Progress In Electromagnetics Research, PIER 63, 51–73, 2006

THE METHOD OF IMAGES IN VELOCITY-DEPENDENT SYSTEMS

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Abstract—This study investigates the application of image methodology to velocity-dependent wave systems. Special Relativity is used for the analysis of waves scattered by arbitrary moving objects in the presence of a perfectly-conducting plane-interface. The various scenarios considered involve geometrical, material, and kinematic symmetries. Cases discussed include free-space, material media at-rest, and material media in motion, with respect to the plane-interface boundary. The last configuration is elaborated for two different scenarios: the first assumes the same medium velocity throughout space when the plane boundary is removed; the second introduces two symmetrical velocity-fields in the half-spaces involved, with a jump in flow direction at the interface.

Where the method applies it simplifies the analysis, and the results enrich our yet limited repertoire of canonical problems for relativistic scattering.

1. INTRODUCTION

The method of images constitutes a useful albeit limited technique facilitating solutions of the Laplace equation, e.g., in electrostatics and magnetostatics, and to a lesser extent of the vector or scalar Helmholtz wave equations. Essentially, the approach seeks to replace given boundaries and the conditions on them, by virtual sources or scatterers which maintain the original boundary conditions, hence also preserve the fields in the initial regions of space. Additional fields are created in regions of space which are irrelevant to the original problem. The ensuing configurations are usually easier to analyze.

Presently image techniques are studied in wave systems involving scatterers and interfaces in motion. This requires the use of Special Relativity in order to facilitate the transformation of fields and wave parameters from one inertial reference-system to another, i.e., taking into account both kinematical and dynamical effects of motion. In addition to the material and geometrical symmetry considerations involved with static configurations, here problems also involve kinematical symmetries dependent on velocities.

In the following, we consider three reference-systems: $\mathbf{\Gamma}$ is associated with the plane boundary at-rest; $\mathbf{\Gamma}'$ characterizes the scattering object at-rest; $\mathbf{\Gamma}''$ involves the image object at-rest. A feature common to all scenarios discussed below is a plane perfect mirror, i.e., a perfect conductor, or alternatively a perfect magnetic wall, characterized by $\varepsilon \to \infty$, $\mu \to \infty$, respectively. The plane-interface, at-rest in $\mathbf{\Gamma}$, is defined by its orientation in space, specified by a unit normal vector $\hat{\boldsymbol{n}}$, and by the origin $\boldsymbol{r} = 0$ located on it. In view of the symmetries involved, this choice of origin is conducive to simpler expressions. For simplicity only the case of a perfectly-conducting boundary is considered, the analogous case of a magnetic mirror leads to similar results. Furthermore, examples are limited to two-dimensional geometries involving cylindrical scatterers.

The symmetrical situations discussed subsequently involve pairs of vectors. We deal with a pair of vectors, say $\boldsymbol{a}$, $\boldsymbol{b}$, symmetrical with respect to the interface $\hat{\boldsymbol{n}}$, satisfying

$$\tilde{\boldsymbol{N}} \cdot (\boldsymbol{a} - \boldsymbol{b}) = 0, \quad \tilde{\boldsymbol{N}} = \tilde{\boldsymbol{I}} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \tag{1}$$

In (1) the dyadic, $\tilde{\boldsymbol{N}} = \tilde{\boldsymbol{I}} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} = -\hat{\boldsymbol{n}} \times \hat{\boldsymbol{n}} \times \tilde{\boldsymbol{I}}$, with the unit dyadic $\tilde{\boldsymbol{I}}$, sorts out the components parallel to the plane, i.e., perpendicular with respect to $\hat{\boldsymbol{n}}$. Obviously this requires $\boldsymbol{a}$, $\boldsymbol{b}$, to be co-planar, otherwise the tangential component vectors will not be identical.

The complementary case, where the components parallel with respect to the boundary add to zero, is given by pairs of vectors, say $\boldsymbol{A}$, $\boldsymbol{B}$, satisfying

$$\tilde{\boldsymbol{N}} \cdot (\boldsymbol{A} + \boldsymbol{B}) = 0 \tag{2}$$

Below we also deal with many cases where the components perpendicular to the plane are equal in length, augmenting (1), (2), with $\hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\boldsymbol{a} + \boldsymbol{b}) = 0$, $\hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\boldsymbol{A} - \boldsymbol{B}) = 0$, respectively. This will happen in isotropic media, applying in our case to free space and material media at-rest with respect to the boundary in reference-systems $\mathbf{\Gamma}$.

2. RELATIVISTIC ELECTRODYNAMICS

Consider an inertial reference-system $\mathbf{\Gamma}$, characterized by a quadruplet of spatiotemporal coordinates

$$\boldsymbol{R} = (\boldsymbol{r}, ict) = (x, y, z, ict) \tag{3}$$

where c is the universal constant, usually referred to as "the speed of light in vacuum". Mathematically, $\boldsymbol{R}$ denotes the location four-vector in the Minkowski four-dimensional space. For early references see e.g., Sommerfeld [1].

The Maxwell equations for source-free regions, e.g., see Stratton [2], are given by

$$\partial_{\boldsymbol{r}} \times \boldsymbol{E} = -\partial_t \boldsymbol{B}, \quad \partial_{\boldsymbol{r}} \times \boldsymbol{H} = \partial_t \boldsymbol{D}, \quad \partial_{\boldsymbol{r}} \cdot \boldsymbol{D} = 0, \quad \partial_{\boldsymbol{r}} \cdot \boldsymbol{B} = 0 \tag{4}$$

see also [3, 4] for present notation. In general, all fields in (4) depend on space and time, i.e., $\boldsymbol{E} = \boldsymbol{E}(\boldsymbol{R})$, etc. Einstein's so-called "principle of relativity" [5, 6] asserts that the Maxwell equations are form-invariant in all inertial reference-systems. Thus in a reference system $\mathbf{\Gamma}^{(*)}$, we have like in (4)

$$\begin{aligned} \partial_{\boldsymbol{r}^{(*)}} \times \boldsymbol{E}^{(*)} &= -\partial_{t^{(*)}} \boldsymbol{B}^{(*)}, \ \partial_{\boldsymbol{r}^{(*)}} \times \boldsymbol{H}^{(*)} = \partial_{t^{(*)}} \boldsymbol{D}^{(*)} \\ \partial_{\boldsymbol{r}^{(*)}} \cdot \boldsymbol{D}^{(*)} &= 0, \quad \partial_{\boldsymbol{r}^{(*)}} \cdot \boldsymbol{B}^{(*)} = 0 \end{aligned} \tag{5}$$

with $\boldsymbol{E}^{(*)} = \boldsymbol{E}^{(*)}(\boldsymbol{R}^{(*)})$, etc.

The spatiotemporal coordinates of $\mathbf{\Gamma}^{(*)}$ are related to those of $\mathbf{\Gamma}$ by the Lorentz transformation

$$\begin{aligned} \boldsymbol{r}^{(*)} &= \tilde{\boldsymbol{U}}^{(*)} \cdot (\boldsymbol{r} - \boldsymbol{v}^{(*)} t), \quad t^{(*)} = \gamma^{(*)}(t - \boldsymbol{v}^{(*)} \cdot \boldsymbol{r}/c^2) \\ \gamma^{(*)} &= (1 - \beta^{(*)2})^{-1/2}, \quad \beta^{(*)} = v^{(*)}/c, \quad v^{(*)} = |\boldsymbol{v}^{(*)}| \\ \tilde{\boldsymbol{U}}^{(*)} &= \tilde{\boldsymbol{I}} + (\gamma^{(*)} - 1)\hat{\boldsymbol{v}}^{(*)}\hat{\boldsymbol{v}}^{(*)}, \quad \hat{\boldsymbol{v}}^{(*)} = \boldsymbol{v}^{(*)}/v^{(*)} \end{aligned} \tag{6}$$

In (6), when choosing $\boldsymbol{r}^{(*)} = 0$ we obtain $\boldsymbol{r} = \boldsymbol{v}^{(*)} t$, or $\boldsymbol{v}^{(*)} = d\boldsymbol{r}/dt$, hence $\boldsymbol{v}^{(*)}$ is the velocity of the origin of $\mathbf{\Gamma}^{(*)}$ as observed from $\mathbf{\Gamma}$. The dyadic $\tilde{\boldsymbol{U}}^{(*)}$ multiplies the vector components parallel to the velocity by $\gamma^{(*)}$. Let us symbolize (6) by $\boldsymbol{R}^{(*)} = \boldsymbol{R}^{(*)}[\boldsymbol{R}]$. Its inverse $\boldsymbol{R} = \boldsymbol{R}[\boldsymbol{R}^{(*)}]$ is readily derived by simple substitution

$$\boldsymbol{r} = \tilde{\boldsymbol{U}}^{(*)} \cdot (\boldsymbol{r}^{(*)} + \boldsymbol{v}^{(*)} t^{(*)}), \quad t = \gamma^{(*)}(t^{(*)} + \boldsymbol{v}^{(*)} \cdot \boldsymbol{r}^{(*)}/c^2) \tag{7}$$

Replacing in (7) $\boldsymbol{v}^{(*)} = -\boldsymbol{v}$ yields the same functional form as in (6). This invertibility is an important property of Einstein's theory, often referred to by the somewhat vague phrase "all inertial systems of reference are equivalent". As a corollary to (6), (7), the application of the chain rule of calculus yields

$$\partial_{\boldsymbol{r}^{(*)}} = \tilde{\boldsymbol{U}}^{(*)} \cdot (\partial_{\boldsymbol{r}} + \boldsymbol{v}^{(*)}\partial_t/c^2), \quad \partial_{t^{(*)}} = \gamma^{(*)}(\partial_t + \boldsymbol{v}^{(*)} \cdot \partial_{\boldsymbol{r}}) \tag{8}$$

consistently denoted by $\partial_{\boldsymbol{R}^{(*)}} = \partial_{\boldsymbol{R}^{(*)}}[\partial_{\boldsymbol{R}}]$. Similarly to (6), (7). The inverse of (8) $\partial_{\boldsymbol{R}} = \partial_{\boldsymbol{R}}[\partial_{\boldsymbol{R}^{(*)}}]$ readily follows. Thus new Minkowski four-vector differential operators are defined, $\partial_{\boldsymbol{R}} = (\partial_{\boldsymbol{r}}, -\frac{i}{c}\partial_t)$, $\partial_{\boldsymbol{R}^{(*)}} = (\partial_{\boldsymbol{r}^{(*)}}, -\frac{i}{c}\partial_{t^{(*)}})$, in $\boldsymbol{\Gamma}$, $\boldsymbol{\Gamma}^{(*)}$, respectively.

