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# NOVEL GABOR-BASED GAUSSIAN BEAM EXPANSION FOR CURVED APERTURE RADIATION IN DIMENSION TWO

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**Abstract**—In this article, we propose to apply the Gabor expansion to describe magnetic and electric currents given on a regular curved interface in dimension 2. From this description, we show that the computation of the current radiation can be performed by the introduction of a new kind of gaussian beams. We call them the conformal gaussian beams. Their analytic formulation is obtained using an asymptotic evaluation of the radiation integrals. Their properties are discussed and an application example is presented.

## 1. INTRODUCTION

Many electromagnetic problems deal with the propagation of waves and their interaction with large metallic and / or dielectric objects. The expansion of fields into basis elements is an approach to handle these complex problems. Two expansions are widely used. The first one, the spectral representation, allows to express fields as a combination of plane waves. The second one, the Kirchhoff representation, uses point source excitation to expand field radiation. The properties of these basis elements are totally opposite since plane waves are spatially wide

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and spectrally narrow while the opposite is true for point sources. Since realistic fields have often finite spatial and spectral widths, new basis elements owning these properties have been investigated.

Gaussian beams seem to be appropriate solutions because their parameters allow to control their spectral and spatial widths. Moreover, analytical expressions give their amplitude and phase variations from the near field zone to the far field zone. Different theories have recently been introduced to describe an electromagnetic field as a discrete sum of gaussian beams.

One possibility is the vectorial multimodal orthogonal Gauss-Hermite (or Gauss-Laguerre) basis, which has been successfully used to express the fields scattered by metallic [1] or dielectric [2] objects. However, this expansion strongly depends on the paraxial approximation which requires that the described fields are only weakly diverging along their main propagation direction. Another solution is the Gabor expansion which expresses a signal as a combination of gaussian functions placed on a doubly infinite spectral-spatial discrete lattice [3]. P. D. Eiziger and L. B. Felsen et al. [4] have used this expansion to represent a planar aperture radiation as a sum of beams shifted both in location and in propagation direction. However, the non-orthogonality of the Gaussian functions leads to numerical difficulties in the determination of the expansion coefficients. Two different solutions have been developed. J. J. Maciel and L. B. Felsen propose "Gabor-based narrow waisted gaussian beam algorithm" [5]. The expansion coefficients are directly obtained by sampling the initial field. D. Lugara and C. Letrou replace the Gabor basis by the Gabor frame which corresponds to an over-sampling of the elementary beams [6]. The expansion coefficients are then obtained with the frame adjoint functions, providing numerical stability. The Gabor expansion however presents other limitations: the expansion surface has to be planar and the analytical formulation of the elementary beams doesn't always correspond to the conventional gaussian beams. Three other expansion techniques have also been developed to express a field as a linear combination of gaussian beams, shifted both in location and in propagation direction, on a planar [7], on a semi-spherical surface [8] and on a moderate curved interface [9]. Although these expansions do not use any base or any frame, but a set of gaussian beams, they provide good results.

To treat a more general class of initial fields and interfaces, we propose to use the Gabor expansion with respect to the curvilinear coordinate of a regular interface in dimension two. This new expansion requires to develop a new kind of gaussian beams: the conformal gaussian beams.

This paper is organised as follows. In the next section, we present the Gabor expansion applied to the description of electric and/or magnetic currents on a regular interface. Then, an analytical formulation of conformal gaussian beam is developed and the properties of the electromagnetic radiated fields are discussed. An application example is presented in the last section.

## 2. GABOR EXPANSION OF MAGNETIC AND ELECTRIC CURRENTS

In this section, we aim to use the Gabor expansion to express electric and magnetic currents $\vec{J}^i$ and $\vec{M}^i$, along a curved interface as a combination of elementary gaussian functions. In a two dimensional TE configuration, we consider an infinite unclosed regular curve $(C)$ splitting the space into two zones denoted 1 and 2 (Figure 1).

