

## **SCATTERING COMPUTATION FROM THE TARGET WITH LOSSY BACKGROUND**

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**Abstract**—The accurate analysis of scattering from objects with dimensions large compared to the wavelength using rigorous methods (finite element, FDTD, method of moments) with a personal computer is almost impractical. In asymptotic methods, physical optics (PO), geometrical theory of diffraction (GTD), the accurate modeling of the object's boundary is too cumbersome. The parabolic equation method gives accurate results in calculation of scattering from objects with dimensions ranging from one to tens of wavelengths. Solving parabolic equation with the marching method needs limited computer storage even for scattering calculations of large targets. In this paper, first the calculation procedure of radar cross section using parabolic equation is studied and the necessary equations are derived. The parabolic equation and the model of reflecting facet is utilized for calculation of the scattered fields in the forward and backward directions. In order to model the lossy background, the impedance boundary condition is utilized in lower boundary. Finally the scattered fields and RCS of a ship and a tank are calculated as two examples of targets with lossy background.

### **1. INTRODUCTION**

Parabolic equation is an approximation of the wave equation which models energy propagating in a cone centered on a preferred direction, the paraxial direction. The parabolic equation was first introduced by Leontovich and Fock in order to study the diffraction of radiowaves

around the earth [1]. By the advent of advanced computers closed form solution of the parabolic equation was replaced with numerical solutions. Since then, the parabolic equation is being applied to radar, sonar, acoustic and wave propagation. The parabolic equation has been recently used in scattering and RCS calculations.

## 2. THE PARABOLIC EQUATION FRAMEWORK

In all equations of this paper, the time dependence of the fields is assumed as  $\exp(-j\omega t)$ . For horizontal polarization, the electric field  $\vec{E}$  only has non-zero component  $E_y$ , while for vertical polarization, the magnetic field  $\vec{H}$  only has one non-zero component  $H_y$ . The reduced function  $u$  is defined as

$$u(x, z) = \psi(x, z)e^{-ikx} \quad (1)$$

In which  $\psi(x, z)$  is the  $E_y$  component for horizontal polarization and  $H_y$  component for vertical polarization. The paraxial direction is assumed along the  $x$  axis. Assuming the refractive index of the medium,  $n$ , the field component  $\psi$  satisfies the following two dimensional wave equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} + k^2 n^2 \psi = 0 \quad (2)$$

Using equations (1) and (2), the wave equation in terms of  $u$  is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + 2ik \frac{\partial u}{\partial x} + k^2(n^2 - 1)u = 0 \quad (3)$$

Considering  $Q = \sqrt{\frac{1}{k^2} \frac{\partial^2}{\partial z^2} + n^2}$ , (3) is reduced to

$$\frac{\partial^2 u}{\partial x^2} + 2ik \frac{\partial u}{\partial x} + k^2(Q^2 - 1)u = 0 \quad (4)$$

which can be written as

$$\left[ \frac{\partial}{\partial x} + ik(1 + Q) \right] \left[ \frac{\partial}{\partial x} + ik(1 - Q) \right] u = 0 \quad (5)$$

Decomposing equation (5), the following pair of equations is obtained.

$$\frac{\partial u}{\partial x} = -ik(1 - Q)u \quad (6a)$$

$$\frac{\partial u}{\partial x} = -ik(1 + Q)u \quad (6b)$$

The solution to (6a) corresponds to the forward propagating waves while that of (6b) concerns the backward propagating waves.

### 3. COMPUTATION OF SCATTERING WAVES

The simplest approximation of (6a) is obtained using the first order expansion of Taylor series. Using this approximation, the standard parabolic equation is obtained. We assume  $Q$  as

$$Q = \sqrt{Z + 1} \quad (7)$$

In which  $Z = \frac{1}{k^2} \frac{\partial^2}{\partial z^2} + n^2$ . Using the first order Taylor series of (7) we have

$$Q \simeq 1 + \frac{Z}{2} \quad (8)$$

Substituting (8) in (6a) yields

$$\frac{\partial u}{\partial x} = ik \frac{Z}{2} u \quad (9)$$

With regard to the definition of  $Q$ , and the relation (7), equation (9) is reduced to the following form

$$\frac{\partial^2 u}{\partial z^2} + 2ik \frac{\partial u}{\partial x} + k^2(n^2 - 1)u = 0 \quad (10)$$