Combining (4)–(8), Einstein [2, 5] has derived the field transformations

$$\begin{aligned} \boldsymbol{E}^{(*)} &= \tilde{\boldsymbol{V}}^{(*)} \cdot (\boldsymbol{E} + \boldsymbol{v}^{(*)} \times \boldsymbol{B}), \quad \boldsymbol{B}^{(*)} = \tilde{\boldsymbol{V}}^{(*)} \cdot (\boldsymbol{B} - \boldsymbol{v}^{(*)} \times \boldsymbol{E}/c^2) \\ \boldsymbol{D}^{(*)} &= \tilde{\boldsymbol{V}}^{(*)} \cdot (\boldsymbol{D} + \boldsymbol{v}^{(*)} \times \boldsymbol{H}/c^2), \; \boldsymbol{H}^{(*)} = \tilde{\boldsymbol{V}}^{(*)} \cdot (\boldsymbol{H} - \boldsymbol{v}^{(*)} \times \boldsymbol{D}) \\ \tilde{\boldsymbol{V}}^{(*)} &= \gamma^{(*)}\tilde{\boldsymbol{I}} + (1 - \gamma^{(*)})\hat{\boldsymbol{v}}^{(*)}\hat{\boldsymbol{v}}^{(*)} \end{aligned} \tag{9}$$

which may be generically symbolized by $\boldsymbol{F}^{(*)} = \boldsymbol{F}^{(*)}[\boldsymbol{F}]$. The dyadic $\tilde{\boldsymbol{V}}^{(*)}$ multiplies the vector component perpendicular to the velocity by $\gamma^{(*)}$. Either by manipulating (9) or directly from the principle of relativity, the inverse formulas of (9), $\boldsymbol{F} = \boldsymbol{F}[\boldsymbol{F}^{(*)}]$, readily follows, e.g., $\boldsymbol{E} = \tilde{\boldsymbol{V}}^{(*)} \cdot (\boldsymbol{E}^{(*)} - \boldsymbol{v}^{(*)} \times \boldsymbol{B}^{(*)})$.

By substitution from (5) into (9), differential operators were defined and used [4, 7–9]

$$\begin{aligned} &\boldsymbol{E}^{(*)} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}^{(*)},\boldsymbol{\Gamma}} \cdot \boldsymbol{E}, \; \boldsymbol{H}^{(*)} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}^{(*)},\boldsymbol{\Gamma}} \cdot \boldsymbol{H} \\ &\tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}^{(*)},\boldsymbol{\Gamma}} = \tilde{\boldsymbol{V}}^{(*)} \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}^{(*)} \times \partial_t^{-1}\partial_{\boldsymbol{r}} \times \tilde{\boldsymbol{I}}) \end{aligned} \tag{10}$$

relating the fields in $\boldsymbol{\Gamma}$ to those in $\boldsymbol{\Gamma}^{(*)}$ in a very compact notation. In (10) ∂_t^{-1} denotes the inverse time derivative, which is either the time integral, or disappears by multiplying all fields in (10) by ∂_t.

Sacrificing generality for simplicity, we choose here simple constitutive relations, e.g., in a material medium at-rest in $\boldsymbol{\Gamma}^{(*)}$,

$$\boldsymbol{D}^{(*)} = \varepsilon^{(*)}\boldsymbol{E}^{(*)}, \quad \boldsymbol{B}^{(*)} = \mu^{(*)}\boldsymbol{H}^{(*)} \tag{11}$$

where in (11) the scalars $\varepsilon^{(*)}$, $\mu^{(*)}$, are the material parameters. Only in free-space (vacuum) we have $\varepsilon^{(*)} = \varepsilon_0$, $\mu^{(*)} = \mu_0$ for all inertial reference-systems.

In material media (9), (11), prescribe the Minkowski constitutive relations [1], e.g., for fields measured in $\mathbf{\Gamma}$, and a medium at-rest in $\mathbf{\Gamma'}$, we have

$$\begin{aligned} \boldsymbol{D} + \boldsymbol{v}' \times \boldsymbol{H}/c^2 &= \varepsilon'(\boldsymbol{E} + \boldsymbol{v}' \times \boldsymbol{B}) \\ \boldsymbol{B} - \boldsymbol{v}' \times \boldsymbol{E}/c^2 &= \mu'(\boldsymbol{H} - \boldsymbol{v}' \times \boldsymbol{D}) \end{aligned} \tag{12}$$

degenerating in free-space into (11) in the form $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E},\ \boldsymbol{B} = \mu_0 \boldsymbol{H}$. In general, the forms (12) are much more difficult to handle [1].

Substitution from (5) into (12) and noting that $\tilde{\boldsymbol{U}} \cdot \tilde{\boldsymbol{V}} = \gamma \tilde{\boldsymbol{I}}$ yields differential operators

$$\begin{aligned} \boldsymbol{D} &= \varepsilon' \tilde{\boldsymbol{M}} \cdot \boldsymbol{E} - \boldsymbol{v}' \times \boldsymbol{H}/c^2,\ \boldsymbol{B} = \mu' \tilde{\boldsymbol{M}} \cdot \boldsymbol{H} + \boldsymbol{v}' \times \boldsymbol{E}/c^2 \\ \tilde{\boldsymbol{M}} &= (\tilde{\boldsymbol{I}} - \boldsymbol{v}' \times \partial_t^{-1} \partial_{\boldsymbol{r}} \times \tilde{\boldsymbol{I}}) = \tilde{\boldsymbol{U}}' \cdot \tilde{\boldsymbol{W}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} / \gamma' \end{aligned} \tag{13}$$

displaying the dependence of $\boldsymbol{D}$ on $\boldsymbol{E}$ and $\boldsymbol{H}$, and similarly the dependence of $\boldsymbol{B}$ on $\boldsymbol{H}$ and $\boldsymbol{E}$. Again, for $\boldsymbol{v}' = 0$ the constitutive relations degenerate to the free-space case in all reference-systems.

Alternatively, the Minkowski constitutive relations can be expressed as

$$\begin{aligned} \tilde{\boldsymbol{N}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} \cdot \boldsymbol{H} &= \varepsilon' \tilde{\boldsymbol{W}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} \cdot \boldsymbol{E},\ \tilde{\boldsymbol{N}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} \cdot \boldsymbol{E} = -\mu' \tilde{\boldsymbol{W}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} \cdot \boldsymbol{H} \\ \tilde{\boldsymbol{N}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} &= \tilde{\boldsymbol{V}}' \cdot (\partial_t^{-1} \partial_{\boldsymbol{r}} \times \tilde{\boldsymbol{I}} + \boldsymbol{v}' \times \tilde{\boldsymbol{I}}/c^2) \end{aligned} \tag{14}$$

and once again for $\boldsymbol{v}' = 0$ the constitutive relations (14) degenerate to the free-space case in all reference-systems.

3. PLANE WAVE SCENARIOS

The following simple examples illustrate scenarios investigated below. We start with a monochromatic plane wave given in a medium at-rest in $\mathbf{\Gamma}$

$$\{\boldsymbol{E}, \boldsymbol{H}\} = \{\hat{\boldsymbol{E}} e_0, \hat{\boldsymbol{H}} h_0\} e^{i\boldsymbol{K} \cdot \boldsymbol{R}},\ \boldsymbol{K} = (\boldsymbol{k}, i\omega/c),\ \boldsymbol{K} \cdot \boldsymbol{R} = \boldsymbol{k} \cdot \boldsymbol{r} - \omega t \tag{15}$$

where in (15) and throughout the electrical and magnetic fields are written together, i.e., symbols in braces apply correspondingly; $\boldsymbol{K}$ denotes the spectral (propagation-vector and frequency) Minkowski four-vector. In an isotropic medium at-rest waves are transversal, displaying the properties

$$\begin{aligned} &\hat{\boldsymbol{E}} \times \hat{\boldsymbol{H}} = \hat{\boldsymbol{k}},\ \hat{\boldsymbol{E}} \cdot \hat{\boldsymbol{H}} = \hat{\boldsymbol{E}} \cdot \hat{\boldsymbol{k}} = \hat{\boldsymbol{H}} \cdot \hat{\boldsymbol{k}} = 0 \\ &\omega/k = (\mu\varepsilon)^{-1/2},\ e_0/h_0 = Z = (\mu/\varepsilon)^{1/2} \end{aligned} \tag{16}$$

and in free-space $\mu = \mu_0, \ \varepsilon = \varepsilon_0$.

We consider the initial wave (15) in the half space denoted by $\{\mathit{1}\}$. To satisfy the boundary conditions at the perfectly-conducting plane-interface, a reflected wave must exist in $\{\mathit{1}\}$, given by

$$\{\overline{\boldsymbol{E}}, \overline{\boldsymbol{H}}\} = \{\hat{\overline{\boldsymbol{E}}}\,\overline{e}_0, \hat{\overline{\boldsymbol{H}}}\,\overline{h}_0\} e^{i\overline{\boldsymbol{K}}\cdot\boldsymbol{R}}, \quad \overline{\boldsymbol{K}} = (\overline{\boldsymbol{k}}, i\overline{\omega}/c) \tag{17}$$

where in an isotropic medium we have $v_{ph} = \overline{\omega}/\overline{k} = \omega/k$, denoting the phase velocity which in free-space becomes $v_{ph} = c$, and the transversality properties are similar to (16).

The total tangential electric field must vanish at all points $\tilde{\boldsymbol{N}} \cdot \boldsymbol{r}$ on the boundary, at all times t. This prescribes for the phases, i.e., wave propagation vectors and frequencies, and for the amplitudes, respectively,

$$\omega - \overline{\omega} = 0, \quad \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k} - \overline{\boldsymbol{k}}) = 0, \quad \tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{E}} e_0 + \hat{\overline{\boldsymbol{E}}}\overline{e}_0) = 0 \tag{18}$$

For the case of a magnetic wall the condition on the amplitude is $\tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{H}} h_0 + \hat{\overline{\boldsymbol{H}}}\,\overline{h}_0) = 0$. The symmetry condition $\tilde{\boldsymbol{N}} \cdot (\boldsymbol{k} - \overline{\boldsymbol{k}}) = 0$ is usually referred to as Snell's law, obviously it is not a law, but a result of applying boundary conditions.

Inasmuch as the pair of plane waves (15), (17), subject to (18), satisfies the boundary conditions, it is feasible to extend the waves into region $\{\mathit{2}\}$, the half-space initially shielded by the perfectly-conducting plane-interface, and remove the perfectly-conducting plane-interface, without altering the original waves (15), (17), in region $\{\mathit{1}\}$. The tangential components of both the electric and magnetic fields are continuous across the interface, therefore no equivalent surface charge and current density sources are required. Thusly $\overline{\boldsymbol{E}}$ can be called the image of $\boldsymbol{E}$, constituting the simplest example for the application of the method of images.

In order to observe the fields in a different reference-system, (10) is applied to (15), (17), yielding in $\boldsymbol{\Gamma}'$

$$\begin{aligned}
&\{\boldsymbol{E}', \boldsymbol{H}'\} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}',\boldsymbol{\Gamma}} \cdot \{\boldsymbol{E}, \boldsymbol{H}\} = \{\hat{\boldsymbol{E}}' e_0', \hat{\boldsymbol{H}}' h_0'\} e^{i\boldsymbol{K}'\cdot\boldsymbol{R}'} \\
&\{\overline{\boldsymbol{E}}', \overline{\boldsymbol{H}}'\} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}',\boldsymbol{\Gamma}} \cdot \{\overline{\boldsymbol{E}}, \overline{\boldsymbol{H}}\} = \{\hat{\overline{\boldsymbol{E}}}'\overline{e}_0', \hat{\overline{\boldsymbol{H}}}'\overline{h}_0'\} e^{i\overline{\boldsymbol{K}}'\cdot\boldsymbol{R}'} \\
&\quad \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}',\boldsymbol{\Gamma}} = \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}' \times \partial_t^{-1} \partial_{\boldsymbol{r}} \times \tilde{\boldsymbol{I}}), \ \boldsymbol{K}' = (\boldsymbol{k}', i\omega'/c), \ \overline{\boldsymbol{K}}' = (\overline{\boldsymbol{k}}', i\overline{\omega}'/c)
\end{aligned} \tag{19}$$

where in (19) the phase is obtained subject to the phase-invariance principle discussed below, and $\boldsymbol{v}'$ is the velocity of $\boldsymbol{\Gamma}'$ as observed from $\boldsymbol{\Gamma}$. Once $\boldsymbol{E}', \boldsymbol{H}', \overline{\boldsymbol{E}}', \overline{\boldsymbol{H}}'$, are computed, the associated fields are found from the pertinent Maxwell equations (5)

$$\begin{aligned} \boldsymbol{B}' &= -\partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \boldsymbol{E}', \quad \boldsymbol{D}' = \partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \boldsymbol{H}' \\ \overline{\boldsymbol{B}}' &= -\partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \overline{\boldsymbol{E}}', \quad \overline{\boldsymbol{D}}' = \partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \overline{\boldsymbol{H}}' \end{aligned} \tag{20}$$

As long as we deal with free-space, the operations in (20) are trivial and in $\boldsymbol{\Gamma}'$ we have transversal plane wave as well.

