In this case, not only the stored electric and magnetic field energy in the near zone but also the radiated electric and magnetic field energy in the far zone are identical. Therefore, the generalized resonance may take on the phenomena of strong and sharp fields in the near zone and the large frequency sensitivity in the far zone of the antenna system. If an optimal excitation and design of gain were made, the antenna system would realize the super-directive characteristics [9, 21].

## 4. GENERALIZED FOSTER REACTANCE THEOREM AND RADIATION $Q$ FOR ANTENNAS

An important parameter specifying selectivity, and performance in general, of a resonant system is the quality factor, $Q$. A general definition of $Q$ applicable to all resonant systems is [22]

$$Q = \frac{\omega(\text{time} - \text{average energy stored in system})}{\text{energy loss per second in system}} \tag{23}$$

For the antennas open system, we introduce radiation $Q$ to further describe the generalized resonance behaviors, which can be defined as

$$Q = \begin{cases} \dfrac{2\omega W'_e}{P_{rad}} & W'_m \leq W'_e \\ \dfrac{2\omega W'_m}{P_{rad}} & W'_m \geq W'_e \end{cases} \tag{24}$$

where $W'_e$ and $W'_m$ are the time-average, nonpropagating, stored electric and magnetic energy respectively, and $P_{rad}$ is the radiated power. Since the generalized resonance condition requires $W'_m = W'_e = W'$ and $GRF = 0$, the generalized resonance $Q$ is expressed as

$$Q_G = \frac{2\omega_0 W'}{P_{rad}} \tag{25}$$

where $\omega_0$ is the generalized resonant frequency. $P_{rad}$ and $W'$ can be calculated by (6) and (12), we rewrite them as

$$P_{rad} = \text{Re}\sum_{i=1}^{N}\int_{S_i}\frac{1}{2}(\vec{E}\times\vec{H}^*)\cdot d\vec{s} \tag{26}$$

$$W' = \frac{1}{2}\int_{V_\infty - \sum_{i=1}^{N} V_i}(w_m + w_e)dv - \frac{r_\infty}{2c}P_{rad} \tag{27}$$

It is well known that the Foster reactance theorem is usually stated for a lossless network and is a very important tool for the synthesis of networks. From the viewpoint of network theory, an antenna is essentially equivalent to a one-port lossy network. The loss represents the radiated power from the antenna, which prohibits the direct use of the Foster reactance theorem, because the slope versus frequency of a reactance or susceptance function for the antenna can be negative [19, 25]. In the following section, we will derive strictly the generalized Foster reactance theorem for antennas and lossy networks, for the purpose of clarifying the physical characteristics of slope of reactance or susceptance. Taking the frequency derivatives of Maxwell's equations in a passive lossless isotropic medium, we have

$$\nabla\cdot\left(\frac{\partial\vec{E}}{\partial\omega}\times\vec{H}^* - \frac{\partial\vec{H}}{\partial\omega}\times\vec{E}^*\right) = -j\mu\vec{H}\cdot\vec{H}^* - j\varepsilon\vec{E}\cdot\vec{E}^* \tag{28}$$

Taking the integration of (28) over the passive region $V_\infty - \sum_{i=1}^{N} V_i$, as shown in Fig. 2, and using divergence theorem to the left term, we get

$$\int_{S_\infty + \sum\limits_{i=1}^{N} S_i} \left( \frac{\partial \vec{E}}{\partial \omega} \times \vec{H}^* - \frac{\partial \vec{H}}{\partial \omega} \times \vec{E}^* \right) \cdot d\vec{S} = -j4 \int_{V_\infty - \sum\limits_{i=1}^{N} V_i} (w_m + w_e) dv \tag{29}$$

