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FULL-WAVE MODELING OF STRIPLINE STRUCTURES IN MULTILAYER DIELECTRICS

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Abstract—A novel computational method based on full-wave analysis of stripline planar structures with vertical interconnects in multilayer dielectric media is presented. The method is based on the electric-field integral-equation solved with the Method of Moments (MoM). The special characteristics of stripline structures facilitate the extensive use of semi-analytical techniques to analyze the multilayer structures, limiting significantly the use of purely numerical techniques. The accuracy of the proposed modeling method is examined thoroughly with extensive numerical tests and the results are compared with results generated by commercial simulators for simple stripline structures.

1. INTRODUCTION

The modeling of microstrip structures in multilayer dielectrics (or dielectric media) has been thoroughly investigated in the past, and several methods have already been introduced. The research has been based on various methods such as Finite Elements (FE), Finite Difference Time Domain (FDTD) and Method of Moments (MoM) [1, 2]. The latter is the most widely employed for the modeling of planar geometries, mainly due to the fact that it does not require the meshing of the whole volume of the structure. Several methods can be found in the literature for the analysis of multilayer microstrip

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topologies, either for shielded [3–8] or unshielded structures [9–13]. Moreover, some methodologies have been proposed to take into account the vertical interconnects between planar structures. [11, 12]. However, to the authors' knowledge literature is very thin on topics regarding the modeling of planar multilayer geometries with vertical interconnects in a stripline structure. As this kind of technology profile is becoming more and more popular with the development of Radio Frequency (RF) modules implemented in materials such as Low Temperature Co-fired Ceramic (LTCC), Bismaleimide-triazine (BT) resin and other organic multilayer laminates, a robust and efficient method for the analysis of such structures becomes imperative.

This paper presents an electromagnetic (EM) modeling approach of stripline planar structures, with vertical interconnects, in multilayer media (like LTCC), using the MoM method in the Galerkin form. The formulation of the theoretical problem is based on the well-known Electrical Field Integral Equation (EFIE) in the spatial domain. The characteristics of stripline structures have been taken into account in the theoretical analysis. As a result, extensive analytical solutions become feasible and the need for complex and time-consuming numerical solutions is minimized or avoided. The problem is solved using the MoM method, and the network parameters of the structure (Y or S parameters) are calculated.

In comparison with existing methods (2.5D MoM, Full-3D FDTD/FE) the proposed methodology is inherently 3D and using a MoM-Galerkin method based on piecewise basis and testing functions, guarantees fine accuracy, stability and minimum computational requirements (memory size, computational time).

Finally, in order to test the validity of our methodology, the simulated results are compared to the theoretically expected ones and to the simulation results generated by commercially available EM solvers.

2. THEORETICAL ANALYSIS

It is a well-established fact that the first step in the Method of Moments is the calculation of the dyadic Green's function for the structure under consideration. The generalized structure under study is shown in Figure 1.

The structure consists of several dielectric layers with metallic plates on the top and bottom sides of the structure, thus resulting in a stripline profile. The circuit topology involves planar conductors on each of the layers and vertical interconnects in the form of vias providing connectivity between them. The planar conductors are

