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ONE-DIMENSIONAL SIMULATION OF REFLECTED EM PULSES FROM OBJECTS VIBRATING AT DIFFERENT FREQUENCIES

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Abstract—In this report one-dimensional simulation of Electromagnetic pulses reflected from moving and/or vibrating perfectly conducting surfaces is presented. The computational results are obtained through the application of the method of characteristics with the aid of the characteristic variable and the relativistic boundary conditions. The reflecting perfect surface is set to constantly travel at relatively high speed and/or sinusoidally vibrate with very high frequency in order to easily observe the relativistic effects on the reflected pulses. To validate the numerical method, the reflected electric fields and the corresponding spectra are demonstrated side-by-side for comparisons with the theoretical Doppler shift values. It is found that the computational results and the theoretical values are in good agreement.

1 Introduction

2 Boundary Condition Treatment

3 The Problem

4 Results

5 Conclusion

References

1. INTRODUCTION

The main purpose of this report is to apply a newly developed numerical method to the solution of EM scattering problem involved with moving/vibrating boundary. It is also of interest to predict and observe the relativistic effects of the moving/vibrating perfect boundary on the reflected EM fields. The scheme performance is investigated by comparing the computational results with the theoretical values of Doppler shifts. The analytical theory of electromagnetic wave scattering from moving conductors have been developed and can be found in many literatures, such as references [1–3]. For the past half-century many numerical techniques have been developed for modeling the EM scattering problems. Among them, the two most well known and widely used approaches are the method of moments (MoM) and the finite-difference time-domain (FDTD) technique. From the above mentioned researches the following observations can be found: the familiar Doppler shifts can be observed in the reflected fields from perfect planes experiencing translational motion; the oscillation of object results in changes in phase and magnitude of the scattered fields.

Besides MoM and FDTD, a recently developed approach for direct approximation of the time-domain Maxwell's equations is the method of characteristics (MOC). This characteristic-based method was originally applied to the solutions of the Navier-Stokes equations for the fluid dynamic problems by Whitfield and Janus in the 80s [4]. Shang solves the time-domain Maxwell's equations by using the explicit finite-difference formulation in MOC in the early of 90s [5]. By elaborating with lower-upper approximate factorization method, the implicit formulation was developed for the direct time domain computation of the Maxwell curl equations and found in good agreement with results generated by FDTD [6]. Different from the MoM and FDTD approach where field components are allocated in different nodes, the characteristic-based method positions all field variables in the grid cell center. Each field variable in MOC is then the averaged value over the computational cell. MOC is consequently considered to be a suitable approach for problems involving time-varying cell, such as cases where the object or boundary is in motion. To solve the governing equations, MOC first casts the Maxwell's equations in the form of Euler equation, transforms them into a curvilinear coordinate system, and then directly approximate them by balancing the net flux within each computational cell.

2. BOUNDARY CONDITION TREATMENT

The time-domain Maxwell curl equations constitute a hyperbolic system meaning that in order to obtain a unique solution of the problem one must specify initial values and satisfy appropriate boundary conditions (BC). To specify initial values implies that both the electric and magnetic fields are defined before the numerical procedure progresses. The realization of the boundary conditions means the follows: layers of computational cell around the object's surfaces and the truncated computational boundaries field variables must be manipulated according to physics during the process. For instance, on the surface of a stationary perfect conductor the tangential component of the electric field intensity must vanish and there's no penetration of the field into the conductor. On the other hand, on the interface between media, both the tangential component of field intensities and the normal component of field flux densities must be continuous. On the outer computational boundaries, the Sommerfeld's radiation condition must hold, the proper BC's ensure that this layer of cell should not bounce the outward going fields.

