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NEW MAGNETIC FIELD INTEGRAL EQUATION FOR ANTENNA SYSTEM

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Abstract—The traditional magnetic field integral equation has been generalized to the study of antenna radiation and coupling problems with the feeding lines included. A rigorous proof of the uniqueness of the new magnetic field integral equation has been presented. Some numerical examples have been expounded to demonstrate the validity of the new magnetic field integral equation formulation.

1. INTRODUCTION

The integral equation method has been widely used in electromagnetic engineering for years and has been investigated by many authors [1–33]. A good summary on the application of integral equation techniques in electromagnetics can be found in [1–8]. Although the integral equations have been widely used in the analysis of scattering problems, they suffer from a major drawback that the scattering solution is not unique at the interior resonant frequencies [8]. To overcome this difficulty, several methods have been proposed in the literature including combining the electric field integral field equation (EFIE) and magnetic field integral equation (MFIE) and augmenting the equations with the normal components [21, 22]. A comparison of the proposed methods can be found in [23, 24]. Since the matrix obtained from the discretization of integral equations becomes ill-conditioned at the interior resonant frequencies [25], the matrix condition number can be used to detect the degree of ill-conditioning, thus providing an indicator for interior resonant frequency. Another method of avoiding the non-uniqueness problem is to use the extended boundary condition (EBC) integral equation [26–30]. EBC is defined as the requirement that a set of field quantities vanishes over an observation domain in the zero-field region. The observation domain could be a closed surface, a portion of plane, or a portion of line in the zero-field region. A rigorous treatment of uniqueness problem of integral equations can be found in [31, 32].

Despite the rich literature on the application of integral equation techniques to the scattering problem, one rarely sees an integral equation formulation for an antenna connected to a feeding line. When the integral equations are applied to solve an antenna problem, approximations of the source region are usually adopted by ignoring the antenna feeding lines. For example a linear antenna is usually characterized by an integral equation in which the feeding line is not involved and the source region is usually modeled by a delta gap [1, 9]. Such an approximation gives rise to a serious problem, i.e., the solution based on delta gap cannot be checked experimentally since every experimental setup in practice involves a feeding line [34, 35]. In addition, the integral equation based on the delta gap is only valid for thin wires or low frequency problems. When the frequency is high or wire is thick the integral equation cannot produce reasonable results, especially for the antenna input impedance. Therefore a practical integral equation formulation for the antenna should consider the influences of the feeding lines.

In this paper, the well-known MFIE for describing the scattering problem from an isolated conducting object has been generalized to a metal antenna system with each metal scatter connected to a feeding line. Since the metal part of the antenna surface does not form a closed surface due to the introduction of the antenna input terminal, the MFIE contains both electric current and magnetic current as unknowns while the latter is only distributed on the antenna terminal, resulting in an integral equation which is underdetermined. In order to eliminate the magnetic current on the antenna terminal, the field expansions in the antenna feeding lines have been used to find an expression for the magnetic current in terms of electric current. It has been rigorously proved in the paper that the new MFIE so obtained has a unique solution. To validate the formulation, the new MFIE has been applied to the study of antenna radiation and coupling problems, and excellent numerical results have been obtained.

2. NEW MFIE FOR MULTIPLE METAL ANTENNA SYSTEM

Let us assume that the antenna system consists of N_A metal antennas. To get a universal integral equation for any operating conditions, the metal antenna system is assumed to include all possible sources, as shown in Fig. 1. Each antenna may be in transmitting mode, receiving mode or in a mode that the antenna transmits and receives at the

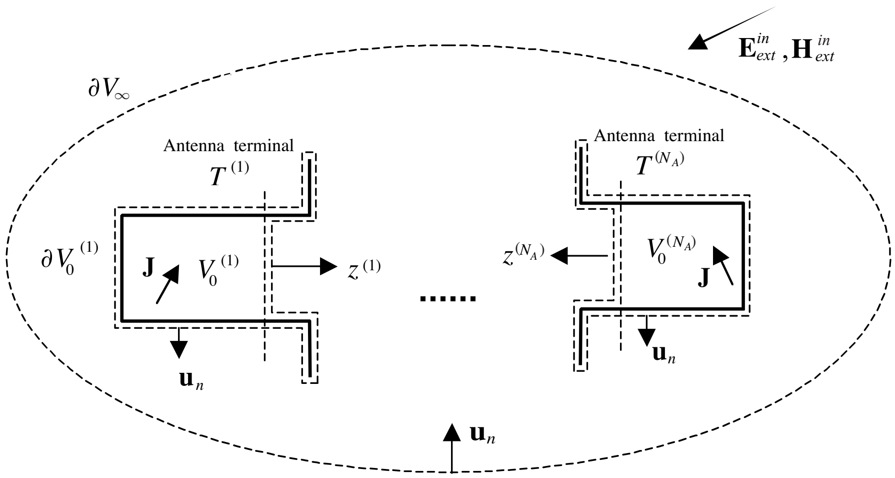

Figure 1. An arbitrary multiple metal antenna system.