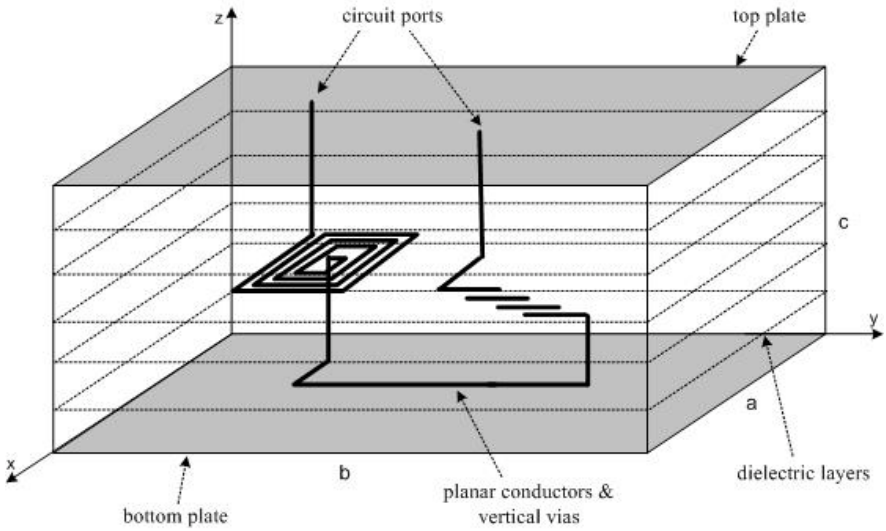

Figure 1. The typical structure of interest.

considered to have zero thickness and infinite conductivity.

In structures like the one described above, it is reasonable to assume that there is no radiation from the sidewalls, if the distance between the top and bottom plates is less than $\lambda/10$. For frequencies in the range of 0–20 GHz the value of $\lambda/10$ is higher than 1.5 mm. On the other hand, most of the microwave structures constructed in multilayer dielectric media rarely exceed the height of 1.5 mm. For example the typical value for the height of one LTCC layer is 40–200 um, so it would take at least 10 layers to reach a maximum structure height of 2 mm. Therefore, the assumption that there is no radiation from the sidewalls of the structure becomes valid for nearly all cases of practical interest.

Based on the above assumption the typical structure of Figure 1 can be considered as a "shielded cavity" with two electric (top and bottom) and four magnetic (sidewalls) walls. As a result, the boundary conditions for the structure of Figure 1 are the following:

$$\hat{n} \times \vec{E} = 0, \quad \text{for } z = 0, c \tag{1}$$

$$\hat{n} \times \vec{H} = 0, \quad \text{for } x = 0, a \tag{2}$$

$$\hat{n} \times \vec{H} = 0, \quad \text{for } y = 0, b \tag{3}$$

The above boundary conditions have a major role in the

calculation of Green's function (4):

$$\overline{\overline{G}}(\vec{r},\vec{r}\,') = \begin{bmatrix} E_{xx} & E_{xy} & E_{xz} \\ E_{yx} & E_{yy} & E_{yz} \\ E_{zx} & E_{zy} & E_{zz} \end{bmatrix} \tag{4}$$

For this calculation a unitary excitation is imposed along each of the x, y, z axes, and the respective electrical field is calculated. Each excitation yields one column of (4). Due to the boundary conditions (1)–(3) each of the elements of (4) can be expressed as a series eigenmode expansion, including both propagating and evanescent TE and TM modes [14]. For example, in the case of two segments parallel to x axis the respective element of Green's function is calculated by:

$$E_{xx} = \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} XX(n,m)\cdot \sin\left(\frac{n\pi x}{a}\right)\cos\left(\frac{m\pi y}{b}\right) \cdot \sin\left(\frac{n\pi x'}{a}\right)\cos\left(\frac{m\pi y'}{b}\right) \begin{cases} \dfrac{\sinh(\gamma_1 z)\sinh(\gamma_1(z'-c))}{\sinh(\gamma_1 c)}, & z<z' \\ \dfrac{\sinh(\gamma_1(z-c))\sinh(\gamma_1 z')}{\sinh(\gamma_1 c)}, & z>z' \end{cases} \tag{5}$$

where: $XX(n,m) = \frac{4}{ab(1+\delta_{m0})}\cdot\frac{\omega^2\varepsilon\mu-\left(\frac{n\pi}{a}\right)^2}{\gamma_1\cdot j\omega\varepsilon}$, $\gamma_1^2 = \left(\frac{n\pi}{a}\right)^2+\left(\frac{m\pi}{b}\right)^2-\omega^2\varepsilon\mu$.