Since the boundary is relativistically moving and/or vibrating, the relativistic boundary conditions have to be taken into account. The boundary conditions used in the present method are the combination of the characteristic variable (CV) boundary conditions and the relativistic boundary conditions. The former are inherited from the nature of MOC. Based on the definition, CV is the product of the instantaneous variable vector and one row eigenvector associated with one particular eigenvalue [4]. Since each eigenvalue designates the direction and velocity of the information propagating across the cell face, the helpfulness of CV for the evaluation of boundary variables is evident, which bears more accurate interpretation of physics. The CV arriving on the boundary is the one carries information propagating across the cell face from the adjacent cell and given by

$$CV^* = \boldsymbol{n} \times \boldsymbol{B}^a + \eta_0 \boldsymbol{D}^a \tag{1}$$

with η_0 being the impedance of free space and both $\boldsymbol{B}^a$ and $\boldsymbol{D}^a$ are the variables of the cell immediate adjacent to the boundary, and $\boldsymbol{n}$ is the unit normal vector of the surface. In order to incorporate the relativistic effects due to the motion of perfect boundary, the relativistic relation is considered and written as

$$\boldsymbol{n} \times \boldsymbol{E}^* = (\vec{v} \cdot \boldsymbol{n})\boldsymbol{B}^* \tag{2}$$

where $\vec{v}$ is the velocity to the perfect surface, and $\boldsymbol{E}^*$ and $\boldsymbol{B}^*$ are the electric field intensity and magnetic flux density evaluated right on the boundary, respectively. The boundary values $\boldsymbol{E}^*$ and $\boldsymbol{B}^*$ then can be directly solved from (1) and (2). It is therefore predictable that the reflected fields would be impressed with noticeable double-Doppler shifts if the reflecting surface experiences a motion at relatively high speed or instantaneous velocity.

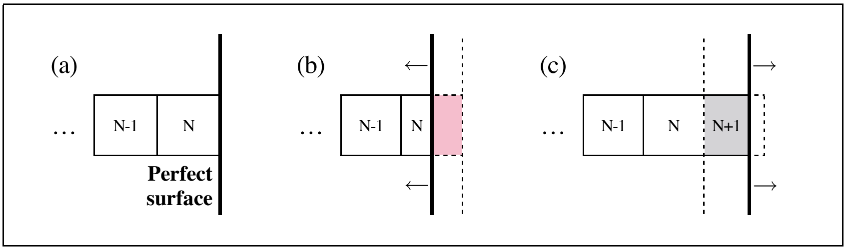

Figure 1. Computational cell indexing: (a) stationary grid system, (b) The Nth cell is partially truncated, (c) The $(N+1)$th fractional cell is introduced.

A stationary grid system will be used for problems involving stationary boundary where both the cell number and cell size are time-invariant and alike as depicted in Figure 1(a). If the boundary is in motion, as in the present simulation, both cell number and cell size are functions of time. If the boundary moves to the left, portion of the Nth cell is truncated as in Figure 1(b). Alternatively, as shown in Figure 1(c), when the boundary travels to the right, an extra fractional cell, the $(N+1)$th cell, is introduced into the grid system. It is understood that the total number of cell eliminated or added may be multiple and depends on the oscillation amplitude and the grid density, and that the determination of the numerical time step is quite important. One has to ensure that the propagation of the numerical field would not to pace or skip any grid during the simulation, which is achievable by cautiously updating the effective cell and the corresponding numerical time step.

3. THE PROBLEM

The incident excitation used in the model is a plane EM pulse having a Gaussian distribution profile and only components D_z and B_y, or equivalently E_z and H_y where the electric field intensity is normalized to unity. The incident is initially set to propagate in the positive x-direction in free space, and normally illuminates upon a perfect plane that is either at rest or in motion. For practical reason, a rectangular window is applied to the Gaussian pulse and has a cutoff level of

100 dB with respect to the peak. The EM pulse is assumed to have a temporal width of about 1.9024 ns measured from the center to $e^{-0.5}$ in magnitude and a highest frequency content of about 404 MHz in the spectrum. The whole spatial span of such pulse is approximately 6 meters. The number of grid cell for the Gaussian pulse is 1000 points and the numerical time step is set so that the numerical EM pulse takes ten steps for one grid cell. Note and recall that, under the above assumption, the CV^* in equation (1) is tangential to the perfect surface. The final forms of the boundary variables are respectively given by