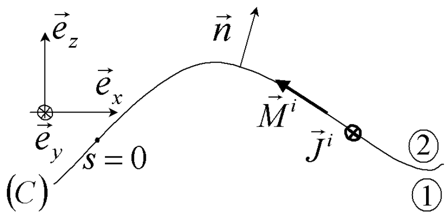


**Figure 1.** Initial configuration.

Along $(C)$, we denote $s$ the curvilinear coordinate with an arbitrary origin $s = 0$. Our TE configuration leads to:

$$\begin{aligned} \vec{J}^i &= J^i \vec{e}_y \\ \vec{M}^i &= M^i \vec{n} \wedge \vec{e}_y \end{aligned} \quad (1)$$

$\vec{n}$ stands for the unit normal vector oriented from 1 to 2. $J^i$ and $M^i$ correspond to the scalar components of the electric and magnetic currents. The Gabor expansion with respect to $s$ can be used to describe both of these scalar components. It allows to express them as a combination of elementary gaussian functions placed on a doubly infinite spatial-spectral discrete lattice [3]: the gaussian functions are translated both in the spatial and spectral domain. It yields the following expressions:

$$J^i(s) = \sum_{m,n=-\infty}^{+\infty} a_{m,n}^{Ji} w_{m,n}(s) \quad \text{and} \quad M^i(s) = \sum_{m,n=-\infty}^{+\infty} a_{m,n}^{Mi} w_{m,n}(s) \quad (2)$$

$w_{m,n}(s)$ are the translated gaussian functions:

$$w_{m,n}(s) = w(s - ml)\exp(-jn\beta s) \tag{3}$$

$w(s)$ stands for the normalized gaussian function:

$$w(s) = \sqrt{\frac{\sqrt{2}}{L}}\exp\left(-\pi\frac{s^2}{L^2}\right) \tag{4}$$

$L$ is the gaussian width. $l$ and $\beta$ denote respectively the spatial and spectral resolution while $m$ and $n$ stand for the spatial and spectral order.

The spatial and spectral resolution $l$ and $\beta$ satisfy:

$$\beta l = 2\pi\nu \tag{5}$$

$\nu$ is an over-sampling parameter. The lower $\nu$ is, i.e., the narrower the spatial-spectral lattice is, the larger the number of beams becomes. The expansion is feasible as soon as $\nu \leq 1$. For $\nu = 1$, the $w_{m,n}$ set forms a basis. In this case, the computation of the expansion coefficients however requires the use of unbounded and non-continued biorthogonal functions, leading to numerical instability. For $\nu < 1$, the $w_{m,n}$ set is not a basis any more but it becomes a frame which is a particular kind of complete family. The use of the dual frame to compute the expansion coefficients ensures the numerical stability. The coefficients are then given by:

$$\begin{aligned} a_{m,n}^{Ji} &= \left\langle J^i, \tilde{w}_{m,n}\right\rangle = \int_{-\infty}^{+\infty} J^i(s)\tilde{w}_{m,n}^*(s)ds \quad \text{and} \\ a_{m,n}^{Mi} &= \left\langle M^i, \tilde{w}_{m,n}\right\rangle = \int_{-\infty}^{+\infty} J^i(s)\tilde{w}_{m,n}^*(s)ds \end{aligned} \tag{6}$$

$\tilde{w}_{m,n}$ stand for the Gabor dual frame which can be obtained by the spatial and spectral translation of an elementary function:

$$\tilde{w}_{m,n}(s) = \tilde{w}(s - ml)\exp(-jn\beta s) \tag{7}$$

$\tilde{w}$ can be computed thanks to an iterative algorithm [10]. In order to optimise the convergence of this algorithm, we will assume in the following parts that [11]:

$$l = \sqrt{\nu}L \tag{8}$$

The radiation of the elementary gaussian electric and magnetic currents defines a new family of gaussian beam that we call the conformal gaussian beams (CGB). There are two kinds of CGB depending on whether the current is electric or magnetic. Their electric radiated fields, polarised along the y direction, are respectively denoted $E^J_{m,n}(P)$ and $E^M_{m,n}(P)$.