Now if we assume that the antenna surface is perfectly conducting, $(\frac{\partial \vec{E}}{\partial \omega} \times \vec{H}^*) \cdot d\vec{s}$ and $(\frac{\partial \vec{H}}{\partial \omega} \times \vec{E}^*) \cdot d\vec{s}$ vanish everywhere over $S_0$ except over the input terminals. The integral on the left-hand side of (29) can be related to the input voltage $[V]$ and current $[I]$ of the antennas, and thus (29) can be changed into as follows:

$$\begin{aligned} [I]^+ \frac{\partial [V]}{\partial \omega} + [V]^+ \frac{\partial [I]}{\partial \omega} &= \int_{S_\infty} \left( \frac{\partial \vec{E}}{\partial \omega} \times \vec{H}^* - \frac{\partial \vec{H}}{\partial \omega} \times \vec{E}^* \right) \cdot d\vec{s} \\ &\quad + j4 \int_{V_\infty - \sum\limits_{i=1}^{N} V_i} (w_m + w_e) dv \end{aligned} \tag{30}$$

where $[V]$ and $[I]$ are the voltage and current vectors at the antenna array terminals. In the far field region, we can write $\vec{E} = \vec{E}_\infty(\omega) \frac{e^{-jkr}}{r}$, $\vec{H} = \vec{H}_\infty(\omega) \frac{e^{-jkr}}{r}$, where $\vec{E}_\infty(\omega)$ and $\vec{H}_\infty(\omega)$ are independent of the position $r$, and $\vec{E}_\infty = \eta \cdot \vec{H}_\infty \times \hat{n}$. Substituting these into (30), we obtain

$$\begin{aligned} [I]^+ \frac{\partial [V]}{\partial \omega} + [V]^+ \frac{\partial [I]}{\partial \omega} &= \int_{S_\infty} \frac{1}{r^2} \left( \frac{\partial \vec{E}_\infty}{\partial \omega} \times \vec{H}_\infty^* - \frac{\partial \vec{H}_\infty}{\partial \omega} \times \vec{E}_\infty^* \right) \cdot d\vec{s} \\ &\quad + j \left[ \begin{array}{l} 4 \int_{V_\infty - \sum\limits_{i=1}^{N} V_i} (w_m + w_e) dv \\ -\frac{4r_\infty}{c} \mathrm{Re} \int_{S_\infty} \frac{1}{2} (\vec{E} \times \vec{H}^*) \cdot d\vec{s} \end{array} \right] \end{aligned} \tag{31}$$

From (11), we know that the net term in square brackets of (31) represents the total time-average, nonpropagating, stored electromagnetic fields energy, which is convergent. Introducing $\vec{E}_\infty = \eta \cdot \vec{H}_\infty \times \hat{n}$ and $\vec{E}_\infty(\omega) = |\vec{E}_\infty(\omega)| e^{j\theta_{E\infty}}$ into (31), where $\theta_{E_\infty}$ denotes the phase of $\vec{E}_\infty$, $\hat{n}$ is the unit normal to $S_\infty$ and that $\hat{n} \cdot \vec{E}_\infty = 0$, we get

$$[I]^+ \frac{\partial [V]}{\partial \omega} + [V]^+ \frac{\partial [I]}{\partial \omega} = \frac{\partial (2P_{rad})}{\partial \omega} + j4 \left( W_m + W_e - \frac{r_\infty}{c} P_{rad} \right)$$

$$+ j4 \int_{S_\infty} \vec{S} \cdot \hat{n} \frac{\partial \theta_{E_\infty}}{\partial \omega} ds \qquad (32)$$

where $\vec{S} = \frac{1}{2}\vec{E} \times \vec{H}^*$ is the complex Poynting vector, $P_{rad} = \int_{S_\infty} \vec{S} \cdot \hat{n} ds = \int_{S_\infty} \frac{1}{2\eta r^2} |\vec{E}_\infty(\omega)|^2 ds$ is the radiated power, and $c$ is the speed of light. Introduce the impedance matrix of the antenna array $[Z] = [R] + j[X]$ and let

$$[I] = \begin{bmatrix} e^{j\theta_{I_1}} & & & 0 \\ & e^{j\theta_{I_2}} & & \\ & & \ddots & \\ 0 & & & e^{j\theta_{I_N}} \end{bmatrix} \cdot \begin{bmatrix} |I_1| \\ |I_1| \\ \vdots \\ |I_N| \end{bmatrix} = e^{j[\theta_I]}[|I|] \qquad (33)$$

where $|I_1|$, $|I_2|$, $\cdots$, $|I_N|$ denote the magnitude of the $N$-port currents, and $\theta_{I_1}$, $\theta_{I_2}$, $\cdots$, $\theta_{I_N}$ denote the phase of the $N$-port currents and $[\theta_I]$ represents the current phase diagonal matrix of $N \times N$ dimensions.