same time (e.g., antenna is in transmitting mode but interfered by an arbitrary incident field from the outside of antenna). The source region $V_0^{(q)}(q = 1, 2, \cdots, N_A)$ of the ith antenna is chosen in such a way that its boundary $\partial V_0^{(q)}$ is coincident with the antenna surface, which is assumed to be a perfect conductor (except for cross sectional portion $\Omega^{(q)}$ where $\partial V_0^{(q)}$ crosses the antenna terminal). Let ∂V_∞ be a large surface that encloses the whole antenna system. From the representation theorem for electromagnetic fields, the total magnetic field in the region bounded by $\partial V_0 = \sum\limits_{i=1}^{N_A} \partial V_0^{(q)}$ and ∂V_∞ can then be expressed as

$$\begin{aligned}\mathbf{H}(\mathbf{r}) \;=\; & -j\frac{k}{\eta}\int\limits_{\partial V_0} G(\mathbf{r},\mathbf{r}')\mathbf{J}_{ms}(\mathbf{r}')ds(\mathbf{r}') + \int\limits_{\partial V_0} \mathbf{J}_s(\mathbf{r}') \times \nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') \\ & -\frac{1}{jk\eta}\int\limits_{\partial V_0} \nabla'_s \cdot \mathbf{J}_{ms}(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') + \mathbf{H}_{ext}^{in}(\mathbf{r})\end{aligned}$$

where $\mathbf{J}_s = \mathbf{u}_n \times \mathbf{H}$; $\mathbf{J}_{ms} = -\mathbf{u}_n \times \mathbf{E}$; $G(\mathbf{r},\mathbf{r}') = e^{-jk|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$ is the Green's function in free space; ∇_s represents the surface divergence; and

$$\mathbf{H}_{ext}^{in}(\mathbf{r}) = -j\frac{k}{\eta}\int\limits_{\partial V_\infty} G(\mathbf{r},\mathbf{r}')\mathbf{J}_{ms}(\mathbf{r}')ds(\mathbf{r}') + \int\limits_{\partial V_\infty} \mathbf{J}_s(\mathbf{r}') \times \nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}')$$

$$-\frac{1}{jk\eta}\int\limits_{\partial V_\infty} \nabla_s' \cdot \mathbf{J}_{ms}(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}')$$

stands for the external incident field. By letting the observation point $\mathbf{r}$ approach the boundary of the source region ∂V_0 from the interior of $\partial V_0 + \partial V_\infty$ and use the jump relations [8], one obtains

$$\begin{aligned}\mathbf{H}(\mathbf{r}) \;=\; & -j\frac{k}{\eta}\int\limits_{\partial V_0} G(\mathbf{r},\mathbf{r}')\mathbf{J}_{ms}(\mathbf{r}')ds(\mathbf{r}') + \int\limits_{\partial V_0}\mathbf{J}_s(\mathbf{r}')\times\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') \\ & -\frac{1}{jk\eta}\int\limits_{\partial V_0} \nabla_s'\cdot\mathbf{J}_{ms}(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') + \mathbf{H}_{ext}^{in}(\mathbf{r}) \\ & +\frac{1}{2}\mathbf{J}_s(\mathbf{r})\times\mathbf{u}_n(\mathbf{r}) - \frac{1}{j2k\eta}\mathbf{u}_n(\mathbf{r})\nabla_s\cdot\mathbf{J}_{ms}(\mathbf{r})\end{aligned}$$

Multiplying both sides of the above equations by $\mathbf{u}_n$ gives

$$\begin{aligned}\frac{1}{2}\mathbf{J}_s(\mathbf{r}) \;=\; & -j\frac{k}{\eta}\mathbf{u}_n(\mathbf{r})\times\int\limits_{\partial V_0} G(\mathbf{r},\mathbf{r}')\mathbf{J}_{ms}(\mathbf{r}')ds(\mathbf{r}') + \mathbf{u}_n(\mathbf{r}) \\ & \times\int\limits_{\partial V_0}\mathbf{J}_s(\mathbf{r}')\times\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') - \frac{1}{jk\eta}\mathbf{u}_n(\mathbf{r}) \\ & \times\int\limits_{\partial V_0}\nabla_s'\cdot\mathbf{J}_{ms}(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') + \mathbf{u}_n(\mathbf{r})\times\mathbf{H}_{ext}^{in}(\mathbf{r})\end{aligned}$$