Finally, the radiation of the currents $\vec{J}^i$ and $\vec{M}^i$ at an observation point $P$ corresponds to the superposition of the CGB produced by the expansion:

$$E^i(P) = \sum_{m,\,n=-\infty}^{+\infty} a^{Ji}_{m,n} E^J_{m,n}(P) + a^{Mi}_{m,n} E^M_{m,n}(P) \qquad (9)$$

We now briefly explain the physical properties of this expansion (Figure 2). First, we note that an elementary current associated with the order $(m, n)$ radiates a field toward both zone 1 and 2. The spatial translation $m$ provides the beam centre which is located at $s = ml$. The spectral translation $n$ produces a linear phase evolution of the gaussian current with respect to $s$. Therefore, we can assume the following properties. For $n\beta/k < 1$, the field radiated by the current propagates and its main propagation axis has an angular shift respectively of $\arcsin(n\beta/k)$ in zone 1 and of $\pi - \arcsin(n\beta/k)$ in zone 2 with respect to the normal vector at $s = ml$. For $n\beta/k > 1$, the beam doesn't propagate and its amplitude rapidly decreases.

Finally, in this section we have shown that the Gabor expansion allows to express the current radiation as a sum of a new kind of gaussian beams.

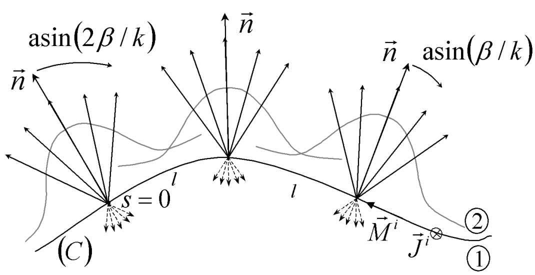


**Figure 2.** Gabor expansion on a curved interface.

## 3. THE CONFORMAL GAUSSIAN BEAM

### 3.1. Analytical Formulation

In this section, we study the formulation of the conformal gaussian beams. First, we show that they can utterly be determined by a reference and four other parameters. Then, we propose an analytic expression thanks to an asymptotic evaluation of the radiation integrals.

CGB are defined from elementary electric and magnetic currents with a gaussian amplitude distribution and a linear phase on a regular curve. Without loss of generality their expression can be reduced to:

$$u(s) = u_0 \exp\left(-\frac{s^2}{W_0^2}\right) \exp\left(-j\beta_0 s\right) \tag{10}$$

$u$ stands either for the electric or for the magnetic current. The dimension of the constant $u_0$ depends on the nature of the current. From this expression, we note that a CGB is determined by its amplitude $u_0$, its waist size $W_0$, its phase term $\beta_0$ and by the curve behaviour about $s = 0$.

Using a translation of $ml$, we easily check that this expression corresponds to the elementary Gabor functions (3) by taking:

$$\begin{aligned} w_{m,n}(s) = u(s - ml) \qquad & u_0 \equiv \sqrt{\frac{\sqrt{2}}{L}} \exp(-jmn\beta l) \\ W_0 = \frac{L}{\sqrt{\pi}} \qquad & \beta_0 = n\beta \end{aligned} \tag{11}$$

To obtain an analytical expression, a second order approximation of the curve about $s = 0$ is used. $(C)$ is then characterised by its local reference $(O_0, \vec{\tau}_0, \vec{n}_0)$ and its curvature $c_0$. $O_0$, $\vec{\tau}_0$, and $\vec{n}_0$ respectively correspond to the point, the tangent vector and the normal vector at $s = 0$. This approximation is illustrated on Figure 3 and its accuracy will increase for small $W_0$ values. Finally, a CGB is completely characterised by its reference $(O_0, \vec{\tau}_0, \vec{n}_0)$ and by its four parameters $u_0$, $W_0$, $\beta_0$, and $c_0$.