$$\begin{aligned} [I]^+ \frac{\partial [V]}{\partial \omega} + [V]^+ \frac{\partial [I]}{\partial \omega} &= [I]^+ \frac{\partial [Z]}{\partial \omega}[I] + 2[I]^+[R]\frac{\partial [I]}{\partial \omega} \\ &= \frac{\partial (2P_{rad})}{\partial \omega} + j[|I|]^T \left( \frac{\partial [X]}{\partial \omega} + 2[R]\frac{\partial [\theta_I]}{\partial \omega} \right) [|I|] \end{aligned} \qquad (34)$$

where $P_{rad} = \frac{1}{2}[|I|]^T[R][|I|]$, and superscript $^T$ denotes transposition. Comparing (32) with (34), we finally obtain

$$\begin{aligned} [|I|]^T \frac{\partial [X]}{\partial \omega}[|I|] &= 4\left( W_m + W_e - \frac{r_\infty}{c} P_{rad} \right) + 4 \int_{S_\infty} \vec{S} \cdot \hat{n} \frac{\partial \theta_{E_\infty}}{\partial \omega} ds \\ &\quad - 2[|I|]^T \left( [R]\frac{\partial [\theta_I]}{\partial \omega} \right) [|I|] \end{aligned} \qquad (35)$$

The above expression states the generalized Foster reactance theorem for an antenna array system, which is also applicable to a multi-port lossy network. It is well known that whatever the stored reactive energy or radiated energy is always positive. But the frequency derivatives of far-field phase and input-current phase can be negative, therefore the second and third terms of the right-hand of (35) do not represent the electromagnetic fields energy in principle, which result from the radiated power loss. In Fante's and Rhode's works [18, 23], the third term of the right-hand of (35) was omitted. It had been shown that the third term and the phase relation between current and the parallel component of $\vec{E}$ are very important in explaining the behaviors of a reactance function of the antennas system. It can be seen that the slope of the reactance do not completely relate to the normalized stored energy any more and is not always positive. It is worthwhile to point out that the generalized Foster reactance theorem will reduce to the traditional Foster reactance theorem when the loss vanishes.

From (25), (27) and (35), we can get the radiation $Q$ for the $N$-antenna system as follows

$$Q_G = \frac{2\omega_0 W'}{P_{rad}} = \frac{1}{2[|I|]^T[R][|I|]} \cdot \left\{[|I|]^T\omega_0\frac{\partial[X]}{\partial\omega}[|I|] + 2[|I|]^T\omega_0\left([R]\frac{\partial[\theta_I]}{\partial\omega}\right)[|I|] - 4\omega_0\delta\right\} \tag{36}$$

where $\delta = \int_{S_\infty} \vec{S}\cdot\hat{n}\frac{\partial\theta_{E_\infty}}{\partial\omega}ds$. In available work, most of the researchers directly define and evaluate antenna $Q$ by using the first term in brackets, and the last two terms of (36) are missing. For some special antennas, e.g., linear radiators with a symmetric current distribution, or for very small antennas, the last two terms will cancel out or have a little contribution to radiation $Q$. But for a general case, they also play an important role in determining the radiation $Q$ and the slop of reactance.

From the above analysis, it can be seen that the accurate calculation of antenna $Q$ is very difficult because of the presence of the frequency derivatives of the current phase and far-field phase. The complex frequency method combined with the model-based parametric estimation (MBPE) technique has been presented by Li and Liang [24], which is used to calculate the quality factor of antennas and scattering resonance systems successfully. By constructing the generalized system function $H(s)$ in complex frequency domain and analyzing the pole-zero characteristics, we can effectively analyze the resonance behaviors and the radiation $Q$ of an arbitrary antenna system. In this paper, the complex frequency method is employed to compute the radiated $Q$ of the generalized resonance system.