$$B_y^* = \frac{1}{\vec{v}-1} CV^* \tag{3}$$

$$E_z^* = \frac{\vec{v}}{\vec{v}-1} CV^*. \tag{4}$$

For the purpose of easy examination on the effects of the moving object on the reflected pulses, the perfect boundary is set to move as specified below. The perfect boundary is uniformly traveling at a speed of 10 percent of the light speed (C) and/or sinusoidally vibrates with constant frequency and amplitude so that the extreme instantaneous velocity equals to ± 0.1 C around the equilibrium position of the vibration motion. If the vibration frequency and amplitude are 1 GHz and 9.5493 mm peak-to-peak, then it results in a speed of 0.1 C near the equilibrium position. Alternatively, if the vibration frequency is 0.5 GHz then the corresponding amplitude is 19.0986 mm to maintain a speed of 0.1 C at the equilibrium position. The resultant instantaneous velocities may be superposed when the perfect surface travels and vibrates at the same time. Furthermore, in this report symbols β_τ and β_v are used to represent the ratios of the translational velocity and oscillatory velocity to the light speed, respectively. Since β_v ranges between -0.1 and $+\,0.1$, $|\beta_v|$ is used to neglect the change of direction. The sign of β's is positive if the boundary and the incident move in the same direction and negative if they are approaching each other.

The variation in the reflected electric field strength is investigated by $|E_i|\frac{1-\beta_c}{1+\beta_c}$ where β_c is the combined velocity and $|E_i|$ is the normalized incident electric field intensity. The change in the highest frequency content is examined and compared with $f_0\sqrt{\frac{1-\beta_\tau}{1+\beta_\tau}}$ where f_0 is the highest frequency content of the incident excitation.

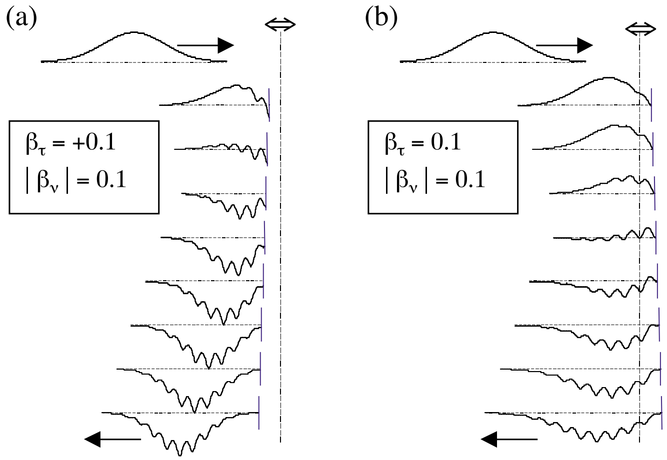

Figure 2. EM pulses reflected from moving and vibrating perfect planes: (a) Vibrating and moving to the left, (b) Vibrating and moving to the right.

4. RESULTS

In order to observe how the EM fields interact with the moving and vibrating perfect boundary, two time-sequences of the electric field intensity are illustrated in Figure 2. The reflected electric fields reveal not only the oscillatory behavior of the perfect surface but also the Doppler effects on the pulse widths. Closer observation on the electric field on the surface found that the electric fields are not always zero in magnitude owing to the application of the relativistic boundary conditions on the boundary.

Plotted in Figure 3 are two pure vibration cases where the EM pulses are reflected by vibrating surfaces with three different vibration frequencies (f_v), namely 0, 0.5 and 1 GHz. Also shown are the field strengths along with the theoretical values in parenthesis. It is noticed that the simulation results are quite accurate and that at the center of the reflected pulses where the EM pulse peak almost coincides with the equilibrium position of the vibration, as pointed by arrows in the plot, the extreme instantaneous velocity is equal to 0.1 C when $f_v = 0.5\,\text{GHz}$ while it is -0.1 C when $f_v = 1.0\,\text{GHz}$. The evidence of the vibration frequency of the boundary is obvious by simply calculating the temporal separations between two consecutive peaks or valleys. The two calculated periods are 2 ns and 1 ns, respectively. Examinations on the effects of the translational movement of the