Making use of the boundary conditions on the metal part of the antenna, the above equations can be written as

$$\begin{aligned}& -\frac{1}{2}\mathbf{J}_s(\mathbf{r}) + \mathbf{u}_n(\mathbf{r})\times\int\limits_{\partial V_0}\mathbf{J}_s(\mathbf{r}')\times\nabla' G(\mathbf{r},\mathbf{r}')ds(\mathbf{r}') \\ & \qquad = -\mathbf{u}_n(\mathbf{r})\times\left[\mathbf{H}_{int}^{in}(\mathbf{r}) + \mathbf{H}_{ext}^{in}(\mathbf{r})\right],\ \ \mathbf{r}\in\partial V_0 \qquad (1)\end{aligned}$$

where

$$\begin{aligned}\mathbf{H}_{int}^{in}(\mathbf{r}) \;=\; & \sum_{q=1}^{N_A}\left[-j\frac{k}{\eta}\int\limits_{\Omega^{(q)}} G(\mathbf{r},\mathbf{r}')\mathbf{J}_{ms}(\mathbf{r}')d\Omega(\mathbf{r}')\right. \\ & \left. -\frac{1}{jk\eta}\int\limits_{\Omega^{(q)}}\nabla_s'\cdot\mathbf{J}_{ms}(\mathbf{r}')\nabla' G(\mathbf{r},\mathbf{r}')d\Omega(\mathbf{r}')\right] \qquad (2)\end{aligned}$$

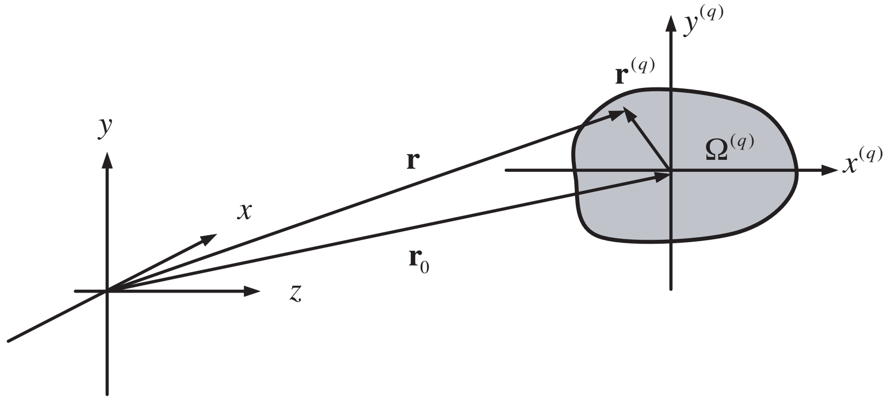

Figure 2. The local coordinates for the feeding line and the global coordinates.

Note that $\mathbf{H}_{ext}^{in}$ is zero if there is no incident wave from the infinity. $\mathbf{H}_{int}^{in}(\mathbf{r})$ is determined by the equivalent surface magnetic current $\mathbf{J}_m = -\mathbf{u}_{z^{(q)}} \times \mathbf{E}$ on the antenna input reference planes $T^{(q)}(q = 1, 2, \cdots, N_A)$. In order to determine the equivalent magnetic current on the reference planes, one may make use of the field expressions in the waveguide. According to the waveguide theory the electric field and magnetic field in the ith feeding waveguide may be expressed as

$$\begin{aligned} -\mathbf{u}_{z^{(q)}} \times \mathbf{E}(\mathbf{r}^{(q)}) &= -\sum_{n=1}^{\infty} \mathbf{u}_{z^{(q)}} \times \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) V_n^{(q)}(z^{(q)}), \ \mathbf{r}^{(q)} \in \Omega^{(q)} \\ \mathbf{u}_{z^{(q)}} \times \mathbf{H}(\mathbf{r}^{(q)}) &= -\sum_{n=1}^{\infty} \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) I_n^{(q)}(z^{(q)}), \qquad \mathbf{r}^{(q)} \in \Omega^{(q)} \end{aligned} \tag{3}$$

where $\mathbf{r}^{(q)} = \mathbf{r} - \mathbf{r}_0$ is the local coordinate system for the qth feeding line, as shown in Fig. 2, and

$$\begin{aligned} V_n^{(q)}(z^{(q)}) &= A_n^{(q)} e^{-j\beta_n^{(q)} z^{(q)}} + B_n^{(q)} e^{j\beta_n^{(q)} z^{(q)}} \\ I_n^{(q)}(z^{(q)}) &= \left(A_n^{(q)} e^{-j\beta_n^{(q)} z^{(q)}} - B_n^{(q)} e^{j\beta_n^{(q)} z^{(q)}} \right) / Z_{wn}^{(q)} \\ \beta_n^{(q)} &= \begin{cases} k, & \text{TEM mode} \\ \sqrt{k^2 - k_{cn}^{(q)2}}, & \text{TE or TM mode} \end{cases} \end{aligned}$$

$$Z_{wn}^{(q)} = \begin{cases} \eta, & \text{TEM mode} \\ \eta k/\beta_n^{(q)}, & \text{TE mode} \\ \eta\beta_n^{(q)}/k, & \text{TM mode} \end{cases}$$

with $\eta = \sqrt{\mu/\varepsilon}$.