## 5. APPLICATIONS AND DISCUSSION

Firstly consider the two parallel dipoles system shown in Fig. 3. Dipole 1 will be excited by the ideal voltage source, and the terminal of dipole 2 shorted. The length both of them is $L = 7.4\,\mathrm{m}$, with radius $a = 0.01\,\mathrm{m}$. The distance between them is $d = 1.0\,\mathrm{m}$. The operational frequency range is from 15 MHz to 25 MHz. The observation points

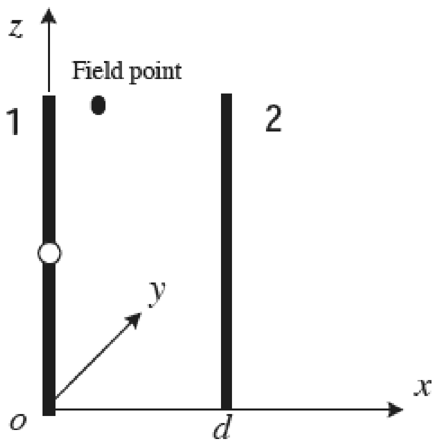



**Figure 3.** Two Parallel dipoles system.

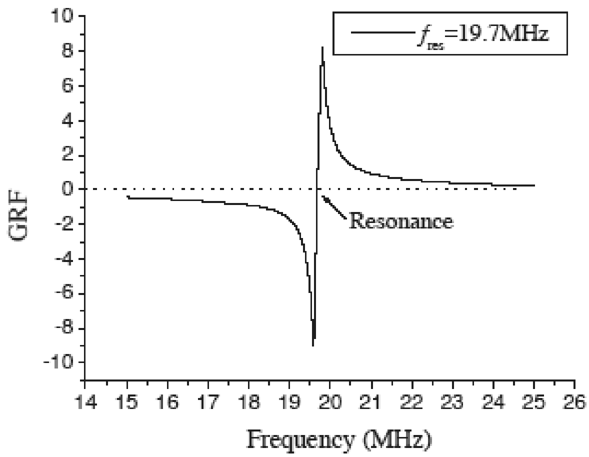



**Figure 4.** The frequency response of $GRF$.

locate at $(x, y, z) = (0.3, 0.0, 7.4)$ in the near field zone and at $(\theta, \varphi) = (90^\circ, 0^\circ)$ in the far field zone, respectively. A numerically rigorous method of moments (MoM) is used to calculate $GRF$ and $E$-fields at the near-zone and far-zone observation points, with the basis functions chosen to be pulses and the testing functions being delta functions. Clearly, a more refined choice can be made if desired. The numerical results are shown in Figs. 4 and 5 respectively.

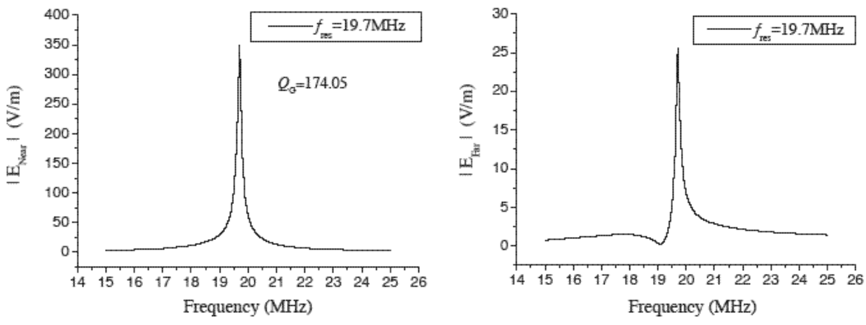



**Figure 5.** The special phenomena of the generalized resonance. (a) Strong and sharp field in the near region. (b) Large frequency sensitivity in the far region.