In the above equations ω is the radial velocity, ε, μ are the properties of the dielectric material, a, b, c are the structure's dimensions and δ_{nm} is the well-known Krönecker delta: $\delta_{nm} = \begin{cases} 1, & \text{if } n=m \\ 0, & \text{if } n\neq m \end{cases}$.

A detailed description of Green's function calculation can be found in [15].

After the calculation of Green's function the electrical field equation of the problem is derived from the boundary conditions on the surface of the circuit conductors, symbolized by S, assuming the presence of an incident field $\vec{E}_{inc}$

$$\vec{E}_{inc}(\vec{r}) = \int_S \overline{\overline{G}}(\vec{r},\vec{r}\,')\cdot\vec{J}(\vec{r}\,')dS' \tag{6}$$

where $\overline{\overline{G}}(\vec{r},\vec{r}\,')$ is the structure's dyadic Green's function and $\vec{J}(\vec{r}\,')$ the surface current density of the metallization layers.

3. NUMERICAL ANALYSIS

The next step in the Method of Moments procedure is to approximate the surface current density of the metallization using a set of expansion functions (also known as basis functions) with unknown coefficients:

$$\vec{J}(\vec{r}) = \sum_{i=1}^{N} a_i \cdot \vec{u}_i(\vec{r}) \tag{7}$$

The type of the selected expansion functions is crucial for the accuracy and speed of the method. In the proposed methodology the metallization layers are discretized into N rectangular segments, and the representation of the surface currents is done by overlapping sinusoidal expansion functions, as shown in Figure 2.

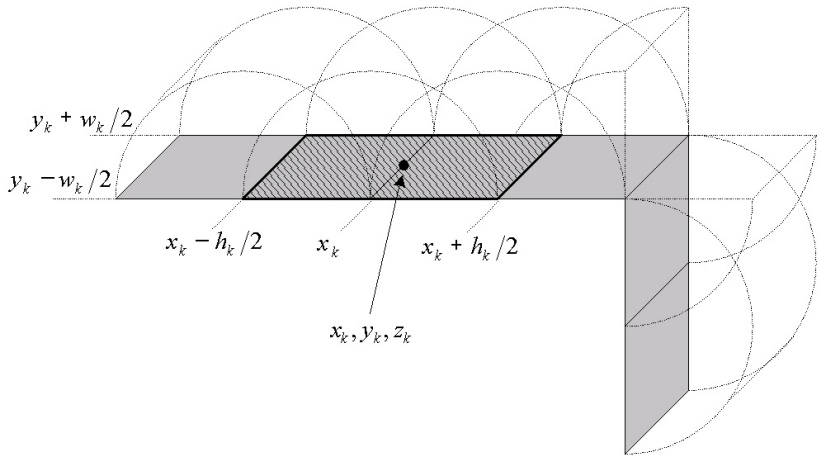

Figure 2. Overlapping sinusoidal expansion functions.

The expansion function that corresponds to the highlighted segment of Figure 2 is given by:

$$\vec{u}_k(\vec{r}) = \hat{x} \cdot \cos\left(\frac{n(x - x_k)}{h_k}\right), \quad \text{for } |x - x_k| < \frac{h_k}{2},\ |y - y_k| < \frac{w_k}{2} \tag{8}$$

where (x_k, y_k, z_k) is the geometrical center of the segment, h_k its length and w_k its width. The current flow of this segment is considered to be parallel to the x-axis. In order to achieve a better approximation of the continuity of currents, half segments are used at the corners of the metallization, as well as at the source terminals.

The expansion functions have been chosen as the testing functions of the MoM method thus resulting to a Galerkin Method [2]. After the application of the selected expansion and testing functions, the integral equation (6) is transformed into the following matrix equation:

$$\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \tag{9}$$

The elements of the $[a]$ vector are the unknown coefficients of (7) and their computation practically leads to the solution of the integral equation (6).

