MUTUAL COUPLING EFFECT ON THERMAL NOISE IN MULTI-ELEMENT ANTENNA SYSTEMS

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Abstract—In this paper, we investigate the thermal noise behavior of the multi-antenna communication systems, when antenna elements are closely spaced. We analyze the mutual coupling effect on thermal noise. We apply the Nyquist's thermal noise theorem to determine thermal noise power in the multi-antenna system and to confirm the partial correlation of thermal noise for antenna spacing lower then one wavelength. Simulation results confirm the decrease of thermal noise power level when antenna spacings drop below a half wavelength.

1. INTRODUCTION

MULTIPLE-INPUT multiple output (MIMO) wireless systems, characterized by multiple antennas at the transmitter and receiver, have demonstrated the potential for increased capacity by exploiting the spatial properties of the multipath channel [1]. If the channel matrix coefficients are i.i.d (independent identically distributed)

complex Gaussian variables, then linear increase in capacity with the number of antenna is possible. The mutual independence of channel coefficients is generally achieved by wide inter-element spacing in the multi-antenna system. However, wide antenna spacing is not always achievable due to physical size constraints of subscriber units in wireless communication systems.

As the consequence of the small antenna spacings, correlation arises among the antenna elements. The impact of the correlation due to mutual coupling effect on the antenna arrays has been evaluated by examining its influence on adaptive array performance in [2]. Then, the model for evaluation the mutual coupling effect on MIMO channel capacity was presented in [3]. Additionally, the mutual coupling effect on MIMO channel capacity through the radiated power and the power collection capability was assessed in [4].

While above studies present important contributions concerning the effect of array mutual coupling on MIMO system performance, they neglect mutual coupling effect on thermal noise. The quantitative analysis of the mutual coupling effect on thermal noise is a missing puzzle to complement the analysis of the mutual coupling effect on the performance of the multi-element antenna systems. The real insight into the physical phenomenon of thermal noise behavior for coupled antennae can be obtained by investigating the thermal noise correlation due to the mutual coupling effect.

In this paper, we investigate the mutual coupling effect on thermal noise in the multi-antenna systems with closely spaced antenna elements. The Nyquist's thermal noise theorem [5] enables to identify the correlated part of the thermal noise from the total thermal noise and to confirm the correlation of noise for antenna spacing below one wavelength. We calculate noise correlation coefficients to additionally assure the existence of the noise correlation.

The rest of this paper is organized as follows. In Section 2, the effect of mutual coupling in the multiple antenna element system is presented. In Subsection 3.1, the mutual coupling on thermal noise is elaborated. Subsection 3.2 is devoted to thermal noise power evaluation in two-dipole array. Simulation results are presented in Section 4. While, concluding remarks are given in Section 5.

2. MUTUAL COUPLING EFFECT

The principal feature of an antenna is to convert an electromagnetic field into an induced voltage or current. However, for closely spaced antenna element, the total (measured) voltage on each antenna elements is a function not only of the excited field but also of the

voltages on the other elements. The phenomenon is known as the mutual coupling effect and can be including in the received voltage model, by inserting the coupling matrix [3]

$$y = MH_cx + n \tag{1}$$

Thus, the new channel matrix that considers the electromagnetic behavior of antennae is $H' = MH_c$ [3]. Using fundamental electromagnetic and circuit theory, the coupling matrix of an array antenna can be written as [2]

$$M = (Z_A + Z_T)(\mathbf{Z} + Z_T\mathbf{I})^{-1} \tag{2}$$

where Z_A is the antenna impedance, Z_T is the impedance of the measurement equipment at each element and $\mathbf{Z}$ is the mutual impedance matrix.

3. MUTUAL COUPLING EFFECT ON THERMAL NOISE

The electromagnetic properties of thermal noise is discussed in [8]. Also, thermal noise radiation is classified into self-radiated thermal noise (self-noise) and induced thermal noise of radiated body in antenna element [8]. Then, the partially correlated thermal noise in two closely spaced antennae with isolated receivers is discussed in [6], based on Nyquist's thermal noise theorem [5].