By analyzing the frequency characteristics of $GRF$, we can accurately determine the generalized resonant frequency, which is the frequency satisfied with $GRF = 0$, as shown in Fig. 4. Fig. 5a shows the frequency response of the observation field point in the near zone of the two parallel dipole antennas. It is observed that there is a strong and sharp electric field occurred at the resonant frequency, 19.7 MHz, which is mutilative to the electric devices located near the antenna. Therefore, the generalized resonance phenomena must be effectively considered in EMC analysis and design.

The radiation $Q$ is calculated by using the complex frequency method and MBPE technique. In this case, one of the complex pole is $s = -\alpha + j\beta = -0.05657 + j19.6921\,(\text{MHz})$, and the value of $Q$ is very large and $Q = \frac{\beta}{2\alpha} = 174.05$. The detailed discussion concerning the complex frequency theory for calculating Q is given in [24]. These results obtained from complex frequency domain also demonstrate the validation of $GRF$ and the essentials of the generalized resonance. In addition, Fig. 5b shows a large frequency sensitivity in the far region of the antenna system close to the generalized resonant frequency. If we make optimal excitation and gain design, the antenna system may realize super-directive characteristics [9, 21].

The effects of the antenna structures on the generalized resonance are discussed here in order to obtain some engineering analysis and design guidelines for EMC. Assume the observation point still locating at $(x, y, z) = (0.3, 0.0, 7.4)$ in the near field zone, the variations in the length of dipole 2 and the distance between antennas will change the generalized resonant behaviors.

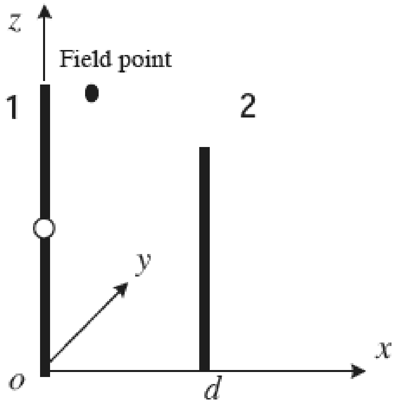
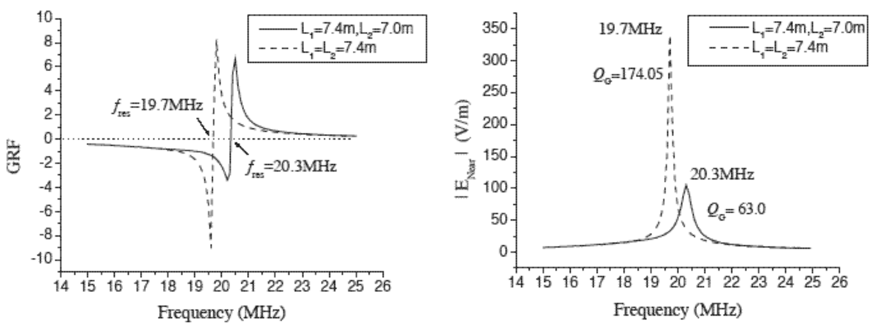



**Figure 6.** Variation in dipole2 length.




(a) (b)

**Figure 7.** Comparisons of $GRF$ and near $E$-field magnitude of the original antenna system with the new antenna system of dipole2 length variation. (a) Frequency response of $GRF$. (b) Frequency response of the near $E$-field magnitude.

- **Case 1.** Variation in length of dipole 2, $L_1 = 7.4\,\text{m}$, $L_2 = 7.0\,\text{m}$, $a = 0.01\,\text{m}$, $d = 1.0\,\text{m}$;

  As shown in Fig. 6, the length of dipole 2 is shortened, i.e., $L_2 = 7.0\,\text{m}$, and other parameters are kept the same as in Fig. 3. It is observed from Figs. 7a and 7b that if the length of dipole 2 is shortened, the generalized resonance frequency will increase. The strong and sharp field in near region becomes weak and the radiation $Q$ decreases.