The elements of the $[Z]$ matrix are calculated through the implementation of the expansion functions in (6). For example consider two segments with x-directed current flows, centered at x_i, y_i, z_i and x_j, y_j, z_j, with dimensions h_i, w_i and h_j, w_j respectively. If $z_i < z_j$ then the mutual coupling Z_{ij} between these two segments is given by:

$$Z_{ij} = \int_{x_i-h_h/2}^{x_i+h_h/2} \int_{y_i-w_h/2}^{y_i+w_h/2} \int_{x_j-h_j/2}^{x_j+h_j/2} \int_{y_j-w_j/2}^{y_j+w_j/2} E_{xx} \cdot \vec{u}_i(\vec{r}) \cdot \vec{u}_j(\vec{r}\,') dx dy dx' dy' \tag{10}$$

In the above equation $\vec{u}_i(\vec{r})$, $\vec{u}_j(\vec{r}\,')$ are the expansion and testing functions defined on each segment, as shown in Figure 2 and equation (8). Substituting in (10) the expressions from (5) and (8) yields:

$$\begin{aligned} Z_{ij} \;=\; & \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} XX(n,m) \cdot \int_{x_i-h_i/2}^{x_i+h_i/2} \sin\left(\frac{n\pi x}{a}\right) \cdot \cos\left(\frac{n(x-x_i)}{h_i}\right) dx \\ & \cdot \int_{y_i-w_i/2}^{y_i+w_i/2} \cos\left(\frac{m\pi y}{b}\right) dy \cdot \int_{x_j-h_j/2}^{x_j+h_j/2} \sin\left(\frac{n\pi x'}{a}\right) \cdot \cos\left(\frac{\pi(x'-x_j)}{h_j}\right) dx' \\ & \cdot \int_{y_j-w_j/2}^{y_j+w_j/2} \cos\left(\frac{m\pi y'}{b}\right) dy' \cdot \frac{\sin(\gamma_1 z_i) \cdot \sinh(\gamma_1 (z_j - c))}{\sinh(\gamma_1 c)} \end{aligned} \tag{11}$$

The computation of all the integrals in the Z matrix calculation can be performed analytically, without employing time- and memory-consuming numerical techniques. For electrically small geometries the time to calculate the Z matrix, also known as "matrix filling time", is

the dominant part of the overall calculation time. In nearly all cases of practical interest the structures we analyze with this methodology can be considered electrically small. For example, most of the structures in LTCC rarely exceed the dimensions $a = b = 20\,\text{mm}$ and $c = 2\,\text{mm}$. Therefore the analytical calculation is expected to be faster and more efficient than other methods employing complex numerical techniques.

For the definition of the elements of vector $[V]$, which correspond to the excitation of the circuit, the delta-gap excitation scheme is used [11]. All the points of excitation are considered to be located either at the top or the bottom plate of the structure. This assumption is valid since all structures under consideration are passive topologies and the only connections of the structures are located at the top or bottom of the structure. (i.e., interconnects to chip or PCB). According to the delta-gap excitation scheme, at the location of port P_n, a voltage is applied over an infinitesimal gap of length $\delta \to 0$ between the top or bottom plate and the nth tip of the metallization that approaches the ground plate at the location of the considered port. The applied voltage is given by:

$$V_n = \hat{z} \cdot \delta(\vec{r} - \vec{r}\,') \tag{12}$$

After the application of the testing functions on the integral equation the V matrix elements are calculated by:

$$\begin{aligned} V_n &= \int_{y_j - w_j/2}^{y_j + w_j/2} \hat{z} \cdot \delta(\vec{r} - \vec{r}\,')dz \Rightarrow V_n \\ &= \begin{cases} w, & \text{at the segments where ports are defined} \\ 0, & \text{at the rest of the circuit segments} \end{cases} \end{aligned} \tag{13}$$