In this part, all the expressions are related to the $(O_0, \vec{\tau}_0, \vec{n}_0)$ reference associated with the beam. The electric fields can be expressed from the radiation integrals in two dimension. They are given by:

$$\begin{aligned} \vec{E}^J(P) &= \frac{k\eta}{4} \int_{-\infty}^{+\infty} \vec{J}(s) H_0^{(1)}(kr) ds \quad \text{and} \\ \vec{E}^M(P) &= -\frac{j}{4} \vec{\nabla} \wedge \int_{-\infty}^{+\infty} \vec{M}(s) H_0^{(1)}(kr) ds \end{aligned} \tag{12}$$

In these expressions, $P$ is the observation point (Its cylindrical coordinates are denoted $R$ and $\theta$), $r = ||\vec{r}||$ with $\vec{r}$ the vector from a point $I$ of curvilinear coordinate $s$ on $(C)$ to the observation point.

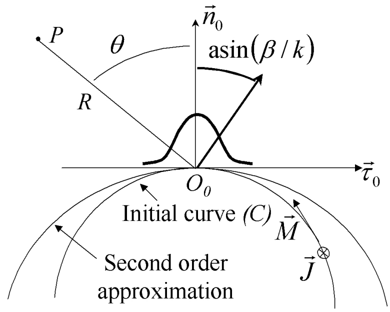


**Figure 3.** Conformal gaussian beam characteristics.

$k$ and $\eta$ are respectively the medium wavenumber and impedance. The first kind Hankel function of order 0 $H_0^{(1)}$ comes from the two dimensional free space Green function. Assuming that the observation point $P$ is not in the very near field radiation zone and using the currents orientation in (1), we obtain:

$$\begin{aligned} E^J(P) &= \sqrt{\frac{k}{2\pi}}\eta\frac{e^{j\pi/4}}{2}\int_{-\infty}^{+\infty} u(s)\frac{\exp(-jkr)}{\sqrt{r}}ds \\ E^M(P) &= -\sqrt{\frac{k}{2\pi}}\frac{e^{j\pi/4}}{2}\int_{-\infty}^{+\infty} \vec{n}.\vec{r}_1 u(s)\frac{\exp(-jkr)}{\sqrt{r}}ds \end{aligned} \quad (13)$$

With $\vec{r}_1 = \vec{r}/r$. Thanks to the second order approximation, the coordinates of the point $I$ on $(C)$ are given by:

$$\overrightarrow{O_0 I} = \begin{bmatrix} s \\ \frac{c_0}{2}s^2 \end{bmatrix} \quad (14)$$

It yields:

$$\vec{n} = \frac{1}{\sqrt{1+(c_0 s)^2}}\begin{bmatrix} -c_0 s \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{r} = \overrightarrow{IP} = \begin{bmatrix} R\sin\theta - s \\ R\cos\theta - \frac{c_0}{2}s^2 \end{bmatrix} \quad (15)$$

Finally, the two radiation integrals can be written:

$$E^U(P) = \int_{-\infty}^{+\infty} E_0^U(s)\exp(g(s))ds \quad (16)$$

$U$ represents either $J$ or $M$. The $E_0^U$ functions are given by:

$$E_0^J(s) = u_0\sqrt{\frac{k}{2\pi}}\eta\frac{e^{j\pi/4}}{2}\frac{1}{\sqrt{r}} \quad \text{and} \quad E_0^M(s) = -u_0\sqrt{\frac{k}{2\pi}}\frac{e^{j\pi/4}}{2}\frac{\vec{n}.\vec{r}_1}{\sqrt{r}} \tag{17}$$