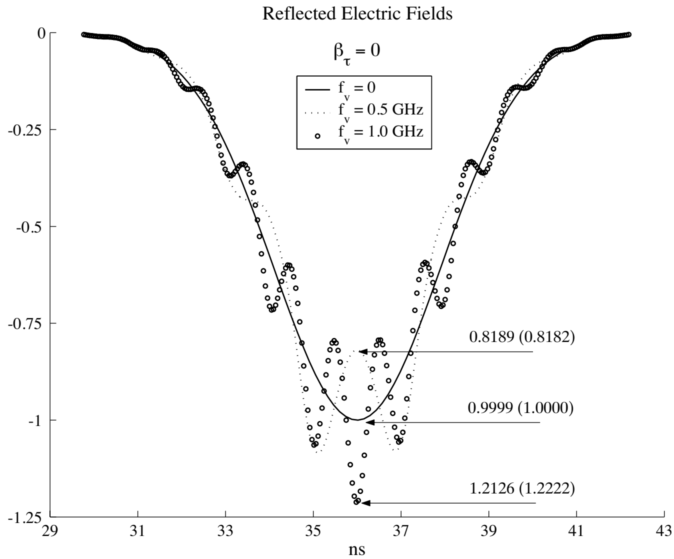

Figure 3. Reflected electric fields from perfect planes vibrating at various frequencies.

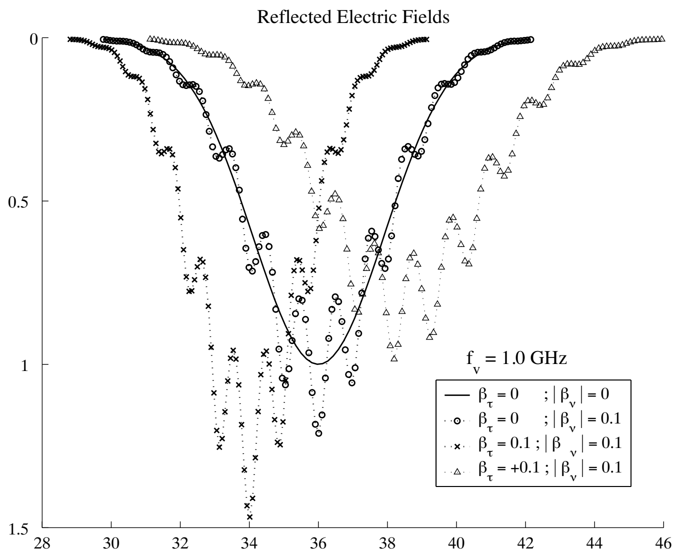

Figure 4. Reflected electric fields from perfect planes vibrating and moving at various velocities.

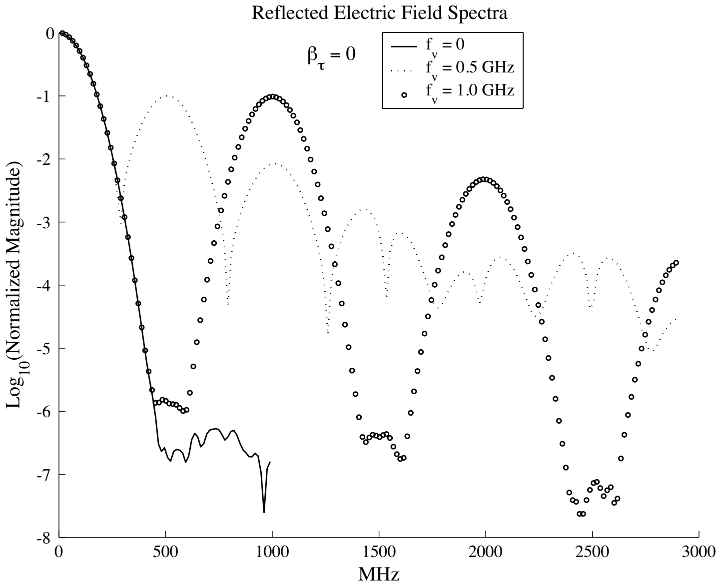

Figure 5. Reflected electric fields spectra of those in Figure 3.