This equation is the standard parabolic equation. The limitations of the standard parabolic equation are due to its bad behavior at large propagation angles. In order to solve the parabolic equation, the finite difference method and Split-Step/Fourier is utilized [3, 4]. To model the complex boundaries of the objects in scattering problems, the finite difference method should be used. In order to solve the parabolic equation we will use the Crank-Nicolson finite difference equations which easily models arbitrary boundaries [4]. The computational domain is constant in the vertical direction and the distance between nodes along  $z$  is  $\Delta z$  and along  $x$  is  $\Delta x$ . The domain  $(x_m = m\Delta x, z)$  is defined as the range  $m$ . We shall use the marching technique to solve the discrete equations. In this method the fields within the range  $m$  are computed versus the fields within the range  $m - 1$ . To extract the discrete form of equation (10), the point  $\xi_m$  is assumed exactly in the middle of range  $m$  and range  $m - 1$  as

$$\xi_m = \frac{x_{m-1} + x_m}{2} \quad (11)$$

Now by using central difference, the discrete form of the equation (10) is obtained as

$$\frac{u(\xi_m, z_{j+1}) + u(\xi_m, z_{j-1}) - 2u(\xi_m, z_j)}{\Delta z^2} + 2ik \frac{u(x_m, z_j) - u(x_{m-1}, z_j)}{\Delta x} + k^2(n^2 - 1)u(\xi_m, z_j) = 0 \quad (12)$$

Approximating  $u$  in  $x = \xi_m$  as average of  $u$  within the domain  $x_{m-1}$  and  $x_m$ , yields

$$u_j^m \left( -2 + b + a_j^m \right) + u_{j+1}^m + u_{j-1}^m = u_j^{m-1} \left( 2 + b - a_j^m \right) - u_{j+1}^{m-1} - u_{j-1}^{m-1} \quad (13)$$

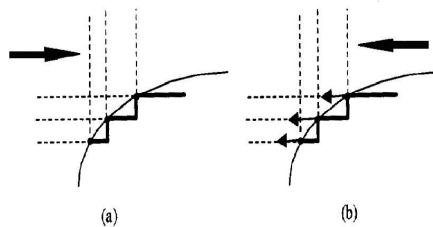
In which

$$u_j^m = u(x_m, z_j), \quad b = 4ik \frac{\Delta z^2}{\Delta x}, \quad a_j^m = k^2(n(x_m, z_j)^2 - 1)\Delta z^2 \quad (14)$$

In order to calculate the scattered field from the target with lossy background we used the perfect transmitting boundary condition [5] and impedance boundary conditions [6] in the upper and lower domain of the computation region respectively. To obtain the field in the whole points of computation region, the fields at  $x = 0$  should be determined first. The incident field is assumed as a plane wave with unit amplitude as follows

$$u(x, z) = \exp(ik(x(\cos \theta - 1) + z \sin \theta)) \quad (15)$$

In which  $\theta$  is the angle of incident plane wave. In order to compute the scattering fields from the object, we discretize the object boundary using a rectangular grid. It is assumed that the object is not penetrable and we calculate the wave outside the object using appropriate boundary condition. The parabolic equation in the forward direction, computes the sum of the incident field  $u_i$  and scattered field  $u_s$ . For computing the backward scattered field the object is considered as series of reflecting faces which act as sources for the backward propagating energy [7, 8]. In this case, the analysis is started from some range beyond the object, setting the initial field to zero. Boundary conditions on each facet are given by the appropriate polarization dependent reflection coefficients, which may vary along the scattering object. The technique is illustrated schematically in Figure 1.



**Figure 1.** Modeling of the object boundaries within the parabolic equation for scattering calculations a) forward and b) backward [7].