The function $g(s)$ expression is:

$$g(s) = -\frac{s^2}{W_0^2} - j\beta_0 s - jkr \tag{18}$$

The integral in (16) has the suitable form for an asymptotic evaluation by the steepest descent path method [12]. Assuming there is only one saddle point $s_s$ defined by $g'(s_s) = 0$, the integral can be asymptotically evaluated by:

$$E^U(P) = \sqrt{2\pi}\frac{j}{\sqrt{g''(s_s)}}E_0^U(s_s)\exp(g(s_s)) \tag{19}$$

In order to find the saddle point, we do a second order approximation on $r$ in $g(s)$ assuming $R = \| \overrightarrow{OP} \|$ is large. This assumption, denoted as the large distance approximation (not so tough as the far field approximation), yields:

$$g(s) = -\frac{k}{2z_0R}(R+jz_0\cos\theta(\cos\theta-c_0R))s^2+j(k\sin\theta-\beta_0)s-jkR \tag{20}$$

The definition of $z_0$ is similar to the Rayleigh distance of a conventional gaussian beam:

$$z_0 = \frac{kW_0^2}{2} \tag{21}$$

From (20), we find an analytical expression for the saddle point given by:

$$s_s = \frac{jz_0R}{k}\frac{(k\sin\theta - \beta_0)}{R + jz_0\cos\theta(\cos\theta - c_0R)} \tag{22}$$

Finally, we obtain the large distance analytical formulations of the CGB. For an electric current, we have:

$$\begin{aligned} E^J(P) &= u_0\eta\frac{e^{j\pi/4}}{2}\frac{1}{\sqrt{r_s}}\sqrt{\frac{z_0R}{R + jz_0\cos\theta(\cos\theta - c_0R)}} \\ &\quad\times \exp\left(-\frac{z_0R(k\sin\theta - \beta_0)^2}{2k(R + jz_0\cos\theta(\cos\theta - c_0R))}\right)\exp\left(-jkR\right) \end{aligned} \tag{23}$$

For a magnetic current, we have similarly:

$$\begin{aligned} E^M(P) &= -u_0 \frac{e^{j\pi/4}}{2} \frac{\vec{n}.\vec{r}_{1s}}{\sqrt{r_s}} \sqrt{\frac{z_0 R}{R + jz_0 \cos\theta(\cos\theta - c_0 R)}} \\ &\quad \times \exp\left(-\frac{z_0 R(k\sin\theta - \beta_0)^2}{2k(R + jz_0\cos\theta(\cos\theta - c_0 R))}\right) \exp\Big(-jkR\Big) \end{aligned} \tag{24}$$

$\vec{n}_s$, $r_s$, $\vec{r}_s$ correspond to the expressions (15) evaluated on the saddle point. In the far field zone, the field expressions can be reduced and become:

$$\begin{aligned} E^J(P) &= u_0 \eta \frac{e^{j\pi/4}}{2} \sqrt{\frac{z_0}{1 - jz_0 c_0 \cos\theta}} \\ &\quad \times \exp\left(-\frac{z_0(k\sin\theta - \beta_0)^2}{2k(l - jz_0 c_0\cos\theta)}\right) \frac{\exp(-jkR)}{\sqrt{R}} \\ E^M(P) &= \frac{-u_0 e^{j\pi/4}}{2} \cos\theta \sqrt{\frac{z_0}{1 - jz_0 c_0 \cos\theta}} \\ &\quad \times \exp\left(-\frac{z_0(k\sin\theta - \beta_0)^2}{2k(l - jz_0 c_0\cos\theta)}\right) \frac{\exp(-jkR)}{\sqrt{R}} \end{aligned} \tag{25}$$

### 3.2. Example of Conformal Gaussian Beams

In this part, we focus both on the properties of CGB and on the test of their analytical expressions.

