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EXACT FORMULAS FOR THE LATERAL ELECTROMAGNETIC PULSES FROM A HORIZONTAL ELECTRIC DIPOLE ON THE BOUNDARY BETWEEN AN ISOTROPIC MEDIUM AND ONE-DIMENSIONALLY ANISOTROPIC MEDIUM

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Abstract—In this paper, the exact formulas are derived for the time-domain electromagnetic field generated by a delta-function current in a horizontal electric dipole located on the planar boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium. Similar to the isotropic case, the amplitude of the tangential pulsed electric field along the boundary is $1/\rho^2$, which is characteristic of the surface-wave or lateral pulse. The tangential electric field consists of a delta-function pulse travelling in Region 2 (anisotropic medium), a oppositely directed delta-function travelling in Region 1 (isotropic medium), and a final static electric field due to the charges left on the dipole. It is seen that the pulsed electromagnetic field components consist of the first and second pulsed in the two regions with different velocities.

1. INTRODUCTION

The frequency-domain electromagnetic (EM) fields from horizontal and vertical electric dipoles located on or near the planar interface between two electrically different media like earth and air or sea water and rock have many useful applications in subsurface and closed-to-the-surface communication, radar, and geophysical prospecting and diagnostics [1–11]. A historical account and extensive list of references have been summarized in the monograph by King, Owens and Wu [11]. Also of interests are the properties and possible applications of the transient EM field due to a dipole source on or near the boundary between two dielectrics. Almost half century ago, Van der Pol [12] first formulated the transient EM field of a vertical electric dipole with a delta-function excitation on the boundary between two half-spaces by invoking the Hertz potential. Lately, the same problem was visited by many investigators [13–29].

In [27, 28], the approximate formulas have been obtained for lateral EM pulses from vertical and horizontal electric dipoles with delta excitation and Gaussian pulse excitation near or on the boundary between two dielectrics. Recently, the approximate formulas are derived for lateral EM pulses from a horizontal electric dipole on the surface of one-dimensionally anisotropic medium [29]. It is well known that it is very difficult to present the *exact* solution of the EM field from a dipole source near or on the boundary. Fortunately, the important progress on this problem has been made by Wu and King [19], and the *exact* formulas were derived in detail for the components E_z and B_ϕ of the transient EM field generated by a delta-function current in a vertical electric dipole on the boundary between two dielectrics. Similar to the case of the vertical dipole [19], the *exact* formulas have been obtained for the transient EM field generated by a horizontal dipole with delta-function excitation on the boundary between two dielectrics [26].

When a horizontal dipole is located on the planar boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium, the problem of the *exact* solution on the transient EM field will be in general more complicated. The relevant geometry and Cartesian coordinate system are illustrated in Fig. 1, where a unit horizontal electric dipole in the $\hat{x}$ direction is located at $(0, 0, -d)$. The half-space $z \leq 0$ (Region 1) is filled with a homogeneous isotropic medium, and the rest space $z \geq 0$ (Region 2) is with a medium characterized by a permittivity tensor of the form

$$\hat{\varepsilon}_2 = \varepsilon_0 \begin{bmatrix} \varepsilon_T & 0 & 0 \\ 0 & \varepsilon_T & 0 \\ 0 & 0 & \varepsilon_L \end{bmatrix}. \tag{1}$$

It is assumed that both Regions 1 and 2 are nonmagnetic so that

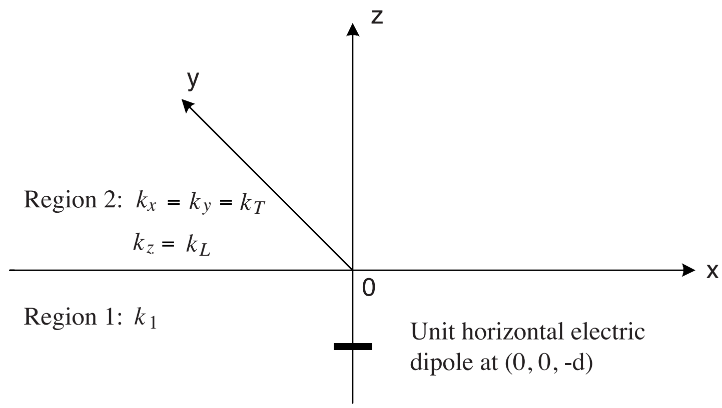

Figure 1. Geometry of a $\hat{x}$-directed horizontal electric dipole on the boundary between one homogeneous isotropic medium and a one-dimensionally anisotropic medium.

$\mu_1 = \mu_2 = \mu_0$. The wave numbers of the two regions are

$$k_1 = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_1} = \frac{\omega\sqrt{\varepsilon_1}}{c}, \tag{2}$$

$$k_T = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_T} = \frac{\omega\sqrt{\varepsilon_T}}{c}, \tag{3}$$

$$k_L = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_L} = \frac{\omega\sqrt{\varepsilon_L}}{c}. \tag{4}$$

In this paper, it is assumed that the dipole source and the observation point approach the boundary between a homogeneous isotropic medium and a anisotropic medium from below ($d \to 0^+$) and from above ($z \to 0^+$), respectively. The *exact* formulas in terms of elementary functions will be obtained for the three time-dependent components E_ρ, E_ϕ, and B_z of the transient EM field due to a delta-function current in a horizontal electric dipole.

2. FORMAL REPRESENTATIONS OF TIME-INDEPENDENT FIELD DUE TO A UNIT HORIZONTAL ELECTRIC DIPOLE ON THE BOUNDARY BETWEEN A ISOTROPIC MEDIUM AND ONE-DIMENSIONALLY ANISOTROPIC MEDIUM

From the available results addressed in [9, 11], with the time dependence of $e^{-i\omega t}$, the Fourier-Bessel representations for the EM field in the cylindrical coordinates (ρ, ϕ, z) with $x = \rho\cos\phi$ and $y = \rho\sin\phi$ $(0 \le \phi < 2\pi)$ can be simplified greatly. They are

$$\begin{aligned}\widetilde{E}_{2\rho}(\rho,\phi;\omega) &= \widetilde{E}_{1\rho}(\rho,\phi;\omega)\\ &= -\frac{\omega\mu_0}{4\pi}\int_0^\infty d\lambda\ \lambda\Bigg\{\frac{\sqrt{k_1^2-\lambda^2}\sqrt{k_L^2-\lambda^2}}{k_T k_L\sqrt{k_1^2-\lambda^2}+k_1^2\sqrt{k_L^2-\lambda^2}}\\ &\quad\times[J_0(\lambda\rho)-J_2(\lambda\rho)]\\ &\quad+\frac{1}{\sqrt{k_T^2-\lambda^2}+\sqrt{k_1^2-\lambda^2}}[J_0(\lambda\rho)+J_2(\lambda\rho)]\Bigg\}\cos\phi, \quad (5)\end{aligned}$$

$$\begin{aligned}\widetilde{E}_{2\phi}(\rho,\phi;\omega) &= \widetilde{E}_{1\phi}(\rho,\phi;\omega)\\ &= \frac{\omega\mu_0}{4\pi}\int_0^\infty d\lambda\ \lambda\Bigg\{\frac{\sqrt{k_1^2-\lambda^2}\sqrt{k_L^2-\lambda^2}}{k_T k_L\sqrt{k_1^2-\lambda^2}+k_1^2\sqrt{k_L^2-\lambda^2}}\\ &\quad\times[J_0(\lambda\rho)+J_2(\lambda\rho)]\\ &\quad+\frac{1}{\sqrt{k_T^2-\lambda^2}+\sqrt{k_1^2-\lambda^2}}[J_0(\lambda\rho)-J_2(\lambda\rho)]\Bigg\}\sin\phi, \quad (6)\end{aligned}$$

$$\begin{aligned}\widetilde{E}_{2z}(z,\phi;\omega) &= \frac{k_1^2}{k_L^2}\widetilde{E}_{1z}(\rho,\phi;\omega)\\ &= \frac{i\omega\mu_0}{4\pi k_L^2}\int_0^\infty d\lambda\ \lambda^2\frac{k_T k_L\sqrt{k_1^2-\lambda^2}-k_1^2\sqrt{k_L^2-\lambda^2}}{k_T k_L\sqrt{k_1^2-\lambda^2}+k_1^2\sqrt{k_L^2-\lambda^2}}\\ &\quad\times J_1(\lambda\rho)\cos\phi, \quad (7)\end{aligned}$$

$$\begin{aligned}\widetilde{B}_{2\rho}(\rho,\phi;\omega) &= \widetilde{B}_{1\rho}(\rho,\phi;\omega)\\ &= -\frac{\mu_0}{8\pi}\int_0^\infty d\lambda\ \lambda\Bigg\{\frac{k_T k_L\sqrt{k_1^2-\lambda^2}-k_1^2\sqrt{k_L^2-\lambda^2}}{k_T k_L\sqrt{k_1^2-\lambda^2}+k_1^2\sqrt{k_L^2-\lambda^2}}\\ &\quad\times[J_0(\lambda\rho)+J_2(\lambda\rho)]\\ &\quad+\frac{\sqrt{k_T^2-\lambda^2}-\sqrt{k_1^2-\lambda^2}}{\sqrt{k_T^2-\lambda^2}+\sqrt{k_1^2-\lambda^2}}[J_0(\lambda\rho)-J_2(\lambda\rho)]\Bigg\}\sin\phi, \quad (8)\end{aligned}$$

$$\begin{aligned}\widetilde{B}_{2\phi}(\rho,\phi;\omega) &= \widetilde{B}_{1\phi}(\rho,\phi;\omega)\\ &= -\frac{\mu_0}{8\pi}\int_0^\infty d\lambda\ \lambda\Bigg\{\frac{k_T k_L\sqrt{k_1^2-\lambda^2}-k_1^2\sqrt{k_L^2-\lambda^2}}{k_T k_L\sqrt{k_1^2-\lambda^2}+k_1^2\sqrt{k_L^2-\lambda^2}}\\ &\quad\times[J_0(\lambda\rho)-J_2(\lambda\rho)]\\ &\quad+\frac{\sqrt{k_T^2-\lambda^2}-\sqrt{k_1^2-\lambda^2}}{\sqrt{k_T^2-\lambda^2}+\sqrt{k_1^2-\lambda^2}}[J_0(\lambda\rho)+J_2(\lambda\rho)]\Bigg\}\cos\phi, \quad (9)\end{aligned}$$

$$\begin{aligned}\widetilde{B}_{2z}(z,\phi;\omega) &= \widetilde{B}_{1z}(\rho,\phi;\omega)\\ &= \frac{i\mu_0}{2\pi}\int_0^\infty d\lambda\ \lambda^2\frac{1}{\sqrt{k_T^2-\lambda^2}+\sqrt{k_1^2-\lambda^2}}J_1(\lambda\rho)\sin\phi, \quad (10)\end{aligned}$$

where J_0, J_1, and J_2 are the Bessel functions of orders 0, 1, and 2, respectively.

It is convenient to express the components of the EM field in terms of ω instead of wave numbers. With $k_1 = \omega{\varepsilon_1}^{1/2}/c$, $k_T = \omega{\varepsilon_T}^{1/2}/c$, $k_L = \omega{\varepsilon_L}^{1/2}/c$, $\lambda' = c\lambda$, and $\rho' = \rho/c$, where $c = (\mu_0\varepsilon_0)^{-1/2}$ is the velocity of light, and taking into account the following relations,

$$J_0(\lambda\rho)+J_2(\lambda\rho) = \frac{2}{\lambda\rho}J_1(\lambda\rho), \quad (11)$$

$$J_0(\lambda\rho)-J_2(\lambda\rho) = 2J_0(\lambda\rho)-\frac{2}{\lambda\rho}J_1(\lambda\rho), \quad (12)$$

we can write the Fourier-Bessel representations for the six components of the EM field in explicit forms.

