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INTERFERENCE SUPRESSION OF THE LINEAR ANTENNA ARRAYS CONTROLLED BY PHASE WITH USE OF SQP ALGORITHM

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Abstract—The performance of mobile cellular radio networks is limited by the level of cochannel interference that can be tolerated. The use of antennas arrays is very helpful in enhancing the performance and capacity of the wireless communication system. This paper presents a method for antenna pattern synthesis that suppress multiple interfering narrow or wide band signals while receiving the desired signal by controlling only the phase. Excitation phases are computed using the Sequential Quadratic Programming (SQP) technique. This method transforms the nonlinear minimization (or maximization) problem to a sequence of quadratic subproblems, based on a quadratic approximation of the Lagrangian function.

1. INTRODUCTION

The capacity of a network depends on the number of telephone subscribers that the owner can serve at his cellular station, by carrier and sector. With the multiplication of the subscribers on the bandwidth, the receiver of the station can quickly more not manage to detect the signal emanating from such or such subscriber. And the operators seek constantly how to optimize the exploitation of the radioelectric spectrum as well as increasing the flow and the ray of their networks. A solution consists in using adaptive antennas arrays [1, 2]. Antennas arrays offer several advantages: they allow reducing cochannel interferences, increasing the capacity of communication system and reducing channel fading by using the spatial diversity of the antennas arrays.

An adaptive antenna consists of two or more antennas (the elements of the array) spatially arranged and electrically interconnected to produce a directional radiation pattern [1–3, 5, 6]. In a phased array the phases of the exciting currents in each element antenna of the array are adjusted to change the pattern of the array or suppress interfering signals from prescribed directions while receiving desired signal from a chosen direction, in order to improve the performance of a communication, sonar or radar system. Although the amplitudes of the currents can also be varied, the phase adjustment is of particular interest in pattern null steering and it is responsible for beam steering [4].

An adaptive antenna has the potential to reduce multipath interferences, to increase signal to noise ratio and introduce frequency reuse [1, 2, 6]. However, several challenges remain in the development of these adaptive systems and one of these is the technique of suppression of interference. Several approaches can be used for pattern synthesis with narrow and wide null steering by phase-only control.

In this work, we propose an efficient method of synthesis based on the SQP algorithm to determine phases which must be applied to each elements of the array in order to obtain a null of radiation in the directions of the interfering signals while to maximize of radiation in the direction of the useful signal (to track the desired users).

2. PROBLEM FORMULATION

Consider a uniform excited linear array of N equispaced isotropic elements positioned along the x-axis with interelement spacing of d. These antennas are supplied with same current amplitude A and with a gradient of phase φ_n, $(n = 1, 2, \cdots, N-1)$ as shown in Figure 1.

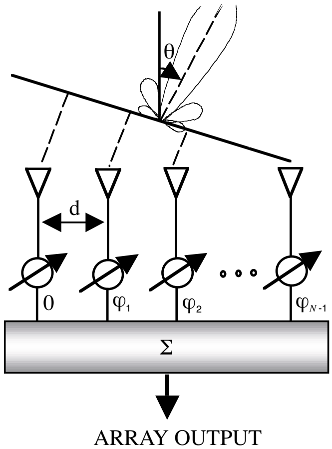

Figure 1. The uniform linear array configuration.

The array factor ($F(\theta)$) can be obtained by considering the elements to be point source which is given by [4, 7]:

$$F(\theta) = \sum_{n=0}^{N-1} e^{j(kd_n \sin\theta + \varphi_n)} \tag{1}$$

Where $\varphi_n \in [\varphi_0, \varphi_1, \cdots, \varphi_{N-1}]$ represents the phase excitation of the nth element (the antenna in the beginning is taken as reference of phase: $\varphi_0 = 0$), d_n is the position of the nth element, k is the wave number, and θ is the scanning angle from broadside.