First, we consider a beam associated with magnetic currents and with parameters: $u_0 = 1$, $W_0 = 2\lambda$, $\beta_0 = k\sin(60°)$, $c_0 = 1/(10\lambda)$. On Figure 4, we represent its far field radiation pattern. The maxima at 60° and 120° confirm the effect of $\beta_0$ on the beam propagation direction. Besides, this figure compares the analytical expression (25) with the numerical computation of the radiation integrals (13) (denoted as the reference solution). We notice a very good agreement between the two results.

On Figure 5a, we observe the beam amplitude in the near field zone obtained with (23). As expected, the field propagates toward both sides of the curve and has a finite extent. Due to the interface curvature, we also note that the currents don't radiate the same way in zone 1 and 2. Figure 5b representing the reference solution, and Figure 5c, representing the difference $|E^M_{CGB}(P) - E^M_{Ref}(P)|_{dB}$ between both previous results, show the accuracy of the

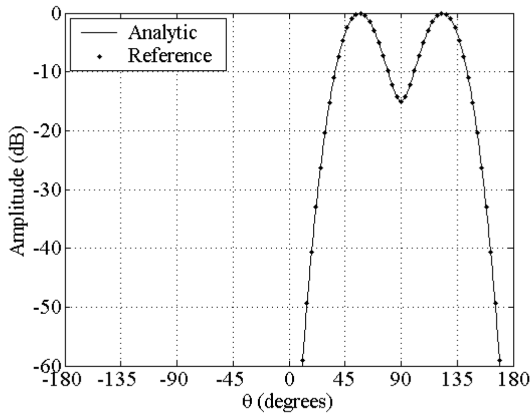


**Figure 4.** Far field radiation pattern of a CGB.

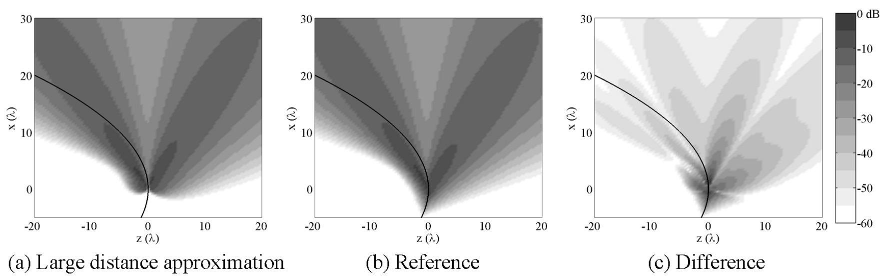


**Figure 5.** Near field distribution of a CGB.

analytical formulation. Although (23) depends on a large distance approximation, the differences are very small except near the beam centre.

From the formulation of the currents (10), we have seen that $W_0$ and $\beta_0$ respectively affect the beam width and the propagation direction. We now briefly look at the effect of $c_0$ on the beam properties. On Figure 6, we represent the far field radiation pattern of the previous beam modifying only its $c_0$ value. We notice that this parameter impacts significantly the fields: a large curvature implies a wide angular width.

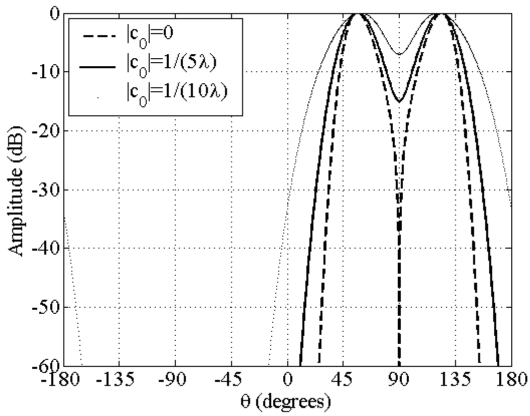


**Figure 6.** Effect of the curvature $c_0$.

## 4. EXAMPLE OF EXPANSION ON AN ELLIPSE CURVE

In this section we use our technique to expand currents distributed on an elliptic interface and to describe their radiated field in the near field zone and in the far field zone. In order to show the abilities of our technique with regards to existing gaussian beam expansions, we choose a configuration presenting both high incidences and large curvature variations.