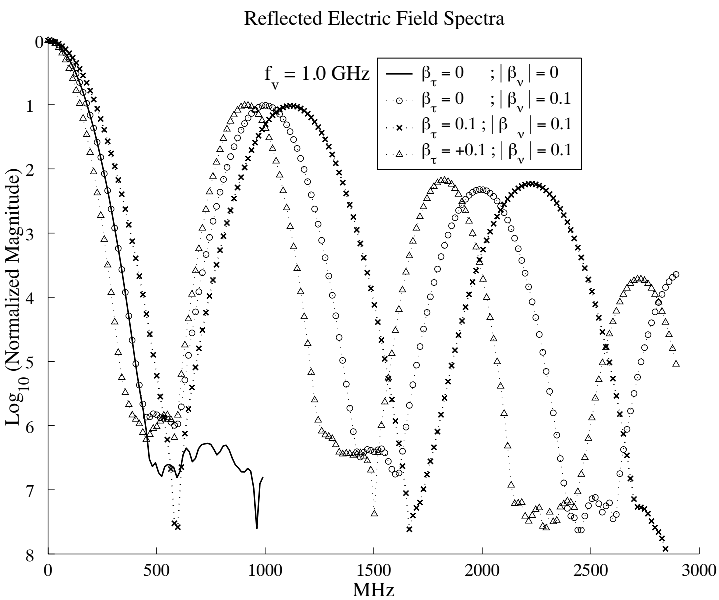

Figure 6. Reflected electric fields spectra of those in Figure 4.

Table 1. Doppler shifts in the side-lobes of the reflected spectrum (as in Figure 6).

Velocities		1st side lobe (GHz)		2nd side lobe (GHz)	
β_τ	$\lvert\beta_v\rvert$	Calculated	Theoretical	Calculated	Theoretical
0	0.1	0.9980	1.0000	1.9833	2.0000
– 0.1	0.1	1.1176	1.1055	2.2137	2.2111
+ 0.1	0.1	0.9137	0.9045	1.8196	1.8091

boundary on the reflected were carried out by setting $f_v = 1.0$ GHz and β_T is one of the following values: -0.1, 0, and $+0.1$. Once more the characteristics of the boundary motion is impressed on the reflected. The Doppler effect in both the magnitude and the pulse duration are rather clear from the plot in Figure 4.

Further investigations on the spectrum are given in Figures 5 and 6. Locations of side-lobes in the spectrum release the oscillatory behavior of the boundary and the relativistic effects due to the motion of the perfect reflector along the propagation direction of the EM pulse. The calculated and theoretical frequencies of side-lobes are listed in Table 1 for comparisons.

5. CONCLUSION

The method of characteristics has been shown to successfully solve EM scattering problems in one-dimension. The worked problems are featured by moving and/or vibrating perfect planes and numerically solved by applying the relativistic and characteristic variable boundary conditions. The computational results are quite accurate compared to the exact values. To develop the existing code for problems with objects of finite dimension is our future goal.

REFERENCES

1. Harfoush, F., A. Taflove, and G. Kriegsmann, "A numerical technique for analyzing electromagnetic wave scattering from moving surfaces in one and two dimensions," *IEEE Trans. Antennas Propagation*, Vol. 37, 55–63, January 1989.
2. Kleinman, R. E. and R. B. Mack, "Scattering by linearly vibrating objects," *IEEE Trans. Antennas Propagation*, Vol. AP-27, No. 3, 344–352, May 1979.

3. Cooper, J., "Scattering of electromagnetic fields by a moving boundary: the one-dimensional case," *IEEE Trans. Antennas Propagation*, Vol. AP-28, No. 6, 791–795, November 1980.
4. Whitfield, D. L. and J. M. Janus, "Three-dimensional unsteady Euler equations solution using flux splitting," AIAA Paper No. 84-1552, June 1984.
5. Shang, J. S., "A characteristic-based algorithm for solving 3-D time-domain Maxwell equations," *Electromagnetics*, Vol. 10, 127, 1990.
6. Donohoe, J. P., J. H. Beggs, and M. Ho, "Comparison of finite-difference time-domain results for scattered EM fields: Yee algorithm vs. a characteristic based algorithm," *27th IEEE Southeastern Symposium on System Theory*, March 1995.

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