The problem of interference suppression in antenna arrays consists in determining the phases excitations φ_n ($n = 1, \cdots, N-1$) which one must apply to each element of the array in order to obtain a minimum of radiation in directions θ_i (direction of the interfering signals, with $i = 1, \cdots, I$) while maintaining the maximum of radiation (main lobe) in direction θ_0 (direction of the useful signal).

Mathematically, the numerical optimization technique is formulated as minimizing an objective function (to maximize a objective function $f(x)$ is equivalent to minimize $-f(x)$) subject to a set of constraints. In our case, the constraints can simply be as en forcing the required array pattern to be δ_i ($i = 1, \cdots, m_e$ and δ_i is the levels in the regions of the suppressed sectors) at the direction of interferences.

The nonlinear programmation problem can be formulated as:

$$\begin{array}{rcll} \text{min}imize - f_{\theta_0}(\varphi) & & & \\ \text{subject to } \ f_{\theta_i}(\varphi) & = & \delta_i & i = 1, \cdots m_e \\ f_{\theta_j}(\varphi) & \leq & \delta_j & j = m_e + 1, \cdots, m \\ -2\pi & \leq & \varphi \leq 2\pi & \end{array} \quad (2)$$

where: $f_\theta(\varphi) = \left| 1 + \sum_{n=1}^{N-1} e^{j(kd_n \sin\theta + \varphi_n)} \right|^2$; $\theta_0, \theta_i, \theta_j, \delta_i$, and δ_i and are the direction of the desired signal, the ith directions of interfering signals, the side lobe region, the levels in the regions of the suppressed sectors, and the level in the side lobe region, respectively and m is the number of the sampled angular direction.

3. OPTIMIZATION PROCEDURE SQP

We want to find the solution of (2).

There has been a tremendous amount of research among the nonlinear programming (NLP) community on finding efficient algorithms for (NLP). The Sequential Quadratic Programming or SQP [8] methods belong to the most powerful nonlinear programming (NLP) algorithms we know today for solving differentiable nonlinear programming problems of the form (2). The theoretical background is described e.g. in Stoer [9] in form of a review, or in Spellucci [10] in form of an extensive text book. From the more practical point of view SQP methods are also introduced briefly in the books of Papalambros, Wilde [11] and Edgar, Himmelblau [12]. Their excellent numerical performance was tested and compared with other methods in Schittkowski [13], and since many years they belong to the most frequently used algorithms to solve practical nonlinear optimization problems.

The basic idea is to formulate and solve a quadratic programming subproblem at each iteration which is obtained by linearizing the constraints and approximating the Lagrangian function

$$L(\varphi, \lambda) = -f_{\theta_0}(\varphi) + \sum_{k=1}^{m} \lambda_k \left(f_{\theta_k}(\varphi) - \delta_k \right) \quad (3)$$

quadratically, where $\varphi \in R^{N-1}$, and where $\lambda = (\lambda_1, \cdots, \lambda_m)^T \in R^m$ is the vector of the Lagrange multiplier.

To formulate the quadratic programming subproblem, we proceed from given iterates $\varphi_k \in R^{N-1}$, an approximation of the solution, and the matrix $M_k \in R^{N-1 \times N-1}$, an approximation of the Hessian matrix of the Lagrangian function. Then one has to solve the following quadratic programming problem:

$$\begin{array}{lll} \text{min}\textit{imize} & -\nabla f_{\theta_0}(\varphi_k)^T d + \frac{1}{2} d^T M_k d & \\ \text{subject to} & \nabla f_{\theta_i}(\varphi_k)^T d + f_{\theta_i}(\varphi_k) = \delta_i & i = 1, \cdots, m_e \\ & \nabla f_{\theta_j}(\varphi_k)^T d + f_{\theta_j}(\varphi_k) \leq \delta_j & j = m_e + 1, \cdots, m \\ & d \in R^{N-1} & \end{array} \tag{4}$$

Where: $M_k \simeq \nabla^2_{\varphi} L(\varphi_k, \lambda_k)$.