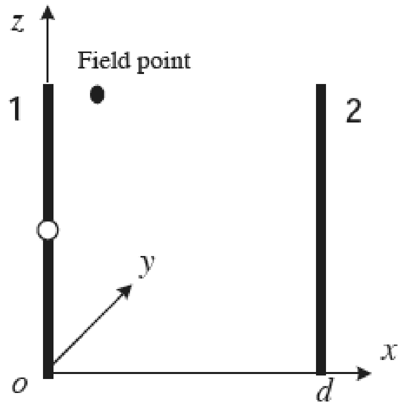
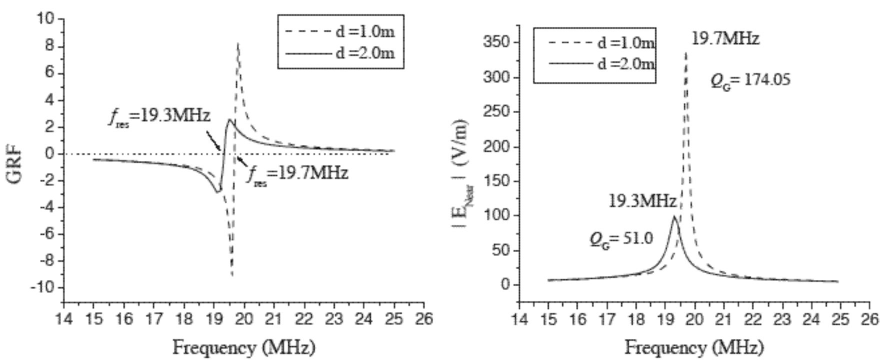



**Figure 8.** Variation in distance between two dipoles.




(a) (b)

**Figure 9.** Comparisons of *GRF* and near field magnitude of the original antenna system with the new antenna system of double distance. (a) Frequency response of *GRF*. (b) Frequency response of the near *E*-field magnitude.

- **Case 2.** Variation in distance between two dipoles, $L_1 = 7.4\,\text{m}$, $L_2 = 7.4\,\text{m}$, $a = 0.01\,\text{m}$, $d = 2.0\,\text{m}$.

  The interaction and mutual coupling of two dipoles are mainly affected by the distance between them. In this case, the distance is increased, i.e., $d = 2.0\,\text{m}$, and the antenna length, radius and exciting source are kept the same as the original antenna system, as shown in Fig. 8. It is observed from Figs. 9a and 9b that when

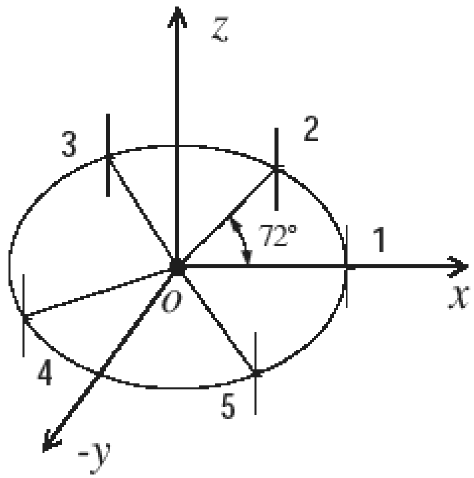


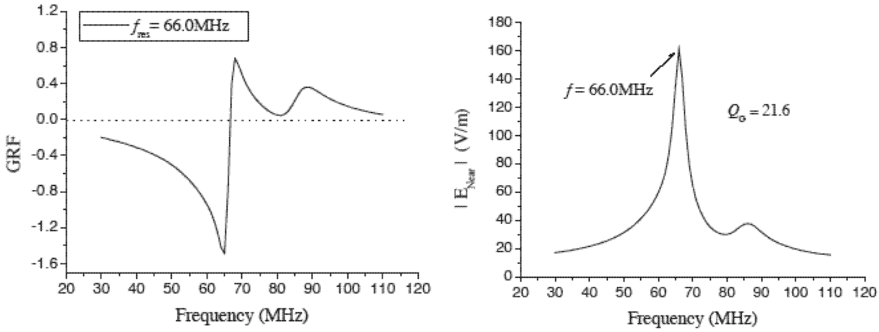

**Figure 10.** Five-element circular array for direction finding system.

the distance is increased, the generalized resonance frequency and the value of $Q$ will decrease.

