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GL METHOD AND ITS ADVANTAGES FOR RESOLVING HISTORICAL DIFFICULTIES

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Abstract—In this paper, we propose two types of new electromagnetic (EM) integral equation systems and their dual integral equation systems. Based on the EM integral equation systems, we propose a GL EM modeling and inversion algorithms. We make finite step iterations to exactly solve these integral equation systems or the EM and seismic differential integral equations in finite sub domains. The Global EM wave field is improved successively by the Local scattering EM wave field in the sub domains. Only 3×3 or 6×6 small matrices need to be solved in the GL method. There is no artificial boundary for infinite domain in the method; In the GL method, the cylindrical and spherical coordinate singularities are resolved; Our method combines the analytic and asymptotic method and numerical method. It is more accurate than FEM and FD method and Born likes approximation, the GL method is available for all frequencies and high contrast materials. Its solution has $O(h^2)$ convergent rate. If the Gaussian integrals are used, the field has $O(h^4)$ super convergence. The method is a high perform parallel algorithm with intrinsic self parallelization properties. The method has very simple scheme or no scheme or half scheme such that it has half mesh and no mesh. In the method, we can use both of Riemann and Lebesgue integral that induces a meshless method. We have developed software for 3D/2.5D EM, seismic, acoustic, flow dynamic, and QEM modeling and inversion.

1. INTRODUCTION

The existing EM theory and analytical and numerical methods are published in many books and journals. However, there are historical difficulties in EM and other field modeling and inversion. The large matrix equation, inaccurate and complex absorption conditions on artificial boundary, the cylindrical and spherical coordinate singularities, and ill posed in the modeling and inversion are historical difficulties. The Born approximation can be only used for low contrast material. In this paper, we propose the GL method "Global and Local field modeling and inversion" for resolving these historical difficulties. Our GL method is completely different from the FEM, FD, Born approximation methods.

We consider the EM, seismic, acoustic, quantum, flow and other field equations on finite inhomogeneous domain that is imbedded into an infinite domain. The analytical incident field and Green field in the background domain are called an initial global field. The inhomogeneous domain is divided into mesh or meshless sub domains. The global field is changing by local scattering field successively in each sub domain. The GL method processes will be finished when the Global field is passing through the all Local sub domains with inhomogeneous material. The abstracts of our GL method have been published in Piers 2005 in Hangzhuo. [1, 4–9], and in the GL Geophysical Laboratory reports [2, 3].

The method has the following advantages: (1) There is no large matrix to solve, only 3×3 or 6×6 small matrices need to be solved; (2) There is no artificial boundary for infinite domain; (3) The GL method combines the analytic and asymptotic method and numerical method consistently. It is more accurate than FEM and FD method and Born likes approximation; (4) The modeling solution has $O(h^2)$ convergent rate. In particular, if the Gaussian integrals are used, the solution has $O(h^4)$ super convergence; (5) The cylindrical and spherical coordinate singularities are resolved; (6) It is available for all frequencies and high contrast materials; (7) the method has very simple or no scheme, it has half mesh or no mesh; (8) In the method, we can use both Riemann and Lebesgue integrals that induce meshless methods; (9) The method can couple consistently with AGILD, FEM, and FD method; (10) The method is an intrinsic self parallel algorithm in parallel T3E and PC cluster.

The plan of this paper is as follows: The introduction is presented in the section 1. In the Section 2, we propose the EM integral equation systems. We propose the 3D/2D GL EM modeling based on the EM integral equation system in the Section 3. In Section 4, we propose the 3D/2D GL EM modeling based on the EM differential integral equation and electric and magnetic field integral equations. We propose the GL EM inversion in the Section 5. In Section 6, we prove the fundamental theorems of the GL method. We describe applications of the GL method in the Section 7. The conclusions are described in the Section 8.