#### 4. IMPEDANCE BOUNDARY CONDITION

In order to model the lossy background we used impedance boundary condition in the parabolic equation. The impedance boundary condition is as follows [6]

$$\hat{n} \times \vec{E} = Z_s(\hat{n} \times \hat{n} \times \vec{H}) \tag{16}$$

$\hat{n}$  is a unit vector, perpendicular to the lossy background and  $Z_s$  is computed as follows

$$Z_s = \sqrt{\frac{\mu}{\varepsilon + i\frac{\sigma}{\omega}}} \tag{17}$$

In which  $\varepsilon$  and  $\sigma$  are the lossy background parameters. Considering the horizontal polarization and extraction of (16) at  $z = 0$  we have

$$\frac{\partial E_y(x, z = 0)}{\partial z} - \frac{ikZ_0}{Z_s} E_y(x, z = 0) = 0 \tag{18}$$

Using (1), equation (18) is reduced to

$$\frac{\partial u(x, z = 0)}{\partial z} - \frac{ikZ_0}{Z_s} u(x, z = 0) = 0 \tag{19}$$

In order to extract the finite difference form of (19) and write it as the form of (13), first we obtain the finite difference expression for the second order derivative of  $z$  at  $(x, 0)$ , using one-way approximation as follows

$$\frac{\partial^2 u}{\partial z^2}(x, 0) \sim 2 \frac{\frac{\partial u}{\partial z}(x, \Delta z/2) - \frac{\partial u}{\partial z}(x, 0)}{\Delta z} \tag{20}$$

In which  $\alpha = \frac{-ikZ_0}{Z_s}$ . Substituting equation (19) into the standard parabolic equation, finally we have

$$u_0^m (-1 + \alpha\Delta z + b + a_0^m) + u_1^m = u_0^{m-1} (1 - \alpha\Delta z + b - a_0^m) - u_1^{m-1} \tag{21}$$

#### 5. COMPUTATION OF RADAR CROSS SECTION

After the calculation of fields over the entire computational domain, we can compute the fields within any arbitrary domain  $x$  as a function of the fields in the domain  $x_0$  in free space i.e.,

$$u(x, z) = \frac{ik}{2} e^{-ik(x-x_0)} \int_{-\infty}^{+\infty} u(x_0, z') \frac{x-x_0}{\rho(z')} H_1^{(1)}(k\rho(z')) dz' \tag{22}$$

In which

$$\rho(z') = \sqrt{(x - x_0)^2 + (z - z')^2} \quad (23)$$

The radar cross section is defined as

$$\sigma(\theta) = \lim_{\rho \rightarrow \infty} 2\pi\rho \left| \frac{u_s}{u_i} \right|^2 \quad (24)$$

Tending  $(x, z)$  to infinity along a given direction in (22), and assuming a unit amplitude for the incident wave, (24) yields

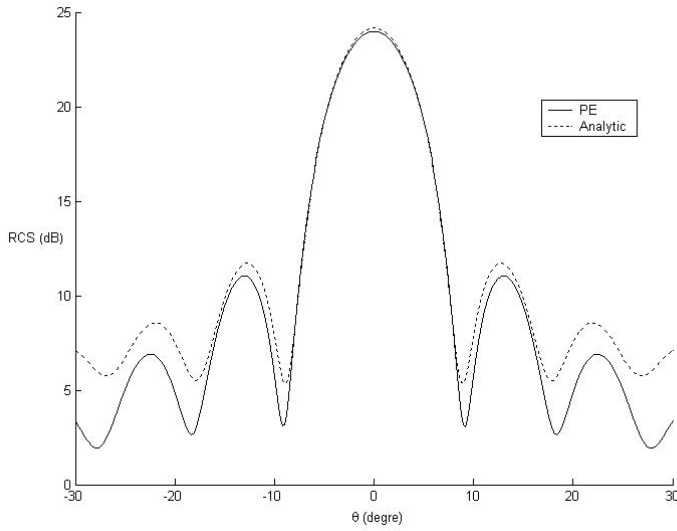
$$\sigma(\theta) = k^2 \cos^2 \theta \left| \int_{-\infty}^{+\infty} u_s(x_0, z') e^{-ikz' \sin \theta} dz' \right|^2 \quad (25)$$

In which  $u_s(x, z)$  is the scattered field.