$$\begin{aligned}\widetilde{E}_{2\rho}(\rho',0;\omega) &= \widetilde{E}_{1\rho}(\rho',0;\omega)\\ &= -\frac{\omega\mu_0}{2\pi c}\int_0^\infty d\lambda'\lambda'\Bigg\{\frac{\sqrt{\omega^2\varepsilon_1-\lambda'^2}\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}\\ &\quad\times\left[J_0(\lambda'\rho')-\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right]\\ &\quad+\frac{1}{\sqrt{\omega^2\varepsilon_T-\lambda'^2}+\sqrt{\omega^2\varepsilon_1-\lambda'^2}}\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\Bigg\}, \quad (13)\end{aligned}$$

$$\begin{aligned}\widetilde{E}_{2\phi}(\rho',\pi/2;\omega) &= \widetilde{E}_{1\phi}(\rho',\pi/2;\omega)\\ &= \frac{\omega\mu_0}{2\pi c}\int_0^\infty d\lambda'\lambda'\Bigg\{\frac{\sqrt{\omega^2\varepsilon_1-\lambda'^2}\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}\\ &\quad\times\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')+\frac{1}{\sqrt{\omega^2\varepsilon_T-\lambda'^2}+\sqrt{\omega^2\varepsilon_1-\lambda'^2}}\\ &\quad\times\left[J_0(\lambda'\rho')-\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right]\Bigg\},\end{aligned}\tag{14}$$

$$\begin{aligned}\widetilde{E}_{2z}(\rho',0;\omega) &= \frac{\varepsilon_1}{\varepsilon_L}\widetilde{E}_{1z}(\rho',0;\omega)\\ &= \frac{i\mu_0}{4\pi\omega\varepsilon_L c}\int_0^\infty d\lambda'\lambda'^2\frac{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}-\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}\\ &\quad\times J_1(\lambda'\rho'),\end{aligned}\tag{15}$$

$$\begin{aligned}\widetilde{B}_{2\rho}(\rho',\pi/2;\omega) &= \widetilde{B}_{1\rho}(\rho',\pi/2;\omega)\\ &= -\frac{\mu_0}{4\pi c^2}\int_0^\infty d\lambda'\lambda'\Bigg\{\frac{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}-\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}\\ &\quad\times\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')+\frac{\sqrt{\omega^2\varepsilon_T-\lambda'^2}-\sqrt{\omega^2\varepsilon_1-\lambda'^2}}{\sqrt{\omega^2\varepsilon_T-\lambda'^2}+\sqrt{\omega^2\varepsilon_1-\lambda'^2}}\\ &\quad\times\left[J_0(\lambda'\rho')-\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right]\Bigg\},\end{aligned}\tag{16}$$

$$\begin{aligned}\widetilde{B}_{2\phi}(\rho',0;\omega) &= \widetilde{B}_{1\phi}(\rho',0;\omega)\\ &= -\frac{\mu_0}{4\pi c^2}\int_0^\infty d\lambda'\lambda'\Bigg\{\frac{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}-\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}\\ &\quad\times\left[J_0(\lambda'\rho')-\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right]\\ &\quad+\frac{\sqrt{\omega^2\varepsilon_T-\lambda'^2}-\sqrt{\omega^2\varepsilon_1-\lambda'^2}}{\sqrt{\omega^2\varepsilon_T-\lambda'^2}+\sqrt{\omega^2\varepsilon_1-\lambda'^2}}\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\Bigg\},\end{aligned}\tag{17}$$

$$\begin{aligned}\widetilde{B}_{2z}(\rho',\pi/2;\omega) &= \widetilde{B}_{1z}(\rho',\pi/2;\omega)\\ &= \frac{i\mu_0}{2\pi c^2}\int_0^\infty d\lambda'\lambda'^2\frac{1}{\sqrt{\omega^2\varepsilon_T-\lambda'^2}+\sqrt{\omega^2\varepsilon_1-\lambda'^2}}J_1(\lambda'\rho').\end{aligned}\tag{18}$$

3. TIME-DEPENDENT COMPONENT $E_{2\rho}$ DUE TO A HORIZONTAL DIPOLE WITH A DELTA-FUNCTION EXCITATION

3.1. The Integrated Formula for Time-dependent Component $E_{2\rho}$

If the exciting current in a horizontal electric dipole is a delta-function current with a unit amplitude, we can write the time-dependent component $E_{2\rho}$ by using the Fourier transform.

$$E_{2\rho}(\rho', 0; t) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty d\omega\ e^{-i\omega t}\widetilde{E}_{2\rho}(\rho', 0; \omega). \tag{19}$$

Substituting (13) into (19), we find

$$\begin{aligned} E_{2\rho}(\rho', 0; t) = & -\frac{\mu_0}{2\pi^2 c}\int_0^\infty d\omega e^{-i\omega t}\omega \\ & \times\mathrm{Re}\int_0^\infty d\lambda'\lambda'\Bigg\{\frac{\sqrt{\omega^2\varepsilon_1-\lambda'^2}\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}} \\ & \times\left[J_0(\lambda'\rho')-\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right] \\ & +\frac{1}{\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\sqrt{\omega^2\varepsilon_T-\lambda'^2}}\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\Bigg\}. \end{aligned} \tag{20}$$

With the definition $\lambda' = \omega\xi$, $d\lambda' = \omega d\xi$, (20) changes as

$$\begin{aligned} E_{2\rho}(\rho', 0; t) = & \frac{\mu_0}{2\pi^2 c}\mathrm{Re}\int_0^\infty \xi d\xi\Bigg\{\frac{\sqrt{\varepsilon_1-\xi^2}\sqrt{\varepsilon_L-\xi^2}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+\varepsilon_1\sqrt{\varepsilon_L-\xi^2}} \\ & \times\left[\frac{\partial^2}{\partial t^2}\int_0^\infty d\omega e^{-i\omega t}J_0(\omega\xi\rho')+\frac{i}{\xi\rho'}\frac{\partial}{\partial t}\int_0^\infty d\omega e^{-i\omega t}J_1(\omega\xi\rho')\right] \\ & -\frac{1}{\sqrt{\varepsilon_1-\xi^2}+\sqrt{\varepsilon_T-\xi^2}}\frac{i}{\xi\rho'}\frac{\partial}{\partial t}\int_0^\infty d\omega e^{-i\omega t}J_1(\omega\xi\rho')\Bigg\}. \end{aligned} \tag{21}$$

The integrals in (21) with respect to ω can be obtained by using the infinite integral formula 6.611-1 of [30]. When $t > \xi\rho'$,

$$\int_0^\infty d\omega e^{-i\omega t} J_0(\omega\xi\rho') = -\frac{i}{\sqrt{t^2-\xi^2\rho'^2}}, \tag{22}$$

$$\int_0^\infty d\omega e^{-i\omega t} J_1(\omega\xi\rho') = \frac{1}{\xi\rho'}\left[1-\frac{t}{\sqrt{t^2-\xi^2\rho'^2}}\right]. \tag{23}$$

Then, (21) can be rewritten as

$$E_{2\rho}(\rho', 0; t) = \frac{\mu_0}{2\pi^2\rho' c}\left[I_1 + I_2 + I_3\right]. \tag{24}$$

where

$$I_1 = \frac{\partial^2}{\partial t^2}\,\mathrm{Im}\int_0^\infty \xi d\xi \frac{\sqrt{\varepsilon_1-\xi^2}\sqrt{\varepsilon_L-\xi^2}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+\varepsilon_1\sqrt{\varepsilon_L-\xi^2}}\frac{1}{\sqrt{t^2/\rho'^2-\xi^2}}, \tag{25}$$

$$I_2 = -\frac{\partial}{\partial t}\,\mathrm{Im}\int_0^\infty \xi d\xi \frac{\sqrt{\varepsilon_1-\xi^2}\sqrt{\varepsilon_L-\xi^2}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+\varepsilon_1\sqrt{\varepsilon_L-\xi^2}}\frac{1}{\xi^2\rho'} \times\left[1-\frac{t}{\sqrt{t^2-\xi^2\rho'^2}}\right], \tag{26}$$

$$I_3 = \frac{\partial}{\partial t}\,\mathrm{Im}\int_0^\infty \xi d\xi \frac{1}{\sqrt{\varepsilon_1-\xi^2}+\sqrt{\varepsilon_T-\xi^2}}\frac{1}{\xi^2\rho'}\left[1-\frac{t}{\sqrt{t^2-\xi^2\rho'^2}}\right]. \tag{27}$$

Next, the main tasks are to evaluate the above three integrals.

3.2. Evaluation of I_1

Following the similar manner used for the evaluation of $E_{2z}(\rho, t)$ due to the vertical dipole in [11, 19], I_1 can be evaluated readily. With the branch-cut structure in Fig. 2, it follows that

$$I_1 = 0, \qquad t/\rho' < \sqrt{\varepsilon_L}. \tag{28}$$

The contour of integration is shown in Fig. 3, we can then obtain

$$I_1 = \frac{\partial^2}{\partial t^2}\,\mathrm{Im}\Bigg\{\int_0^{\sqrt{\varepsilon_L}} \xi d\xi \Bigg[\frac{\sqrt{\varepsilon_1-\xi^2}\sqrt{\varepsilon_L-\xi^2}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+\varepsilon_1\sqrt{\varepsilon_L-\xi^2})\sqrt{t^2/\rho'^2-\xi^2}}$$

$$
-\frac{\sqrt{\varepsilon_1-\xi^2}\sqrt{\varepsilon_L-\xi^2}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+\varepsilon_1\sqrt{\varepsilon_L-\xi^2})(-\sqrt{t^2/\rho'^2-\xi^2})}\Bigg]
$$

$$
+\int\limits_{\sqrt{\varepsilon_L}}^{t/\rho'}\xi d\xi\Bigg[\frac{i\sqrt{\varepsilon_1-\xi^2}\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L})\sqrt{t^2/\rho'^2-\xi^2}}
$$

$$
-\frac{i\sqrt{\varepsilon_1-\xi^2}\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L})(-\sqrt{t^2/\rho'^2-\xi^2})}\Bigg]\Bigg\},
$$

$$
\sqrt{\varepsilon_L}<t/\rho'<\sqrt{\varepsilon_1}. \tag{29}
$$

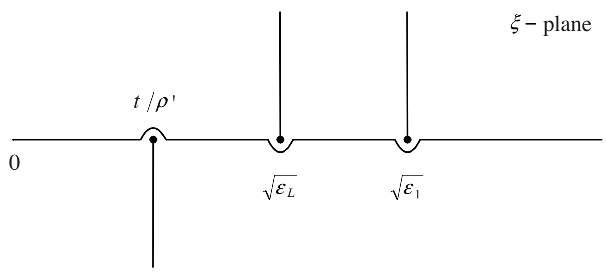

Figure 2. Branch-cut structure for the integrals in (25)–(26).

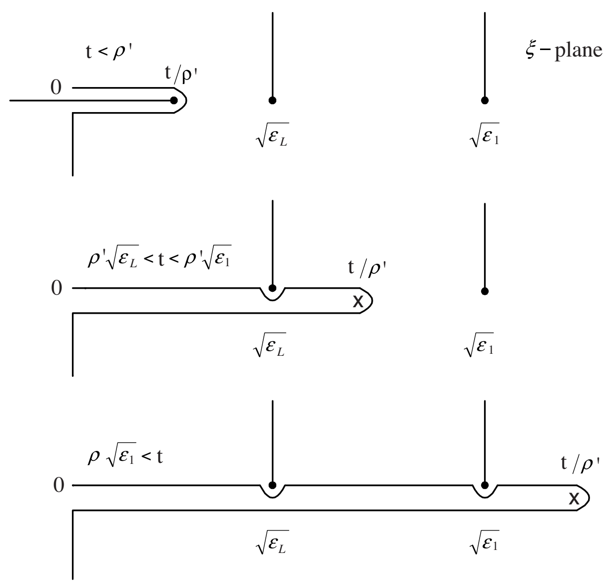

Figure 3. Contours of integration for the integrals in (25)–(26).

Because the integrand of the first integral in (29) is *real*, there is no contribution to the imaginary part. So that

$$I_1 = \frac{\partial^2}{\partial t^2}\ \mathrm{Im} \int_{\sqrt{\varepsilon_L}}^{t/\rho'} \xi d\xi \frac{i2\sqrt{\varepsilon_1 - \xi^2}\sqrt{\xi^2 - \varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1 - \xi^2} + i\varepsilon_1\sqrt{\xi^2 - \varepsilon_L})\sqrt{t^2/\rho'^2 - \xi^2}}. \tag{30}$$

The real and imaginary parts can be separated readily and the result reduces to

$$\begin{aligned} I_1 &= \frac{\partial^2}{\partial t^2} \int_{\sqrt{\varepsilon_L}}^{t/\rho'} \xi d\xi \frac{2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1 - \xi^2)\sqrt{\xi^2 - \varepsilon_L}}{[(\varepsilon^2 - \varepsilon_T\varepsilon_L)\xi^2 - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_L)]\sqrt{t^2/\rho'^2 - \xi^2}} \\ &= -2\sqrt{\varepsilon_T\varepsilon_L}\frac{\partial^2}{\partial t^2} \int_{\sqrt{\varepsilon_L}}^{t/\rho'} \xi^3 d\xi \frac{\sqrt{\xi^2 - \varepsilon_L}}{[(\varepsilon^2 - \varepsilon_T\varepsilon_L)\xi^2 - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \xi^2}} \\ &\quad +2\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}\frac{\partial^2}{\partial t^2} \int_{\sqrt{\varepsilon_L}}^{t/\rho'} \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_L}}{[(\varepsilon^2 - \varepsilon_T\varepsilon_L)\xi^2 - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \xi^2}}. \end{aligned} \tag{31}$$

With the change of the variable $\zeta = \xi^2$, $d\zeta = 2\xi d\xi$, it becomes

$$\begin{aligned} I_1 &= -\sqrt{\varepsilon_T\varepsilon_L}\frac{\partial^2}{\partial t^2} \int_{\varepsilon_L}^{t^2/\rho'^2} \zeta d\zeta \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}} \\ &\quad +\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}\frac{\partial^2}{\partial t^2} \int_{\varepsilon_L}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}. \end{aligned} \tag{32}$$

It is found that the two integrals in (32) need to be evaluated.

$$\vartheta_0^{(1)} = \int_{\varepsilon_L}^{t^2/\rho'^2} \zeta d\zeta \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}, \tag{33}$$

$$\vartheta_0^{(2)} = \int_{\varepsilon_L}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}. \tag{34}$$

The above two integrals are evaluated in Appendix A. The results are

$$\vartheta_0^{(1)} = \frac{\pi}{2(\varepsilon_1^2-\varepsilon_T\varepsilon_L)}\left\{\frac{t^2}{\rho'^2}-\varepsilon_L+\frac{2\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}-\frac{2\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right.$$
$$\left.\times\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}, \tag{35}$$

$$\vartheta_0^{(2)} = \frac{\pi}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)}\left\{1-\sqrt{\varepsilon_T\varepsilon_L}\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}. \tag{36}$$