   Nyquist Thermal Noise Theorem - The theorem states that for passive network in thermal equilibrium it would appear possible to represent the complete thermal-noise behavior by applying Nyquist's theorem independently to each element of the network [5]. In the case of the multi-antenna system these elements are self-impedances and mutual-impedances. In addition, the general nonreciprocal network with a system of internal thermal generators all at absolute temperature T can be represented as the source-free network together with a system of noise current generators I_r and I_s with infinite internal impedance [6]. In that case, the Nyquist's thermal noise theorem[5] states that the nodal current cross-correlation is given by

$$\overline{I_sI_r}df = 2kT(Y_{sr} + Y_{sr}^*)df \tag{3}$$

Alternatively the internal noise sources can be represented by a system of nodal voltage generators V_r and V_s, with zero internal impedance. The correlation of nodal voltage generators is given by:

$$\overline{V_sV_r}df = 2kT(Z_{sr} + Z_{sr}^*)df \tag{4}$$

where Z_{sr} and Y_{sr} are the mutual impedance and admittance, respectively and k is Boltzmann constant. Correlation is zero when the mutual coupling is purely reactive.

Thermal noise coupling matrix - In this subsection, we derive thermal noise coupling matrix for two-antenna array based on the generalized Nyquist's thermal noise theorem. Although, we use this simple model, it still enables significant conceptual insight to be gained into thermal noise behavior of the multiple antenna systems.
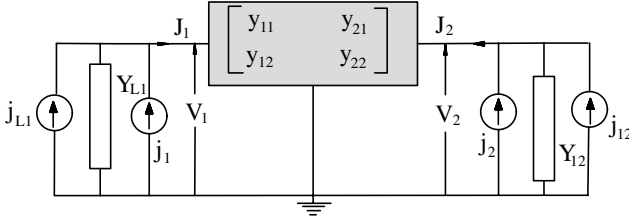

Figure 1. Nodal network representation for two antenna array.

One can write the noise current for two-antenna array from Fig. 1 in terms of its spectral density by

$$
\begin{aligned}
J_1 &= y_{11}V_1 + y_{12}V_2 = j_{L1} + j_1 - Y_{L1}V_1 \\
J_2 &= y_{21}V_1 + y_{22}V_2 = j_{L2} + j_2 - Y_{L2}V_2
\end{aligned}
\tag{5}
$$

where J_i, $i = 1, 2$ are the total noise current spectrum and V_i, $i = 1, 2$ associated noise voltage spectrum of ith antenna element. j_i, $i = 1, 2$ are the nodal noise current spectrum of ith port, j_{Li}, $i = 1, 2$ are the noise current spectrum associated with load admittance of receiver Y_{Li}, $i = 1, 2$ of ith antenna elements.

Following that,

$$
\begin{aligned}
j_{L1} + j_1 &= (y_{11} + Y_{L1})V_1 + y_{12}V_2 \\
j_{L2} + j_2 &= y_{21}V_1 + (y_{22} + Y_{L2})V_2
\end{aligned}
\tag{6}
$$

$$
\begin{bmatrix} j_{L1} + j_1 \\ j_{L2} + j_2 \end{bmatrix} = \begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
\tag{7}
$$

$$
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix}^{-1} \begin{bmatrix} j_{L1} + j_1 \\ j_{L2} + j_2 \end{bmatrix}
\tag{8}
$$

$$
V_1 = \frac{1}{|D|} \left((y_{22} + Y_{L2})(j_{L1} + j_1) - y_{21}(j_{L2} + j_1) \right)
\tag{9}
$$

$$V_2 = \frac{1}{|D|}\left((y_{11} + Y_{L1})(j_{L2} + j_2) - y_{12}(j_{L1} + j_1)\right) \tag{10}$$

where $|D|$ is determinant of the following matrix

$$\begin{pmatrix} y_{11} + Y_{L1} & y_{12} \\ y_{21} & y_{22} + Y_{L2} \end{pmatrix} \tag{11}$$