Let d_k be the optimal solution. The solution is used to form a new iterate

$$\varphi_{k+1} = \varphi_k + \alpha_k d_k \tag{5}$$

where $\alpha_k \in [0, 1]$ is a suitable step length parameter.

We can now define the SQP algorithm:

1. set $k := 1$
2. solve the QP subproblem described on Equation (4) to determine d_k and let λ_{k+1} be the vector of the Lagrange multiplier of the linear constraints obtained from the QP.
3. Compute the length α_k of the step and set $\varphi_{k+1} = \varphi_k + \alpha_k d_k$
4. Compute M_{k+1} from M_k using a quasi-Newton formula
5. Increment k. Stop if a solution is found. Otherwise, go to step 2.

This algorithm is the generalization of the Newton method for the constrained case. It has the same properties. It has, near the optimum, quadratic convergence.

The motivation for the success of SQP methods is found in the following observation: An SQP method is identical to Newton method to solve the necessary optimality conditions, if M_k is the Hessian of the Lagrangian function and if we start sufficiently close to a solution. The statement is easily derived in case of equality constraints only, that is $m_e = m$, but holds also for inequality restrictions. A straightforward analysis shows that if $d_k = 0$ is an optimal solution of (4) and λ_k the corresponding multiplier vector, then φ_k and λ_k satisfy the necessary optimality conditions of (2).

4. NUMERICAL RESULTS

In order to illustrate the performance of the SQP algorithm for steering single, multiple, and broadband nulls in the imposed directions by controlling the phase excitation only, ten examples of uniform excited

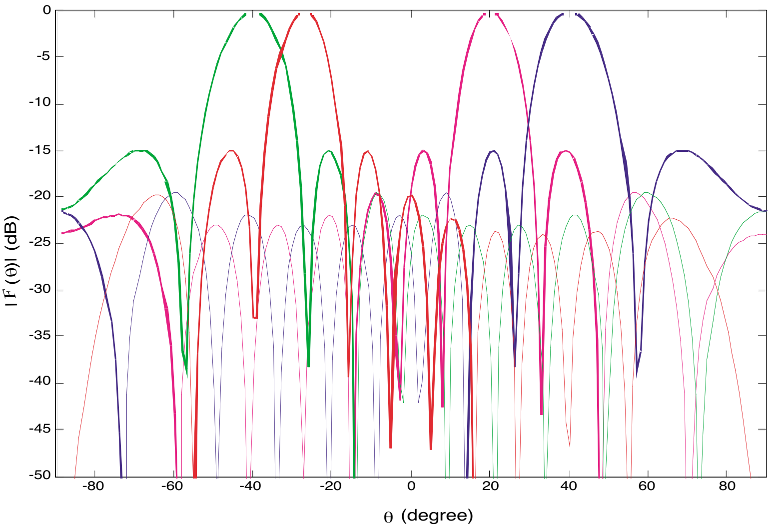

Figure 2. The initial uniform excited linear array, $N = 10d = \lambda/2$.

linear array with 10 one-half wavelength spaced isotropic elements were performed. Initially, the synthesis was made with a uniform excited linear array pattern. This array was made with 10 equispaced elements with 0.5λ interelement spacing. The corresponding radiation patterns are given in Figure 2.

The results of creating multiple suppressed narrow and wide band interferences are presented. Let the template with the mainbeam of uniform Linear array, $N = 10,\ d = \lambda/2$.

The template to be used contains one narrowband interference at the angular directions $-50°$ as shown in Figure 3. The phases of antenna elements are computed as given in column 1 of Table 1.

Figure 4, and 5 show the pattern synthesis with one prescribed wide sector is imposed at $-50°$. The desired signal imposed at $40°$. The corresponding suppressed sector levels are $\delta_1 = -65$ and -95 dB, respectively. The numbers of pattern nulls that are used to realize the suppressed sectors are 2 and 3, respectively, as shown in Figures 4 and 5. The Figures 3, 4, 5 shows that the method makes it possible to suppress narrow and wide band interferences and also control the level of this zero without degradation of the mainbeam (desired signal).