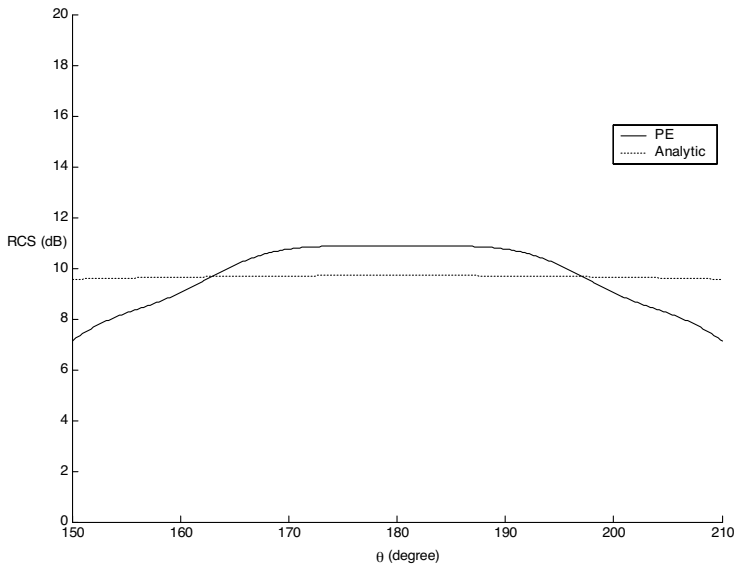
## 6. RESULTS FOR THE SCATTERED FIELDS

In order to show the validity of the parabolic equation, the RCS of a conducting cylinder with radius  $3\lambda$  is calculated in forward and backward directions. The incident wave is a plane wave with horizontal polarization and a wavelength equal to 1 meter (corresponding to 300 MHz). The results are given in Figures 2 and 3. The dotted lines represent the analytical results obtained from the extraction of Hankel functions [10]. As it can be seen, there is a good agreement between the analytical results and the parabolic equation results up to angles about 15 degrees.

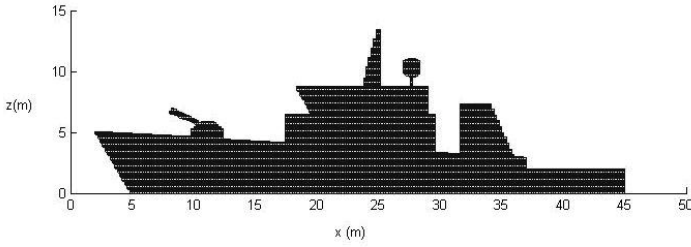
Next the scattered fields and RCS of a ship and a tank with a lossy background are considered. In order to calculate the scattered fields from the ship and tank, their staircase model is utilized. The staircase model of the ship and the tank are shown in Figures 4 and 5 respectively. The dimensions of the computational domain are 50 by 50 meters for the ship and 10 by 20 meters for the tank in the  $x$  and  $z$  directions respectively. For calculating the surface impedance of the sea and the earth,  $\varepsilon_r$  is assumed as 74 and 5 and  $\sigma$  is assumed as 4 and 0.01 respectively. The grid spacing in the  $x$  and  $z$  directions are assumed  $\lambda/10$  and  $\lambda/5$  respectively. The incident wave illuminates the ship and the tank from left with horizontal polarization and frequencies 300 and 1200 MHz respectively. The scattered fields from the ship and the tank in the forward and backward and their RCS results are shown in Figures 6 through 13.



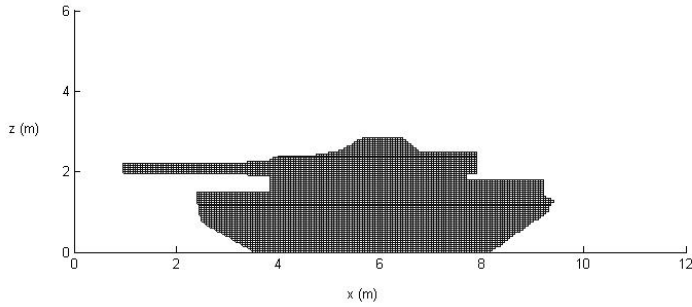
**Figure 2.** RCS of the conducting cylinder in the forward direction with radius  $3\lambda$ .