2. NEW ELECTROMAGNETIC INTEGRAL EQUATION SYSTEMS

In this section, we propose the new EM integral equation systems as follows:

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_\Omega \begin{bmatrix} E_b^J(r',r) & H_b^J(r',r) \\ E_b^M(r',r) & H_b^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix} dr', \quad (1)$$

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix} = \begin{bmatrix} E_b(r) \\ H_b(r) \end{bmatrix} + \int_\Omega \begin{bmatrix} E^J(r',r) & H^J(r',r) \\ E^M(r',r) & H^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E_b(r') \\ H_b(r') \end{bmatrix} dr', \quad (2)$$

where $[D]$ is the EM material parameter variation matrix, for the isotropy materials, $[D]$ is 6×6 diagonal matrix with variance materials $(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)$ and $i\omega(\mu - \mu_b)$, for anisotropy materials the $[D]$ will be 6×6 full matrix. $E(r)$ is the electric field, $H(r)$ is the magnetic field, $E_b(r)$ is incident electric field in the background medium, $H_b(r)$ is incident magnetic field in the background medium, $E_b^M(r',r), \ldots, H_b^M(r',r)$ are electric or magnetic background field Green tensors exciting by the electric or magnetic dipole source respectively, The integral equations (1) and (2) are the dual system of each other.

3. THE 3D/2D NEW GL EM MODELING BASED ON THE ELECTROMAGNETIC INTEGRAL EQUATION SYSTEM

We propose the GL EM modeling based on the EM integral equation system in this section.

(3.1) The domain Ω is divided into a set of n mesh or meshless sub domains $\{\Omega_k\}$, $\Omega = \cup\{\Omega_k\}$.

(3.2) In each Ω_k, we solve the EM Green tensor integral equation system based on the equations (1) and (2). By dual operation, the equation systems are reduced into a 6×6 matrix equations. By solving the 6×6 equations, we obtain Green tensor field E_k^J and H_k^M.

(3.3) We improve the Global EM field $[E_k(r), H_k(r)]$ by the Local scattering field

$$\begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_k = \begin{bmatrix} E(r) \\ H(r) \end{bmatrix}_{k-1} + \int_{\Omega_k} \begin{bmatrix} E_k^J(r',r) & H_k^J(r',r) \\ E_k^M(r',r) & H_k^M(r',r) \end{bmatrix} [D] \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{k-1} dr', \quad (3)$$

$k = 1, 2, \ldots, n$, successively. The $[E_n(r), H_n(r)]$ is the GL solution of the EM integral equations (1) and (2).

4. THE 3D/2D GL EM MODELING BASED ON THE EM DIFFERENTIAL INTEGRAL EQUATION

4.1. The GL EM Modeling Based on the Magnetic Differential Integral Equation

Since 1995, we have proposed the magnetic field differential integral equation (MDI) in the frequency and time domain [10–13]. In this section, we propose the dual magnetic field differential integral equation of our MDI [10–13],

$$H(r) = H_b(r) + \int_{\Omega} \frac{(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)}{(\sigma + i\omega\varepsilon)_b} E^M(r', r) \cdot \nabla \times H_b(r') dr'. \tag{4}$$

Based on the equation (4), the GL magnetic field modeling is as follows:

(4.1) The step (4.1) is the same as (3.1).

(4.2) In each $\Omega_k,\ k = 1, 2, \ldots, n$, we solve the magnetic field differential integral equation to find $E_k^M(r', r)$ successively. By the dual curl operation, only 3×3 matrix equations need to be solved.

(4.3) We improve the Global field $H_k(r)$ by the Local scattering field

$$H_k(r) = H_{k-1}(r) + \int_{\Omega_k} \frac{(\sigma + i\omega\varepsilon) - (\sigma_b + i\omega\varepsilon_b)}{(\sigma + i\omega\varepsilon)_b} E_k^M(r', r) \cdot \nabla \times H_{k-1}(r') dr', \tag{5}$$

$k = 1, 2, \ldots, n$, successively. $H_n(r)$ is the GL magnetic field solution of (4).