Substituting (35) and (36) into (32), the result becomes

$$I_1 = -\frac{\sqrt{\varepsilon_T\varepsilon_L}\pi}{2(\varepsilon_1^2-\varepsilon_T\varepsilon_L)}\frac{\partial^2}{\partial t^2}\left\{\frac{t^2}{\rho'^2}-\varepsilon_L-\frac{2\varepsilon_1^2(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}+\frac{2\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right.$$
$$\left.\times\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}, \quad \sqrt{\varepsilon_L}<\frac{t}{\rho'}<\sqrt{\varepsilon_1}. \tag{37}$$

When $t/\rho' > \sqrt{\varepsilon_1}$, with the contour in Fig. 3, we obtain

$$I_1 = \frac{\partial^2}{\partial t^2}\,\mathrm{Im}\left\{\int_{\sqrt{\varepsilon_L}}^{\sqrt{\varepsilon_1}}\xi d\xi\frac{2\sqrt{\varepsilon_1-\xi^2}i\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L})\sqrt{t^2/\rho'^2-\xi^2}}\right.$$
$$\left.+\int_{\sqrt{\varepsilon_1}}^{t/\rho'}\xi d\xi\frac{i2\sqrt{\xi^2-\varepsilon_1}\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\xi^2-\varepsilon_1}+\varepsilon_1\sqrt{\xi^2-\varepsilon_L})\sqrt{t^2/\rho'^2-\xi^2}}\right\}. \tag{38}$$

The imaginary part is written as follow:

$$I_1 = \frac{\partial^2}{\partial t^2}\left\{\int_{\varepsilon_L}^{\sqrt{\varepsilon_1}}\xi d\xi\frac{2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right.$$
$$\left.+\int_{\sqrt{\varepsilon_1}}^{t/\rho'}\xi d\xi\frac{2[\varepsilon_1(\xi^2-\varepsilon_L)\sqrt{\xi^2-\varepsilon_1}-\sqrt{\varepsilon_T\varepsilon_L}(\xi^2-\varepsilon_1)\sqrt{\xi^2-\varepsilon_L}]}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right\}$$
$$= \frac{\partial^2}{\partial t^2}\left\{\int_{\varepsilon_L}^{t/\rho'}\xi d\xi\frac{2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right.$$

$$+\int_{\sqrt{\varepsilon_1}}^{t/\rho'}\xi d\xi\frac{2\varepsilon_1(\xi^2-\varepsilon_L)\sqrt{\xi^2-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\Bigg\}. \quad (39)$$

With the change of variable $\zeta=\xi^2$, we write

$$\begin{aligned}
I_1 &= \frac{\partial^2}{\partial t^2}\left\{\int_{\varepsilon_L}^{t^2/\rho'^2}d\zeta\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\zeta)\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right.\\
&\quad\left.+\int_{\varepsilon}^{t^2/\rho'^2}d\zeta\frac{\varepsilon_1(\zeta-\varepsilon_L)\sqrt{\zeta-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}\\
&= -\sqrt{\varepsilon_T\varepsilon_L}\frac{\partial^2}{\partial t^2}\int_{\varepsilon_L}^{t^2/\rho'^2}\zeta d\zeta\frac{\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\\
&\quad+\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}\frac{\partial^2}{\partial t^2}\int_{\varepsilon_L}^{t^2/\rho'^2}d\zeta\frac{\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\\
&\quad+\varepsilon_1\frac{\partial^2}{\partial t^2}\int_{\varepsilon_1}^{t^2/\rho'^2}\zeta d\zeta\frac{\sqrt{\zeta-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\\
&\quad-\varepsilon_1\varepsilon_L\frac{\partial^2}{\partial t^2}\int_{\varepsilon_1}^{t^2/\rho'^2}d\zeta\frac{\sqrt{\zeta-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}.
\end{aligned} \quad (40)$$

The evaluations of the first and second integrals in (40) are shown in (35) and (36). Next, the third and fourth integrals need to be evaluated. They are

$$\vartheta_\varepsilon^{(1)} = \int_{\varepsilon_1}^{t^2/\rho'^2}\zeta d\zeta\frac{\sqrt{\zeta-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}, \quad (41)$$

$$\vartheta_\varepsilon^{(2)} = \int_{\varepsilon_1}^{t^2/\rho'^2}d\zeta\frac{\sqrt{\zeta-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}. \quad (42)$$

The above two integrals are evaluated in Appendix B. The results are

$$\vartheta_\varepsilon^{(1)} = \frac{\pi}{2(\varepsilon_1^2-\varepsilon_T\varepsilon_L)}\left\{\frac{t^2}{\rho'^2}-\varepsilon_1+\frac{2\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}-\frac{2\varepsilon_1^2\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right.$$
$$\left.\times\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}, \tag{43}$$

$$\vartheta_\varepsilon^{(2)} = \frac{\pi}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)}\left\{1-\varepsilon_1\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}. \tag{44}$$

With substitutions (35), (36), (43), and (44) into (40), it follows

$$I_1 = \frac{\pi}{2(\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L})}\frac{\partial^2}{\partial t^2}\left[\frac{t^2}{\rho'^2}-\frac{\varepsilon_1^2-\sqrt{\varepsilon_T\varepsilon_L}\varepsilon_L}{\varepsilon_1-\sqrt{\varepsilon_T\varepsilon_L}}+\frac{2\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right],\quad \frac{t}{\rho'}>\sqrt{\varepsilon_1}. \tag{45}$$

Combined with (28), (37), and (45), I_1 can be re-written as follows:

$$I_1 = \frac{\pi}{2}\frac{\partial^2}{\partial t^2}f_1\left(\frac{t}{\rho'}\right). \tag{46}$$

where

$$f_1\left(\frac{t}{\rho'}\right) = \begin{cases} 0, & \frac{t}{\rho'}<\sqrt{\varepsilon_L} \\ -\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\left\{\frac{t^2}{\rho'^2}-\varepsilon_L-\frac{2\varepsilon_1^2(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}+\frac{2\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right. \\ \left.\times\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}, & \sqrt{\varepsilon_L}<\frac{t}{\rho'}<\sqrt{\varepsilon_1}. \\ \frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}}\left[\frac{t^2}{\rho'^2}-\frac{\varepsilon_1^2-\sqrt{\varepsilon_T\varepsilon_L}\varepsilon_L}{\varepsilon_1-\sqrt{\varepsilon_T\varepsilon_L}}+\frac{2\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right], & \frac{t}{\rho'}>\sqrt{\varepsilon_1} \end{cases} \tag{47}$$

It follows that $f_1(\sqrt{\varepsilon_L}-) = f_1(\sqrt{\varepsilon_L}+) = 0$ and $f_1(\sqrt{\varepsilon_1}-) = f_1(\sqrt{\varepsilon_1}+) = \frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)}{(\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L})^2}$. It is seen that $f_1(t/\rho')$ is everywhere continuous.

$$f_1'\left(\frac{t}{\rho'}\right)=\begin{cases}0, & \frac{t}{\rho'}<\sqrt{\varepsilon_L}\\ -\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\frac{2t}{\rho'^2}\left\{1-\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}\right.\right.\\ \quad\left.\left.\times-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-3/2}\right\}, & \sqrt{\varepsilon_L}<\frac{t}{\rho'}<\sqrt{\varepsilon_1}\\ \frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}}\frac{2t}{\rho'^2}, & \frac{t}{\rho'}>\sqrt{\varepsilon_1}\end{cases}. \tag{48}$$

Since $f_1'(\sqrt{\varepsilon_L}-)=0$, $f_1'(\sqrt{\varepsilon_L}+)=2/[\sqrt{\varepsilon_T}\rho']$, there is a step discontinuity of $2/[\sqrt{\varepsilon_T}\rho']$ in $f_1'(t/\rho')$ at $t/\rho'=\sqrt{\varepsilon_L}$. Similarly, $f_1'(\sqrt{\varepsilon_1}-)=-2\sqrt{\varepsilon_T\varepsilon_L}/[\sqrt{\varepsilon_1}(\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L})\rho']$, $f_1'(\sqrt{\varepsilon}+)=2\sqrt{\varepsilon_1}/[(\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L})\rho']$, $f_1'(t/\rho')$ has a step discontinuity of $2/(\sqrt{\varepsilon_1}\rho')$ at $t/\rho'=\sqrt{\varepsilon_1}$. Then,

$$f_1''\left(\frac{t}{\rho'}\right)=\begin{cases}0, & \frac{t}{\rho'}<\sqrt{\varepsilon_L}\\ -\frac{2\sqrt{\varepsilon_T\varepsilon_L}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\rho'^2}\left\{1+\frac{\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)^{3/2}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)^{3/2}}\right.\\ \quad\left.\times\left[\frac{2t^2}{\rho'^2}+\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right]\left[\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right]^{-5/2}\right\}, & \sqrt{\varepsilon_L}<\frac{t}{\rho'}<\sqrt{\varepsilon_1}\\ \frac{2}{(\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L})\rho'^2}, & \frac{t}{\rho'}>\sqrt{\varepsilon_1}\end{cases}. \tag{49}$$

Obviously, the exact formula for I_1 is obtained readily.

$$I_1=\frac{c\pi}{\rho}\left[\frac{1}{\sqrt{\varepsilon_T}}\delta\left(t-\frac{\sqrt{\varepsilon_L}\rho}{c}\right)+\frac{1}{\sqrt{\varepsilon_1}}\delta\left(t-\frac{\sqrt{\varepsilon_1}\rho}{c}\right)\right]+\frac{\sqrt{\varepsilon_T\varepsilon_L}c^2\pi}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\rho^2}$$

$$\times\begin{cases} 0, & \dfrac{ct}{\rho} < \sqrt{\varepsilon_L} \\ -\left(1+\dfrac{\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\varepsilon_L)^{3/2}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)^{3/2}}\left[\dfrac{2c^2t^2}{\rho^2}+\dfrac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right]\right. & \\ \left.\times\left[\dfrac{c^2t^2}{\rho^2}-\dfrac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\right]^{-5/2}\right), & \sqrt{\varepsilon_L} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ \dfrac{1}{\sqrt{\varepsilon_T\varepsilon_L}}(\varepsilon_1-\sqrt{\varepsilon_T\varepsilon_L}), & \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases}. \tag{50}$$

3.3. Evaluation of I_2

With the similar procedures addressed in Subsection 3.2, I_2 can be evaluated readily.

$$I_2 = 0, \qquad t/\rho' < \sqrt{\varepsilon_L}. \tag{51}$$

When $\sqrt{\varepsilon_L} < t/\rho' < \sqrt{\varepsilon_1}$,

$$\begin{aligned} I_2 &= -\frac{\partial}{\partial t}\,\mathrm{Im}\left\{\int_0^\infty \frac{\xi d\xi}{\xi^2\rho'}\left[\frac{i\sqrt{\varepsilon_1-\xi^2}\sqrt{\xi^2-\varepsilon_L}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L}}\right.\right. \\ &\quad \left.\left.-\frac{t}{\rho'}\frac{i\sqrt{\varepsilon_1-\xi^2}\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L})(\sqrt{t^2/\rho'^2-\xi^2})}\right]\right\}. \end{aligned} \tag{52}$$

The imaginary part is re-written as

$$\begin{aligned} I_2 = &-\frac{1}{\rho'}\frac{\partial}{\partial t}\left\{\int_0^\infty \frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\right. \\ &\left.-\frac{t}{\rho'}\frac{\partial}{\partial t}\int_0^\infty \frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right\}. \end{aligned} \tag{53}$$

With the contour in Fig. 3, we get

$$I_2 = \frac{2}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int_{\sqrt{\varepsilon_L}}^{t/\rho'} \frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right\}. \tag{54}$$

With $\zeta=\xi^2$, this becomes

$$I_2 = \frac{1}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int_{\varepsilon_L}^{t^2/\rho'^2}\frac{d\zeta}{\zeta}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\zeta)\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}$$
$$= \frac{\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int_{\varepsilon_L}^{t^2/\rho'^2}\frac{d\zeta}{\zeta}\frac{\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}$$
$$-\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int_{\varepsilon_L}^{t^2/\rho'^2}d\zeta\frac{\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}. \tag{55}$$

The first integral in (55) need to be evaluated and the second one has been evaluated in (36).

$$\vartheta_0^{(3)} = \int_{\varepsilon_L}^{t^2/\rho'^2}\frac{d\zeta}{\zeta}\frac{\sqrt{\zeta-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}. \tag{56}$$

The integral $\vartheta_0^{(3)}$ is evaluated in Appendix C, and the result is

$$\vartheta_0^{(3)} = \frac{\pi}{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\left\{\sqrt{\varepsilon_L}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right.$$
$$\left.-\sqrt{\varepsilon_T\varepsilon_L}\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}. \tag{57}$$

Substituting (36) and (57) into (55), we write

$$I_2 = \frac{\sqrt{\varepsilon_T\varepsilon_L}\pi}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\left[-\frac{1}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}+\frac{1}{\sqrt{\varepsilon_L}(\varepsilon_1-\varepsilon_T)}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right.\right.$$
$$+\sqrt{\varepsilon_T\varepsilon_L}\left(\frac{1}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}-\frac{1}{\varepsilon_L(\varepsilon_1-\varepsilon_T)}\right)$$
$$\left.\left.\times\left(\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right)^{-1/2}\right]\right\}, \quad 1<\frac{t}{\rho'}<\sqrt{\varepsilon}. \tag{58}$$