With the elements of Z matrix calculated and those of the V matrix defined, the matrix equation (9) can be solved for the unknown current expansion coefficients:

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} Z_{11}^{-1} & Z_{12}^{-1} & \cdots & Z_{1N}^{-1} \\ Z_{21}^{-1} & Z_{22}^{-1} & \cdots & Z_{2N}^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1}^{-1} & Z_{N2}^{-1} & \cdots & Z_{NN}^{-1} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} \tag{14}$$

The calculation of the unknown current expansion functions leads to a good approximation of the current distribution at any point of the circuit. However in most cases it is desired to have some sort of network parameters (i.e., scattering or admittance matrices) to describe the

circuit's performance. This kind of information can be easily acquired from the solution of (14). The procedure is analytically described in [6] and [15].

4. EXPERIMENTAL RESULTS

In order to verify the validity of the proposed method, some basic stripline structures are being examined. The vertical cross-section of the structure appears in Figure 3.

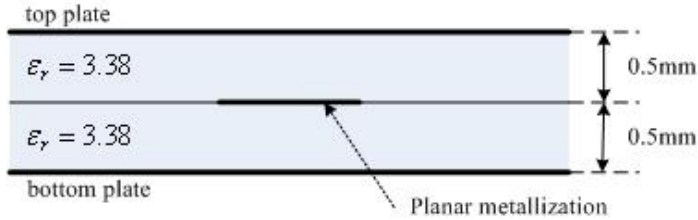

Figure 3. Structure cross-section.

It consists of two dielectric layers, each with 0.5 mm thickness. The dielectric constant of both layers is $\varepsilon_r = 3.38$. The circuit metallization is located in the middle of the layers. Metallic plates are located at the top and bottom of the structure, resulting in a stripline structure. The results of our method are compared to the results of the widely used commercial EM solver, Agilent's Momentum®.

The first circuit that will be examined is the simple transmission line with 10mm length and 0.2 mm width. Figure 4 shows the comparison of simulated results for the S parameters of the transmission line, both for Momentum and the proposed method.

There is a very good agreement between the two approaches as the resonance frequencies are predicted quite correctly with a discrepancy less than 0.2 GHz. Also the maximum magnitude of the discrepancy observed is around 5–6% at the frequency of 14 GHz.

The second test circuit is a band pass-filter that appears in Figure 5.

The filter was designed to have a passband at 6–7 GHz and at least 20 dB rejection at 6 and 10 GHz, following the procedure described in [16]. In Figure 6 the results of the proposed method are compared to those of Momentum's.

Again the results of the new method are considered to correlate quite well. The passband of the filter is correctly predicted and the error in the passband gain is less than 2%. Also, the slope of the

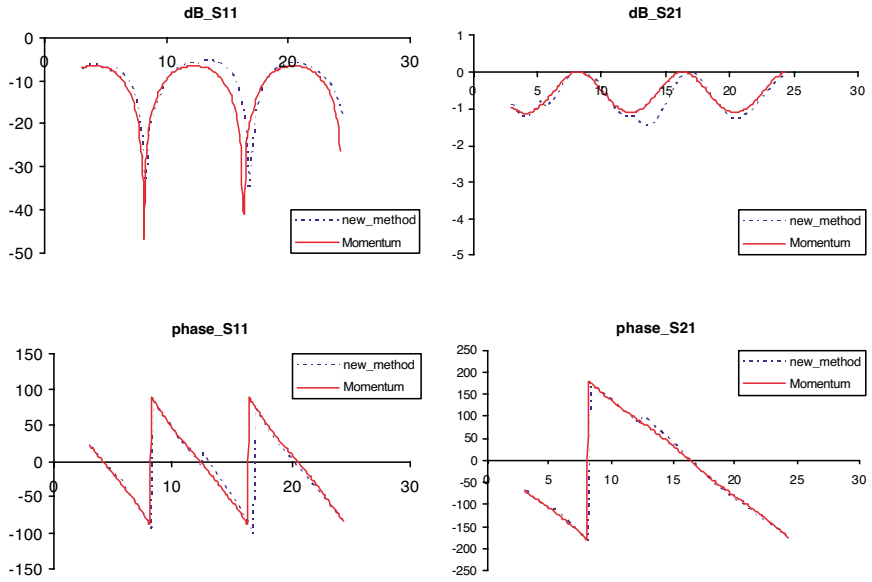

Figure 4. Short stub simulation results.