4.2. GL EM Modeling Based on the Electric Differential Integral Equation

The GL electric field modeling based on the dual electric field differential integral equation of our EDI in 1995 [10–13],

$$E(r) = E_b(r) + \int_{\Omega} \frac{\mu - \mu_b}{\mu_b} H_b^J(r', r) \cdot \nabla \times E(r') dr'. \tag{6}$$

4.3. GL EM Modeling Based on the Electric and Magnetic Integral Equation

The GL methods based on the electric integral equation and the magnetic integral equation are similar to (4.1)–(4.3). Since the electric and magnetic integral equations have divergent Green kernel, a special approach for resolving the divergent singularity is developed.

4.4. GL Modeling for Quantum Field and QEM Field

The GL Schordinger modeling for two hydrogen atoms and interaction between QEM field and atoms is useful for QEM field in nanometer materials. We find GL numerical quanta for very high frequency EM field by GLQEM simulation.

5. THE NEW GL EM INVERSION

The formal logic system and experiments are base of the sciences. Most equations are forward equations. Maxwell equation and elastic equation are forward equation and are not for inversion. The EM integral equation systems (1) and (2) and equations (4) and (6) can be used for both forward and inversion. They are well posed for forward and ill posed for inversion. From essential formal logic in physics, these equations are well posed for forward and ill posed for inversion. How to build a well posed inverse equation is the main project of scientific inversion. Our new idea of the inverse formal logic and inverse experiment in physics motivates us to propose the GL inversion that is a new explicit inversion.

5.1. The GL EM Inversion GLEMI1 for Determining σ, ε, and μ

The following EM integral equation is for increments of EM parameters $\delta\sigma$, $\delta\varepsilon$, $\delta\mu$

$$\begin{bmatrix} \delta E(r) \\ \delta H(r) \end{bmatrix}_k = \int_{\Omega_k} \begin{bmatrix} E_k^J(r',r) & H_k^J(r',r) \\ E_k^M(r',r) & H_k^M(r',r) \end{bmatrix} [\delta D]_k \begin{bmatrix} E(r') \\ H(r') \end{bmatrix}_{k-1} dr'. \tag{7}$$

5.2. The GL EM Inversion GLEMI2 for Determining σ, ε

The following magnetic field differential integral equation is for variation of parameters $\delta\sigma$, $\delta\varepsilon$,

$$\delta H_k(r) = \int_{\Omega_l} \frac{(\delta\sigma + i\omega\delta\varepsilon)_k}{(\sigma + i\omega\varepsilon)_b} E_k^M(r', r) \cdot \nabla \times H_{k-1}(r') dr', \tag{8}$$

5.3. The GL EM Inversion GLEMI3 for Determining μ

The following electric field differential integral equation is for variation of EM parameter $\delta\mu$,

$$\delta E_k(r) = \int_{\Omega_k} \frac{\delta\mu_k}{\mu_b} H_k^M(r', r) \cdot \nabla \times E_{k-1}(r') dr'. \tag{9}$$

The suitable strong and weaker regularizing should be added to (7), (8), and (9) to control inversion being stable and reasonable resolution. In our GL EM inversion, only smaller matrices need to be solved. The resolution is dependent on the data configuration, quality and the regularizing.

6. THE THEORY AND ADVANTAGES OF THE GL METHOD

6.1. Theory

Theorem 1. The GL EM field $[E_n(r), H_n(r)]$ from (3.1)–(3.3) is convergent to exact EM field that satisfies the EM integral equation systems (1) and (2) and the MAXWELL EM equation.

Theorem 2. The GL Magnetic field, $H_n(r)$ from (4.1)–(4.3) is convergent to the exact magnetic field, $H(r)$ that satisfies the magnetic field differential integral equation (4). The GL EM field, $H_n(r)$ is convergent to the exact magnetic field $H(r)$ that satisfies the exact MAXWELL EM equation.

Theorem 3. By Riemann division, the GL EM field $[E_n(r), H_n(r)]$ from (3.1)–(3.3) and the GL magnetic field $H_n(r)$ from (4.1)–(4.3) have $O(h^2)$ convergent if the trapezoid and mid point integrals are used. In particular, if the Gaussian integrals are used, the GL EM field has $O(h^4)$ super convergent rate. Proof: The Theorems 1–3 have proved in [2].