When $t/\rho'>\sqrt{\varepsilon_1}$, with the contour in Fig. 3, I_2 is expressed as follows:

$$I_2 = -\frac{\partial}{\partial t}\,\mathrm{Im}\left\{\int_0^{\sqrt{\varepsilon_1}}\frac{\xi d\xi}{\xi^2\rho'}\frac{i\sqrt{\varepsilon_1-\xi^2}\sqrt{\xi^2-\varepsilon_L}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L}}\right.$$

$$-\frac{t}{\rho'}\int\limits_{0}^{\sqrt{\varepsilon_1}}\frac{\xi d\xi}{\xi^2\rho'}\frac{i\sqrt{\varepsilon_1-\xi^2}\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L})\sqrt{t^2/\rho'^2-\xi^2}}$$
$$+\int\limits_{\sqrt{\varepsilon_1}}^{\infty}\frac{\xi d\xi}{\xi^2\rho'}\frac{i\sqrt{\xi^2-\varepsilon_1}\sqrt{\xi^2-\varepsilon_L}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L}}$$
$$\left.-\frac{t}{\rho'}\int\limits_{\sqrt{\varepsilon_1}}^{\infty}\frac{\xi d\xi}{\xi^2\rho'}\frac{i\sqrt{\xi^2-\varepsilon_1}\sqrt{\xi^2-\varepsilon_L}}{(\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+i\varepsilon_1\sqrt{\xi^2-\varepsilon_L})\sqrt{t^2/\rho'^2-\xi^2}}\right\}. \quad (59)$$

The imaginary part reduces to

$$I_2 = -\frac{1}{\rho'}\frac{\partial}{\partial t}\left\{\int\limits_{0}^{\sqrt{\varepsilon_1}}\frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\right.$$
$$-\frac{t}{\rho'}\int\limits_{0}^{\sqrt{\varepsilon_1}}\frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}$$
$$+\int\limits_{\sqrt{\varepsilon_1}}^{\infty}\frac{\xi d\xi}{\xi^2}\frac{\varepsilon_1(\xi^2-\varepsilon_L)\sqrt{\xi^2-\varepsilon_1}-\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}$$
$$\left.-\frac{t}{\rho'}\int\limits_{\sqrt{\varepsilon_1}}^{\infty}\frac{\xi d\xi}{\xi^2}\frac{\varepsilon_1(\xi^2-\varepsilon_L)\sqrt{\xi^2-\varepsilon_1}-\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right\}$$
$$= -\frac{1}{\rho'}\frac{\partial}{\partial t}\left\{\int\limits_{0}^{\infty}\frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\right.$$
$$-\frac{t}{\rho'}\int\limits_{0}^{\infty}\frac{\xi d\xi}{\xi^2}\frac{\sqrt{\varepsilon_T\varepsilon_L}(\varepsilon_1-\xi^2)\sqrt{\xi^2-\varepsilon_L}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}$$
$$+\int\limits_{\sqrt{\varepsilon_1}}^{\infty}\frac{\xi d\xi}{\xi^2}\frac{\varepsilon_1(\xi^2-\varepsilon_L)\sqrt{\xi^2-\varepsilon_1}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}$$
$$\left.-\frac{t}{\rho'}\int\limits_{\sqrt{\varepsilon}}^{\infty}\frac{\xi d\xi}{\xi^2}\frac{\varepsilon_1(\xi^2-\varepsilon_L)\sqrt{\xi^2-\varepsilon_1}}{[((\varepsilon_1^2-\varepsilon_T\varepsilon_L)\xi^2-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\xi^2}}\right\}. \quad (60)$$

In terms of the variable $\zeta = \xi^2$, $d\zeta = 2\xi d\xi$, this becomes

$$I_2 = -\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int\limits_{\varepsilon_L}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta-\varepsilon_L}}{[((\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}$$
$$-\frac{\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int\limits_{\varepsilon_L}^{t^2/\rho'^2} \frac{d\zeta}{\zeta} \frac{\sqrt{\zeta-\varepsilon_L}}{[((\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}$$
$$+\frac{\varepsilon_1}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int\limits_{\varepsilon_1}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta-\varepsilon_1}}{[((\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}$$
$$-\frac{\varepsilon_1\varepsilon_L}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int\limits_{\varepsilon_1}^{t^2/\rho'^2} \frac{d\zeta}{\zeta} \frac{\sqrt{\zeta-\varepsilon_1}}{[((\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}\right\}. \tag{61}$$

The evaluations of the first, second, and third integrals in (61) have been found in (36), (57), and (44), respectively. Next, the fourth integral will be evaluated. It is

$$\vartheta_\varepsilon^{(3)} = \int\limits_{\varepsilon_1}^{t^2/\rho'^2} \frac{d\zeta}{\zeta} \frac{\sqrt{\zeta-\varepsilon_1}}{[((\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta-\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}. \tag{62}$$

The integral $\vartheta_\varepsilon^{(3)}$ is evaluated in Appendix D, and the result is

$$\vartheta_\varepsilon^{(3)} = \frac{\pi}{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\left[\sqrt{\varepsilon_1}\left(\frac{t^2}{\rho'^2}\right)^{-1/2} - \varepsilon_1\left(\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2} - \frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right)^{-1/2}\right]. \tag{63}$$

Thus,

$$I_2 = \frac{\pi}{\rho'}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\left[\frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}} - \left(\frac{\sqrt{\varepsilon_1}}{\varepsilon_1-\varepsilon_T} - \frac{\sqrt{\varepsilon_T}}{\varepsilon_1-\varepsilon_T}\right)\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right]\right\}, \quad \frac{t}{\rho'} > \sqrt{\varepsilon}. \tag{64}$$

Combined with (51), (58), and (64), the following expressions for I_2 can be obtained readily.

$$I_2 = \frac{\pi}{\rho'}\frac{\partial}{\partial t}\, f_2\left(\frac{t}{\rho'}\right), \tag{65}$$

where

$$f_2\left(\frac{t}{\rho'}\right)=\begin{cases}0, & \frac{t}{\rho'}<\sqrt{\varepsilon_L}\\ \frac{t}{\rho'}\left[-\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}+\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\sqrt{\varepsilon_L}(\varepsilon_1-\varepsilon_T)}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right. & \\ \quad+\left(\frac{\varepsilon_T\varepsilon_L}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}-\frac{\varepsilon_T\varepsilon_L}{\varepsilon_L(\varepsilon_1-\varepsilon_T)}\right) & \\ \left.\left(\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right)^{-1/2}\right], & \sqrt{\varepsilon_L}<\frac{t}{\rho'}<\sqrt{\varepsilon_1}\\ \frac{t}{\rho'}\left[\frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}}-\left(\frac{\sqrt{\varepsilon_1}}{\varepsilon_1-\varepsilon_T}-\frac{\sqrt{\varepsilon_T}}{\varepsilon_1-\varepsilon_T}\right)\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right], & \\ & \frac{t}{\rho'}>\sqrt{\varepsilon_1}.\end{cases}\tag{66}$$

Since $f_2(\sqrt{\varepsilon_L}-)=f_2(\sqrt{\varepsilon_L}+)=0$, $f_2(\sqrt{\varepsilon_1}-)=f_2(\sqrt{\varepsilon_1}+)=\frac{\sqrt{\varepsilon_T\varepsilon_L}}{\sqrt{\varepsilon_1}}\left[-\frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}}+\frac{\sqrt{\varepsilon_1}-\sqrt{\varepsilon_T}}{\sqrt{\varepsilon_L(\varepsilon_1-\varepsilon_T)}}\right]$, it is continuous at $t/\rho'=\sqrt{\varepsilon_L}$ and $t/\rho'=\sqrt{\varepsilon_1}$.

$$f_2'\left(\frac{t}{\rho'}\right)=\begin{cases}0, & \frac{t}{\rho'}<\sqrt{\varepsilon_L}\\ \frac{\sqrt{\varepsilon_T\varepsilon_L}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\rho'}\left[-1+\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}\right. & \\ \left.\times\left(\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right)^{-3/2}\right], & \sqrt{\varepsilon_L}<\frac{t}{\rho'}<\sqrt{\varepsilon_1}\\ \frac{1}{\rho'}\frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}}, & \frac{t}{\rho'}>\sqrt{\varepsilon_1}.\end{cases}\tag{67}$$

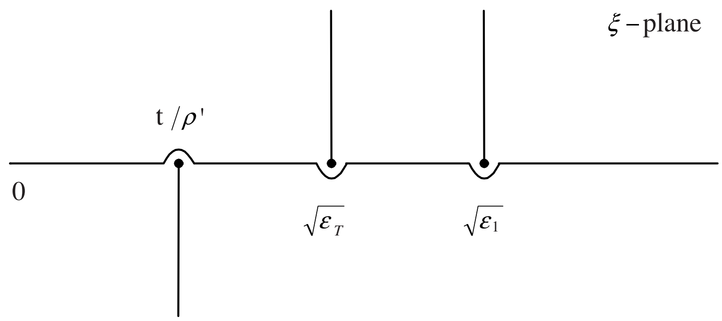

Figure 4. Branch-cut structure for the integral in (27).

Then, the exact formula for I_2 is expressed as follows:

$$I_2 = \frac{c^2\pi}{\rho^2}\begin{cases} 0, & \frac{ct}{\rho} < \sqrt{\varepsilon_L} \\ \frac{\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\Big[-1+\varepsilon_1^2\sqrt{\varepsilon_T\varepsilon_L}\frac{(\varepsilon_1-\varepsilon_L)^{3/2}}{(\varepsilon_1^2-\varepsilon_T\varepsilon_L)^{3/2}} & \\ \times\left(\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right)^{-3/2}\Big], & \sqrt{\varepsilon_L}<\frac{ct}{\rho}<\sqrt{\varepsilon_1} \\ \frac{1}{\varepsilon_1+\sqrt{\varepsilon_T\varepsilon_L}}, & \frac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases} \tag{68}$$

3.4. Evaluation of I_3

Following the same procedures in evaluations of I_2 addressed in Subsection 3.3, I_3 can also be evaluated readily. When $t/\rho' < \sqrt{\varepsilon_T}$,

$$I_3 = 0. \tag{69}$$

When $\sqrt{\varepsilon_T} < t/\rho' < \sqrt{\varepsilon_1}$,

$$\begin{aligned} I_3 \;=\; \frac{\partial}{\partial t}\,\mathrm{Im}\Big\{\int_0^\infty \frac{\xi d\xi}{\xi^2\rho'}\Big[&\frac{1}{\sqrt{\varepsilon_1-\xi^2}+i\sqrt{\xi^2-\varepsilon_T}} \\ &-\frac{t}{\rho'}\frac{1}{(\sqrt{\varepsilon_1-\xi^2}+i\sqrt{\xi^2-\varepsilon_T})(\sqrt{t^2/\rho'^2-\xi^2})}\Big]\Big\}. \end{aligned} \tag{70}$$

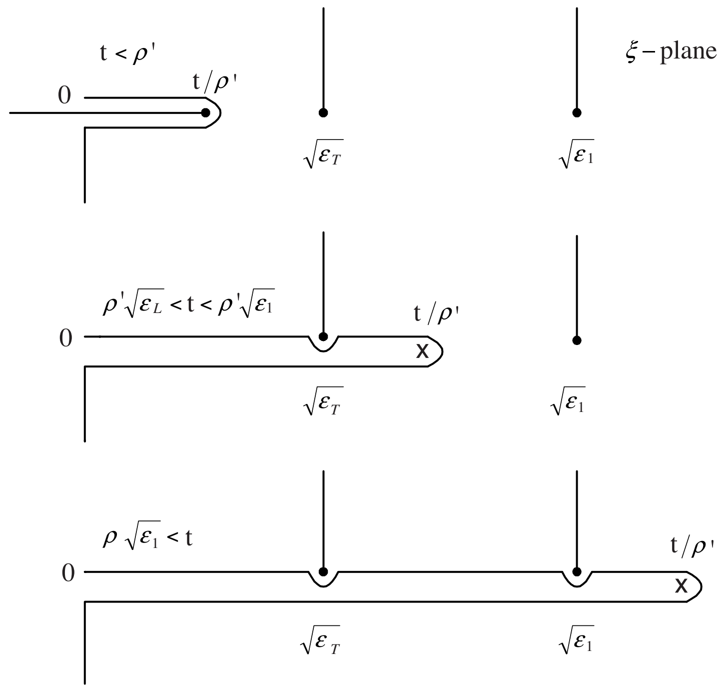

Figure 5. Contours of integration for the integral in (27).