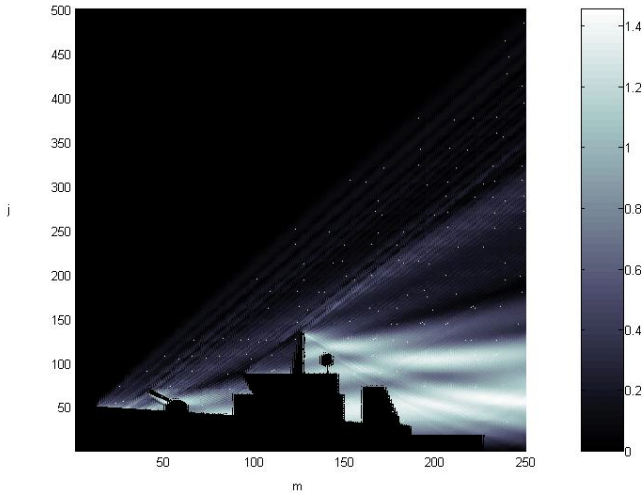
**Figure 3.** RCS of the conducting cylinder in the backward direction with radius  $3\lambda$ .



**Figure 4.** Ship staircase model (dimensions are in meters).

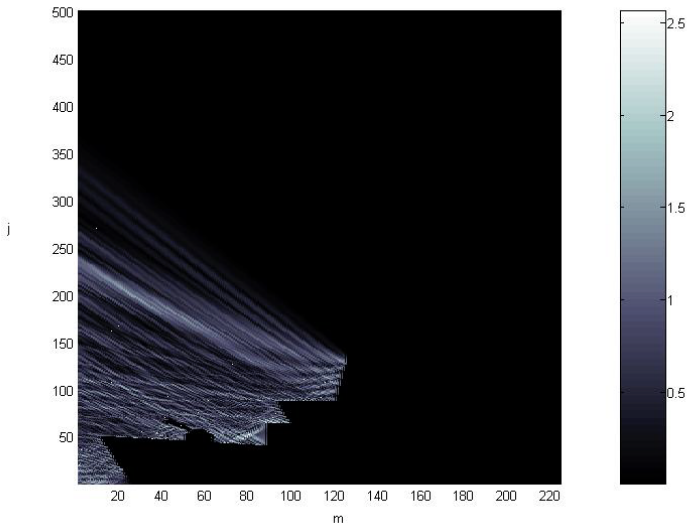


**Figure 5.** Tank staircase model (dimensions are in meters).

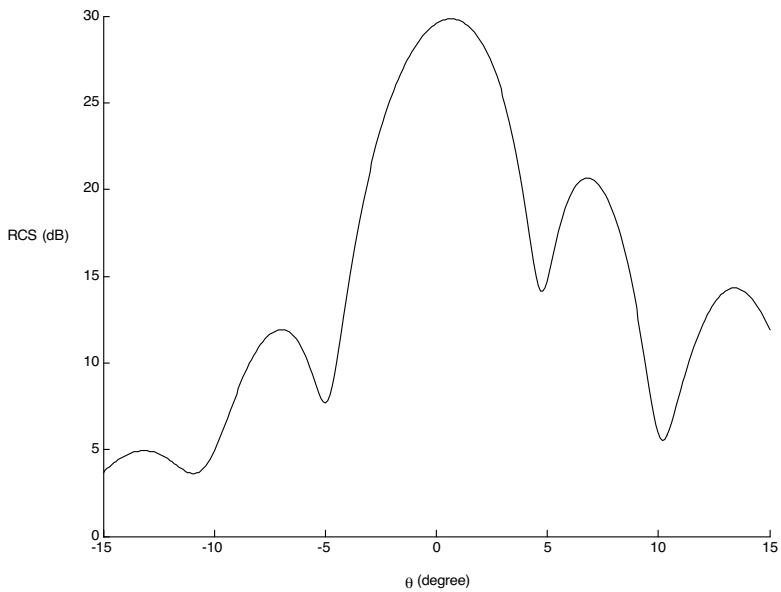