Figures 4, 6 and 7 shows that we can create a null and mainbeam in any directions. The computed element phases for Figures 3, 4, 5 and 6 are given in Table 1.

To show the ability of creating multiple suppressed narrow and

Table 1. Computed element phases φ_n for Figures 3, 4, 5, 6 and 7.

Elem. No.	φ_n (Degree)				
	Fig.3	Fig.4	Fig.5	Fig.6	Fig.7
1	0	0	0	0	0
2	-114.4696	-63.8591	-76.0134	108.3104	-57.0293
3	132.1764	143.2762	119.6518	-122.1083	-132.1477
4	13.9051	24.1199	-17.3237	-24.8184	-174.3008
5	-102.7210	-69.4918	-110.8007	111.2555	-258.0691
6	144.7282	155.3991	140.4686	-155.4868	65.3605
7	28.1020	61.7873	47.0135	-19.4129	-18.4078
8	-90.1693	-57.3690	-89.9209	77.8770	-60.5609
9	156.4767	149.7663	105.7378	-152.5416	-135.6793
10	42.0071	85.9072	29.7330	-44.2313	-192.7086

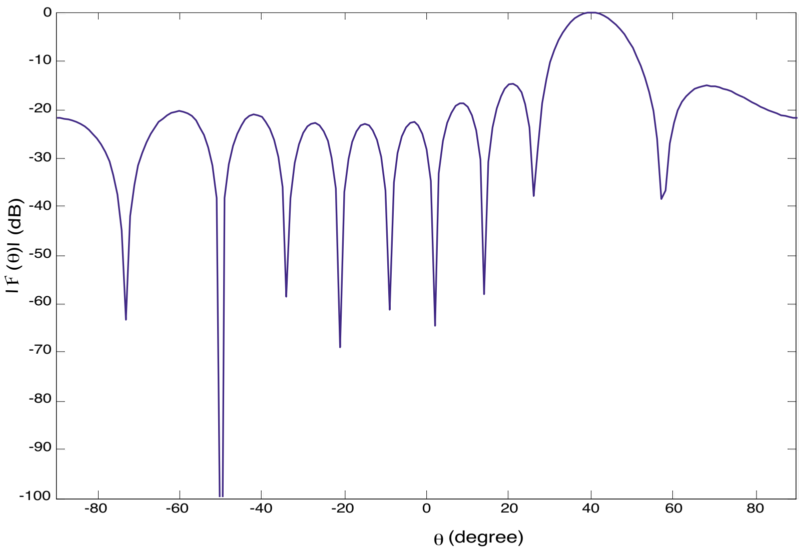

Figure 3. Pattern synthesis with a narrow band null at $-50°$ and steering lobe at $40°$.

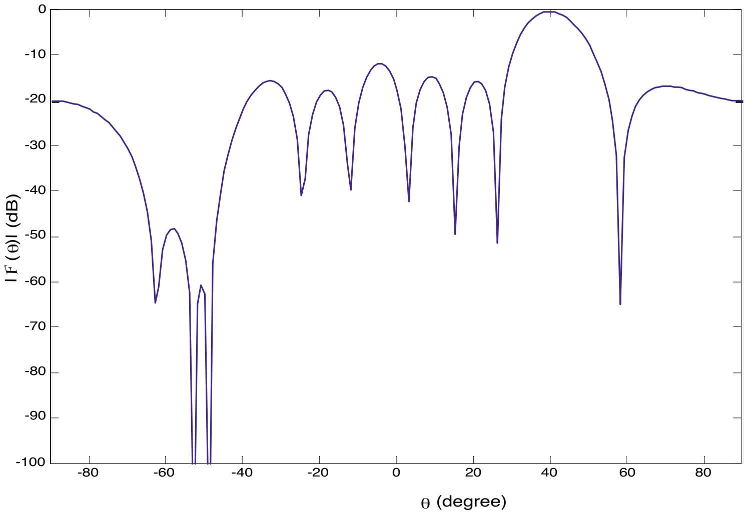

Figure 4. Pattern synthesis with a wide sectors imposed around $-50°$ and steering lobe at $40°$.

