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A MODIFIED THREE-ANTENNA GAIN MEASUREMENT METHOD TO SIMPLIFY UNCERTAINTY ESTIMATION

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Abstract—A modified three-antenna gain measurement method is proposed to significantly simplify, and thereby to reduce the costs of, uncertainty estimation. Unlike the three-antenna method, the new method employs a standard antenna whose gains and uncertainty limits have been established through absolute calibration. The measurements are carried out as in the three-antenna method, with the reference antenna also analysed as presumed unknown. An overall uncertainty figure for the measurement is obtained only from the resulting data in conjunction with the uncertainty on the reference antenna gain values. The proposal is verified by using a rigorous experimental study on the gains of three pyramidal horns in the frequency range 5.8 GHz–8.2 GHz. First, the gains and error limits of the antennas are estimated by using the three-antenna method. One of the antennas is then used as reference antenna and the modified method employed on the measured data to determine the gains of a test antenna. It emerges that the new method substantially simplifies the secondary calibration and gain measurement of antennas.

1. INTRODUCTION

Uncertainty estimation (or error analysis) in any measurement requires the careful consideration of all possible sources of error that might affect it. A thorough error analysis is a challenge and is often considered to be “one of the most difficult challenges” facing RF metrology [1]. The cost and time requirement of rigorous uncertainty analysis is quite high, which in general means that methods that employ such rigorous analysis are normally implemented only by

National Measurement Institutes and are rarely applied to routine testing or calibration [2]. In the light of this, a preliminary proposal of a modified three-antenna gain measurement method to simplify uncertainty estimation in antenna gain estimation and secondary calibration was recently considered [3]. It was pointed out in the concluding section of [3] that it would be desirable to develop the preliminary proposal into a more rigorous concept. Consistent with this remark, in this paper a revised proposal that is more complete is presented such that it can now be believed to be in accordance with the practices recommended for uncertainty estimation by the International Organization for Standardization (ISO) [2, 4, 5]. This proposal is validated by using a rigorous experimental study on the gains of three pyramidal horns in the frequency range 5.8 GHz–8.2 GHz.

The paper proceeds as follows: Section 2 presents the measurement proposal. The experimental validation study and the results are discussed in Section 3. Section 4 discusses the salient features of the method. Section 5 concludes the paper.

2. THE MODIFIED THREE-ANTENNA METHOD

The three-antenna technique [6, 7] does not require *a priori* knowledge of the gain of any of the three antennas involved. On the other hand, in the modified three-antenna method, one of the antennas needs to be a reference antenna whose gains and error limits have been established through absolute calibration. The reference antenna acts as a Reference Material (RM), which is defined as "a material or substance one or more of whose property values are sufficiently homogeneous and well established to be used for the calibration of an apparatus, the assessment of a measurement method, or for assigning values to materials" [5]. In the modified three-antenna method, measurements are done just as in the case of the three-antenna method, with the RM also analysed as presumed unknown. A comparison of the result obtained for the RM with its reference values essentially constitutes a "calibration of the whole measurement process against a traceable reference" and provides useful information on the combined effect of many of the potential sources of uncertainty. The parameter used for this purpose is the so called *bias*, defined as the value obtained for the RM divided by the value expected [2].

An overall uncertainty estimation from the above method requires that two contributions be taken into account at the minimum: (1) the uncertainty associated with the bias, and (2) the precision of the measurement [2]. The bias uncertainty is estimated by combining the standard uncertainty on the RM values and the limiting error associated with the bias. Even when the bias is insignificant or is corrected for, the uncertainty associated with its determination needs to be considered in the overall uncertainty assessment [2]. A measure of the precision, defined as "the closeness of agreement between independent test results obtained under prescribed conditions [5]," is obtained by estimating the standard deviation associated with the measured data on the test antenna and by subsequently estimating the limiting error.

An actual measurement by using the modified three-antenna method would proceed as follows: Power measurements are done and repeated n times using all the pairs of the three antennas, with the RM also analyzed as presumed unknown. In this measurement effort, the antenna combinations were rotated n times, with a single reading taken each time[†]. The gains of the three antennas at a range r and wavelength λ are then determined by using simultaneous equations of the form [8, eq. (6)].

$$G_T G_R = \frac{|1-\Gamma_R\Gamma_L|^2|1-\Gamma_G\Gamma_T|^2}{(1-|\Gamma_R|^2)(1-|\Gamma_T|^2)|1-\Gamma_G\Gamma_L|^2}\left(\frac{4\pi r}{\lambda}\right)^2\left(\frac{{}^{f}P_L}{{}^{i}P_L}\right) \tag{1}$$

where G_T and G_R are the transmitting and receiving antenna gains, ${}^{i}P_L$ is the power delivered to the power meter when the generator and the load are directly connected and ${}^{f}P_L$ is the power delivered when the antennas are connected. The variables Γ_T, Γ_R, Γ_G and Γ_L represent, respectively, the reflection coefficient of transmitting antenna, receiving antenna, generator and power meter. In this work, the reflection coefficient values for the antennas were measured by using the HP 8510 B network analyzer, while for the generator and the power meter, the values given by the manufacturers were employed.