**Figure 6.** Amplitude of the scattered field  $u_s(m, j)$  from the ship in the forward direction in 300 MHz.

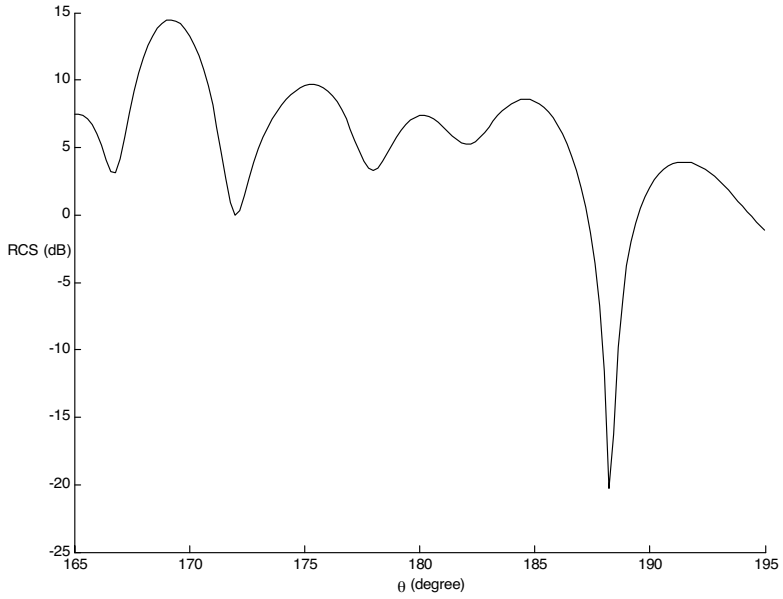




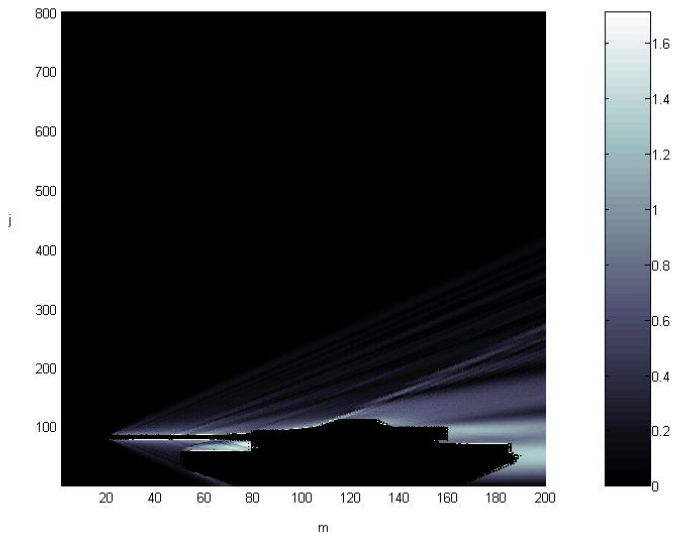
**Figure 7.** Amplitude of the scattered field  $u_s(m, j)$  from the ship in the backward direction in 300 MHz.



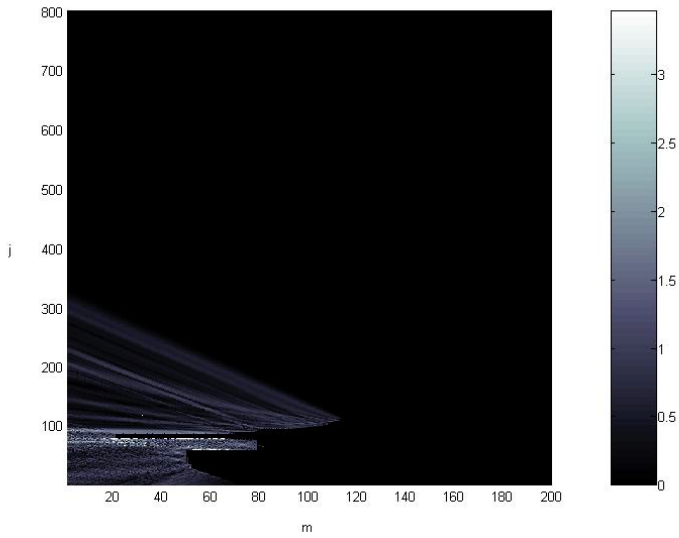
**Figure 8.** RCS results for the ship in the forward direction in 300 MHz.