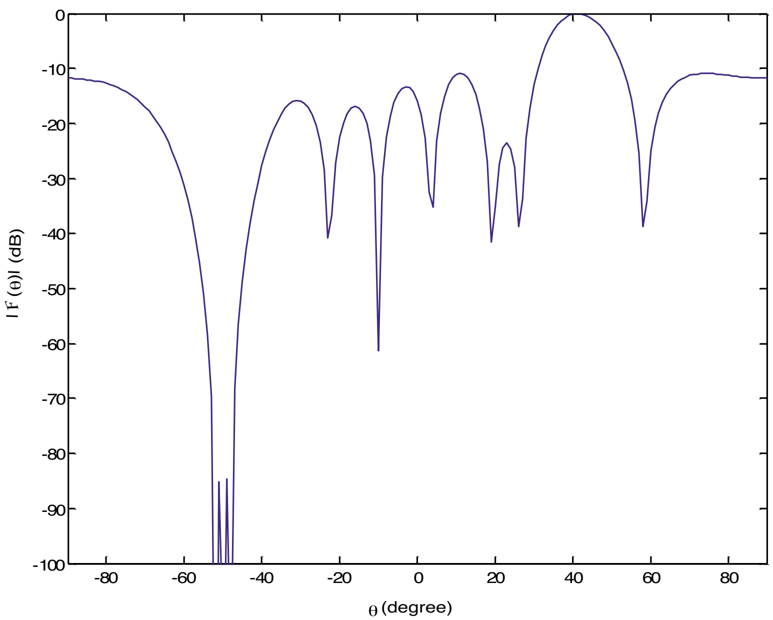

Figure 5. Pattern synthesis with a wide sector imposed around $-50°$ with a suppressed sector level is $\delta_1 = -95\,\text{dB}$ and steering lobe at $40°$.

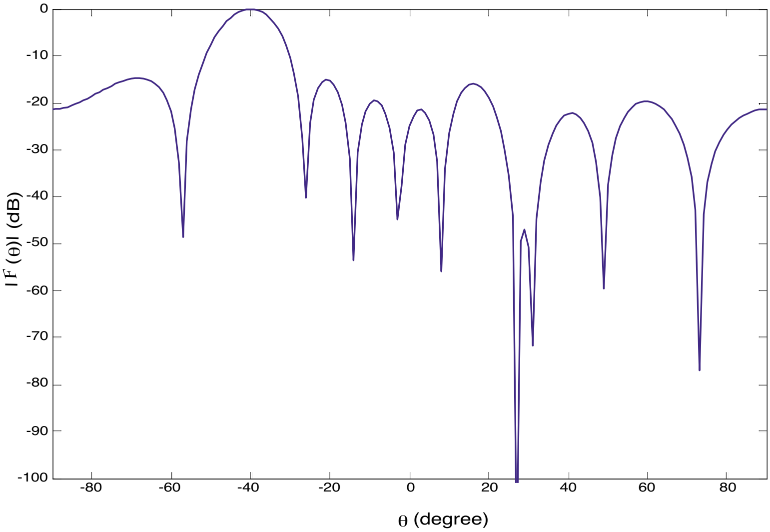

Figure 6. Pattern synthesis with a wide sectors imposed around 27° and steering lobe at −40°.