Inasmuch as $\boldsymbol{K}, \overline{\boldsymbol{K}}, \boldsymbol{R}$ are Minkowski four-vectors, we have for the inner products

$$\boldsymbol{K} \cdot \boldsymbol{R} = \boldsymbol{K}' \cdot \boldsymbol{R}', \quad \overline{\boldsymbol{K}} \cdot \boldsymbol{R} = \overline{\boldsymbol{K}}' \cdot \boldsymbol{R}' \tag{21}$$

also referred to as the phase-invariance principle (already used in (19)), e.g., see [4]. Substituting from (6), (7), into (21) and collecting terms yields

$$\begin{aligned} \boldsymbol{k}' &= \tilde{\boldsymbol{U}}' \cdot (\boldsymbol{k} - \boldsymbol{v}'\omega/c^2), \quad \omega' = \gamma'(\omega - \boldsymbol{v}' \cdot \boldsymbol{k}) \\ \overline{\boldsymbol{k}}' &= \tilde{\boldsymbol{U}}' \cdot (\overline{\boldsymbol{k}} - \boldsymbol{v}'\overline{\omega}/c^2), \quad \overline{\omega}' = \gamma'(\overline{\omega} - \boldsymbol{v}' \cdot \overline{\boldsymbol{k}}) \end{aligned} \tag{22}$$

where in (22) the transformations $\boldsymbol{k}' = \boldsymbol{k}'[\boldsymbol{k}]$, $\overline{\boldsymbol{k}}' = \overline{\boldsymbol{k}}'[\overline{\boldsymbol{k}}]$ are the formulas for the relativistic Fresnel drag phenomenon, and $\omega' = \omega'[\omega]$, $\overline{\omega}' = \overline{\omega}'[\overline{\omega}]$ are usually referred to as the relativistic Doppler effect formulas. Similarly to (6), we symbolize (22) by $\boldsymbol{K}' = \boldsymbol{K}'[\boldsymbol{K}]$, $\overline{\boldsymbol{K}}' = \overline{\boldsymbol{K}}'[\overline{\boldsymbol{K}}]$. The inverse transformations $\boldsymbol{K} = \boldsymbol{K}[\boldsymbol{K}']$, $\overline{\boldsymbol{K}} = \overline{\boldsymbol{K}}[\overline{\boldsymbol{K}}']$ are readily derived.

In $\boldsymbol{\Gamma}$ we have the conditions (18). By transformation into $\boldsymbol{\Gamma}'$, the formulas for reflection of a plane wave from a moving mirror [5] are obtained, often dubbed as the relativistic aberration effect. The interesting aspect of this quite trivial analysis is that when everything is observed from $\boldsymbol{\Gamma}'$, we have now the plane boundary moving at a velocity $-\boldsymbol{v}'$. The effect of the boundary can be replaced by an image wave, namely $\overline{\boldsymbol{E}}'$, (19), whose parameters are determined by the boundary-value problem in $\boldsymbol{\Gamma}$ and the pertinent relativistic transformations, and thus we have defined the image wave in the case of a moving mirror.

We also need to consider waves in a reference-system $\boldsymbol{\Gamma}''$ moving with velocity $\boldsymbol{v}''$ when observed from $\boldsymbol{\Gamma}$. The relevant formulas are obtained from (19)–(22) by replacing the primes with double-primes

$$\begin{aligned} &\{\boldsymbol{E}'', \boldsymbol{H}''\} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}'',\boldsymbol{\Gamma}} \cdot \{\boldsymbol{E}, \boldsymbol{H}\} = \{\hat{\boldsymbol{E}}'' e_0'', \hat{\boldsymbol{H}}'' h_0''\} e^{i\boldsymbol{K}''\cdot\boldsymbol{R}''} \\ &\{\overline{\boldsymbol{E}}'', \overline{\boldsymbol{H}}''\} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}'',\boldsymbol{\Gamma}} \cdot \{\overline{\boldsymbol{E}}, \overline{\boldsymbol{H}}\} = \{\hat{\overline{\boldsymbol{E}}}'' e_0'', \hat{\overline{\boldsymbol{H}}}'' \overline{h}_0''\} e^{i\overline{\boldsymbol{K}}''\cdot\boldsymbol{R}''} \\ &\tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}'',\boldsymbol{\Gamma}} = \tilde{\boldsymbol{V}}'' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}'' \times \partial_t^{-1}\partial_{\boldsymbol{r}} \times \tilde{\boldsymbol{I}}) \end{aligned}$$

$$\boldsymbol{B}'' = -\partial_{t''}^{-1}\partial_{\boldsymbol{r}''}\times\boldsymbol{E}'',\ \boldsymbol{D}'' = \partial_{t''}^{-1}\partial_{\boldsymbol{r}''}\times\boldsymbol{H}'',\ \boldsymbol{K}'' = (\boldsymbol{k}'', i\omega''/c) \quad (23)$$
$$\overline{\boldsymbol{B}}'' = -\partial_{t''}^{-1}\partial_{\boldsymbol{r}''}\times\overline{\boldsymbol{E}}'',\ \overline{\boldsymbol{D}}'' = \partial_{t''}^{-1}\partial_{\boldsymbol{r}''}\times\overline{\boldsymbol{H}}'',\ \overline{\boldsymbol{K}}'' = (\overline{\boldsymbol{k}}'', i\overline{\omega}''/c)$$
$$\boldsymbol{k}'' = \tilde{\boldsymbol{U}}''\cdot(\boldsymbol{k} - \boldsymbol{v}''\omega/c^2),\ \omega'' = \gamma''(\omega - \boldsymbol{v}''\cdot\boldsymbol{k})$$
$$\overline{\boldsymbol{k}}'' = \tilde{\boldsymbol{U}}''\cdot(\overline{\boldsymbol{k}} - \boldsymbol{v}''\overline{\omega}/c^2),\ \overline{\omega}'' = \gamma''(\overline{\omega} - \boldsymbol{v}''\cdot\overline{\boldsymbol{k}})$$

The two reference-systems $\boldsymbol{\Gamma}', \boldsymbol{\Gamma}''$, are moving symmetrically with respect to the plane-interface, i.e.,

$$\tilde{\boldsymbol{N}}\cdot(\boldsymbol{v}'' - \boldsymbol{v}') = 0, \quad \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}\cdot(\boldsymbol{v}'' + \boldsymbol{v}') = 0 \quad (24)$$

Unlike free-space scenarios, material media require more scrutiny. The first scenario involving material media is quite similar to the free-space case: In $\boldsymbol{\Gamma}$ in region $\{1\}$ we now assume a medium at-rest, possessing arbitrary parameters ε, μ. The same waves (15), (17), exist here. The boundary conditions at the perfectly-conducting plane are once again satisfied by (18). Also, by extending the waves and the medium from the initial half-space $\{1\}$ to the half-space $\{2\}$, the fields in $\{1\}$ remain unaltered.

When transformations into $\boldsymbol{\Gamma}'$ are effected, propagation is observed in a moving medium, yielding the waves (19)-(22), revealing the aberration effect for the present case of moving material media. It is noted that in $\boldsymbol{\Gamma}'$ the waves are not transversal any more.

The next scenario assumes in $\{1\}$ a medium moving relative to the perfectly-conducting plane-interface, e.g., see [10]. It must be emphasized that we neglect here the fluid-dynamical aspect of the problem, namely the flow continuity, allowing an arbitrary jump in the material velocity-field at the boundary. One way of looking at it is to assume the plane-interface to be porous in a way that allows the fluid to freely move through it, at the same time we assume the pores to be sufficiently small with respect to the wavelength, such that the interface acts like a "faraday cage" screen, electrically performing as a perfect conductor. The situation is complicated by the fact that when we reside in $\boldsymbol{\Gamma}$, where the boundary is at-rest, we encounter a moving medium, while from the reference-system $\boldsymbol{\Gamma}'$, where the medium is at-rest, the boundary appears to be in motion.

With this in mind, consider a pair of transversal plane waves (19) in the medium at-rest in $\boldsymbol{\Gamma}'$ in $\{1\}$, possessing constitutive parameters ε', μ'. According to (10), the inverse of (19)–(22) is computed. In $\boldsymbol{\Gamma}$ we have

$$\{\boldsymbol{E}, \boldsymbol{H}\} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'}\cdot\{\boldsymbol{E}', \boldsymbol{H}'\} = \{\hat{\boldsymbol{E}}e_0, \hat{\boldsymbol{H}}h_0\}e^{i\boldsymbol{K}\cdot\boldsymbol{R}}$$
$$\{\overline{\boldsymbol{E}}, \overline{\boldsymbol{H}}\} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'}\cdot\{\overline{\boldsymbol{E}}', \overline{\boldsymbol{H}}'\} = \{\hat{\overline{\boldsymbol{E}}}\,\overline{e}_0, \hat{\overline{\boldsymbol{H}}}\,\overline{h}_0\}e^{i\overline{\boldsymbol{K}}\cdot\boldsymbol{R}}$$

$$\begin{aligned}
&\tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'} = \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} + \boldsymbol{v}' \times \partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \tilde{\boldsymbol{I}}) \\
&\boldsymbol{B} = -\partial_t^{-1}\partial_{\boldsymbol{r}} \times \boldsymbol{E},\ \boldsymbol{D} = \partial_t^{-1}\partial_{\boldsymbol{r}} \times \boldsymbol{H} \\
&\overline{\boldsymbol{B}} = -\partial_t^{-1}\partial_{\boldsymbol{r}} \times \overline{\boldsymbol{E}},\ \overline{\boldsymbol{D}} = \partial_t^{-1}\partial_{\boldsymbol{r}} \times \overline{\boldsymbol{H}} \\
&\boldsymbol{k} = \tilde{\boldsymbol{U}}' \cdot (\boldsymbol{k}' + \boldsymbol{v}'\omega'/c^2),\ \omega = \gamma'(\omega' + \boldsymbol{v}' \cdot \boldsymbol{k}') \\
&\overline{\boldsymbol{k}} = \tilde{\boldsymbol{U}}' \cdot (\overline{\boldsymbol{k}}' + \boldsymbol{v}'\overline{\omega}'/c^2),\ \overline{\omega} = \gamma'(\overline{\omega}' + \boldsymbol{v}' \cdot \overline{\boldsymbol{k}}')
\end{aligned} \tag{25}$$

In general, the waves (25) in $\boldsymbol{\Gamma}$ are not simple transversal plane waves, because we are dealing here with propagation in a moving medium. For boundaries at-rest, boundary conditions are independent of the material composition of the medium, including its motion, and are directly derived from the Maxwell equations [11, 12].

In the present case the boundary conditions are evaluated in $\boldsymbol{\Gamma}$, at the plane-interface at-rest, hence the same boundary condition (18) are prescribed here too. However, because of the medium motion, an equivalent anisotropic medium is encountered here, depending on the velocity-field $\boldsymbol{v}'$. It follows that here, unlike the cases of waves in free-space or in a medium at-rest with respect to the boundary, we have

$$\hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\boldsymbol{k} - \overline{\boldsymbol{k}}) \neq 0, \quad \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\hat{\boldsymbol{E}}e_0 + \hat{\overline{\boldsymbol{E}}}\overline{e}_0) \neq 0 \tag{26}$$

i.e., in general the components of the propagation vectors and field vectors perpendicular to the plane-interface are unequal. The values of the unknown vectors in (26) are prescribed by the transversality properties of the waves in the medium's rest-frame $\boldsymbol{\Gamma}'$.