The average power absorbed dissipated in the receiver load of the first antenna is proportional to P_{L1} in the following form

$$P_{L1} = \frac{1}{2}(Y_{L1} + Y_{L1}^*)\overline{V_1 V_1^*} \tag{12}$$

Similarly, for the second antenna:

$$P_{L2} = \frac{1}{2}(Y_{L2} + Y_{L2}^*)\overline{V_2 V_2^*} \tag{13}$$

Substituting the expressions (9) in (12) yields

$$\begin{aligned} P_{L1} = \frac{Y_{L1} + Y_{L1}^*}{2|D||D^*|}&\left((y_{22} + Y_{L2})(y_{22}^* + Y_{L2}^*)(\overline{i_{L1}i_{L1}^*} + \overline{i_1 i_1^*})\right.\\ &\left.-y_{21}(y_{22}^*+Y_{L2}^*)\overline{i_1 i_2^*} - y_{21}^*(y_{22}+Y_{L2})\overline{i_1^* i_2} + y_{21}y_{21}^*(\overline{i_{L2}i_{L2}^*} + \overline{i_2 i_2^*})\right) \end{aligned} \tag{14}$$

Using (3) for nodal current correlation, the expression (14) becomes

$$\begin{aligned} P_{L1} = 2kT\frac{(Y_{L1} + Y_{L1}^*)}{2|D||D^*|}&\left((y_{22}+Y_{L2})(y_{22}^*+Y_{L2}^*)((Y_{L1}+Y_{L1}^*)+(y_{11}+y_{11}^*))\right.\\ &-y_{21}(y_{22}^* + Y_{L2}^*)(y_{12} + y_{12}^*) - y_{21}^*(y_{22} + Y_{L2})(y_{12} + y_{12}^*)\\ &\left.+y_{21}y_{21}^*((Y_{L2} + Y_{L2}^*) + (y_{22} + y_{22}^*))\right) \end{aligned} \tag{15}$$

The spectral density of thermal noise power dissipated in the received load of first antenna P_{L1} consists of two parts, self-noise and induced thermal noise [9]. Self-noise originates in its own resistive element, while induced thermal noise arises from the adjacent antenna element. The induced thermal noise exists due to thermal noise mutual coupling effect.

Similarly, the spectral density of thermal noise power of second antenna is

$$\begin{aligned} P_{L2} = 2kT\frac{Y_{L2} + Y_{L2}^*}{2|D||D^*|}&\left((y_{11}+Y_{L1})(y_{11}^*+Y_{L1}^*)((Y_{L2}+Y_{L2}^*)+(y_{22} + y_{22}^*))\right.\\ &-y_{12}(y_{11}^* + Y_{L1}^*)(y_{21} + y_{21}^*) - y_{12}^*(y_{11} + Y_{L1})(y_{21} + y_{21}^*)\\ &\left.+y_{12}y_{12}^*((Y_{L1} + Y_{L1}^*) + (y_{11} + y_{11}^*))\right) \end{aligned} \tag{16}$$

where k is Boltzmann constant and T is absolute temperature.

Now, the absorbed thermal noise power in the receiver load of first and second antennae over the bandwidth B are, respectively

$$N_{mc}(1) = \int_B P_{L1} df \tag{17}$$

$$N_{mc}(2) = \int_B P_{L2} df \tag{18}$$

The thermal noise matrix of two coupled antennae is then

$$N_{mc} = \begin{pmatrix} N_{mc}(1) & 0 \\ 0 & N_{mc}(2) \end{pmatrix} \tag{19}$$

Here, thermal noise matrix for two-coupled antennae is diagonal matrix, because the receiver loads are isolated. In fact, the coupled antenna system with its mutual impedance matrix translates into the diagonal receiver load matrix. Thus, each loads absorbs thermal noise radiation from its own antenna element and from adjacent antenna, what represents the partially correlated noise. In such a way, we define thermal noise matrix as eigenvalue matrix of one general thermal noise correlated matrix. Here, the correlation is due to mutual coupling effect.

4. SIMULATION EXPERIMENTS

To confirm results of the presented analytical analysis, we use the simulation models consist of the uniform linear arrays (ULA) with two and three half-wave dipoles in multi-antenna system. Mutual- and self impedances are calculated by using SONNET software [7]. This simplified models still enable significant conceptual insight be gained into the multi-antenna system performance in terms of thermal noise power level.