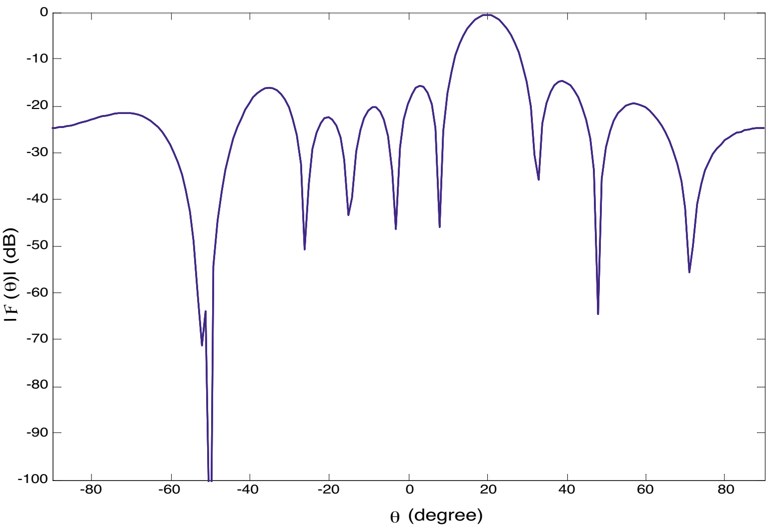

Figure 7. Pattern synthesis with a wide sectors imposed around −50° and steering lobe at 20°.

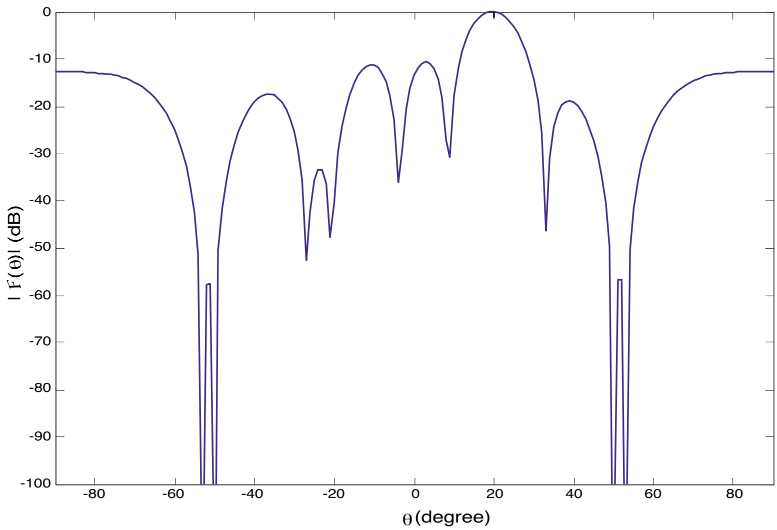

Figure 8. Pattern synthesis with a two wide sectors imposed around $-50°$, $50°$ and steering lobe at $20°$.

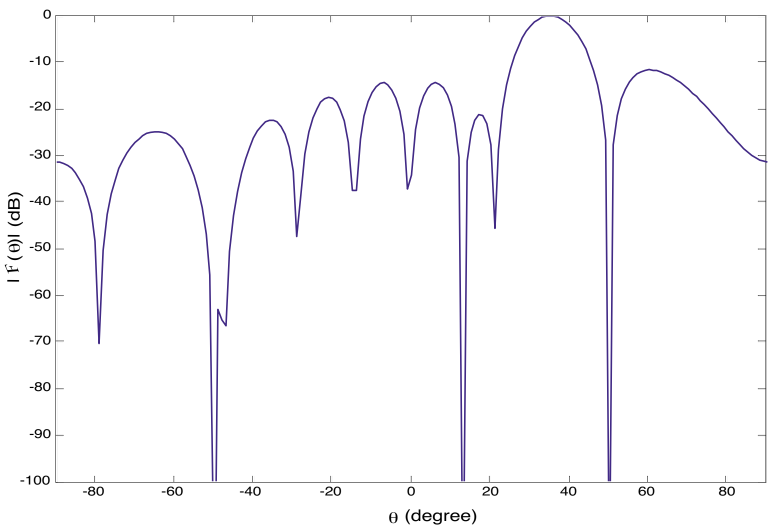

Figure 9. Pattern synthesis with a wide sectors imposed around $-50°$, with a two narrows band nulls imposed at 14 and 50 and steering lobe at $35°$.