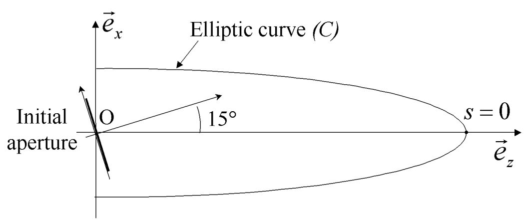


**Figure 7.** Configuration.

The geometrical configuration is shown on Figure 7. The curve $(C)$ is elliptic with semi major axes $10\lambda$ along $x$ and $66.67\lambda$ along $z$.

In a preliminary step, we explain how realistic initial currents on $(C)$ can be obtained. A planar aperture of diameter $8\lambda$ is placed on the ellipse centre $O$ and is rotated of $15°$. Its illumination law is a truncated cosine and its radiated fields are denoted $\vec{H}^i$ and $\vec{E}^i$. The initial electric and magnetic currents on $(C)$ are given by the application of the equivalence theorem to $\vec{H}^i$ and $\vec{E}^i$:

$$\begin{aligned} \vec{J}^i &= \vec{n} \wedge \vec{H}^i \\ \vec{M}^i &= -\vec{n} \wedge \vec{E}^i \end{aligned} \tag{26}$$

These currents radiate zero inside the ellipse and $\vec{H}^i$ and $\vec{E}^i$ outside.

Now, we expand them with our technique. The curvilinear coordinate origin $s = 0$ is located as shown on Figure 7. Concerning the expansion parameters, we choose $\nu = 0.75$ (as recommended in [13]) and $l = \lambda$. The expansion coefficients are computed with (6). Their amplitude is presented on Figure 8. We see that several spectral orders are excited. Hence, CGB with different propagation axes and also evanescent CGB are required to achieve an accurate computation of the radiation.

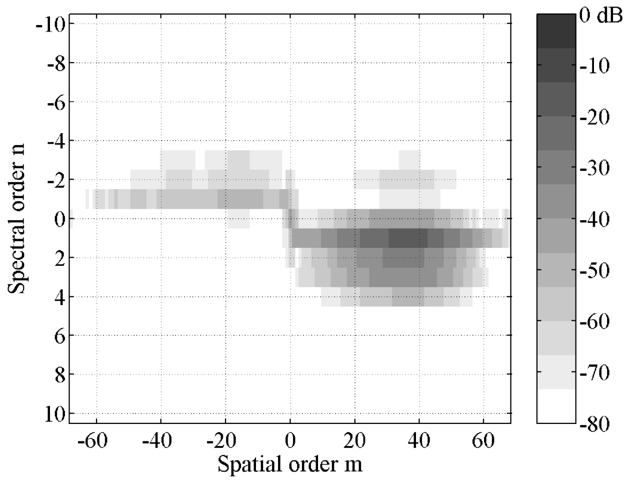


**Figure 8.** Amplitude of the expansion coefficients.

On Figure 9, we compare on $(C)$ the initial equivalent magnetic currents and those computed with the Gabor summation (2). We note a very good agreement between both results. Their difference expressed in dB is always below $-130\,\text{dB}$.

On Figure 10a, we show the distribution of the electric field amplitude obtained with the CGB summation (9). We use here the large distance formulations (23) and (24). As expected, the radiated electric field is equal to zero inside the ellipse. On Figure 10b, we show the distribution of the difference between our result and the reference. Except very close the interface, our method gives very accurate results.