At each frequency, the "true" value G_r of the RM in dB (obtained through absolute calibration, or quoted by the manufacturer) is subtracted from its measured value G_m for each of the n trials. This gives the value of the bias β. The bias is used to correct the estimated (uncorrected) test antenna gains. Thus, if G_u and G respectively represent the uncorrected and corrected gain values, then this operation proceeds as follows:

$$\beta = G_m - G_r \tag{2}$$

$$G = G_u - \beta \tag{3}$$

[†] It is also possible to take all data points for each antenna combination before moving on to the next combination. While doing so will not affect the method, it will result in significantly reduced measurement time and uncertainty due to alignment, and cable connections.

The overall standard uncertainty U for the measurement is then estimated from the following equation:

$$U = \sqrt{\left(\frac{\sigma_\beta^2 + \sigma_G^2}{n}\right) + U_{\mathrm{RM}}^2} \tag{4}$$

where σ_β is the standard deviation of the bias values, σ_G is the standard deviation of the bias-corrected test antenna gain values and U_{RM} is the standard uncertainty on the reference antenna gain values. The division by n in (4) assumes normal distribution. The uncertainty with 99% confidence limits is, of course, given by $3U$ [9].

The measurement is repeated n times so as to be able to account for the random errors and to ensure the sustenance of the uncertainty estimate. As regards the effects of systematic error sources, the use of the same instrumentation throughout the measurement run will ensure their nearly identical effect on the gains of all the antennas. It is desirable that the cables at the generator and load ends are not disturbed during the measurements; this is anyway quite practical, as only the antennas need to be replaced.

3. EXPERIMENTAL VERIFICATION

3.1. Description of the Measurements

The gain measurement of pyramidal horns in the frequency band of 5.8 to 8.2 GHz was considered. A commercially available standard gain horn following Slayton's design [10] and two nominally identical pyramidal horns constituted the set of three antennas. The dimensions of the pyramidal horns used are shown in Table 1, where a_1 and b_1 are the horn aperture dimensions and l_E and l_H are the slant lengths in the E and H planes, respectively. Each of these antennas was fed by a rectangular waveguide of internal dimension 3.5 cm × 1.6 cm.

The test range was a level brail track on the rooftop of the laboratory. The transmitting and receiving antennas were mounted on movable carriages at a height of about 2.5 m. The separation between the antennas was 5 m. Radio absorbing material covered the track in between the transmitting and the receiving towers. Bore sight alignment was ensured by using an optical system mounted on an alignment plate. Vertical polarization was used in mounting the antennas.

Power ratio measurements were performed and repeated using all combinations of the three antennas. Twenty sets of measurements were

Table 1. Dimensions of pyramidal horns.

Horn designation	Dimensions, cm			
	a_1	a_2	l_E	l_H
A1(Standard antenna)	28.85	21.37	47.50	50.84
A2 and A3	28.80	21.35	29.96	32.81

taken, as this is the desired number for obtaining reliable estimates of uncertainty [11].

3.2. Gain Estimation by the Three-Antenna Method

The gains estimated by the three-antenna method are shown in Table 2. These values were obtained as follows: First, at every measurement frequency the gain was estimated for each of the 20 measurement sets. Any questionable data points were eliminated by employing Chauvenet's criterion [11]. Then, the average gain value was determined. The proximity corrections of Chu and Semplak [12] were employed.

The uncertainty in these gain values can be determined by considering the following various sources of error in this method:

1) Stability of equipment, repeatability of electrical connectors.
2) Multipath propagation.
3) Power meter uncertainty.
4) Improper matching of the antennas.
5) Uncertainty in proximity correction.

A detailed uncertainty analysis of the measured data was done [13] by employing the usual statistical formulas [14] to determine the uncertainty due to random effects. The uncertainty due to multipath interference was determined by making use of the data obtained in an earlier measurement effort conducted on the same test-range [15]; this value is expected to represent the worst-case uncertainty due to this effect in the considered frequency range. While the manufacturer-quoted value was taken for uncertainty due to the power meter, for uncertainty due to mismatch and finite-range correction, the conservative criterions in [8, 9] were employed. The final uncertainty figures were estimated by combining in quadrature all the limiting errors thus obtained. The uncertainty figures are summarised in Table 3.