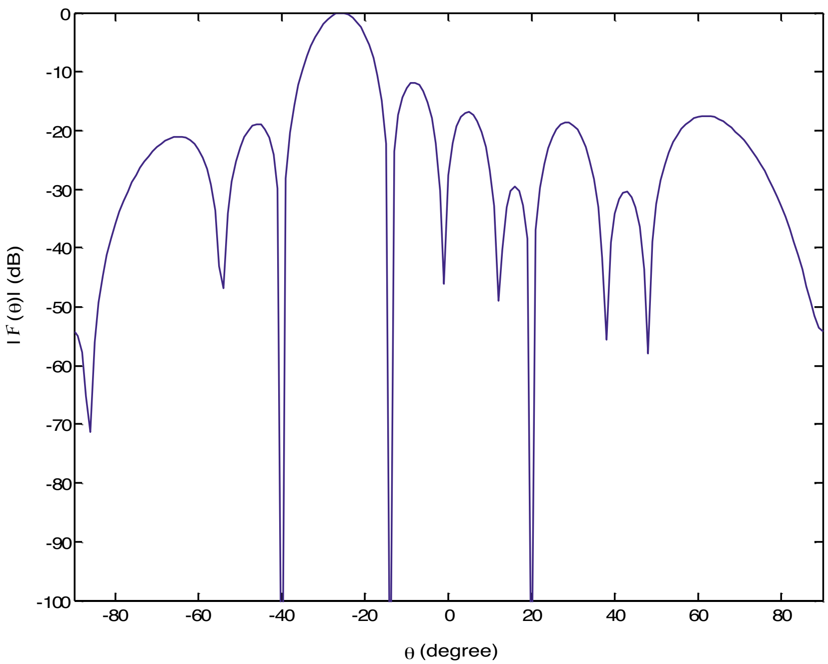

Figure 10. Pattern synthesis with tree narrows band nulls imposed at $-40°$, -14, 20 and steering lobe at $-27°$.

wide sectors in the side lobe region: two prescribed wide sectors are imposed at $-50°$ and $50°$. The specified levels of the two sectors are $\delta_i = -60\,\text{dB}$ as shown in Figure 8. Figure 9 shows the synthesized pattern with mainbeam at $35°$, with two narrow nulls imposed at $15°$ and $50°$, respectively and with wide suppressed sector at $-50°$.

The following computer simulation example demonstrates the capacity of this model to suppress a multiple interference signal. In Figure 10, 11 and 12, we have shown the nulling with triple nulls imposed at $-40°$, $-14°$, 20 and imposed the desired signal at $-27°$, with four nulls imposed in the directions angular -40, -15, -6, 20 and imposed the desired signal at 45, and with four nulls imposed at -40, -14, -4, 20 and imposed the mainbeam at -27, respectively. As can be seen from Figures 10, 11, and 12, all desired nulls are deeper than 100 dB. The computed element phases for Figures are given in Table 2.