6.2. Advantages of the GL Method

We consider EM modeling in infinite domain that involves the finite inhomogeneous boundary domain. When we use FEM or implicit FD method to solve the problem, we need the radiation or absorption boundary condition on the artificial boundary with large enough domain. Solving the large matrix is difficult. The radiation and absorption boundary condition is complicated and inconvenience. In the EM inversion, the FEM and FD EM modeling is used in iterations. The absorption boundary errors will propagate into the internal domain, the noise is enhancing to damage the inversion. In the Section 2, we propose the EM integral equation systems (1) and (2) that are equivalent to the 3D Maxwell EM equation in infinite domain with finite inhomogeneous domain for isotropic and anisotropic materials. Our GL EM modeling does not need any artificial boundary for solving the EM integral equation and the magnetic differential integral equation. Our GL EM modeling only needs to solve 3×3 or 6×6 small matrices, it does not need to solve any large matrix. There are $1/\rho^2$ singularity in the cylindrical coordinate and $1/r^2$, $1/\sin^2\phi$ singularities in the spherical coordinate system for Maxwell equation. These coordinate singularities are difficulties in FEM and FD method. In the EM integral equations (1)–(3) and electric and magnetic differential integral equations (4)–(6) are for the cylindrical and spherical coordinate, the coordinate singularities are resolved. There is no coordinate singularity in the GL method. The GL modeling combines analytical and numerical methods consistent together and has super convergence. The GL method resolve many historical difficulties in traditional FEM, FD, and Born approximation methods.

7. SIMULATIONS AND APPLICATIONS

7.1. Simulations

We have made several GL seismic and EM wave propagation simulations that show the wave excited by internal sources is out going propagation perfectly without any error reflection on the boundary. Because the page limitation, we only use one dimension TE wave propagation to compare GL method and FEM method in the frequency domain. The absorption boundary condition is used for FEM. The $\sigma = \sigma_b = 10^{-5}\,\mathrm{s/m}$, $\mu = \mu_b = 4\pi\times 10^{-7}$, $\varepsilon_b = \varepsilon_0 = 0.88\times 10^{-11}$, $\varepsilon(x) = 100\varepsilon_0$, the inhomogeneous interval [0,6] is divided into 3 sub intervals, [0,2], [2,4], and [4,6], $\varepsilon(x) = 100\varepsilon_0$, when x is in [0,2], $\varepsilon(x) = 200\varepsilon_0$, when x is in [2,4], $\varepsilon(x) = 300\varepsilon_0$, when x is in [4,6], a dipole source is

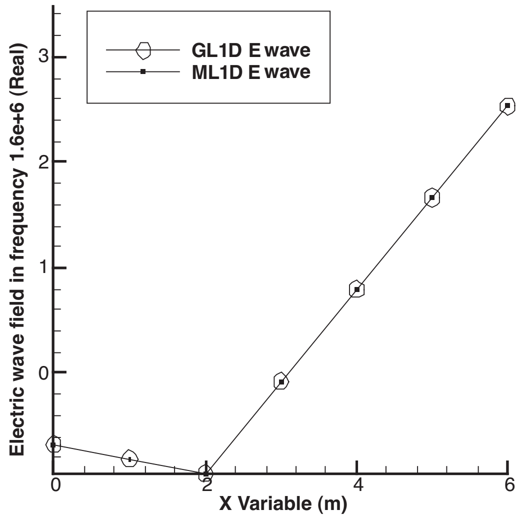

Figure 1. GL and ML electric wave with freq. $1.6e^6$ Hz.

located in the $x_s = 2$, 128 frequencies are used, the minimum frequency is 1 Hz, the maximum frequency is 3.14×10^8 Hz. We obtained excellent wave field results by our GL modeling method. The numerical results show that GL wave is very accurate to match the multiple layer analytic wave for the high frequency 1.6×10^6 (Figure 1) and frequency 1.6×10^8 (Figure 2). The Figures 3 and 4 show that the FEM is fail to approximate the exact wave in the high frequency. The GL and exact total and scattering electric wave are shown in Figure 5 and Figure 6 respectively. They show that the GL electric wave is very accurate to match multiple layer wave, but FEM wave is not. Many 2.5D and 3D GL EM and seismic wave show that GL modeling is accurate, fast and stable. The GL inversion is reasonable high resolution.