Table 2. Gain of horns (dB) estimated by three-antenna method.

f, GHz	Standard Antenna	Antenna A2	Antenna A3
5.8	21.50	18.91	18.81
6.0	22.10	18.51	18.45
6.2	22.15	18.70	18.73
6.4	21.99	18.49	18.38
6.6	22.46	18.35	18.35
6.8	22.49	18.40	18.34
7.0	22.20	18.04	18.00
7.4	22.54	18.02	18.00
7.6	22.63	17.80	17.79
7.8	22.66	17.74	17.74
8.0	22.72	17.54	17.55
8.2	22.87	17.20	17.26

From the table, the maximum 3σ uncertainty in this frequency range is seen to be $\pm 0.5\,\mathrm{dB}$. A comparison of the measured gain values along with the associated uncertainty of the standard antenna with the manufacturer-quoted values verifies the correctness of the measurement.

The gains of the antennas A2 and A3 presented in the Table 2 show a general decreasing trend as a function of frequency. This is due to their axial length being less than that required for "normal" gain behaviour [16].

3.3. Gain Estimation by the Modified Three-Antenna Method

The particularly significant feature of the modified three-antenna gain measurement method is that it substantially simplifies the estimation of uncertainty. We will now use the measured data obtained in this work to verify this feature. Since the "true" gain values of the standard antenna (A1) quoted by the manufacturer are known (with a 3σ uncertainty of $\pm 0.3\,\mathrm{dB}$), we will employ it as the test antenna. We will use the antenna A3, whose gain values have been established by the three-antenna method with a 3σ uncertainty of $\pm 0.5\,\mathrm{dB}$ in this

Table 3. Summary of uncertainty figures.

f, GHz	Uncertainty (dB) due to					Uncertainty(dB)	
	Random effect	Multi-path Interference	Power meter	Mismatch	Proximity correction	1σ	3σ
5.8	0.03	0.12	0.07	0.03	0.06	0.16	0.48
6.0	0.03	0.12	0.07	0.03	0.06	0.16	0.48
6.2	0.03	0.12	0.07	0.03	0.06	0.16	0.48
6.4	0.03	0.12	0.07	0.03	0.07	0.16	0.48
6.6	0.03	0.12	0.07	0.03	0.07	0.16	0.48
6.8	0.03	0.12	0.07	0.03	0.07	0.16	0.48
7.0	0.02	0.12	0.07	0.03	0.08	0.16	0.48
7.4	0.03	0.12	0.07	0.03	0.08	0.17	0.51
7.6	0.03	0.12	0.07	0.03	0.08	0.17	0.51
7.8	0.03	0.12	0.07	0.03	0.09	0.17	0.51
8.0	0.03	0.12	0.07	0.03	0.09	0.17	0.51
8.2	0.04	0.12	0.07	0.03	0.09	0.17	0.51

study, as the reference antenna.

For illustration, we consider the estimation of gain and uncertainty at 8 GHz. We will use all the twenty samples. (Here too, we will eliminate any questionable data points by employing Chauvenet's criterion [11].) The "actual" gain of the reference antenna at this frequency, as measured above by the three-antenna method, is 17.55 dB with a 3σ uncertainty of ± 0.5 dB. Table 4 gives the values of the reference and test antenna gains and the bias. An application of Chauvenet's criterion eliminates data number 8 in this table. Then, the average gain of the test antenna is estimated to be 22.71 dB. The uncertainty estimation is done by an application of (2)–(4) as follows:

Standard uncertainty on reference antenna gain $U_{\mathrm{RM}} = \pm 0.166\,\mathrm{dB}$

Standard deviation of bias values $\sigma_b = 0.135\,\mathrm{dB}$

Standard deviation of corrected test antenna gains $\sigma_G = 0.328\,\mathrm{dB}$

Overall standard uncertainty $\sqrt{\left(\frac{0.135^2+0.328^2}{19}\right) + 0.166^2} = \pm 0.185\,\mathrm{dB}$

Overall 3σ uncertainty $= \pm 0.56\,\mathrm{dB}$

Table 4. Gain estimation using *MTAM* at 8.0 GHz.