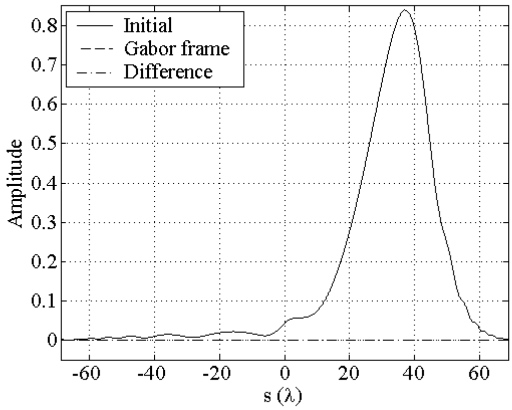


**Figure 9.** Gabor expansion of the magnetic current on $(C)$.

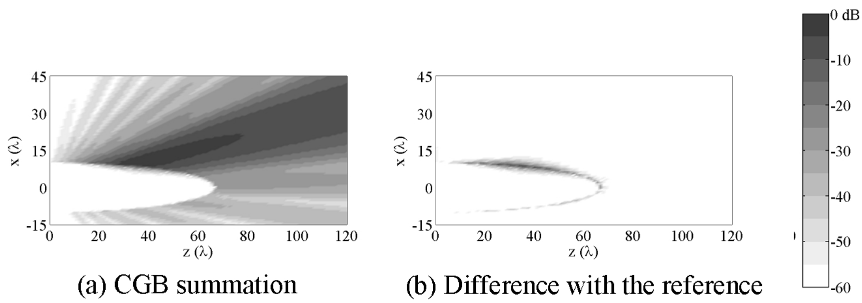


**Figure 10.** Radiated field from the ellipse.

Furthermore, we note that this configuration requires CGB with a wide range of parameters $c_0$ and $\beta_0$. Indeed, the beam curvature varies approximately from $1/(1.5\lambda)$ to $1/(444\lambda)$. and we remind that several spectral orders are excited. On Figure 11, we present the far field radiation pattern obtained with the far field analytical formulations (25). The comparison with the reference solution is once again very satisfactory.

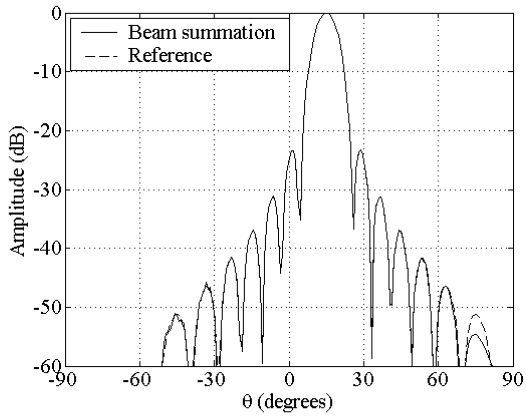


**Figure 11.** Comparison in the far field zone.

## 5. CONCLUSION

Firstly, the Gabor frame has been applied to expand electric and magnetic currents on a two dimensional regular unclosed interface. This expansion has lead to the introduction of the conformal Gaussian beams. They are defined by the radiation of currents having a Gaussian amplitude and a linear phase evolution with respect to the curvilinear coordinate. Thanks to a second order approximation of the interface, an analytical formulation of these new beams has been developed using the asymptotic descent path method combined with a large distance approximation. Finally, we have successfully tested both the analytical formulation and the expansion principle on significant examples.

We are now looking for other analytical formulations depending on the paraxial approximation and offering accurate results near the interface. Furthermore, to reduce the number of beams in the expansion, some improvements are also investigated by taking into account the physical properties of the configuration as we did in the expansion proposed in [9]. We also intend to develop a similar approach to describe currents on an interface in dimension three. Concerning the application outlook, the combination of this expansion with the Gaussian beam transmission and reflection coefficients [14] will certainly provide efficient solutions to compute the radiation of antennas placed under dielectric radomes. We expect in particular good performances for sharp nose aircraft radomes presenting difficulties due to great incidence angles and large variation of the local curvature.

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