We now seek to define conditions, such that the plane-interface can be removed without affecting the waves in $\{1\}$. To that end we have to define appropriate waves in $\boldsymbol{\Gamma}$ in region $\{2\}$. One trivial scenario is suggested by extending the same medium, its velocity, and the waves (15), (17), from region $\{1\}$ into $\{2\}$. As far as removing the plane-interface is concerned, this provides a valid configuration. However, it is immediately realized that when waves are considered in $\boldsymbol{\Gamma}', \boldsymbol{\Gamma}''$, the symmetry (24) for the velocity is violated. Consequently, unlike the case of free space or media at-rest in $\boldsymbol{\Gamma}$ discussed below, it is not feasible to define symmetrically moving image scatterers in $\{2\}$.

We therefore turn to another alternative, where in $\boldsymbol{\Gamma}''$ in $\{2\}$ a medium at-rest is defined, materially identical to the medium at-rest in $\boldsymbol{\Gamma}'$ in $\{1\}$. When observed from $\boldsymbol{\Gamma}$, the medium in $\boldsymbol{\Gamma}''$ in $\{2\}$ is moving with a symmetrical velocity satisfying (24). Similarly to the above argument regarding the flow discontinuity at the plane-interface, when we remove now the plane boundary and assume medium velocity $\boldsymbol{v}''$ in $\{2\}$, the jump in the direction of the velocity field is disregarded.

Similarly to (19), where the wave $\{\boldsymbol{E}', \boldsymbol{H}'\}$ is given in $\boldsymbol{\Gamma}'$ in region $\{\mathit{1}\}$, we now define $\{\overline{\boldsymbol{E}}'', \overline{\boldsymbol{H}}''\}$ according to (23) in $\boldsymbol{\Gamma}''$ in $\{\mathit{2}\}$ in a medium at-rest. Thus we have waves in media at-rest in regions $\{\mathit{1}\}$, $\{\mathit{2}\}$, for $\boldsymbol{\Gamma}'$, $\boldsymbol{\Gamma}''$, correspondingly

$$\begin{aligned}
\{\boldsymbol{E}'_{\{1\}}, \boldsymbol{H}'_{\{1\}}\} &= \{\hat{\boldsymbol{E}}'_{\{1\}} e'_{0\{1\}}, \hat{\boldsymbol{H}}'_{\{1\}} h'_{0\{1\}}\} e^{i\boldsymbol{K}'_{\{1\}}\cdot\boldsymbol{R}'} \\
\boldsymbol{K}'_{\{1\}} &= (\boldsymbol{k}'_{\{1\}}, i\omega'_{\{1\}}/c) \\
\{\overline{\boldsymbol{E}}'_{\{1\}}, \overline{\boldsymbol{H}}'_{\{1\}}\} &= \{\hat{\overline{\boldsymbol{E}}}'_{\{1\}} \overline{e}'_{0\{1\}}, \hat{\overline{\boldsymbol{H}}}'_{\{1\}} \overline{h}'_{0\{1\}}\} e^{i\overline{\boldsymbol{K}}'_{\{1\}}\cdot\boldsymbol{R}'} \\
\overline{\boldsymbol{K}}'_{\{1\}} &= (\overline{\boldsymbol{k}}'_{\{1\}}, i\overline{\omega}'_{\{1\}}/c) \\
\{\boldsymbol{E}''_{\{2\}}, \boldsymbol{H}''_{\{2\}}\} &= \{\hat{\boldsymbol{E}}''_{\{2\}} e''_{0\{2\}}, \hat{\boldsymbol{H}}''_{\{2\}} h''_{0\{2\}}\} e^{i\boldsymbol{K}''_{\{2\}}\cdot\boldsymbol{R}''} \\
\boldsymbol{K}''_{\{2\}} &= (\boldsymbol{k}''_{\{2\}}, i\omega''_{\{2\}}/c) \\
\{\overline{\boldsymbol{E}}''_{\{2\}}, \overline{\boldsymbol{H}}''_{\{2\}}\} &= \{\hat{\overline{\boldsymbol{E}}}''_{\{2\}} \overline{e}''_{0\{2\}}, \hat{\overline{\boldsymbol{H}}}''_{\{2\}} \overline{h}''_{0\{2\}}\} e^{i\overline{\boldsymbol{K}}''_{\{2\}}\cdot\boldsymbol{R}''} \\
\overline{\boldsymbol{K}}''_{\{2\}} &= (\overline{\boldsymbol{k}}''_{\{2\}}, i\overline{\omega}''_{\{2\}}/c)
\end{aligned} \tag{27}$$

In order to have symmetrical waves we impose

$$\begin{aligned}
\omega'_{\{1\}} - \overline{\omega}''_{\{2\}} = 0, \quad \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}'_{\{1\}} - \overline{\boldsymbol{k}}''_{\{2\}}) = 0, \quad \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\boldsymbol{k}'_{\{1\}} + \overline{\boldsymbol{k}}''_{\{2\}}) &= 0 \\
\tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{E}}'_{\{1\}} e'_{0\{1\}} + \hat{\overline{\boldsymbol{E}}}''_{\{2\}} \overline{e}''_{0\{2\}}) = 0, \ \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\hat{\boldsymbol{E}}'_{\{1\}} e'_{0\{1\}} - \hat{\overline{\boldsymbol{E}}}''_{\{2\}} \overline{e}''_{0\{2\}}) &= 0 \\
\tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{H}}'_{\{1\}} h'_{0\{1\}} - \hat{\overline{\boldsymbol{H}}}''_{\{2\}} \overline{h}''_{0\{2\}}) = 0, \ \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\hat{\boldsymbol{H}}'_{\{1\}} h'_{0\{1\}} - \hat{\overline{\boldsymbol{H}}}''_{\{2\}} \overline{h}''_{0\{2\}}) &= 0
\end{aligned} \tag{28}$$

It follows that similarly to (18) we now have in $\boldsymbol{\Gamma}$ on both sides of the perfectly-conducting plane-interface, in the corresponding regions $\{\mathit{1}\}$, $\{\mathit{2}\}$,

$$\begin{aligned}
\omega_{\{1\}} - \overline{\omega}_{\{1\}} = 0, \ \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}_{\{1\}} - \overline{\boldsymbol{k}}_{\{1\}}) &= 0 \\
\tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{E}}_{\{1\}} e_{0\{1\}} + \hat{\overline{\boldsymbol{E}}}_{\{1\}} \overline{e}_{0\{1\}}) &= 0 \\
\omega_{\{2\}} - \overline{\omega}_{\{2\}} = 0, \ \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}_{\{2\}} - \overline{\boldsymbol{k}}_{\{2\}}) &= 0 \\
\tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{E}}_{\{2\}} e_{0\{2\}} + \hat{\overline{\boldsymbol{E}}}_{\{2\}} \overline{e}_{0\{2\}}) &= 0
\end{aligned} \tag{29}$$

and therefore

$$\begin{aligned}
\omega_{\{1\}} = \overline{\omega}_{\{1\}} = \omega_{\{2\}} = \overline{\omega}_{\{2\}}, \ \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}_{\{1\}} - \boldsymbol{k}_{\{2\}}) &= 0 \\
\tilde{\boldsymbol{N}} \cdot (\overline{\boldsymbol{k}}_{\{1\}} - \overline{\boldsymbol{k}}_{\{2\}}) = 0, \ \tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{E}}_{\{1\}} e_{0\{1\}} - \hat{\boldsymbol{E}}_{\{2\}} e_{0\{2\}}) &= 0
\end{aligned}$$

$$\begin{aligned}\tilde{\boldsymbol{N}} \cdot (\hat{\overline{\boldsymbol{E}}}_{\{1\}}\overline{e}_{0\{1\}} - \hat{\overline{\boldsymbol{E}}}_{\{2\}}\overline{e}_{0\{2\}}) &= 0 \\ \tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{H}}_{\{1\}}h_{0\{1\}} - \hat{\boldsymbol{H}}_{\{2\}}h_{0\{2\}}) &= 0 \\ \tilde{\boldsymbol{N}} \cdot (\hat{\overline{\boldsymbol{H}}}_{\{1\}}\overline{h}_{0\{1\}} - \hat{\overline{\boldsymbol{H}}}_{\{2\}}\overline{h}_{0\{2\}}) &= 0\end{aligned} \tag{30}$$

According to (30), in $\boldsymbol{\Gamma}$, on both sides of the perfectly-conducting plane-interface, in the corresponding regions $\{1\}$, $\{2\}$, the frequencies are equal, and the tangential components of the propagation vectors and fields are continuous, implying that the plane-interface can be removed without affecting the original fields in $\boldsymbol{\Gamma}$ in $\{1\}$.

Adhering to our previous convention, fields with, without, upper bar are considered as the incident, the image, waves respectively. It is interesting to note that due to the different velocities $\boldsymbol{v}'$, $\boldsymbol{v}''$, in effect we have now two different anisotropic media in regions $\{1\}$, $\{2\}$. Hence the changes of the propagation vectors $\boldsymbol{k}_{\{1\}}$, $\boldsymbol{k}_{\{2\}}$ and $\overline{\boldsymbol{k}}_{\{1\}}$, $\overline{\boldsymbol{k}}_{\{2\}}$, across the boundary, display how the waves, propagating from one medium into the other, are *refracted* at the interface.

With this the discussion for individual plane waves is completed, facilitating the application of the results to moving scatterers.

4. IMAGES IN VELOCITY-INDEPENDENT SCATTERING

The present section deals with a simple case of wave scattering from an infinite cylinder, in the presence of a perfectly-conducting plane-interface. The choice of a two-dimensional geometry serves to introduce basic concepts, tools, and relevant notation in a simple way. The corresponding three-dimensional analog follows along the same lines, but the mathematical details become more complicated, due to the vector spherical waves and their associated special functions.

In $\boldsymbol{\Gamma}$, where the plane-interface is at-rest, in free-space, an incident plane-wave (15) is assumed in the half-space $\{1\}$. For simplicity, consider $\hat{\boldsymbol{E}} = \hat{\boldsymbol{z}}$ to be polarized along the cylindrical axis. It follows that $\hat{\boldsymbol{H}}$, $\hat{\boldsymbol{k}}$, are in the perpendicular xy-plane, in which a radius-vector $\boldsymbol{r}$ is defined. The plane-interface is defined by the direction of its normal $\hat{\boldsymbol{n}}$, with $\hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{z}} = 0$ for the cylindrical case, and by the origin $\boldsymbol{r} = 0$ located on it.

Define for the scatterer a local right-handed cylindrical coordinate system

$$\boldsymbol{\rho} = \boldsymbol{\rho}(\rho, \psi), \quad \hat{\boldsymbol{\rho}} \times \hat{\boldsymbol{\psi}} = \hat{\boldsymbol{z}} \tag{31}$$

with polar coordinates ρ, ψ. In the corresponding three-dimensional case we would have $\boldsymbol{\rho} = \boldsymbol{\rho}(\rho, \theta, \psi)$, $\hat{\boldsymbol{\rho}} \times \hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\psi}}$. The local coordinate system is located relative to the initial one by means of

$$\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{\rho} \tag{32}$$

The incident wave (15) is translated to the local coordinate system by substituting (32) into the phase $\boldsymbol{K} \cdot \boldsymbol{R}$, yielding

$$\boldsymbol{K} \cdot \boldsymbol{R} = \boldsymbol{k} \cdot \boldsymbol{r} - \omega t = \boldsymbol{k} \cdot \boldsymbol{r}_0 + \boldsymbol{k} \cdot \boldsymbol{\rho} - \omega t \tag{33}$$

The geometry of the cylindrical scatterer is defined in the coordinate system (31) relative to the local origin $\boldsymbol{\rho} = 0$. This geometry, as well as the constitutive parameters are uniform along the cylindrical z-axis, such that no cross-polarization occurs.