The other example is a practical five-element circular array for direction finding (FCADF) system shown in Fig. 10. The antenna system is a circular array with radius 1.2 m, which consists of five same dipoles, with length 2.0 m and radius of dipoles 0.015 m. The working frequency varied from 30 MHz to 110 MHz, and the feed points locate in the $xoy$ plane. The observation point in the near field region locates at $(x, y, z) = (1.1, 0.0, 0.8)$. Similarly, a piece of computer code based on a numerically rigorous method of moments (MoM) is used to calculate the frequency responses of $GRF$ and near $E$-fields in the following two cases.

Fig. 11 shows the $GRF$ and the phenomenon of generalized resonance in the near region with single excitation. It can be seen that the generalized resonance behaviors are also remarkable by virtue of the strong interaction and mutual coupling between the antenna elements in the FCADF system. The value of $Q$ is still large, approximate 21.6. But when the excitations are changed, i.e., all dipoles excited by the ideal voltage sources, the generalized resonant frequency increases and $Q$ factor reduces to 8.34, as shown in Fig. 12. The above analyses show that the generalized resonance can be characterized by the $GRF$ and radiation $Q$, and some effective methods can be adopted to improve the generalized resonance behaviors, which can give some design guidelines for EMC in the complicated electromagnetic environments.

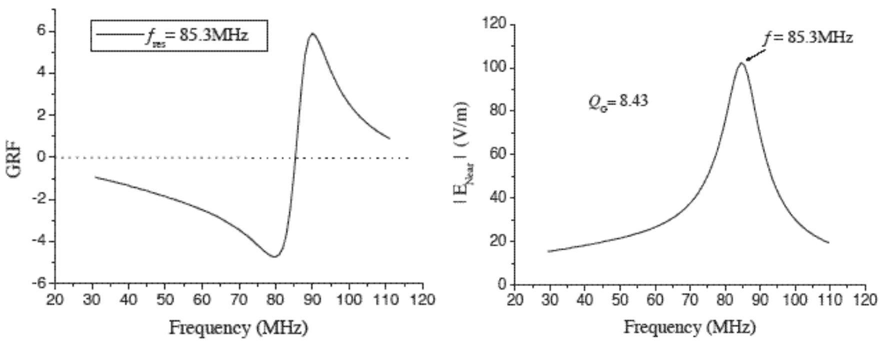
**Case 1.** Dipole 1 excited by the ideal voltage source and others shorted.




(a) (b)

**Figure 11.** *GRF* and the generalized resonance behavior in the near region with single excitation. (a) Frequency response of *GRF*. (b) Frequency response of the near *E*-field magnitude.

**Case 2.** All dipoles excited by the ideal voltage source.




(a) (b)

**Figure 12.** *GRF* and the generalized resonance behaviors in the near region with full excitation. (a) Frequency response of *GRF*. (b) Frequency response of the near *E*-field magnitude.

## 6. CONCLUSION

This paper points out the nature of the generalized resonance is the balance of the electric and magnetic fields energy stored in a multi-antenna system. The phenomena, such as strong and sharp fields in the near zone, large frequency sensitivity in the far zone, are the special behaviors of the generalized resonance. From (20) and (21), we know the generalized resonance relates to not only system itself and modes, but also the complicated excitations and loads. Therefore, we may take some effective methods to improve the generalized resonance according to practical applications, which is an important part of the EMC analysis and design.

In this paper, the generalized Foster reactance theorem for an antenna array is presented, which physically states that the slope versus frequency of the reactance function of the antenna system does not precisely relate to the electromagnetic field energy stored in the antenna system and is possibly negative because of the presence of radiated losses. It is worthwhile to point out that the generalized resonance is analyzed from the viewpoint of an antenna system, but we believe that the similar phenomena and nature may exist in other complicated open systems.

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