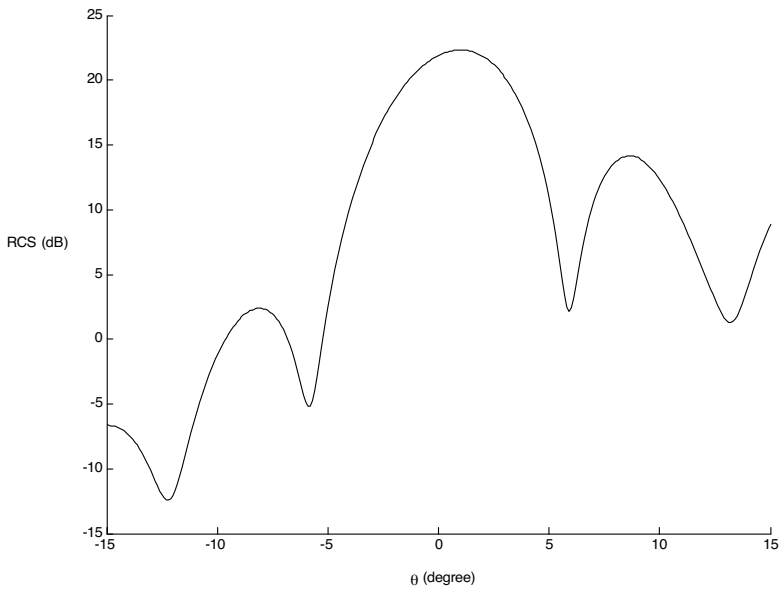
**Figure 9.** RCS results for the ship in the backward direction in 300 MHz.



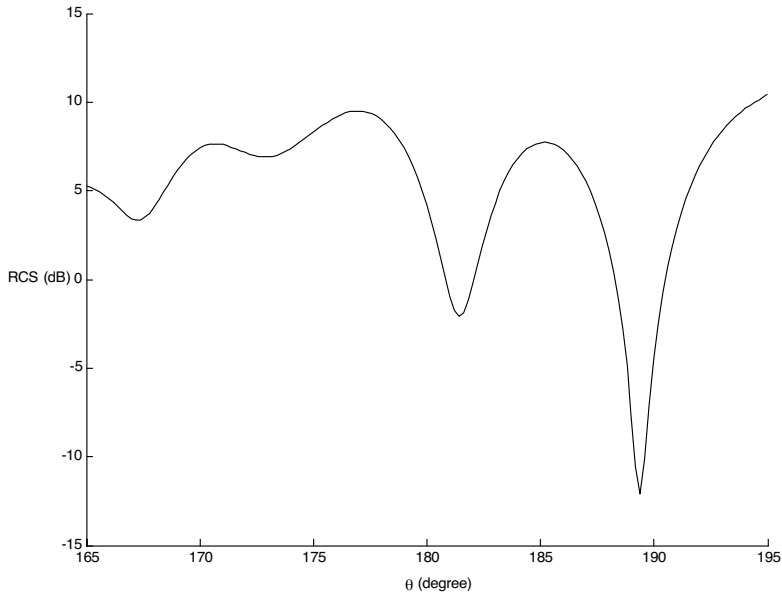
**Figure 10.** Amplitude of the scattered field  $u_s(m, j)$  from the tank in the forward direction in 300 MHz.



**Figure 11.** Amplitude of the scattered field  $u_s(m, j)$  from the tank in the backward direction in 300 MHz.



**Figure 12.** RCS results for the tank in the forward direction in 1200 MHz.



**Figure 13.** RCS results for the tank in the backward direction in 1200 MHz.

## 7. CONCLUSION

In this paper the scattered fields and RCS of a target with a lossy background has been calculated using parabolic equation method. In order to show validity of the method the RCS of a conducting cylinder has been computed using the parabolic equation method and the results has been compared with the analytic results. There has been a good agreement between the two results over a 15 degrees range. Next, considering the sea and the earth as impedance boundaries and discretization of the ship's and tank's boundaries on the rectangular grid, and implementation of a PEC boundary condition, their scattered fields as well as their RCS have been calculated. Note that the analysis of structures which are comparable in size to a wavelength is very time consuming on desktop computers with rigorous methods. The ship dimensions are about  $14\lambda$  by  $40\lambda$  for 300 MHz and the tank dimensions are about  $10\lambda$  by  $32\lambda$  for 1200 MHz, which are both very large in dimensions but could be analyzed by a desktop computer employing the PE method.

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