We wish to restrict the present analysis to outgoing waves only. It has been shown by Twersky [13–15] that the scattered wave is given in terms of outgoing waves at least outside the circumscribing circle (or circumscribing sphere for the corresponding three-dimensional case) of radius $\rho_{\max}$. Hence for the present case we have to ensure that the plane-interface is outside this region, i.e., $\rho_{\max} < |\boldsymbol{r}_0 \cdot \hat{\boldsymbol{n}}|$. Accordingly the scattered wave, due to the excitation (15), is given by

$$\begin{aligned} \boldsymbol{E}_{\hat{\boldsymbol{k}}}(\boldsymbol{\rho}, t) &= \hat{\boldsymbol{z}} e_0 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0} e^{-i\omega t} \Sigma_m i^m a_m(\hat{\boldsymbol{k}}) H_m(\kappa) e^{im\psi} \\ &\sim e_0 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0} H(\kappa) \boldsymbol{g}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{\rho}}),\ \kappa = k\rho = k\hat{\boldsymbol{\rho}} \cdot \boldsymbol{\rho} = \boldsymbol{k}_\rho \cdot \boldsymbol{\rho} \\ H(\kappa) &= (2/i\pi\kappa)^{1/2} e^{i\kappa - i\omega t},\ \boldsymbol{g}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{\rho}}) = \hat{\boldsymbol{z}} \Sigma_m a_m(\hat{\boldsymbol{k}}) e^{im\psi} \end{aligned} \tag{34}$$

and the associated $\boldsymbol{H}$-field is computed by application of the Maxwell equations (4). In (34) H_m denotes the Hankel functions of the first kind, which together with the time factor $e^{-i\omega t}$ guarantee outgoing waves, and $e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0}$ is the extra phase factor resulting from the translation of the incident wave into the local coordinate system $\boldsymbol{\rho}$. The coefficients $a_m(\hat{\boldsymbol{k}})$, for the specific geometry of the cylinder at-hand, depend on the direction of incidence of the excitation wave (15). In the far field $\boldsymbol{E}_{\hat{\boldsymbol{k}}}$ tends asymptotically to an outgoing symmetrical cylindrical wave times the scattering amplitude $\boldsymbol{g}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{\rho}})$, with $\hat{\boldsymbol{k}}$, $\hat{\boldsymbol{\rho}}(\psi)$, indicating directions of incidence and observation, respectively. The notation $\boldsymbol{k}_\rho \cdot \boldsymbol{\rho}$ serves to emphasize the *quasi* plane-wave behavior of the scattered wave in the far field.

Using (32), the far field (34) is translated back to the initial coordinate system, yielding

$$\boldsymbol{E}_{\hat{\boldsymbol{k}}}(\boldsymbol{r}, t) \sim e_0 e^{i(\boldsymbol{k} - \boldsymbol{k}_\rho)\cdot\boldsymbol{r}_0} (2/i\pi\kappa)^{1/2} e^{i\boldsymbol{k}_\rho\cdot\boldsymbol{r} - i\omega t} \boldsymbol{g}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{\rho}}) \tag{35}$$

and in view of the constants and the slowly varying terms in (35), $\boldsymbol{E}_{\hat{\boldsymbol{k}}}(\boldsymbol{r}, t)$ resembles a plane wave with the phase $\boldsymbol{k}_\rho \cdot \boldsymbol{r} - \omega t$.

Similarly to (31), consider now a right-handed local coordinate system defined in $\{2\}$

$$\overline{\boldsymbol{\rho}} = \overline{\boldsymbol{\rho}}(\overline{\rho}, \overline{\psi}), \quad \hat{\overline{\boldsymbol{\rho}}} \times \hat{\overline{\boldsymbol{\psi}}} = \hat{\boldsymbol{z}} \tag{36}$$

For (31), (36), to be symmetrical and have mirror azimuthal angles, implies

$$\begin{gathered} \boldsymbol{r} = \overline{\boldsymbol{r}}_0 + \overline{\boldsymbol{\rho}}, \ \tilde{\boldsymbol{N}} \cdot (\overline{\boldsymbol{r}}_0 - \boldsymbol{r}_0) = 0, \quad \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\overline{\boldsymbol{r}}_0 + \boldsymbol{r}_0) = 0 \\ \tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{\rho}} - \hat{\overline{\boldsymbol{\rho}}}) = 0\Big|_{\overline{\psi}=-\psi}, \quad \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\hat{\boldsymbol{\rho}} + \hat{\overline{\boldsymbol{\rho}}}) = 0\Big|_{\overline{\psi}=-\psi} \\ \tilde{\boldsymbol{N}} \cdot (\hat{\boldsymbol{\psi}} + \hat{\overline{\boldsymbol{\psi}}}) = 0\Big|_{\overline{\psi}=-\psi}, \quad \hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\hat{\boldsymbol{\psi}} - \hat{\overline{\boldsymbol{\psi}}}) = 0\Big|_{\overline{\psi}=-\psi} \end{gathered} \tag{37}$$

Corresponding to the initial scatterer defined in $\{1\}$, a geometrically symmetric mirror-scatterer is now defined in $\{2\}$. While the initial scatterer is excited by (15) in $\{1\}$, the image object is excited by the image plane-wave (17) considered in $\{2\}$. These waves are symmetrical according to (18) and $\hat{\boldsymbol{n}}\hat{\boldsymbol{n}} \cdot (\boldsymbol{k} + \overline{\boldsymbol{k}}) = 0$.

The wave scattered from the image cylinder is (cf. (34)) is

$$\begin{gathered} \boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}}(\overline{\boldsymbol{\rho}}, t) = \hat{\boldsymbol{z}}\overline{e}_0 e^{i\overline{\boldsymbol{k}}\cdot\overline{\boldsymbol{r}}_0} e^{-i\omega t}\Sigma_m i^m \overline{a}_m(\hat{\overline{\boldsymbol{k}}}) H_m(\overline{\kappa}) e^{im\overline{\psi}} \\ \sim \overline{e}_0 e^{i\overline{\boldsymbol{k}}\cdot\overline{\boldsymbol{r}}_0} H(\overline{\kappa}) \overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}, \hat{\overline{\boldsymbol{\rho}}}), \ \overline{\kappa} = k\overline{\rho} = k\hat{\overline{\boldsymbol{\rho}}} \cdot \overline{\boldsymbol{\rho}} = \overline{\boldsymbol{k}}_{\overline{\rho}} \cdot \overline{\boldsymbol{\rho}}, \ \overline{e}_0 = -e_0 \\ H(\overline{\kappa}) = (2/i\pi\overline{\kappa})^{1/2} e^{i\overline{\kappa} - i\omega t}, \ \overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}, \hat{\overline{\boldsymbol{\rho}}}) = \hat{\boldsymbol{z}}\Sigma_m \overline{a}_m(\hat{\overline{\boldsymbol{k}}}) e^{im\overline{\psi}} \end{gathered} \tag{38}$$

The symmetries prescribe for (34), (38)

$$\overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}, \hat{\overline{\boldsymbol{\rho}}}) = \boldsymbol{g}(\hat{\boldsymbol{k}}, \hat{\rho})\Big|_{\overline{\psi}=-\psi}, \quad \overline{a}_m(\hat{\overline{\boldsymbol{k}}}) = a_m(\hat{\boldsymbol{k}}) \tag{39}$$

Similarly to (35), we now have

$$\boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}}(\boldsymbol{r}, t) \sim \overline{e}_0 e^{i(\overline{\boldsymbol{k}} - \overline{\boldsymbol{k}}_{\overline{\rho}})\cdot\overline{\boldsymbol{r}}_0} (2/i\pi\overline{\kappa})^{1/2} e^{i\overline{\boldsymbol{k}}_{\overline{\rho}}\cdot\boldsymbol{r} - i\omega t} \overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}, \hat{\overline{\boldsymbol{\rho}}}), \ \overline{e}_0 = -e_0 \tag{40}$$

Remove now the perfectly-conducting plane-interface and consider the various symmetries together with the expressions (35), (40), at points on the plane-interface. Clearly the original boundary conditions (18) are maintained on both side of the plane-interface, and the fields in $\{1\}$ remain unaltered, for both the incident and reflected waves, and the scattered waves originating in the half-spaces $\{1\}$ and $\{2\}$, respectively.

Plane waves are very convenient for discussing image-method situations in velocity-dependent systems. Accordingly, arbitrary wave functions, for arbitrary distances from the scatterer, can be recast in terms of plane-wave integral representations. For cylindrical geometries in particular, the Sommerfeld integral representations [2, 4, 8] are exploited. Accordingly (34), (38) are recast in terms of a superposition of inhomogeneous plane waves, in the form

$$\begin{aligned}
&\boldsymbol{E}_{\hat{\boldsymbol{k}}}(\boldsymbol{\rho},t)=e_0 e^{i\boldsymbol{k}\cdot\boldsymbol{r}_0}\int_{\psi,\tau} e^{i\boldsymbol{k}_\rho\cdot\boldsymbol{\rho}-i\omega t}\boldsymbol{g}(\hat{\boldsymbol{k}},\hat{\boldsymbol{\rho}})d\tau/\pi,\ \int_{\psi,\tau}=\int_{\tau=\psi-\pi/2+i\infty}^{\tau=\psi+\pi/2-i\infty}\\
&\boldsymbol{g}=\hat{\boldsymbol{z}}g(\hat{\boldsymbol{k}},\hat{\boldsymbol{\rho}})=\hat{\boldsymbol{z}}\Sigma_m a_m(\hat{\boldsymbol{k}})e^{im\tau},\ \boldsymbol{k}_p\cdot\boldsymbol{\rho}=k\hat{\boldsymbol{\rho}}\cdot\boldsymbol{\rho}=k\rho\cos(\psi-\tau) \quad (41)\\
&\boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}}(\overline{\boldsymbol{\rho}},t)=\overline{e}_0 e^{i\overline{\boldsymbol{k}}\cdot\overline{\boldsymbol{r}}_0}\int_{\overline{\psi},\overline{\tau}} e^{i\boldsymbol{k}_{\overline{p}}\cdot\overline{\boldsymbol{\rho}}-i\omega t}\overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}},\hat{\overline{\boldsymbol{\rho}}})d\overline{\tau}/\pi,\ \int_{\overline{\psi},\overline{\tau}}=\int_{\overline{\tau}=\overline{\psi}-\pi/2+i\infty}^{\overline{\tau}=\overline{\psi}+\pi/2-i\infty}\\
&\overline{\boldsymbol{g}}=\hat{\boldsymbol{z}}\overline{g}(\hat{\overline{\boldsymbol{k}}},\hat{\overline{\boldsymbol{\rho}}})=\hat{\boldsymbol{z}}\Sigma_m\overline{a}_m(\hat{\overline{\boldsymbol{k}}})e^{im\overline{\tau}},\ \boldsymbol{k}_{\overline{p}}\cdot\overline{\boldsymbol{\rho}}=k\hat{\overline{\boldsymbol{\rho}}}\cdot\overline{\boldsymbol{\rho}}=k\overline{\rho}\cos(\overline{\psi}-\overline{\tau})
\end{aligned}$$

Although in the integrand (41) the plane waves propagate in complex directions indicated by $\hat{\boldsymbol{p}}$, $\hat{\overline{\boldsymbol{p}}}$, in all other respects they are treated like plane waves with real propagation vectors.