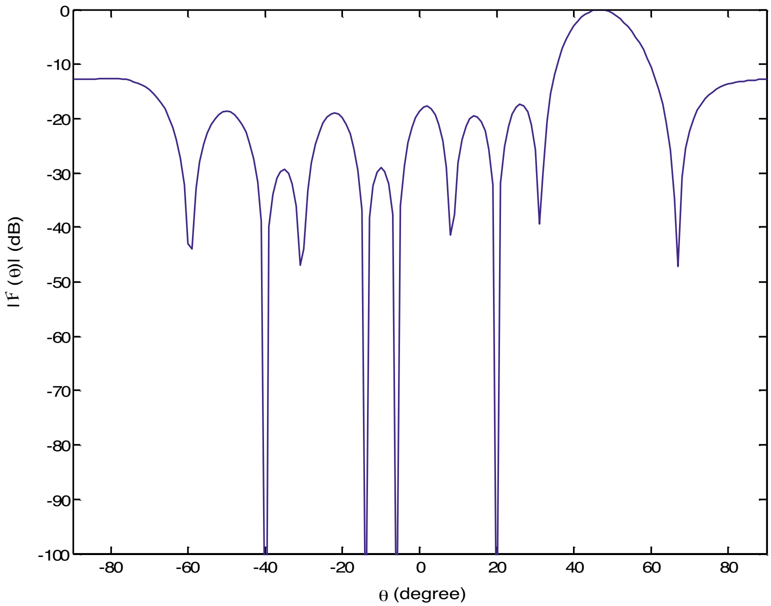

Figure 11. Pattern synthesis with four nulls imposed at $-40°$, -15, -6, 20 and steering lobe at 45.

Table 2. Computed element phases φ_n for Figures 8, 9, 10, 11 and 12.

Elem.	φ_n (Degree)				
No.	Fig.8	Fig.9	Fig.10	Fig.11	Fig.12
1	0	0	0	0	0
2	324.6799	-78.1404	-288.4156	229.5830	-279.5717
3	-76.3654	-219.7272	-222.4559	88.9871	173.5302
4	203.1052	37.6785	213.0409	-47.0404	250.5526
5	104.1566	-53.5796	-45.3551	-156.7721	334.2319
6	98.9719	-149.5689	-349.2146	-305.3921	-319.4783
7	360.0000	-240.8270	112.3895	-55.1239	-235.7991
8	279.4825	16.5786	-172.1136	-191.1490	-158.7767
9	-121.5667	-125.0081	-106.1538	-331.7387	-65.6747
10	-156.8917	156.8515	325.4302	257.8436	-345.2465

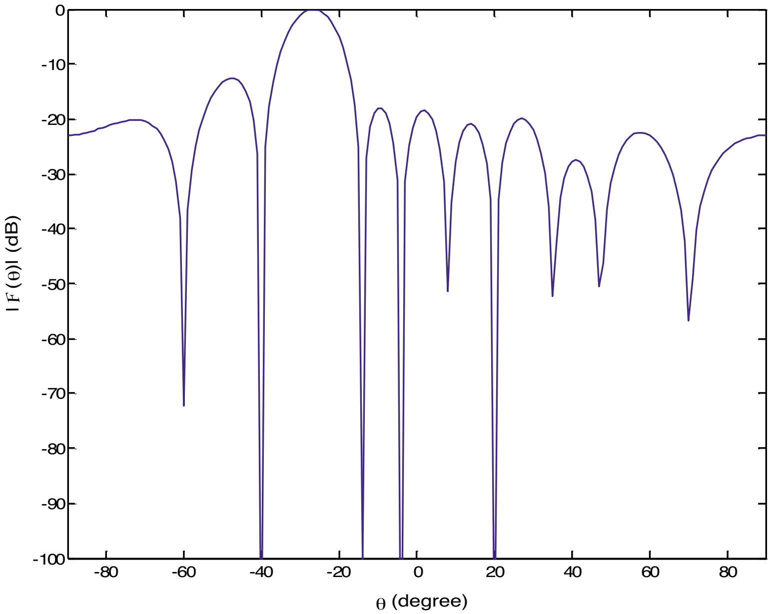

Figure 12. Pattern synthesis with four nulls imposed at $-40°$, -14, -4, 20 and steering lobe at -27.

5. CONCLUSION

A new method using a sequential quadratic programming (SQP) algorithm to suppress a multiple narrow and wide band interferences and track the desired signal by controlling only the phase has been presented. SQP algorithm solves a quadratic programming subproblem in each iteration which is obtained by linearizing the constraints and approximating the Lagrangian function. A good precision on the gradients is necessary, because it determines the direction of descent and intervenes under the conditions stopping of the algorithm. Contrary to many methods which check the constraints at each iteration, the SQP algorithm imposes the respect of the constraints only at the final solution.

The computer simulation results show that the phase-only control using the SQP algorithm is efficient for forming nulls for any prescribed directions.

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