Fig. 2 depicts mutual coupling effect upon thermal noise power in the multiple antenna systems. Simulation analysis shows decrease in thermal noise power for antenna spacing below 0.5λ when mutual coupling of thermal noise (mctn) is considered in comparison with traditional approach which ignores the coupling interaction for thermal noise (nmctn). Results are given for antenna spacing below 0.7λ. The simulation results indicate on the trend that thermal noise power level of one dipole will decrease further as the number of antenna in the multi-antenna elements increases.

Additionally, we estimate the noise correlation coefficients in order to drawn conclusions about mutual coupling effect on thermal
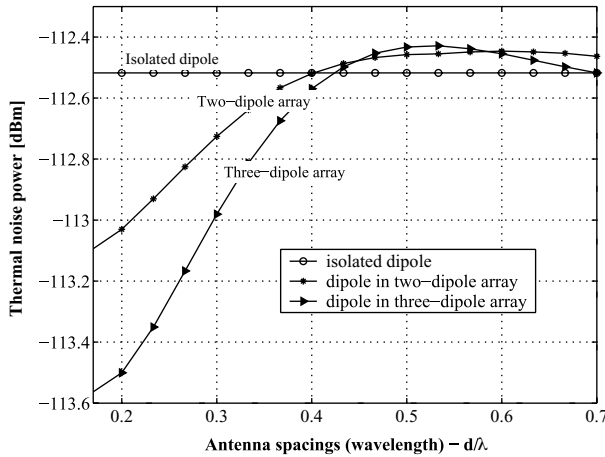
Figure 2. Effect of mutual coupling on thermal noise power in the multiple element antenna systems.

noise. Therefore, we analyze the two- and three-antenna array in order to estimate the correlation coefficients of complex thermal noise voltages within antenna spacing range $[0, 1\lambda]$. The voltage correlation coefficient is computed as $\gamma_{12} = \langle V_1, V_2 \rangle$. Here, V_i, $i = 1, 2$ is voltage at the output port of i_{th} antenna element. Operation $\langle a, b \rangle$ computes the complex correlation coefficient between a and b as

$$\langle a, b \rangle = \frac{E\{[a - E\{a\}][b - E\{b\}]^*\}}{\sqrt{E\{|a - E\{a\}|^2\}E\{|b - E\{b\}|^2\}}} \quad .$$

We calculate thermal noise voltages for two-antenna array by using (9) and (10).

Fig. 3 plots the resulting magnitude of correlation coefficients versus antenna spacing, for both, two-dipole and three-dipole arrays. The correlation coefficients are calculated for the adjacent dipoles (12), (23) (Fig. 3) and dipoles set $2 * d$ apart (13) (Fig. 3). Results from Fig. 3 confirm that thermal noise between the adjacent antenna elements in the multi-antenna system is highly correlated for antenna spacing up to 0.5λ.

The uncorrelated white noise is usually presupposed in antenna array applications, neglecting the radiation characteristics of noise. However, the results from Fig. 3 show that the mutual coupling strongly correlates thermal noise in the closely spaced antenna elements.
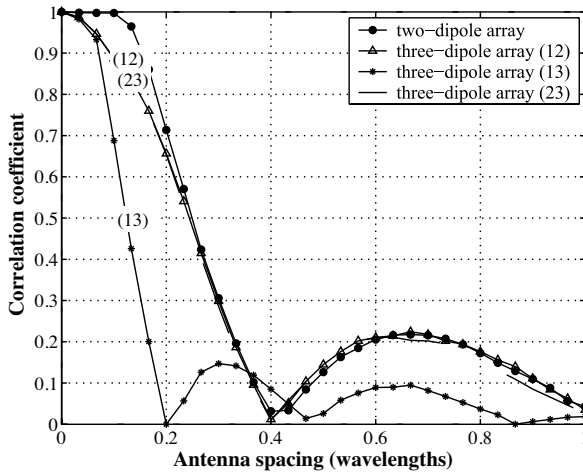
Figure 3. Correlation coefficients of thermal noise voltages due to mutual coupling effect.

5. CONCLUSION

This paper outlines a procedure for thermal noise analysis of the multi-antenna system with coupled antennae. We present a method for thermal noise power calculation in two-antenna array, which can be used to determine thermal noise behavior of the multi-antenna system with small antenna spacing. We confirm the partial correlation of thermal noise for antenna spacing below a wavelength.

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