No.	Reference antenna gain, dB		Test antenna gain, dB	
	Measured	Bias	Uncorrected	Corrected
1	17.34	-0.21	22.32	22.53
2	17.59	+0.04	22.15	22.11
3	17.68	+0.13	22.32	22.19
4	17.37	-0.18	22.53	22.71
5	17.54	-0.01	22.59	22.60
6	17.54	-0.01	23.00	23.01
7	17.67	+0.12	22.61	22.49
8	17.94	+0.39	22.75	22.36
9	17.58	+0.03	22.97	22.94
10	17.56	+0.01	23.04	23.03
11	17.59	+0.04	22.78	22.74
12	17.50	-0.05	23.09	23.14
13	17.42	-0.13	22.94	23.07
14	17.50	-0.05	22.92	22.97
15	17.82	+0.27	22.45	22.18
16	17.48	-0.07	22.99	23.06
17	17.64	+0.09	22.73	22.64
18	17.51	-0.04	22.64	22.68
19	17.80	+0.25	22.69	22.44
20	17.34	-0.21	22.82	23.03

Thus we have estimated by using the modified three-antenna method that the 3σ uncertainty is 0.56 dB. Note that the "true" gain of the test antenna is 22.79 dB with a 3σ uncertainty of 0.3 dB. Thus the "true" gain value of the test antenna is in agreement with the value estimated by the modified three-antenna method within the appropriate uncertainty limits. This approach demonstrates the simplification of uncertainty estimation by the modified three-antenna method.

The final results for the all the frequencies are summarised in Table 5. It is seen that there is in general good agreement between the values estimated by using the modified three-antenna method and the "true" values. It is thus evident that the modified three-antenna method is a viable method for simplifying uncertainty estimation. It is of interest to note that at most of the frequencies the uncertainty figures in Table 5, when rounded off, reduce to ±0.5 dB, which is the uncertainty of the reference antenna gain values. The implication is that measurement accuracy improvement warrants the establishment of *better* reference antennas (with lower uncertainty figures). This, of course, ought to be a continuing goal of antenna metrology.

Table 5. Summary of test antenna gains estimated using *MTAM*.

f, GHz	Test antenna gain estimated from *MTAM*, dB		"True" test antenna gain, dB (3σ Uncertainty = ±0.3 dB)
	Average value	3σ Uncertainty	
5.8	21.50	0.53	21.66
6.0	22.07	0.53	21.82
6.2	22.12	0.53	21.97
6.4	21.99	0.53	22.11
6.6	22.43	0.53	22.24
6.8	22.49	0.54	22.36
7.0	22.18	0.53	22.46
7.4	22.55	0.54	22.64
7.6	22.63	0.53	22.70
7.8	22.67	0.55	22.75
8.0	22.71	0.56	22.79
8.2	22.87	0.55	22.81

4. DISCUSSION

The modified three-antenna method offers a simple technique for uncertainty estimation in antenna gain measurement and secondary calibration. As a significant feature having cost implications, it does not put stringent requirement that the entire instrumentation used for the measurement be calibrated, although it is important that the signal generator gives single and correct frequency, and that the generator and the load are reasonably stable. Also, it is sufficient that all measurements be conducted at a single antenna separation (measurements at several separations are not necessary). With the modified method, a qualitative check of an experimental set-up can be quickly accomplished by taking just one set of measurement, as the bias indicates the quality of the measurement.

If site reflections have negligible effect on the measurements, any restriction on the gains (patterns) of the reference antenna versus the test antennas is not necessary. However, should site reflections be substantial, the gain values of the reference and the test antennas will have to be comparable for the applicability of the method. This being a limitation of the method, another limitation is that it cannot be used for primary calibration, as a reference antenna always acts as the benchmark in this measurement. Also, the overall uncertainty in this method will necessarily be slightly higher than the uncertainty on the values of the reference antenna used.

Could the gain substitution technique be employed for uncertainty estimation in a similar fashion? Possibly not, considering that in this method the reference antenna is automatically assumed to offer its reference gain values in the given measurement environment, which need not necessarily be the case [17].

5. CONCLUSIONS

Error analysis that leads to establishing uncertainty limits for measurements is a demanding task. Towards alleviating this in the case of secondary calibration and gain measurement of antennas, a modified three-antenna gain measurement technique was proposed and rigorously examined in this paper. The gain measurement of pyramidal horns was considered for illustration. The results were demonstrative of the validity of the proposed method.

It is believed that this method would be particularly useful for laboratories that do not find it practical to carry out exhaustive error analysis. Even when resources for such a detailed analysis are available, if the accuracy requirements for the end-use are such that an uncertainty slightly higher than that of the standard antenna would suffice, the method provides an easier, quicker and relatively economical method for the gain measurement and secondary calibration of antennas.

It is the author's belief that the concept introduced in this paper has the potential to be extended to uncertainty estimation in measurements in general.

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