The real and imaginary parts are separated readily and only the imaginary part is retained. Then, the result reduces to

$$I_3 = -\frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'} \frac{\partial}{\partial t} \left\{ \int_0^\infty \frac{\xi d\xi}{\xi^2} \sqrt{\xi^2 - \varepsilon_T} - \frac{t}{\rho'} \int_0^\infty \frac{\xi d\xi}{\xi^2} \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} \right\}. \tag{71}$$

With the contour in Fig. 5, we have

$$I_3 = \frac{2}{(\varepsilon - \varepsilon_T)\rho'} \frac{\partial}{\partial t} \left\{ \frac{t}{\rho'} \int_{\sqrt{\varepsilon_T}}^{t/\rho'} \frac{\xi d\xi}{\xi^2} \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} \right\}. \tag{72}$$

With the change of variable $\zeta = \xi^2$, $d\zeta = 2\xi d\xi$, this becomes

$$I_3 = \frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'} \frac{\partial}{\partial t} \left\{ \frac{t}{\rho'} \int_{\varepsilon_T}^{t^2/\rho'^2} \frac{d\zeta}{\zeta} \frac{\sqrt{\zeta - 1}}{\sqrt{t^2/\rho'^2 - \zeta}} \right\}. \tag{73}$$

The integral in (73) can be evaluated easily.

$$\begin{aligned}\vartheta_0^{(4)} &= \int_{\varepsilon_T}^{t^2/\rho'^2} \frac{d\zeta}{\zeta} \frac{\sqrt{\zeta - 1}}{\sqrt{t^2/\rho'^2 - \zeta}} \\ &= \int_{\varepsilon_T}^{t^2/\rho'^2} \frac{d\zeta}{\sqrt{(t^2/\rho'^2 - \zeta)(\zeta - \varepsilon_T)}} - \int_{\varepsilon_T}^{t^2/\rho'^2} \frac{d\zeta}{\zeta\sqrt{(t^2/\rho'^2 - \zeta)(\zeta - \varepsilon_T)}} \\ &= \pi\left[1 - \sqrt{\varepsilon_T}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right]. \end{aligned} \tag{74}$$

Substituting (74) into (73), we get

$$I_3 = \frac{\pi}{(\varepsilon_1 - \varepsilon_T)\rho'} \frac{\partial}{\partial t}\left[1 - \sqrt{\varepsilon_T}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right], \quad \sqrt{\varepsilon_T} < \frac{ct}{\rho} < \sqrt{\varepsilon_1}. \tag{75}$$

When $t/\rho' > \sqrt{\varepsilon_1}$, with the contour in Fig. 5, we write

$$\begin{aligned} I_3 = \frac{\partial}{\partial t} \,\mathrm{Im}\Bigg\{ &\int_0^{\sqrt{\varepsilon_1}} \frac{\xi d\xi}{\xi^2\rho'^2} \frac{1}{\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T}} \\ &- \frac{t}{\rho'} \int_0^{\sqrt{\varepsilon_1}} \frac{\xi d\xi}{\xi^2\rho'^2} \frac{1}{(\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T})\sqrt{t^2/\rho'^2 - \xi^2}} \\ &+ \int_{\sqrt{\varepsilon_1}}^{\infty} \frac{\xi d\xi}{\xi^2\rho'^2} \frac{1}{i\sqrt{\xi^2 - \varepsilon_1} + i\sqrt{\xi^2 - \varepsilon_T}} \\ &- \frac{t}{\rho'} \int_{\sqrt{\varepsilon_1}}^{\infty} \frac{\xi d\xi}{\xi^2\rho'^2} \frac{1}{(i\sqrt{\xi^2 - \varepsilon_1} + i\sqrt{\xi^2 - \varepsilon_T})\sqrt{t^2/\rho'^2 - \xi^2}} \Bigg\}. \end{aligned} \tag{76}$$

The contributing imaginary part is

$$\begin{aligned} I_3 = -\frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'^2} \frac{\partial}{\partial t} \Bigg\{ &\int_0^{\infty} \frac{\xi d\xi}{\xi^2} \sqrt{\xi^2 - \varepsilon_T} - \frac{t}{\rho'} \int_0^{\infty} \frac{\xi d\xi}{\xi^2} \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} \\ &- \int_{\sqrt{\varepsilon_1}}^{\infty} \frac{\xi d\xi}{\xi^2} \sqrt{\xi^2 - \varepsilon_1} + \frac{t}{\rho'} \int_{\sqrt{\varepsilon_1}}^{\infty} \frac{\xi d\xi}{\xi^2} \frac{\sqrt{\xi^2 - \varepsilon_1}}{\sqrt{t^2/\rho'^2 - \xi^2}} \Bigg\}. \end{aligned} \tag{77}$$

In terms of the variable $\zeta = \xi^2$, the result becomes

$$I_3 = \frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'^2}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int_{\varepsilon_T}^{t^2/\rho'^2}\frac{d\zeta}{\zeta}\frac{\sqrt{\zeta-\varepsilon_T}}{\sqrt{t^2/\rho'^2-\zeta}}\right\}$$
$$-\frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'^2}\frac{\partial}{\partial t}\left\{\frac{t}{\rho'}\int_{\varepsilon_1}^{t^2/\rho'^2}\frac{d\zeta}{\zeta}\frac{\sqrt{\zeta-\varepsilon_T}}{\sqrt{t^2/\rho'^2-\zeta}}\right\}. \quad (78)$$

The evaluation of the first integral in (78) is found in (74). Next, the second integral will be evaluated.

$$\begin{aligned}\vartheta_\varepsilon^{(4)} &= \int_{\varepsilon_1}^{t^2/\rho'^2}\frac{d\zeta}{\zeta}\frac{\sqrt{\zeta-\varepsilon_1}}{\sqrt{t^2/\rho'^2-\zeta}}\\ &= \int_{\varepsilon_1}^{t^2/\rho'^2}\frac{d\zeta}{\sqrt{(t^2/\rho'^2-\zeta)(\zeta-\varepsilon_1)}} - \varepsilon_1\int_{\varepsilon_1}^{t^2/\rho'^2}\frac{d\zeta}{\zeta\sqrt{(t^2/\rho'^2-\zeta)(\zeta-\varepsilon_1)}}\\ &= \pi\left[1-\sqrt{\varepsilon_1}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right]. \quad (79)\end{aligned}$$

With substitution (74) and (79) into (78), we obtain

$$I_3 = \frac{\pi}{(\varepsilon_1 - \varepsilon_T)\rho'}\frac{\partial}{\partial t}\left[\frac{t}{\rho'}(\sqrt{\varepsilon_1}-\sqrt{\varepsilon_T})\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right], \quad \frac{ct}{\rho} > \sqrt{\varepsilon_1}. \quad (80)$$

Combined with (69), (75), and (80), we write

$$I_3 = \frac{\pi}{(\varepsilon_1 - \varepsilon_T)\rho'}\frac{\partial}{\partial t}\, f_3\left(\frac{t}{\rho'}\right), \quad (81)$$

where

$$f_3\left(\frac{t}{\rho'}\right) = \frac{t}{\rho'}\begin{cases} 0, & \frac{ct}{\rho} < \sqrt{\varepsilon_T} \\ \left[1-\sqrt{\varepsilon_T}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right], & \sqrt{\varepsilon_T} < \frac{ct}{\rho} < \sqrt{\varepsilon_1} \\ (\sqrt{\varepsilon_1}-\sqrt{\varepsilon_T})\left(\frac{t^2}{\rho'^2}\right)^{-1/2}, & \frac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases}. \quad (82)$$

Obviously, $f_3(\sqrt{\varepsilon_T}-) = f_3(\sqrt{\varepsilon_T}+) = 0$, $f_3(\sqrt{\varepsilon_1}-) = f_3(\sqrt{\varepsilon}+) = \sqrt{\varepsilon_1} - \sqrt{\varepsilon_T}$, it is continuous at $t/\rho' = 1$ and $t/\rho' = \sqrt{\varepsilon_1}$.

$$f_3'\left(\frac{t}{\rho'}\right) = \begin{cases} 0 \quad , & \dfrac{ct}{\rho} < \sqrt{\varepsilon_T} \\ \dfrac{1}{\rho'} \quad , & \sqrt{\varepsilon_T} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ 0 \quad , & \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases} . \tag{83}$$

Finally, the exact formula for I_3 is obtained readily.

$$I_3 = \frac{c^2\pi}{(\varepsilon_1 - \varepsilon_T)\rho^2} \begin{cases} 0 \quad , & \dfrac{ct}{\rho} < \sqrt{\varepsilon_T} \\ 1 \quad , & \sqrt{\varepsilon_T} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ 0 \quad , & \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases} . \tag{84}$$

3.5. Evaluation of $E_{2\rho}(\rho, 0; t)$

Combined with (24), (50), (68), and (84), the exact formula for the electric field component $E_{2\rho}(\rho, 0; t)$ can be expressed in terms of elementary functions.

$$E_{2\rho}(\rho,0;t) = \frac{1}{2\pi\varepsilon_0 c\rho^2}\left[\frac{1}{\sqrt{\varepsilon_T}}\delta\left(t - \frac{\sqrt{\varepsilon_L}\rho}{c}\right) + \frac{1}{\sqrt{\varepsilon_1}}\delta\left(t - \frac{\sqrt{\varepsilon_1}\rho}{c}\right)\right] + \frac{1}{2\pi\varepsilon_0\rho^3}$$
$$\times \begin{cases} 0, & \dfrac{ct}{\rho} < \sqrt{\varepsilon_L} \\ -\dfrac{2\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon^2 - \varepsilon_T\varepsilon_L} \dfrac{\varepsilon^2\varepsilon_T\varepsilon_L(\varepsilon_1 - \varepsilon_L)^{3/2}}{(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)^{5/2}} \left[\dfrac{c^2t^2}{\rho^2} + \dfrac{2\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon^2 - \varepsilon_T\varepsilon_L}\right] & \\ \quad \cdot \left[\dfrac{c^2t^2}{\rho^2} - \dfrac{\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\right]^{-5/2}, & \sqrt{\varepsilon_L} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_T} \\ \dfrac{1}{\varepsilon_1 - \varepsilon_T} - \dfrac{2\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon^2 - \varepsilon_T\varepsilon_L} \dfrac{\varepsilon^2\varepsilon_T\varepsilon_L(\varepsilon_1 - \varepsilon_L)^{3/2}}{(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)^{5/2}} \left[\dfrac{c^2t^2}{\rho^2} + \dfrac{2\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon^2 - \varepsilon_T\varepsilon_L}\right] & \\ \quad \cdot \left[\dfrac{c^2t^2}{\rho^2} - \dfrac{\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\right]^{-5/2}, & \sqrt{\varepsilon_T} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ \dfrac{2}{\varepsilon_1 + \sqrt{\varepsilon_T\varepsilon_L}}, & \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases} . \tag{85}$$

From (85), it is seen that the amplitude of the pulsed field along the boundary is $1/\rho^2$, which is characteristic of the surface-wave or lateral pulse. In particular, it is found that the first pulse arrives at $t = \sqrt{\varepsilon_L}\rho/c$ has travelled along the boundary in Region 2 (anisotropic medium) with the velocity $c/\sqrt{\varepsilon_L}$ and the second pulse arrives at $t = \sqrt{\varepsilon_1}\rho/c$ has travelled along the boundary in Region 1 with the velocity $c/\sqrt{\varepsilon_1}$.

4. TIME-DEPENDENT COMPONENT $E_{2\phi}$ DUE TO A HORIZONTAL DIPOLE WITH A DELTA-FUNCTION EXCITATION

4.1. The Integration for Time-dependent Component $E_{2\phi}$

Similar to the time-dependent component $E_{2\rho}$ addressed in Section 3, the time-dependent component $E_{2\phi}$ due to a horizontal dipole with a delta-function excitation can be written as follows:

$$E_{2\phi}(\rho', \pi/2; t) = \frac{1}{\pi}\text{Re}\int_0^\infty e^{-i\omega t}\widetilde{E}_{2\phi}(\rho', \pi/2; \omega)d\omega. \tag{86}$$

With substitution (14) into (86), we write

$$\begin{aligned}E_{2\phi}(\rho', \pi/2; t) = &\frac{\mu_0}{2\pi^2 c}\int_0^\infty d\omega e^{-i\omega t}\omega \\ &\times\text{Re}\int_0^\infty d\lambda'\lambda'\left\{\frac{\sqrt{\omega^2\varepsilon_1-\lambda'^2}\sqrt{\omega^2\varepsilon_L-\lambda'^2}}{\omega^2\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\omega^2\varepsilon_1\sqrt{\omega^2\varepsilon_L-\lambda'^2}}\frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right. \\ &\left.+\frac{1}{\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\sqrt{\omega^2\varepsilon_T-\lambda'^2}}\left[J_0(\lambda'\rho') - \frac{1}{\lambda'\rho'}J_1(\lambda'\rho')\right]\right\}. \end{aligned}\tag{87}$$

With the definition $\lambda' = \omega\xi$, $d\lambda' = \omega d\xi$, (87) reads as

$$\begin{aligned}E_{2\phi}(\rho', \pi/2; t) = &\frac{\mu_0}{2\pi^2 c}\text{Re}\int_0^\infty \xi d\xi\left\{\frac{\sqrt{\varepsilon_1-\xi^2}\sqrt{\varepsilon_L-\xi^2}}{\sqrt{\varepsilon_T\varepsilon_L}\sqrt{\varepsilon_1-\xi^2}+\varepsilon_1\sqrt{\varepsilon_L-\xi^2}}\frac{i}{\xi\rho'}\right. \\ &\times\partial t\int_0^\infty d\omega e^{-i\omega t}J_1(\omega\xi\rho') - \frac{1}{\sqrt{\varepsilon_1-\xi^2}+\sqrt{\varepsilon_T-\xi^2}} \\ &\left.\times\left[\frac{i}{\xi\rho'}\frac{\partial}{\partial t}\int_0^\infty d\omega e^{-i\omega t}J_1(\omega\xi\rho') + \frac{\partial^2}{\partial t^2}\int_0^\infty d\omega e^{-i\omega t}J_0(\omega\xi\rho')\right]\right\}. \end{aligned}\tag{88}$$

Taking into account the relationships in (22) and (23), (88) can be rewritten as follows:

$$E_{2\phi}(\rho', \pi/2; t) = \frac{\mu_0}{2\pi^2 \rho' c} \left[I_2 + I_3 + I_4 \right], \tag{89}$$

where I_2 and I_3 are expressed in (26) and (27), and I_4 is expressed as follows:

$$I_4 = -\frac{\partial^2}{\partial t^2} \,\mathrm{Im} \int_0^\infty \xi d\xi \frac{1}{\sqrt{\varepsilon_1 - \xi^2} + \sqrt{\varepsilon_T - \xi^2}} \frac{1}{\sqrt{t^2/\rho'^2 - \xi^2}}. \tag{90}$$

It is seen that the evaluations for I_2 and I_3 have been found in (68) and (84), respectively. In the next step, we will evaluate I_4.