7.2. Applications

We develop 3D and 2.5D GL EM, seismic, acoustic, flow, QEM modeling software and some GL EM and seismic inversion software. These GL EM softwares are useful for geophysical EM and seismic exploration; earthquake EM and seismic exploration; forest EM and seismic exploration; environment; EM field in nanometer materials and superconductivity [6]; nondestructive testing imaging [5]; airborne EM exploration; the stress and displacement analysis in dam. rock, underground structure; the EM stirring and flow for caster [7]; GPR, radar, and weather imaging; Naiver Stocks weather simulation, etc.

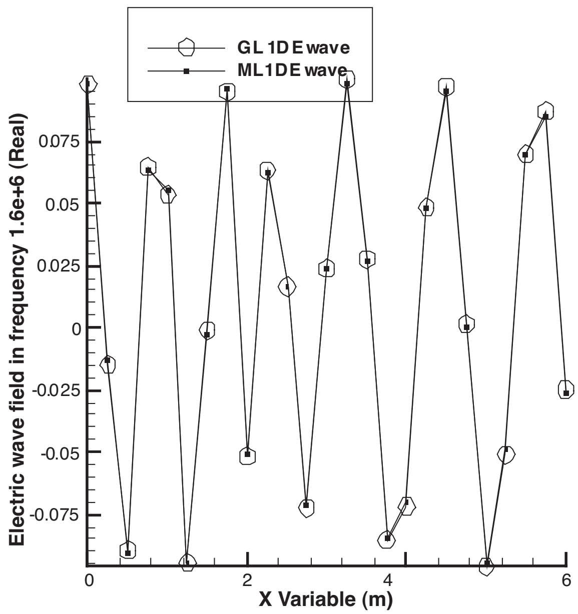

Figure 2. GL and ML electric wave with freq. $1.6e^8$ Hz.

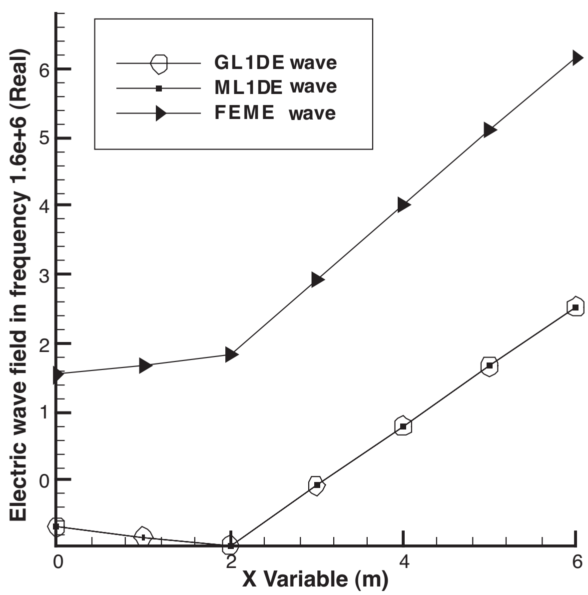

Figure 3. GL, ML and FEM electric wave with freq. $1.6e^6$ Hz.