The waves (41) are now translated back to their initial coordinate systems, yielding

$$\begin{aligned}
\boldsymbol{E}_{\hat{\boldsymbol{k}}}(\boldsymbol{r},t)&=e_0\int_{\psi,\tau} e^{i(\boldsymbol{k}-\boldsymbol{k}_p)\cdot\boldsymbol{r}_0}e^{i\boldsymbol{k}_p\cdot\boldsymbol{r}-i\omega t}\boldsymbol{g}(\hat{\boldsymbol{k}},\hat{\boldsymbol{p}})d\tau/\pi\\
\boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}}(\boldsymbol{r},t)&=\overline{e}_0\int_{\overline{\psi},\overline{\tau}} e^{i(\overline{\boldsymbol{k}}-\boldsymbol{k}_{\overline{p}})\cdot\overline{\boldsymbol{r}}_0}e^{i\boldsymbol{k}_{\overline{p}}\cdot\boldsymbol{r}-i\omega t}\overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}},\hat{\overline{\boldsymbol{p}}})d\overline{\tau}/\pi
\end{aligned} \quad (42)$$

Similarly to the argument following (40), in the integrands in (42), at the boundary, pairs of plane waves with symmetrical directions $\hat{\boldsymbol{p}}$, $\hat{\overline{\boldsymbol{p}}}$, can be identified, combining to satisfy the boundary conditions at the plane-interface.

In retrospect, the same conclusions could have been obtained by directly using the exact series in (34), (38), however the concept of plane-wave representations, which applies to three-dimensional situations as well, is more general [2, 4, 13–15], applicable to arbitrary wave functions. This approach will also be needed here in the sequel.

Finally, we have to pay attention to the multiple-scattering aspects of the problems at hand. The configuration of a scatterer and a plane boundary, or alternatively, a scatterer and its image, constitutes a many-body system in which successive scattering of waves bouncing from one object to another takes place. Thus in addition to the scattering processes discussed above, the waves reflected by the boundary (or scattered by the image scatterer), also excite the original scatterer, etc. A closed self-consistent multiple-scattering formalism has been devised by Twersky [13–15], but in velocity-dependent systems, in general, only a successive-scattering formalism is feasible, enumerating one scattering process after its predecessor. This is due to the fact that new frequencies are created with each successive-scattering process [8], and the new frequencies are unknown before the preceding modes are computed. The problem of dealing with higher-order modes becomes increasingly complicated [8, 16]. Subsequently only a limited number of interactions will be considered, namely, the interactions of the excitation plane-wave with the scatterer, and the reflected plane-wave with the image scatterer, as demonstrated above for the velocity-independent case.

5. VELOCITY-DEPENDENT IMAGES IN FREE-SPACE

The relatively simple case of free-space scattering is considered first. As in the previous section, for simplicity we deal with cylindrical scatterers, oriented along the cylindrical z-axis, and perpendicular velocities in the xy-plane.

Consider a scatterer at-rest in the reference-system $\mathbf{\Gamma}'$, defined relative to $\mathbf{\Gamma}$ according to (6) with $\boldsymbol{v}^{(*)} = \boldsymbol{v}'$. In $\mathbf{\Gamma}$ an incident wave (15) is given, with $\hat{\boldsymbol{E}} = \hat{\boldsymbol{z}}$, corresponding in $\mathbf{\Gamma}'$ to the wave given by (19). By inspecting (34), (41), (42), and judiciously modifying the pertinent notation, the scattered wave in $\mathbf{\Gamma}'$ is

$$\begin{aligned}\boldsymbol{E}'_{\hat{\boldsymbol{k}}'}(\boldsymbol{\rho}', t') &= \hat{\boldsymbol{z}} e'_0 e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'_0} e^{-i\omega' t'} \Sigma_m i^m a_m(\hat{\boldsymbol{k}}') H_m(\kappa') e^{im\psi'} \\ &= e'_0 e^{i\boldsymbol{k}'\cdot\boldsymbol{r}'_0} \int_{\psi',\tau'} e^{i\boldsymbol{k}'_{p'}\cdot\boldsymbol{\rho}' - i\omega' t'} \boldsymbol{g}'(\hat{\boldsymbol{k}}', \hat{\boldsymbol{p}}') d\tau'/\pi \\ \boldsymbol{E}'_{\hat{\boldsymbol{k}}'}(\boldsymbol{r}', t') &= e'_0 \int_{\psi',\tau'} e^{i(\boldsymbol{k}'-\boldsymbol{k}'_{p'})\cdot\boldsymbol{r}'_0} e^{i\boldsymbol{k}'_{p'}\cdot\boldsymbol{r}' - i\omega' t'} \boldsymbol{g}'(\hat{\boldsymbol{k}}', \hat{\boldsymbol{p}}') d\tau'/\pi\end{aligned} \tag{43}$$

where in (43) it is assumed that $\boldsymbol{g}'(\hat{\boldsymbol{k}}', \hat{\boldsymbol{r}}')$, hence also $a_m(\hat{\boldsymbol{k}}')$, are already available, e.g., by solving the boundary-value scattering problem in $\mathbf{\Gamma}'$. The last expression (43) results from the translation back into the initial $\boldsymbol{r}'$, t', coordinate system in $\mathbf{\Gamma}'$.

The plane waves in the integrand (43) individually satisfy (21), (22), hence the transformation from $\mathbf{\Gamma}'$ into $\mathbf{\Gamma}$ according to (25), (cf. (42)), yields

$$\begin{aligned}&\boldsymbol{E}_{\hat{\boldsymbol{k}}'} = \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}'} \cdot \boldsymbol{E}'_{\hat{\boldsymbol{k}}'}(\boldsymbol{r}', t') = e'_0 \int_{\psi',\tau'} e^{i(\boldsymbol{k}'-\boldsymbol{k}'_{p'})\cdot\boldsymbol{r}'_0} e^{i\boldsymbol{k}_{p'}\cdot\boldsymbol{r} - i\omega_{p'} t} \boldsymbol{g}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{p}}') d\tau'/\pi \\ &\boldsymbol{k}_{p'}\cdot\boldsymbol{r} - \omega_{p'} t = \boldsymbol{k}'_{p'}\cdot\boldsymbol{r}' - \omega' t', \ \ \boldsymbol{g}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{p}}') = \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}'} \cdot \boldsymbol{g}'(\hat{\boldsymbol{k}}', \hat{\boldsymbol{p}}')\end{aligned} \tag{44}$$

$$\tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}'} = \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} + \boldsymbol{v}' \times \partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \tilde{\boldsymbol{I}}) = \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}' \times \hat{\boldsymbol{k}}'_{p'} \times \tilde{\boldsymbol{I}}/c)$$
$$= \hat{\boldsymbol{z}}\gamma'(1 + \boldsymbol{v}' \cdot \hat{\boldsymbol{k}}'_{p'}/c)$$

where in (44) in free space $|\boldsymbol{k}'_{p'}| = \omega'/c,\ |\boldsymbol{k}_{p'}| = \omega_{p'}/c$. The plane-wave phase is represented in terms of the $\boldsymbol{\Gamma}$ parameters, but everything else is left in terms of $\boldsymbol{\Gamma}'$ parameters for convenience. Only when explicit computations are performed, further substitutions may be advised.

Note that in $\boldsymbol{\Gamma}'$ a single excitation frequency ω' is present, with a real value ω'. According to (21), and the inverse of (22) $\boldsymbol{K} = \boldsymbol{K}[\boldsymbol{K}']$, the phase $\boldsymbol{k}_{p'} \cdot \boldsymbol{r} - \omega_{p'}t$, in terms of $\boldsymbol{\Gamma}$ coordinates $\boldsymbol{r}$, t, is obtained from $\boldsymbol{k}'_{p'} \cdot \boldsymbol{r}' - \omega' t'$, which is in terms of $\boldsymbol{\Gamma}'$ coordinates. For the frequency in particular we have

$$\omega_{p'} = \gamma'(\omega' + \boldsymbol{v}' \cdot \boldsymbol{k}'_{p'}) \tag{45}$$

Inasmuch as (45) involves both real ω' and complex $\boldsymbol{k}'_{p'}$, we now have complex frequencies $\omega_{p'}$. Thus $e^{-\omega_p \cdot t}$ manifests the time dependent variation of the wave amplitudes due to the receding or approaching scatterer, as observed in $\boldsymbol{\Gamma}$.

In (44) we have a superposition (integral) of plane waves, for which we seek a corresponding set of reflected waves, or their images. This can be achieved by assuming a symmetrical image-object moving in the half-space region $\{2\}$, at-rest in the image reference-system $\boldsymbol{\Gamma}''$. This reference-system is related to $\boldsymbol{\Gamma}$ according to (6), with the velocity $\boldsymbol{v}^{(*)} = \boldsymbol{v}''$. The two reference-frames $\boldsymbol{\Gamma}'$, $\boldsymbol{\Gamma}''$, considered with respect to the boundary at-rest in $\boldsymbol{\Gamma}$, are moving symmetrically according to (24).

By inspection of (38), (41), (43), the wave scattered from the image-cylinder, at-rest in $\boldsymbol{\Gamma}''$, is given by

$$\begin{aligned}
\boldsymbol{E}''_{\hat{\overline{\boldsymbol{k}}}''}(\overline{\boldsymbol{\rho}}'', t'') &= \hat{\boldsymbol{z}} e''_0 e^{i\overline{\boldsymbol{k}}''\overline{\boldsymbol{r}}''_0} e^{-i\overline{\omega}''t''} \Sigma_m i^m \overline{a}_m(\hat{\overline{\boldsymbol{k}}}'') H_m(\overline{\kappa}'') e^{im\overline{\psi}''} \\
&= \overline{e}''_0 e^{i\overline{\boldsymbol{k}}''\cdot\overline{\boldsymbol{r}}''_0} \int_{\overline{\psi}'',\overline{\tau}''} e^{i\overline{\boldsymbol{k}}''_{\overline{p}''}\cdot\overline{\boldsymbol{\rho}}'' - i\overline{\omega}''t''} \overline{\boldsymbol{g}}''(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') d\overline{\tau}''/\pi \qquad (46) \\
\overline{\boldsymbol{g}}'' &= \hat{\boldsymbol{z}}\overline{g}''(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') = \hat{\boldsymbol{z}}\Sigma_m \overline{a}_m(\hat{\overline{\boldsymbol{k}}}'') e^{im\overline{\tau}''},\ \overline{\boldsymbol{\rho}}'' = \overline{\boldsymbol{\rho}}''(\overline{\rho}'', \overline{\psi}'') \\
\boldsymbol{k}''_{\overline{p}''} \cdot \overline{\boldsymbol{\rho}}'' &= k''\hat{\overline{\boldsymbol{p}}}'' \cdot \overline{\boldsymbol{\rho}}'' = k''\overline{\rho}'' \cos(\overline{\psi}'' - \overline{\tau}'')
\end{aligned}$$

and similarly to (43), translation to the initial coordinates $\boldsymbol{r}''$, t'', of reference-frame $\boldsymbol{\Gamma}''$ yields

$$\boldsymbol{E}''_{\hat{\overline{\boldsymbol{k}}}''}(\boldsymbol{r}'', t'') = \overline{e}''_0 \int_{\overline{\psi}'',\overline{\tau}''} e^{i(\overline{\boldsymbol{k}}'' - \overline{\boldsymbol{k}}''_{\overline{p}''})\cdot\overline{\boldsymbol{r}}''_0} e^{i\overline{\boldsymbol{k}}''_{\overline{p}''}\cdot\boldsymbol{r}'' - i\overline{\omega}_{\overline{p}''}t} \overline{\boldsymbol{g}}''(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') d\overline{\tau}''/\pi \tag{47}$$