4.2. Evaluation of I_4

Following the similar procedures in the evaluations of I_2 and I_3, the evaluation of I_4 can be carried out readily. When $t/\rho' < \sqrt{\varepsilon_T}$,

$$I_4 = 0. \tag{91}$$

When $\sqrt{\varepsilon_T} < t/\rho' < \sqrt{\varepsilon_1}$, with the contour in Fig. 5, we have

$$\begin{aligned} I_4 = -\frac{\partial^2}{\partial t^2} \,\mathrm{Im} \Bigg\{ & \int_0^{\sqrt{\varepsilon_T}} \xi d\xi \Bigg[\frac{1}{(\sqrt{\varepsilon_1 - \xi^2} + \sqrt{\varepsilon_T - \xi^2})\sqrt{t^2/\rho'^2 - \xi^2}} \\ & - \frac{1}{(\sqrt{\varepsilon_1 - \xi^2} + \sqrt{\varepsilon_T - \xi^2})(-\sqrt{t^2/\rho'^2 - \xi^2})} \Bigg] \\ & + \int_{\sqrt{\varepsilon_T}}^{t/\rho'} \xi d\xi \Bigg[\frac{1}{(\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T})\sqrt{t^2/\rho'^2 - \xi^2}} \\ & - \frac{1}{(\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T})(-\sqrt{t^2/\rho'^2 - \xi^2})} \Bigg] \Bigg\}. \end{aligned} \tag{92}$$

Because the integrand of the first integral is *real*, the contributing imaginary part is

$$I_4 = -\frac{\partial^2}{\partial t^2} \,\mathrm{Im} \int_{\sqrt{\varepsilon_T}}^{t/\rho'} \xi d\xi \frac{2}{(\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T})\sqrt{t^2/\rho'^2 - \xi^2}}. \tag{93}$$

The imaginary part becomes,

$$I_4 = \frac{2}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \int_{\sqrt{\varepsilon_T}}^{t/\rho'} \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}}. \tag{94}$$

With $\zeta = \xi^2$, it follows

$$I_4 = \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \int_{\varepsilon_T}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \zeta}}. \tag{95}$$

Let $T' = t^2/\rho'^2 - \varepsilon_T$, and $x' = \zeta - \varepsilon_T$, the integral in (95) can be evaluated as follows:

$$\begin{aligned} \vartheta_0^{(4)} &= \int_{\varepsilon_T}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \zeta}} = \int_0^{T'} \frac{x' dx'}{\sqrt{(T' - x')x'}} \\ &= \frac{\pi}{2}\left(\frac{t^2}{\rho'^2} - \varepsilon_T\right). \end{aligned} \tag{96}$$

Thus,

$$I_4 = \frac{\pi}{2(\varepsilon_1 - \varepsilon_T)} \frac{\partial^2}{\partial t^2} \left(\frac{t^2}{\rho'^2} - \varepsilon_T\right), \quad \sqrt{\varepsilon_T} < \frac{t}{\rho'} < \sqrt{\varepsilon_1}. \tag{97}$$

When $t/\rho' > \sqrt{\varepsilon_1}$, with the contour in Fig. 5, we have

$$\begin{aligned} I_4 = -\frac{\partial^2}{\partial t^2} \operatorname{Im}\Bigg\{ &\int_{\sqrt{\varepsilon_T}}^{\sqrt{\varepsilon_1}} \xi d\xi \frac{2}{(\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T})\sqrt{t^2/\rho'^2 - \xi^2}} \\ &+ \int_{\sqrt{\varepsilon_1}}^{\frac{t}{\rho'}} \xi d\xi \frac{2}{(i\sqrt{\xi^2 - \varepsilon_1} + i\sqrt{\xi^2 - \varepsilon_T})\sqrt{t^2/\rho'^2 - \xi^2}} \Bigg\}. \end{aligned} \tag{98}$$

The contributing imaginary part is

$$\begin{aligned} I_4 &= \frac{2}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \left\{ \int_{\sqrt{\varepsilon_T}}^{\sqrt{\varepsilon_1}} \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} + \int_{\sqrt{\varepsilon_1}}^{t/\rho'} \xi d\xi \frac{\sqrt{\xi^2 - \sqrt{\varepsilon_T}} - \sqrt{\varepsilon_1 - \xi^2}}{\sqrt{t^2/\rho'^2 - \xi^2}} \right\} \\ &= \frac{2}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \left\{ \int_{\sqrt{\varepsilon_T}}^{t/\rho'} \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} - \int_{\sqrt{\varepsilon_1}}^{t/\rho'} \xi d\xi \frac{\sqrt{\varepsilon_1 - \xi^2}}{\sqrt{t^2/\rho'^2 - \xi^2}} \right\}. \end{aligned} \tag{99}$$

In terms of the variable $\zeta = \xi^2$, it follows

$$I_4 = \frac{1}{\varepsilon_1 - \varepsilon_T}\frac{\partial^2}{\partial t^2}\left\{\int\limits_{\varepsilon_T}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \zeta}} - \int\limits_{\varepsilon_1}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\varepsilon_1 - \zeta}}{\sqrt{t^2/\rho'^2 - \zeta}}\right\}. \quad (100)$$

The first integral in (100) has been evaluated in (96), and the second one in (100) need to be evaluated. Let $T_\varepsilon = t^2/\rho'^2 - \varepsilon_1$ and $y' = \zeta - \varepsilon_1$, the second integral in (100) is written as

$$\vartheta_\varepsilon^{(4)} = \int\limits_{\varepsilon_1}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_1}}{\sqrt{t^2/\rho'^2 - \zeta}} = \int\limits_0^{T_\varepsilon} \frac{y' dy'}{\sqrt{(T_\varepsilon - y')y'}}. \quad (101)$$

The above integral can be evaluated readily.

$$\vartheta_\varepsilon^{(4)} = \frac{\pi}{2}\left(\frac{t^2}{\rho'^2} - \varepsilon_1\right). \quad (102)$$

Substituting (96) and (102) into (100), we have

$$I_4 = \frac{\pi}{2}, \quad \frac{t}{\rho'} > \sqrt{\varepsilon}. \quad (103)$$

From (91), (97), and (103), it follows

$$I_4 = \frac{\pi}{2}\frac{\partial^2}{\partial t^2} f_4\left(\frac{t}{\rho'}\right), \quad (104)$$

where

$$f_4\left(\frac{t}{\rho'}\right) = \begin{cases} 0 & , \quad \dfrac{t}{\rho'} < \sqrt{\varepsilon_T} \\ \dfrac{1}{\varepsilon - 1}\left(\dfrac{t^2}{\rho'^2} - \varepsilon_T\right) & , \quad \sqrt{\varepsilon_T} < \dfrac{t}{\rho'} < \sqrt{\varepsilon_1} \\ 1 & , \quad \dfrac{t}{\rho'} > \sqrt{\varepsilon_1} \end{cases}. \quad (105)$$

Obviously, $f_4(\sqrt{\varepsilon_T}-) = f_4(\sqrt{\varepsilon_T}+) = 0$ and $f_4(\sqrt{\varepsilon_1}-) = f_4(\sqrt{\varepsilon_1}+) = 1$, it follows that $f_4(t/\rho')$ is everywhere continuous.

$$f_4'\left(\frac{t}{\rho'}\right) = \begin{cases} 0 & , \quad \dfrac{t}{\rho'} < \sqrt{\varepsilon_T} \\ \dfrac{1}{\varepsilon_1 - \varepsilon_T}\dfrac{2t}{\rho'^2} & , \quad \sqrt{\varepsilon_T} < \dfrac{t}{\rho'} < \sqrt{\varepsilon_1} \\ 0 & , \quad \dfrac{t}{\rho'} > \sqrt{\varepsilon_1} \end{cases}. \quad (106)$$

Since $f_4'(1-) = 0$, $f_4'(1+) = 2\sqrt{\varepsilon_T}/(\varepsilon_1 - \varepsilon_T)\rho'$, $f_1'(t/\rho')$ has a step discontinuity of $2\sqrt{\varepsilon_T}/(\varepsilon_1 - \varepsilon_T)\rho'$ at $t/\rho' = \sqrt{\varepsilon_T}$. Similarly, $f_4'(\sqrt{\varepsilon}-) = 2\sqrt{\varepsilon_1}/(\varepsilon_1 - \varepsilon_T)\rho'$, $f_4'(\sqrt{\varepsilon}+) = 0$, $f_4'(t/\rho')$ has a step discontinuity of $-2\sqrt{\varepsilon_1}/(\varepsilon_1 - \varepsilon_T)\rho'$ at $t/\rho' = \sqrt{\varepsilon_1}$. Thus,

$$f_4''\left(\frac{t}{\rho'}\right) = \begin{cases} 0 & , \quad \dfrac{t}{\rho'} < \sqrt{\varepsilon_T} \\ \dfrac{2}{(\varepsilon_1 - \varepsilon_T)\rho'^2} & , \quad \sqrt{\varepsilon_T} < \dfrac{t}{\rho'} < \sqrt{\varepsilon_1} \\ 0 & , \quad \dfrac{t}{\rho'} > \sqrt{\varepsilon_1} \end{cases} . \tag{107}$$

Then, the exact formula for I_4 is written as follows:

$$\begin{aligned} I_4 = {} & \frac{c\pi}{(\varepsilon_1 - \varepsilon_T)\rho}\left[\sqrt{\varepsilon_T}\delta\left(t - \frac{\rho}{c}\right) - \sqrt{\varepsilon_1}\delta\left(t - \frac{\sqrt{\varepsilon}\rho}{c}\right)\right] \\ & + \frac{c^2\pi}{(\varepsilon_1 - \varepsilon_T)\rho^2}\begin{cases} 0 & , \quad \dfrac{ct}{\rho} < \sqrt{\varepsilon_T} \\ 1 & , \quad \sqrt{\varepsilon_T} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ 0 & , \quad \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases} . \end{aligned} \tag{108}$$

4.3. Evaluation of $E_{2\phi}(\rho, \pi/2; t)$

Substituting (68), (84), and (108) into (89), the exact formulas for $E_{2\phi}$ can be expressed in terms of elementary functions.

$$\begin{aligned} E_{2\phi}(\rho, \pi/2; t) = {} & \frac{1}{2\pi\varepsilon_0(\varepsilon_1 - \varepsilon_T)c\rho^2}\left[\sqrt{\varepsilon_T}\delta\left(t - \frac{\sqrt{\varepsilon_T}\rho}{c}\right) - \sqrt{\varepsilon_1}\delta\left(t - \frac{\sqrt{\varepsilon_1}\rho}{c}\right)\right] \\ & + \frac{1}{2\pi\varepsilon_0\rho^3}\begin{cases} 0, & \dfrac{ct}{\rho} < \sqrt{\varepsilon_L} \\ -\dfrac{\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} + \dfrac{\varepsilon_1^2\varepsilon_T\varepsilon_L(\varepsilon_1 - \varepsilon_L)^{3/2}}{(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)^{5/2}}\left[\dfrac{c^2t^2}{\rho^2} - \dfrac{\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\right]^{-3/2}, & \sqrt{\varepsilon_L} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_T} \\ \dfrac{2}{\varepsilon_1 - \varepsilon_T} - \dfrac{\sqrt{\varepsilon_T\varepsilon_L}}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} + \dfrac{\varepsilon_1^2\varepsilon_T\varepsilon_L(\varepsilon_1 - \varepsilon_L)^{3/2}}{(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)^{5/2}} \\ \quad \times \left[\dfrac{c^2t^2}{\rho^2} - \dfrac{\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\right]^{-3/2}, & \sqrt{\varepsilon_T} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ \dfrac{1}{\varepsilon_1 + \sqrt{\varepsilon_T\varepsilon_L}}, & \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases} . \end{aligned} \tag{109}$$

It is seen that the first pulse has travelled at $t = \sqrt{\varepsilon_T}\rho/c$ along the boundary in Region 2 with the velocity $c\sqrt{\varepsilon_T}$ and the second one has travelled at $t = \sqrt{\varepsilon_1}\rho/c$ along the boundary in Region 1 with the velocity $c\sqrt{\varepsilon_1}$.