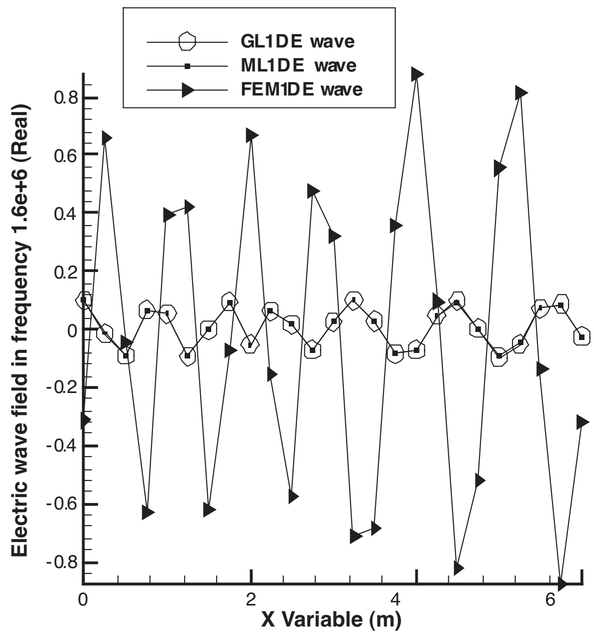

Figure 4. GL, ML and FEM electric wave with freq. $1.6e^8$ Hz.

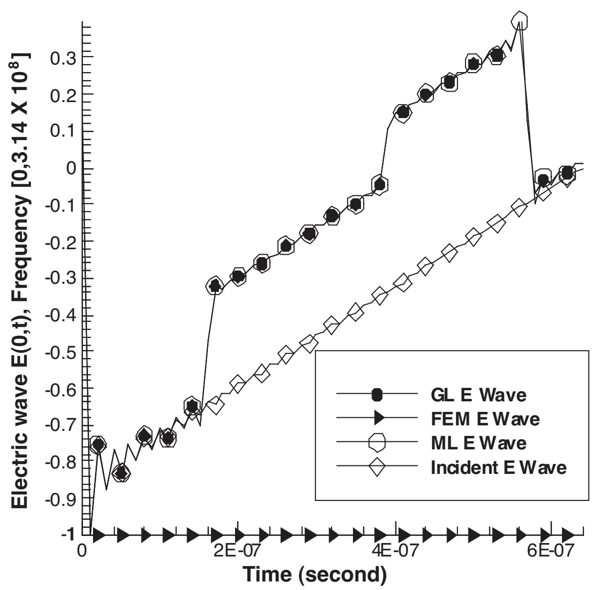

Figure 5. GL, ML and FEM electric wave $E(0, t)$ in time domain.

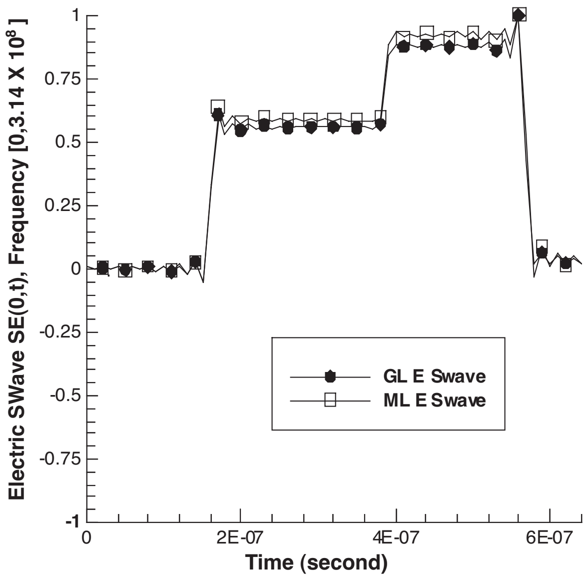

Figure 6. GL and ML scat. electric swave $SE(0,t)$ on time.

8. CONCLUSIONS

Simulations show that the GL modeling is very fast, low cost and accurate. The GL inversion is stable and high resolution. The GL EM field is fast convergent to exact EM field for high frequency and contrast, while FEM method fails to simulate wave field in the high frequency. The GL methods resolve historical difficulties. GL Geophysical Laboratory and authors have reserved patents of 3D/2.5D/2D GL EM, seismic, flow, acoustic QEM modeling and inversion algorithms and have reserved patents of the GL software.

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