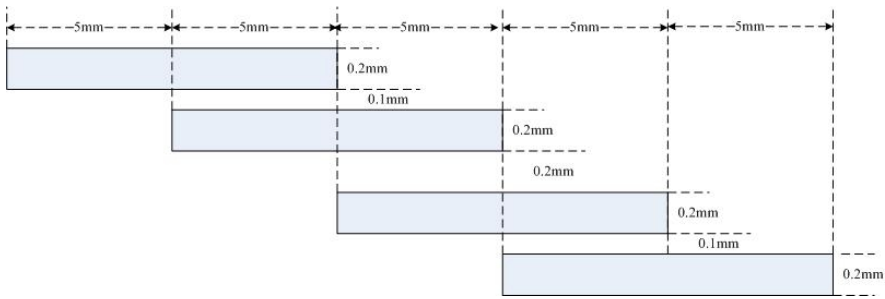

Figure 5. Band-pass filter top view.

filter stopband is again predicted well. The results exhibit larger discrepancies in the stopband, both for magnitude and phase. However these discrepancies are observed at levels lower than −30 dB where the same level of accuracy is extremely difficult to be obtained and the effects on the performance of our circuit are limited.

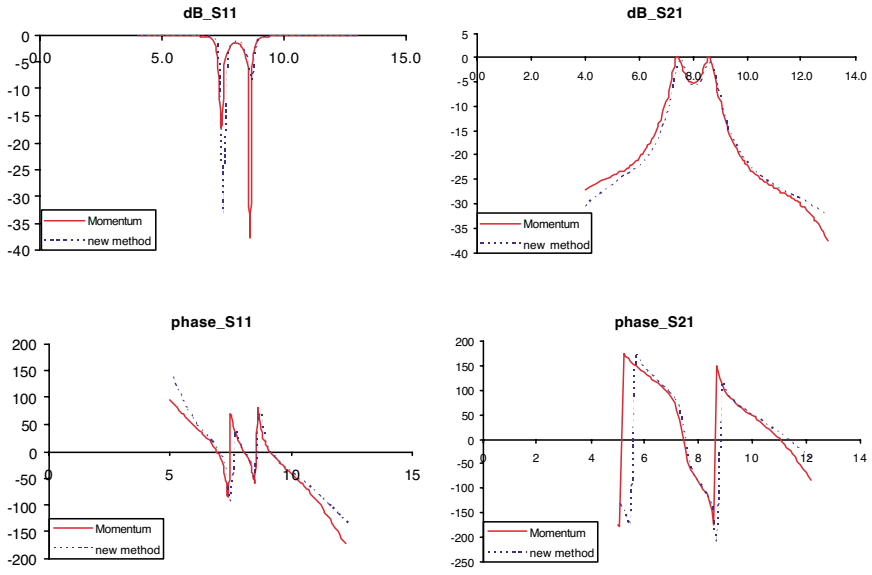

Figure 6. Filter simulation results.

5. CONCLUSION

A novel methodology for the electromagnetic simulation of stripline structures with vertical interconnects has been presented. It is based on the MoM method in its Galerkin form and takes advantage of the special characteristics of stripline structure, to allow the analytical computation of the impedance matrix. The need for complex time and memory consuming numerical calculations is minimized. The comparison of the simulated results versus the theoretical and experimental ones shows that this methodology can lead to a very accurate and efficient tool for the modeling of stripline structures with vertical interconnects.

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