5. TIME-DEPENDENT COMPONENT B_{2Z} DUE TO A HORIZONTAL DIPOLE WITH A DELTA-FUNCTION EXCITATION

When the horizontal electric dipole is radiated by a unit moment that is a delta-function pulse in time, the vertical magnetic field, which is *real*, can also be given by the following Fourier transform.

$$B_{2z}(\rho', \pi/2; t) = \frac{1}{\pi}\mathrm{Re}\int_0^\infty d\omega\ e^{-i\omega t}\widetilde{B}_{2z}(\rho', \pi/2; \omega). \tag{110}$$

With (18), it follows that

$$B_{2z}(\rho', \pi/2; t) = \mathrm{Re}\frac{i\mu}{2\pi^2 c^2}\int_0^\infty d\omega e^{-i\omega t}\int_0^\infty d\lambda'\lambda'^2\frac{J_1(\lambda'\rho')}{\sqrt{\omega^2\varepsilon_1-\lambda'^2}+\sqrt{\omega^2\varepsilon_T-\lambda'^2}}. \tag{111}$$

With the definition $\lambda' = \omega\xi$, $d\lambda' = \omega d\xi$, we write

$$B_{2z}(\rho', \pi/2; t) = \mathrm{Re}\frac{i\mu_0}{2\pi^2 c^2}\frac{\partial^2}{\partial t^2}\int_0^\infty\frac{\xi^2 d\xi}{\sqrt{\varepsilon_1-\xi^2}+\sqrt{\varepsilon_T-\xi^2}}\int_0^\infty d\omega\ e^{-i\omega t}J_1(\omega\xi\rho'). \tag{112}$$

Taking into account the relationship of (23), the result becomes

$$\begin{aligned} B_{2z}(\rho', \pi/2; t) &= \mathrm{Re}\frac{i\mu_0}{2\pi^2 c^2}\frac{\partial^2}{\partial t^2}\int_0^\infty\frac{\xi^2 d\xi}{\sqrt{\varepsilon_1-\xi^2}+\sqrt{\varepsilon_T-\xi^2}}\frac{1}{\xi\rho'}\left(1-\frac{t}{\sqrt{t^2-\xi^2\rho'^2}}\right)\\ &= -\frac{\mu_0}{2\pi^2 c^2\rho'}\frac{\partial^2}{\partial t^2}\mathrm{Im}\left[\frac{t}{\rho'}\int_0^\infty\frac{\xi d\xi}{(\sqrt{\varepsilon_1-\xi^2}+\sqrt{\varepsilon_T-\xi^2})\sqrt{t^2/\rho'^2-\xi^2}}\right]. \end{aligned} \tag{113}$$

Following the same procedures in the evaluations of I_2 and I_3, the evaluation of (113) can be carried out readily. When $t/\rho' < \sqrt{\varepsilon_T}$,

$$B_{2z}(\rho', \pi/2; t) = 0. \tag{114}$$

When $\sqrt{\varepsilon_T} < t/\rho' < \sqrt{\varepsilon_1}$, with the contour in Fig. 5, it becomes

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{2\pi^2 c^2 \rho'} \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \int_0^\infty \xi d\xi \frac{t}{\rho'} \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}}. \quad (115)$$

With $\zeta = \xi^2$, we get

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{2\pi^2 c^2 \rho'} \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \left[\frac{t}{\rho'} \int_{\varepsilon_T}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \zeta}} \right]. \quad (116)$$

The integral in (116) has been evaluated in (96). Thus

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{4\pi c^2 \rho'} \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \left[\frac{t}{\rho'} \left(\frac{t^2}{\rho'^2} - \varepsilon_T \right) \right], \quad \sqrt{\varepsilon_T} < \frac{t}{\rho'} < \sqrt{\varepsilon_1}. \quad (117)$$

When $t/\rho' > \sqrt{\varepsilon_1}$, with the contour in Fig.5, the result is

$$\begin{aligned} B_{2z}(\rho', \pi/2; t) \; = \; & \frac{\mu_0}{2\pi^2 c^2 \rho'} \frac{\partial^2}{\partial t^2} \operatorname{Im} \Bigg[\int_0^{\sqrt{\varepsilon_1}} \frac{\xi d\xi}{\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T}} \\ & - \frac{t}{\rho'} \int_0^{\sqrt{\varepsilon_1}} \frac{\xi d\xi}{(\sqrt{\varepsilon_1 - \xi^2} + i\sqrt{\xi^2 - \varepsilon_T}) \cdot \sqrt{t^2/\rho'^2 - \xi^2}} \\ & + \int_{\sqrt{\varepsilon_1}}^\infty \frac{\xi d\xi}{i\sqrt{\xi^2 - \varepsilon_1} + i\sqrt{\xi^2 - \varepsilon_T}} \\ & - \frac{t}{\rho'} \int_{\sqrt{\varepsilon_1}}^\infty \frac{\xi d\xi}{(i\sqrt{\xi^2 - \varepsilon_1} + i\sqrt{\xi^2 - \varepsilon_T}) \cdot \sqrt{t^2/\rho'^2 - \xi^2}} \Bigg] \\ = \; & -\frac{\mu_0}{2\pi^2 c^2 \rho'} \cdot \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \Bigg[\int_0^{\sqrt{\varepsilon_1}} \xi d\xi \sqrt{\xi^2 - \varepsilon_T} \\ & - \frac{t}{\rho'} \int_0^{\sqrt{\varepsilon_1}} \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} + \int_{\sqrt{\varepsilon_1}}^\infty \xi d\xi \left(\sqrt{\xi^2 - \varepsilon_T} - \sqrt{\xi^2 - \varepsilon_1} \right) \\ & - \frac{t}{\rho'} \int_{\sqrt{\varepsilon_1}}^\infty \xi d\xi \frac{(\sqrt{\xi^2 - \varepsilon_T} - \sqrt{\xi^2 - \varepsilon_1})}{\sqrt{t^2/\rho'^2 - \xi^2}} \Bigg]. \end{aligned} \quad (118)$$

Then,

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{2\pi^2 c^2 \rho'} \cdot \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \times \left[\frac{t}{\rho'} \int_0^\infty \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \xi^2}} - \frac{t}{\rho'} \int_{\sqrt{\varepsilon_1}}^\infty \xi d\xi \frac{\sqrt{\xi^2 - \varepsilon_1}}{\sqrt{t^2/\rho'^2 - \xi^2}} \right]. \tag{119}$$

In terms of the variable $\zeta = \xi^2$, $d\zeta = 2\xi d\xi$, it follows

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{2\pi^2 c^2 \rho'} \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \times \left[\frac{t}{\rho'} \left(\int_{\varepsilon_T}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_T}}{\sqrt{t^2/\rho'^2 - \zeta}} - \int_{\varepsilon_1}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_1}}{\sqrt{t^2/\rho'^2 - \zeta}} \right) \right]. \tag{120}$$

The two integrals in (120) have been evaluated in (96) and (102). Then, we get

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{4\pi c^2 \rho'} \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{\partial^2}{\partial t^2} \left[\frac{t}{\rho'} (\varepsilon - 1) \right], \quad \frac{t}{\rho'} > \sqrt{\varepsilon_1}. \tag{121}$$

With substitutions (114), (117), and (121) into (113), it follows

$$B_{2z}(\rho', \pi/2; t) = \frac{\mu_0}{4\pi c^2 \rho'} \frac{\partial^2}{\partial t^2} f_5\left(\frac{t}{\rho'}\right), \tag{122}$$

where

$$f_5\left(\frac{t}{\rho'}\right) = \frac{1}{\varepsilon_1 - \varepsilon_T} \frac{t}{\rho'} \begin{cases} 0 & , \quad \frac{t}{\rho'} < \sqrt{\varepsilon_T} \\ \frac{t^2}{\rho'^2} - \varepsilon_T & , \quad \sqrt{\varepsilon_T} < \frac{t}{\rho'} < \sqrt{\varepsilon_1} \\ \varepsilon_1 - \varepsilon_T & , \quad \frac{t}{\rho'} > \sqrt{\varepsilon_1} \end{cases}. \tag{123}$$

Obviously, $f(\sqrt{\varepsilon_T}-) = f(\sqrt{\varepsilon_T}+) = 0$ and $f(\sqrt{\varepsilon_1}-) = f(\sqrt{\varepsilon_1}+) = \sqrt{\varepsilon_1}$, $f_5(t/\rho')$ is everywhere continuous.

$$f_5'\left(\frac{t}{\rho'}\right) = \frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'} \begin{cases} 0 & , \quad \frac{t}{\rho'} < \sqrt{\varepsilon_T} \\ \frac{3t^2}{\rho'^2} - \varepsilon_T & , \quad \sqrt{\varepsilon_T} < \frac{t}{\rho'} < \sqrt{\varepsilon_1} \\ \varepsilon_1 - \varepsilon_T & , \quad \frac{t}{\rho'} > \sqrt{\varepsilon_1} \end{cases}. \tag{124}$$

Since $f_5'(\sqrt{\varepsilon_T}-) = 0$, $f_5'(\sqrt{\varepsilon_T}+) = 2\varepsilon_T/[(\varepsilon_1 - \varepsilon_T)\rho']$, $f'(t/\rho')$ has a step discontinuity of $2\varepsilon_T/[(\varepsilon_1 - \varepsilon_T)\rho']$ at $t/\rho' = \sqrt{\varepsilon_T}$. Similarly, $f'(\sqrt{\varepsilon_1}-) = (3\varepsilon_1 - \varepsilon_T)/[(\varepsilon_1 - \varepsilon_T)\rho']$, $f'(\sqrt{\varepsilon_1}+) = 1/\rho'$, $f'(t/\rho')$ has a step discontinuity of $-2\varepsilon_1/[(\varepsilon_1 - \varepsilon_T)\rho']$ at $t/\rho' = \sqrt{\varepsilon_1}$. Thus

$$f_7''\left(\frac{t}{\rho'}\right) = \frac{1}{(\varepsilon_1 - \varepsilon_T)\rho'^3}\begin{cases} 0 & , \quad \dfrac{t}{\rho'} < \sqrt{\varepsilon_T} \\ 6t & , \quad \sqrt{\varepsilon_T} < \dfrac{t}{\rho'} < \sqrt{\varepsilon_1} \\ 0 & , \quad \dfrac{t}{\rho'} > \sqrt{\varepsilon_1} \end{cases}. \tag{125}$$

With substitution (125) into (122) and $\rho' = \rho/c$, the exact formula for the vertical magnetic field is expressed in terms of elementary functions.

$$\begin{aligned} B_{2z}(\rho, \pi/2; t) \;=\; & \frac{1}{2\pi\varepsilon_0 c^2\rho^2}\frac{1}{\varepsilon_1 - \varepsilon_T}\left[\varepsilon_T\delta\left(t - \frac{\sqrt{\varepsilon_T}\rho}{c}\right) - \varepsilon_1\delta\left(t - \frac{\sqrt{\varepsilon_1}\rho}{c}\right)\right] \\ & - \frac{1}{2\pi\varepsilon_0 c\rho^3}\frac{1}{\varepsilon_1 - \varepsilon_T}\begin{cases} 0 & , \quad \dfrac{ct}{\rho} < \sqrt{\varepsilon_T} \\ \dfrac{3ct}{\rho} & , \quad \sqrt{\varepsilon_T} < \dfrac{ct}{\rho} < \sqrt{\varepsilon_1} \\ 0 & , \quad \dfrac{ct}{\rho} > \sqrt{\varepsilon_1} \end{cases}. \end{aligned} \tag{126}$$

6. DISCUSSIONS AND CONCLUSIONS

The *exact* formulas in terms of elementary functions are obtained in time domain for the components of the EM field from a horizontal electric dipole on the boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium. Similar to the isotropic case, the amplitude of the tangential pulsed electric field along the boundary is $1/\rho^2$, which is characteristic of the surface-wave or lateral pulse. Both the tangential electric field and the vertical magnetic field consist of a delta-function pulse travelling in Region 2 (anisotropic medium), a oppositely directed delta-function travelling in Region 1 (isotropic medium), and a final static electric field due to the charges left on the dipole. The pulsed field consists of the first pulse in Region 2 with the velocity $c/\sqrt{\varepsilon_L}$ and the second pulse in Region 1 with the velocity $c/\sqrt{\varepsilon_1}$ for the component $E_\rho(\rho, 0; t)$. Similarly, the pulsed fields consist of a first pulse in Region 2 with the velocity $c\sqrt{\varepsilon_T}$ and the second pulse in Region 1 with the velocity $c\sqrt{\varepsilon_1}$ for the components $E_\phi(\rho, \pi/2; t)$ and $B_z(\rho, \pi/2; t)$. Also it should be noted that the three time-dependent components $E_z(\rho, 0; t)$, $B_\rho(\rho, \pi/2; t)$, and $B_\phi(\rho, 0; t)$ of a horizontal dipole with a delta-function excitation are not expressible in terms of elementary functions.