As in (44), the scattered wave (46) is now transformed into $\mathbf{\Gamma}$

$$\begin{aligned} \boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}''} &= \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}''} \cdot \boldsymbol{E}''_{\hat{\overline{\boldsymbol{k}}}''}(\boldsymbol{r}'', t'') \\ &= \overline{e}''_0 \int_{\overline{\psi}'', \overline{\tau}''} e^{i(\overline{\boldsymbol{k}}'' - \overline{\boldsymbol{k}}''_{\overline{p}''}) \cdot \overline{\boldsymbol{r}}''_0} e^{i\overline{\boldsymbol{k}}_{\overline{p}''} \cdot \boldsymbol{r} - i\overline{\omega}_{\overline{p}''} t} \overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') d\overline{\tau}''/\pi \qquad (48) \\ \overline{\boldsymbol{k}}_{\overline{p}''} \cdot \boldsymbol{r} - \overline{\omega}_{\overline{p}''} t &= \overline{\boldsymbol{k}}''_{\overline{p}''} \cdot \boldsymbol{r}'' - i\overline{\omega}'' t'', \ \overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') = \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}''} \cdot \overline{\boldsymbol{g}}''(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') \\ \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}''} &= \tilde{\boldsymbol{V}}'' \cdot (\tilde{\boldsymbol{I}} + \boldsymbol{v}'' \times \partial_{t''}^{-1} \partial_{\boldsymbol{r}''} \times \tilde{\boldsymbol{I}}) = \tilde{\boldsymbol{V}}'' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}'' \times \hat{\overline{\boldsymbol{k}}}''_{\overline{p}''} \times \tilde{\boldsymbol{I}}/c) \\ &= \hat{\boldsymbol{z}} \gamma''(1 + \boldsymbol{v}'' \cdot \hat{\overline{\boldsymbol{k}}}''_{\overline{p}''}/c), \ |\overline{\boldsymbol{k}}''_{\overline{p}''}| = \overline{\omega}''/c, \ |\overline{\boldsymbol{k}}_{\overline{p}''}| = \overline{\omega}_{\overline{p}''}/c \end{aligned}$$

Once again (48) displays the phenomenon encountered in (45), namely that in $\mathbf{\Gamma}$ the frequency $\overline{\omega}_{\overline{p}''}$ for each complex direction indicated by $\hat{\overline{\boldsymbol{p}}}''$ is complex, and $e^{-\overline{\omega}_{\overline{p}''} t}$ manifests the time dependence of the amplitude due to the motion.

Inasmuch as we are now dealing with plane waves in $\mathbf{\Gamma}$, we can take pairs of waves from the integrands of (44), (48), and impose on them the boundary conditions at the plane-interface in $\mathbf{\Gamma}$. Thus from the symmetries (24), (39), see also (43), (46), we have

$$\begin{aligned} &(\boldsymbol{k}' - \boldsymbol{k}'_{p'}) \cdot \boldsymbol{r}'_0 = (\overline{\boldsymbol{k}}'' - \overline{\boldsymbol{k}}''_{\overline{p}''}) \cdot \overline{\boldsymbol{r}}''_0 \Big|_{\overline{\psi}'' = -\psi'} \\ &\omega_{p'} = \overline{\omega}_{\overline{p}''} \Big|_{\overline{\psi}'' = -\psi'}, \ \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}_{p'} - \overline{\boldsymbol{k}}_{\overline{p}''}) = 0 \Big|_{\overline{\psi}'' = -\psi'} \qquad (49) \\ &\boldsymbol{g}(\hat{\boldsymbol{k}}', \hat{\boldsymbol{p}}') = \overline{\boldsymbol{g}}(\hat{\overline{\boldsymbol{k}}}'', \hat{\overline{\boldsymbol{p}}}'') \Big|_{\overline{\psi}'' = -\psi'} \end{aligned}$$

which finally allows us to remove the perfectly-conducting plane-interface, and consider the image problem as solved.

Observation from $\mathbf{\Gamma}'$, where the plane boundary is moving with velocity $\boldsymbol{v}'$ follows upon transforming (46) from $\mathbf{\Gamma}$ to $\mathbf{\Gamma}'$, by applying the differential operator taken from (19)

$$\boldsymbol{E}'_{\hat{\overline{\boldsymbol{k}}}''} = \tilde{\boldsymbol{W}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} \cdot \boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}''}, \quad \tilde{\boldsymbol{W}}^{\mathbf{\Gamma}',\mathbf{\Gamma}} = \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}' \times \partial_t^{-1} \partial_{\boldsymbol{r}} \times \tilde{\boldsymbol{I}}) \qquad (50)$$

constituting a very complicated computation, whose detail should be left to a numerical simulation project. In principle, each of the plane waves in the integrand, which in $\mathbf{\Gamma}$ satisfy the conditions of equal frequencies and the Snell law in (49), now will behave according to Einstein's aberration formula [5], as remarked above after (22), for the single plane wave.

6. VELOCITY-DEPENDENT IMAGES IN MATERIAL MEDIA

Once again we start with a material medium at-rest in $\mathbf{\Gamma}$ in region $\{1\}$, possessing arbitrary parameters ε, μ. The waves (15), (17), are assumed, satisfying the boundary conditions prescribed by (18) at the perfectly-conducting plane. The waves transform into $\mathbf{\Gamma}'$ according to (19)–(22). Similarly, according to (23), the waves can be transformed into reference-system $\mathbf{\Gamma}''$, moving with velocity $\boldsymbol{v}''$ when observed from $\mathbf{\Gamma}$, with $\mathbf{\Gamma}'$, $\mathbf{\Gamma}''$, related by the symmetry condition (24).

It must be born in mind that in $\mathbf{\Gamma}'$, $\mathbf{\Gamma}''$, we presently deal with waves in moving media. The wave scattered by an object at-rest in the moving medium in $\mathbf{\Gamma}'$, excited by $\boldsymbol{E}'$ in (19), is in general given by the integral representation in (43). However, since we are dealing here with a moving medium having a preferred direction prescribed by the velocity, the medium is effectively anisotropic. Consequently the scattered wave will not submit to a Hankel-Fourier series representation as in (43). The Fresnel drag effect $\boldsymbol{k}' = \boldsymbol{k}'[\boldsymbol{k}]$ given in (22) must be taken into account, prescribing that for the present case $\boldsymbol{k}'_{p'}$ in the last two lines of (43) is velocity-dependent. Therefore, for each plane wave in the integrand, we are dealing with an individual effective phase-velocity v'_{ph}. The evaluation of specific scattering problems of this kind is complicated. Some relevant work for scatterers moving relative to the ambient medium have been discussed before, e.g., see [11, 12, 17].

Transforming the plane waves in the integrand (43) into $\mathbf{\Gamma}$ yields (44). Similarly, we have the plane waves in the integrand (46), (47), scattered by an image-object at-rest in the moving medium in $\mathbf{\Gamma}''$, subject to (24), excited by $\overline{\boldsymbol{E}}''$, (23). Similarly to (44), the scattered wave (46), (47), is transformed into $\mathbf{\Gamma}$, yielding (48).

Obviously the present scenario follows closely along the lines of the free-space and velocity-independent cases. Thus in (42), pairs of plane waves with symmetrical directions were sought, each pair satisfying the boundary conditions at the plane-interface. Here the same situation applies to the plane waves in the integrands of (44) and (48). Hence the boundary conditions are satisfied, and the perfectly-conducting plane-interface may be removed without altering the fields in region $\{1\}$.

By transforming all waves into $\mathbf{\Gamma}'$, we are dealing with a scatterer at-rest embedded in a moving material medium, in the presence of a plane-interface at-rest in $\mathbf{\Gamma}$, observed from $\mathbf{\Gamma}'$ to be moving at velocity $-\boldsymbol{v}'$. This is the same situation which for the single plane-wave is described by (19)–(22), associated with the aberration phenomenon.

We are now ready to discuss the scenario involving a material medium and a scatterer, both at-rest in $\mathbf{\Gamma}'$ in $\{\mathit{1}\}$, in the presence of a perfectly-conducting plane-interface at-rest in $\mathbf{\Gamma}$. In $\mathbf{\Gamma}$ a medium moving relative to the boundary is encountered, therefore the above remarks regarding the flow continuity apply here too. We seek to replace this configuration by an image medium and an image scatterer at-rest in $\mathbf{\Gamma}''$ in $\{\mathit{2}\}$. The evaluation of the boundary-value problems for the scatterers at-rest with respect to the media in $\mathbf{\Gamma}'$, $\mathbf{\Gamma}''$, is classical and need not be further discussed.

The excitation wave, and its reflection which is equivalent to an image wave, have been discussed above, (27)–(30). As shown in (27), we need to represent the waves in both regions $\{\mathit{1}\}$ and $\{\mathit{2}\}$, because in $\mathbf{\Gamma}$ we encounter two different effective anisotropic media in those regions, whose properties are governed by the motion. This is the key to understanding and solving the problem. Corresponding to the first line in (27), we have the scattered wave (43), in $\mathbf{\Gamma}'$ in $\{\mathit{1}\}$ given by

$$\begin{aligned}
\boldsymbol{E}'_{\hat{\boldsymbol{k}}'\{1\}}(\boldsymbol{\rho}',t') &= \hat{\boldsymbol{z}} e'_{0\{1\}} e^{i\boldsymbol{k}'_{\{1\}}\cdot\boldsymbol{r}'_0} e^{-i\omega'_{\{1\}}t'} \Sigma_m i^m a_{m\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}}) H_m(\kappa'_{\{1\}}) e^{im\psi'} \\
&= e'_{0\{1\}} e^{i\boldsymbol{k}'_{\{1\}}\cdot\boldsymbol{r}'_0} \int_{\psi',\tau'} e^{i\boldsymbol{k}'_{p'\{1\}}\cdot\boldsymbol{\rho}'-i\omega'_{\{1\}}t'} \boldsymbol{g}'_{\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}},\hat{\boldsymbol{p}}') d\tau'/\pi \\
\boldsymbol{E}'_{\hat{\boldsymbol{k}}'\{1\}}(\boldsymbol{r},t') &= e'_{0\{1\}} \int_{\psi',\tau'} e^{i(\boldsymbol{k}'_{\{1\}}-\boldsymbol{k}'_{p'\{1\}})\cdot\boldsymbol{r}'_0} e^{i\boldsymbol{k}'_{p'\{1\}}\cdot\boldsymbol{r}'-i\omega'_{\{1\}}t'} \\
&\quad \cdot \boldsymbol{g}'_{\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}},\hat{\boldsymbol{p}}') d\tau'/\pi
\end{aligned} \tag{51}$$

and according to (44), the wave (51) is transformed into $\mathbf{\Gamma}$ in $\{\mathit{1}\}$, yielding

$$\begin{aligned}
\boldsymbol{E}_{\hat{\boldsymbol{k}}'\{1\}} &= \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}'} \cdot \boldsymbol{E}'_{\hat{\boldsymbol{k}}'\{1\}}(\boldsymbol{r}',t') \\
&= e'_{0\{1\}} \int_{\psi',\tau'} e^{i(\boldsymbol{k}'_{\{1\}}-\boldsymbol{k}'_{p'\{1\}})\cdot\boldsymbol{r}'_0} e^{i\boldsymbol{k}_{p'\{1\}}\cdot\boldsymbol{r}-i\omega_{p'\{1\}}t} \boldsymbol{g}_{\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}},\hat{\boldsymbol{p}}') d\tau'/\pi \\
\boldsymbol{k}_{p'\{1\}}\cdot\boldsymbol{r} - \omega_{p'\{1\}}t &= \boldsymbol{k}'_{p'\{1\}}\cdot\boldsymbol{r}' - \omega'_{\{1\}}t', \\
\boldsymbol{g}_{\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}},\hat{\boldsymbol{p}}') &= \tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}'} \cdot \boldsymbol{g}'_{\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}},\hat{\boldsymbol{p}}') \\
\tilde{\boldsymbol{W}}^{\mathbf{\Gamma},\mathbf{\Gamma}'} &= \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} + \boldsymbol{v}' \times \partial_{t'}^{-1}\partial_{\boldsymbol{r}'} \times \tilde{\boldsymbol{I}}) = \tilde{\boldsymbol{V}}' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}' \times \hat{\boldsymbol{k}}'_{p'\{1\}} \times \tilde{\boldsymbol{I}}/c) \\
&= \hat{\boldsymbol{z}}\gamma'(1 + \boldsymbol{v}'\cdot\hat{\boldsymbol{k}}'_{p'\{1\}}/c)
\end{aligned} \tag{52}$$