APPENDIX A. THE EVALUATIONS OF $\vartheta_0^{(1)}$ AND $\vartheta_0^{(2)}$

The two integrals $\vartheta_0^{(1)}$ and $\vartheta_0^{(2)}$ need to be evaluated.

$$\vartheta_0^{(1)} = \int_{\varepsilon_L}^{t^2/\rho'^2} \zeta d\zeta \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}, \quad \text{(A1)}$$

$$\vartheta_0^{(2)} = \int_{\varepsilon_L}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}. \quad \text{(A2)}$$

Let $T = t^2/\rho'^2 - \varepsilon_L$ and $x' = \zeta - \varepsilon_L$, then

$$\vartheta_0^{(1)} = \int_0^T \frac{x'(x' + \varepsilon_L)dx'}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)(x' + \varepsilon_L) - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{(T - x')x'}}, \quad \text{(A3)}$$

$$\vartheta_0^{(2)} = \int_0^T \frac{x'dx'}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)(x' + \varepsilon_L) - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{(T - x')x'}}. \quad \text{(A4)}$$

Let $E = \frac{\varepsilon_T\varepsilon_L(\varepsilon_1 - \varepsilon_L)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}$ and $x = x' + E$, then

$$\vartheta_0^{(1)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \int_E^{T+E} \frac{(x - E)(x + \varepsilon_L - E)dx}{x\sqrt{(T + E - x)}\sqrt{x - E}}, \quad \text{(A5)}$$

$$\vartheta_0^{(2)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \int_E^{T+E} \frac{(x - E)dx}{x\sqrt{(T + E - x)}\sqrt{x - E}}. \quad \text{(A6)}$$

Let $X = (T + E - x)(x - E) = -E(T + E) + (T + 2E)x - x^2$, then

$$\vartheta_0^{(1)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \left\{ \int_E^{T+E} \frac{xdx}{X^{1/2}} + (\varepsilon_L - 2E) \times \int_E^{T+E} \frac{dx}{X^{1/2}} - E(\varepsilon_L - E) \int_E^{T+E} \frac{dx}{xX^{1/2}} \right\}, \quad \text{(A7)}$$

$$\vartheta_0^{(2)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\left\{ \int_E^{T+E} \frac{dx}{X^{1/2}} - E\int_E^{T+E} \frac{dx}{xX^{1/2}} \right\}. \qquad \text{(A8)}$$

So that

$$\vartheta_0^{(1)} = \frac{\pi}{2(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)}\left\{ \frac{t^2}{\rho'^2} - \varepsilon_L + \frac{2\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} - \frac{2\varepsilon_1\sqrt{\varepsilon_T\varepsilon_L}\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \right.$$
$$\left. \times\left[\frac{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}{\varepsilon_1 - \varepsilon_L}\frac{t^2}{\rho'^2} - \frac{\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1 - \varepsilon_L}\right]^{-1/2}\right\}, \qquad \text{(A9)}$$

$$\vartheta_0^{(2)} = \frac{\pi}{(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)}\left\{ 1 - \sqrt{\varepsilon_T\varepsilon_L}\left[\frac{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}{\varepsilon_1 - \varepsilon_L}\frac{t^2}{\rho'^2} - \frac{\varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)}{\varepsilon_1 - \varepsilon_L}\right]^{-1/2}\right\}. \qquad \text{(A10)}$$

APPENDIX B. THE EVALUATIONS OF $\vartheta_\varepsilon^{(1)}$ AND $\vartheta_\varepsilon^{(2)}$

The two integrals $\vartheta_\varepsilon^{(1)}$ and $\vartheta_\varepsilon^{(2)}$ need to be evaluated.

$$\vartheta_\varepsilon^{(1)} = \int_{\varepsilon_1}^{t^2/\rho'^2} \zeta d\zeta \frac{\sqrt{\zeta - \varepsilon_1}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}, \qquad \text{(B1)}$$

$$\vartheta_\varepsilon^{(2)} = \int_{\varepsilon_1}^{t^2/\rho'^2} d\zeta \frac{\sqrt{\zeta - \varepsilon_1}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}. \qquad \text{(B2)}$$

Let $T_\varepsilon = t^2/\rho'^2 - \varepsilon_1$ and $y' = \zeta - \varepsilon_1$, then

$$\vartheta_\varepsilon^{(1)} = \int_0^{T_\varepsilon} \frac{y'(y' + \varepsilon_1)dy'}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)(y' + \varepsilon_1) - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{(T - y')y'}}, \qquad \text{(B3)}$$

$$\vartheta_\varepsilon^{(2)} = \int_0^{T_\varepsilon} \frac{y'dy'}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)(y' + \varepsilon_1) - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{(T - y')y'}}. \qquad \text{(B4)}$$

Let $E_\varepsilon = \frac{\varepsilon_1^2(\varepsilon_1 - \varepsilon_L)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}$ and $y = y' + E_\varepsilon$, then

$$\vartheta_\varepsilon^{(1)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{(y-E_\varepsilon)(y+\varepsilon_L-E_\varepsilon)dy}{y\sqrt{(T_\varepsilon+E_\varepsilon-y)}\sqrt{y-E_\varepsilon}}, \tag{B5}$$

$$\vartheta_\varepsilon^{(2)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{(y-E_\varepsilon)dy}{y\sqrt{(T_\varepsilon+E_\varepsilon-y)}\sqrt{y-E_\varepsilon}}. \tag{B6}$$

Let $Y = (T_\varepsilon + E_\varepsilon - y)(y - E_\varepsilon) = -E_\varepsilon(T_\varepsilon + E_\varepsilon) + (T_\varepsilon + 2E_\varepsilon)y - y^2$, then

$$\begin{aligned}\vartheta_\varepsilon^{(1)} &= \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\left\{ \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{ydy}{Y^{1/2}} + (\varepsilon_1 - 2E_\varepsilon) \right.\\ &\quad \left. \times \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{dy}{Y^{1/2}} - E_\varepsilon(\varepsilon_1 - E_\varepsilon) \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{dy}{yY^{1/2}} \right\},\end{aligned} \tag{B7}$$

$$\vartheta_\varepsilon^{(2)} = \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\left\{ \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{dy}{Y^{1/2}} - E_\varepsilon \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{dy}{yY^{1/2}} \right\}. \tag{B8}$$

So that

$$\begin{aligned}\vartheta_\varepsilon^{(1)} &= \frac{\pi}{2(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)}\left\{ \frac{t^2}{\rho'^2} - \varepsilon_1 + \frac{2\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} - \frac{2\varepsilon_1^2\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L} \right.\\ &\quad \left. \times \left[\frac{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}{\varepsilon_1 - \varepsilon_L}\frac{t^2}{\rho'^2} - \frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L} \right]^{-1/2} \right\},\end{aligned} \tag{B9}$$

$$\vartheta_\varepsilon^{(2)} = \frac{\pi}{(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)}\left\{ 1 - \varepsilon_1 \left[\frac{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}{\varepsilon_1 - \varepsilon_L}\frac{t^2}{\rho'^2} - \frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L} \right]^{-1/2} \right\}. \tag{B10}$$

APPENDIX C. THE EVALUATION OF $\vartheta_0^{(3)}$

The integral $\vartheta_0^{(3)}$ will be evaluated.

$$\vartheta_0^{(3)} = \int_{\varepsilon_L}^{t/\rho'} \frac{d\zeta}{\zeta} \frac{\sqrt{\zeta - \varepsilon_L}}{[(\varepsilon_1^2 - \varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1 - \varepsilon_T)]\sqrt{t^2/\rho'^2 - \zeta}}, \tag{C1}$$

Let $x' = \zeta - \varepsilon_L$, $T = t^2/\rho'^2 - \varepsilon_L$, and $E = \frac{\varepsilon_T\varepsilon_L(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}$, then

$$\begin{aligned}
\vartheta_0^{(3)} &= \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\int_0^T \frac{x'dx'}{(x'+\varepsilon_L)(x'+E)\sqrt{(T-x')x'}} \\
&= \frac{1}{\varepsilon_1^2 - \varepsilon_T\varepsilon_L}\frac{1}{\varepsilon_L - E}\left\{\int_0^T \frac{x'dx'}{(x'+E)\sqrt{(T-x')x'}}\right. \\
&\quad \left. -\int_0^T \frac{x'dx'}{(x'+\varepsilon_L)\sqrt{(T-x')x'}}\right\} \\
&= \frac{1}{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\left\{\int_0^T \frac{x'dx'}{(x'+E)\sqrt{(T-x')x'}}\right. \\
&\quad \left. -\int_0^T \frac{x'dx'}{(x'+\varepsilon_L)\sqrt{(T-x')x'}}\right\}.
\end{aligned} \tag{C2}$$

Let $x = x' + E$ and $X_1 = (T + E - x)(x - E)$, then

$$\begin{aligned}
\int_0^T \frac{x'dx'}{(x'+E)\sqrt{(T-x')x'}} &= \int_E^{T+E} \frac{(x-E)dx}{x\sqrt{(T+E-x)(x-E)}} \\
&= \int_E^{T+E} \frac{dx}{X_1^{1/2}} - E\int_E^{T+E} \frac{dx}{xX_1^{1/2}} \\
&= \pi\left\{1-\sqrt{\varepsilon_T\varepsilon_L}\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}.
\end{aligned} \tag{C3}$$

Similarly, let $x = x' + \varepsilon_L$ and $X_2 = (T + \varepsilon_L - x)(x - \varepsilon_L)$, then

$$\begin{aligned}
\int_0^T \frac{x'dx'}{(x'+\varepsilon_L)\sqrt{(T-x')x'}} &= \int_{\varepsilon_L}^{T+\varepsilon_L} \frac{(x-\varepsilon_L)dx}{x\sqrt{(T+\varepsilon_L-x)(x-\varepsilon_L)}} \\
&= \int_{\varepsilon_L}^{T+\varepsilon_L} \frac{dx}{X_2^{1/2}} - \int_{\varepsilon_L}^{T+\varepsilon_L} \frac{dx}{xX_2^{1/2}} \\
&= \pi\left[1-\sqrt{\varepsilon_L}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right].
\end{aligned} \tag{C4}$$

With substitutions (C3) and (C4) into (C2), we get

$$\vartheta_0^{(3)} = \frac{\pi}{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\left[\sqrt{\varepsilon_L}\left(\frac{t^2}{\rho'^2}\right)^{-1/2} - \sqrt{\varepsilon_T\varepsilon_L}\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2} - \frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right]. \quad \text{(C5)}$$

APPENDIX D. THE EVALUATION OF $\vartheta_\varepsilon^{(3)}$

The integral $\vartheta_\varepsilon^{(3)}$ will be evaluated.

$$\vartheta_\varepsilon^{(3)} = \int_{\varepsilon_L}^{t/\rho'} \frac{d\zeta}{\zeta}\frac{\sqrt{\zeta-\varepsilon_1}}{[(\varepsilon_1^2-\varepsilon_T\varepsilon_L)\zeta - \varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)]\sqrt{t^2/\rho'^2-\zeta}}, \quad \text{(D1)}$$

Let $y' = \zeta - \varepsilon_1$, $T_\varepsilon = t^2/\rho'^2 - \varepsilon_1$, and $E_\varepsilon = \frac{\varepsilon_1^2(\varepsilon_1-\varepsilon_L)}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}$, then

$$\begin{aligned}
\vartheta_\varepsilon^{(3)} &= \frac{1}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\int_0^{T_\varepsilon}\frac{y'dy'}{(y'+\varepsilon_1)(y'+E_\varepsilon)\sqrt{(T_\varepsilon-y')y'}} \\
&= \frac{1}{\varepsilon_1^2-\varepsilon_T\varepsilon_L}\frac{1}{\varepsilon_1-E_\varepsilon}\left\{\int_0^{T_\varepsilon}\frac{y'dy'}{(y'+E_\varepsilon)\sqrt{(T_\varepsilon-y')y'}} - \int_0^{T_\varepsilon}\frac{y'dy'}{(y'+\varepsilon_1)\sqrt{(T_\varepsilon-y')y'}}\right\} \\
&= \frac{1}{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\left\{\int_0^{T_\varepsilon}\frac{y'dy'}{(y'+E_\varepsilon)\sqrt{(T_\varepsilon-y')y'}} - \int_0^{T_\varepsilon}\frac{y'dy'}{(y'+\varepsilon_1)\sqrt{(T_\varepsilon-y')y'}}\right\}. \quad \text{(D2)}
\end{aligned}$$

Let $y = y' + E_\varepsilon$ and $Y_1 = (T\varepsilon + E_\varepsilon - y)(y - E_\varepsilon)$, then

$$\int_0^{T_\varepsilon}\frac{y'dy'}{(y'+E_\varepsilon)\sqrt{(T_\varepsilon-y')y'}} = \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon}\frac{(y-E_\varepsilon)dy}{y\sqrt{(T_\varepsilon+E_\varepsilon-y)(y-E_\varepsilon)}}$$

$$= \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{dy}{Y_1^{1/2}} - E_\varepsilon \int_{E_\varepsilon}^{T_\varepsilon+E_\varepsilon} \frac{dy}{yY_1^{1/2}}$$

$$= \pi\left\{1-\varepsilon_1\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}. \quad \text{(D3)}$$

Similarly, let $y = y' + \varepsilon_1$ and $Y_2 = (T_\varepsilon + \varepsilon_1 - y)(y - \varepsilon_1)$, then

$$\int_0^{T_\varepsilon} \frac{y'dy'}{(y'+\varepsilon_1)\sqrt{(T_\varepsilon - y')y'}} = \int_{\varepsilon_1}^{T_\varepsilon+\varepsilon_1} \frac{(y-\varepsilon_1)dy}{y\sqrt{(T_\varepsilon+\varepsilon_1-y)(y-\varepsilon_1)}}$$

$$= \int_{\varepsilon_1}^{T_\varepsilon+\varepsilon_1} \frac{dy}{Y_2^{1/2}} - \int_{\varepsilon_1}^{T_\varepsilon+\varepsilon_1} \frac{dy}{yY_2^{1/2}}$$

$$= \pi\left[1-\sqrt{\varepsilon_1}\left(\frac{t^2}{\rho'^2}\right)^{-1/2}\right]. \quad \text{(D4)}$$

With substitutions (D3) and (D4) into (D2), we obtain

$$\vartheta_\varepsilon^{(3)} = \frac{\pi}{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}\left\{\sqrt{\varepsilon_1}\left(\frac{t^2}{\rho'^2}\right)^{-1/2} -\varepsilon_1\left[\frac{\varepsilon_1^2-\varepsilon_T\varepsilon_L}{\varepsilon_1-\varepsilon_L}\frac{t^2}{\rho'^2}-\frac{\varepsilon_1\varepsilon_L(\varepsilon_1-\varepsilon_T)}{\varepsilon_1-\varepsilon_L}\right]^{-1/2}\right\}. \quad \text{(D5)}$$

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