Each plane wave in (52) must be associated with a reflected wave, such that together, according to the first line (29), the boundary conditions are satisfied

$$\begin{aligned} \omega_{p'\{1\}} - \overline{\omega}_{p'\{1\}} = 0, \quad \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}_{p'\{1\}} - \overline{\boldsymbol{k}}_{p'\{1\}}) &= 0 \\ \tilde{\boldsymbol{N}} \cdot (\boldsymbol{g}_{\{1\}}(\hat{\boldsymbol{k}}'_{\{1\}}, \hat{\boldsymbol{p}}') e'_{0\{1\}} + \overline{\boldsymbol{g}}_{\{1\}}(\hat{\overline{\boldsymbol{k}}}'_{\{1\}}, \hat{\boldsymbol{p}}') \overline{e}'_{0\{1\}}) &= 0 \end{aligned} \tag{53}$$

Therefore in $\boldsymbol{\Gamma}$ in $\{1\}$, the wave reflected from the plane-interface is given by

$$\boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}'\{1\}} = \overline{e}'_{0\{1\}} \int_{\psi',\tau'} e^{i(\boldsymbol{k}'_{\{1\}} - \boldsymbol{k}'_{p'\{1\}}) \cdot \boldsymbol{r}'_0} e^{i\overline{\boldsymbol{k}}_{p'\{1\}} \cdot \boldsymbol{r} - i\overline{\omega}_{p'\{1\}} t} \overline{\boldsymbol{g}}_{\{1\}}(\hat{\overline{\boldsymbol{k}}}'_{\{1\}}, \hat{\boldsymbol{p}}') d\tau'/\pi \tag{54}$$

Once again, in order to remove the perfectly-conducting interface, we need to define a medium at rest in $\boldsymbol{\Gamma}''$ in $\{2\}$, materially identical to the medium at-rest in $\boldsymbol{\Gamma}'$ in $\{1\}$. Observed in $\boldsymbol{\Gamma}$, the medium is moving with a symmetrical velocity $\boldsymbol{v}''$ satisfying (24). Consistently, we must have waves in $\boldsymbol{\Gamma}$ in $\{2\}$, such that the second line (29) is satisfied, i.e., similarly to (53) we now have

$$\begin{aligned} \omega_{p'\{2\}} - \overline{\omega}_{p'\{2\}} = 0, \quad \tilde{\boldsymbol{N}} \cdot (\boldsymbol{k}_{p'\{2\}} - \overline{\boldsymbol{k}}_{p'\{2\}}) &= 0 \\ \tilde{\boldsymbol{N}} \cdot (\boldsymbol{g}_{\{2\}}(\hat{\boldsymbol{k}}''_{\{2\}}, \hat{\boldsymbol{p}}'') e''_{0\{2\}} + \overline{\boldsymbol{g}}_{\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}, \hat{\boldsymbol{p}}') \overline{e}''_{0\{2\}}) &= 0 \end{aligned} \tag{55}$$

according to the subsequent expressions.

Similarly to (46), (47), the wave scattered by the image object in $\boldsymbol{\Gamma}''$ in $\{2\}$ (cf. (51)) is given by

$$\begin{aligned} \boldsymbol{E}''_{\hat{\overline{\boldsymbol{k}}}''\{2\}}(\overline{\boldsymbol{\rho}}'', t'') &= \hat{\boldsymbol{z}} \overline{e}''_{0\{2\}} e^{i\overline{\boldsymbol{k}}''_{\{2\}} \overline{\boldsymbol{r}}''_0} e^{-i\overline{\omega}''_{\{2\}} t''} \Sigma_m i^m \overline{a}_{m\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}) H_m(\overline{\kappa}''_{\{2\}}) e^{im\overline{\psi}''} \\ &= \overline{e}''_{0\{2\}} e^{i\overline{\boldsymbol{k}}''_{\{2\}} \cdot \overline{\boldsymbol{r}}''_0} \int_{\overline{\psi}'', \overline{\tau}''} e^{i\overline{\boldsymbol{k}}''_{\overline{p}''\{2\}} \cdot \overline{\boldsymbol{\rho}}'' - i\overline{\omega}''_{\{2\}} t''} \overline{\boldsymbol{g}}''_{\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}, \hat{\overline{\boldsymbol{p}}}'') d\overline{\tau}''/\pi \\ \boldsymbol{E}''_{\hat{\overline{\boldsymbol{k}}}''\{2\}}(\boldsymbol{r}'', t'') &= \overline{e}''_{0\{2\}} \int_{\overline{\psi}'', \overline{\tau}''} e^{i(\overline{\boldsymbol{k}}''_{\{2\}} - \overline{\boldsymbol{k}}''_{\overline{p}''\{2\}}) \cdot \overline{\boldsymbol{r}}''_0} e^{i\overline{\boldsymbol{k}}''_{\overline{p}''\{2\}} \boldsymbol{r}'' - i\overline{\omega}''_{\{2\}} t''} \\ &\cdot \overline{\boldsymbol{g}}''_{\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}, \hat{\overline{\boldsymbol{p}}}'') d\overline{\tau}''/\pi \end{aligned} \tag{56}$$

Transforming (56) into $\boldsymbol{\Gamma}$ in $\{2\}$, according to (48) (cf. (52)), yields

$$\begin{aligned} &\boldsymbol{E}_{\hat{\overline{\boldsymbol{k}}}''\{2\}} = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma}, \boldsymbol{\Gamma}''} \boldsymbol{E}''_{\hat{\overline{\boldsymbol{k}}}''\{2\}}(\boldsymbol{r}'', t'') \\ &= \overline{e}''_{0\{2\}} \int_{\psi'', \tau''} e^{i(\overline{\boldsymbol{k}}''_{\{2\}} - \overline{\boldsymbol{k}}''_{\overline{p}''\{2\}}) \cdot \overline{\boldsymbol{r}}''_0} e^{i\overline{\boldsymbol{k}}_{\overline{p}''\{2\}} \cdot \boldsymbol{r} - i\overline{\omega}_{\overline{p}''\{2\}} t} \overline{\boldsymbol{g}}_{\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}, \hat{\overline{\boldsymbol{p}}}'') d\tau''/\pi \\ &\overline{\boldsymbol{k}}_{\overline{p}''\{2\}} \cdot \boldsymbol{r} - \overline{\omega}_{\overline{p}''\{2\}} t = \overline{\boldsymbol{k}}''_{\overline{p}''\{2\}} \cdot \boldsymbol{r}'' - i\overline{\omega}''_{\{2\}} t'' \end{aligned}$$

$$\overline{\boldsymbol{g}}_{\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}, \hat{\overline{\boldsymbol{p}}}'') = \tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}''} \cdot \overline{\boldsymbol{g}}''_{\{2\}}(\hat{\overline{\boldsymbol{k}}}''_{\{2\}}, \hat{\overline{\boldsymbol{p}}}'') \tag{57}$$

$$\tilde{\boldsymbol{W}}^{\boldsymbol{\Gamma},\boldsymbol{\Gamma}''} = \tilde{\boldsymbol{V}}'' \cdot (\tilde{\boldsymbol{I}} + \boldsymbol{v}'' \times \partial_{t''}^{-1}\partial_{\boldsymbol{r}''} \times \tilde{\boldsymbol{I}}) = \tilde{\boldsymbol{V}}'' \cdot (\tilde{\boldsymbol{I}} - \boldsymbol{v}'' \times \hat{\overline{\boldsymbol{k}}}''_{\overline{p}''\{2\}} \times \tilde{\boldsymbol{I}}/c)$$
$$= \hat{\boldsymbol{z}}\gamma''(1 + \boldsymbol{v}'' \cdot \hat{\overline{\boldsymbol{k}}}''_{\overline{p}''\{2\}}/c)$$

In $\boldsymbol{\Gamma}$ in $\{2\}$ in order to satisfy the boundary conditions (55), reflected waves are needed, constituting the analog of (54)

$$\boldsymbol{E}_{\hat{\boldsymbol{k}}''\{2\}} = e''_{0\{2\}} \int_{\psi'',\tau''} e^{i(\overline{\boldsymbol{k}}''_{\{2\}} - \overline{\boldsymbol{k}}''_{\overline{p}''\{2\}})\cdot\overline{\boldsymbol{r}}''_0} e^{i\boldsymbol{k}_{\overline{p}''\{2\}}\cdot\boldsymbol{r} - i\omega_{\overline{p}''\{2\}}t} \boldsymbol{g}_{\{2\}}(\hat{\boldsymbol{k}}''_{\{2\}}, \hat{\overline{\boldsymbol{p}}}'') d\overline{\tau}''/\pi \tag{58}$$

With all that accomplished, the perfectly-conducting plane-interface can be removed without affecting the initial waves in $\{1\}$. It is noted that on the two sides of the interface, in the different media as defined by the motion, the fields without the upper-bar and those endowed with an upper-bar are different. As for the single plane waves, this phenomenon can be ascribed to refraction at the interface separating the two media, see discussion after (30).

7. CONCLUDING REMARKS

The method of images in velocity-dependent wave systems is investigated. Apparently it works only for a limited class of problems, namely scattering in the presence of magnetic or electric perfect-mirrors, with plane-interface geometry. Problems involving material half-spaces, e.g., [18], must be excluded, but their limiting cases, when the interface becomes a perfect mirror, could be tested against the present method.

A theoretical discussion of the problem is presented here. Further numerical simulations are needed for depicting the new physical phenomena. This subject brings together scattering problems in media at-rest and in-motion, multiple scattering, and special relativity, thus providing an extension of classical method of images ideas.

Plane wave integral representations and the use of differential operators for field transformations facilitate compact notation, allowing for the description of various scenarios.

Free space situations are simpler to analyze, providing the basis for more elaborate cases involving material media, both at-rest and in-motion with respect to the perfectly-conducting plane-interface. When appropriate moving image scatterers and media are provided, this boundary can be removed without affecting the fields in the half-space where the initial problem is stated.

Material media pose the problem of evaluating scattering problems in the presence of moving media: In the scenario where the plane-interface is at-rest with respect to the material medium, the scatterer is embedded in a moving medium. The other case involves a scatterer at-rest with respect to the embedding medium, but the plane-interface is immersed in a moving media. These cases are not simple to analyze, and require sacrificing the mechanical flow-continuation in favor of a manageable definition of electromagnetic problems.

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