Let us assume that the feeding line of antenna is in a single mode operation. Therefore the mode voltage and current may be written as

$$\begin{aligned} V_1^{(q)}(z^{(q)}) &= \delta^{(q)} e^{-j\beta_1^{(q)} z^{(q)}} + B_1^{(q)} e^{j\beta_1^{(q)} z^{(q)}}, \\ V_n^{(q)}(z^{(q)}) &= B_n^{(q)} e^{j\beta_n^{(q)} z^{(q)}}, \quad n \geq 2 \\ I_1^{(q)}(z^{(q)}) &= \frac{1}{Z_{w1}^{(q)}} \left(\delta^{(q)} e^{-j\beta_1^{(q)} z^{(q)}} - B_1^{(q)} e^{j\beta_1^{(q)} z^{(q)}} \right) \\ I_n^{(q)}(z^{(q)}) &= -\frac{1}{Z_{w1}^{(q)}} B_n^{(q)} e^{j\beta_n^{(q)} z^{(q)}}, \quad n \geq 2 \end{aligned}$$

where $\delta^{(q)} = 1$ if the qth antenna is in transmitting mode and excited by dominant mode of unit amplitude, and $\delta^{(q)} = 0$ if the qth antenna is in a receiving mode. Thus on the reference plane $T^{(q)}(z^{(q)} = 0)$, (3) may be written as

$$\begin{aligned} \mathbf{J}_{ms}(\mathbf{r}^{(q)}) &= -\mathbf{u}_{z^{(q)}} \times \mathbf{e}_1^{(q)}(\mathbf{r}^{(q)}) \left(\delta^{(q)} + B_1^{(q)} \right) - \sum_{n=2}^{\infty} \mathbf{u}_{z^{(q)}} \times \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) B_n^{(q)}, \\ & \qquad \mathbf{r}^{(q)} \in \Omega^{(q)} \\ \mathbf{J}_s(\mathbf{r}^{(q)}) &= -\mathbf{e}_1^{(q)}(\mathbf{r}^{(q)}) \left(\delta^{(q)} - B_1^{(q)} \right) / Z_{w1}^{(q)} + \sum_{n=2}^{\infty} \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) B_n^{(q)} / Z_{wn}^{(q)}, \\ & \qquad \mathbf{r}^{(q)} \in \Omega^{(q)} \end{aligned}$$

The expansion coefficients can be determined by the second equation of the above equations

$$\begin{aligned} B_1^{(q)} &= \delta^{(q)} + Z_{w1}^{(q)} \int_{\Omega^{(q)}} \mathbf{J}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_1^{(q)} \left(\mathbf{r}^{(q)} \right) d\Omega \\ B_n^{(q)} &= Z_{wn}^{(q)} \int_{\Omega^{(q)}} \mathbf{J}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) d\Omega \end{aligned}$$

The equivalent magnetic current on the reference plane $T^{(q)}$ may thus be expressed by

$$\mathbf{J}_{ms}(\mathbf{r}^{(q)}) = -2\delta^{(q)} \mathbf{u}_{z^{(q)}} \times \mathbf{e}_1^{(q)}(\mathbf{r}^{(q)}) - \sum_{n=1}^{\infty} \mathbf{u}_{z^{(q)}} \times \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) Z_{wn}^{(q)}$$

$$\times \int_{\Omega^{(q)}} \mathbf{J}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) d\Omega(\mathbf{r}^{(q)}), \quad \mathbf{r}^{(q)} \in \Omega^{(q)} \tag{4}$$

Inserting this into (2) yields

$$\mathbf{H}_{int}^{in}(\mathbf{r}) = \sum_{q=1}^{N_A} \left[2\delta^{(q)} \mathbf{G}_1^{(q)}(\mathbf{r}) + \sum_{n=1}^{\infty} Z_{wn}^{(q)} \mathbf{G}_n^{(q)}(\mathbf{r}) \int_{\Omega^{(q)}} \mathbf{J}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) d\Omega(\mathbf{r}^{(q)}) \right] \tag{5}$$

where

$$\begin{aligned} \mathbf{G}_n^{(q)}(\mathbf{r}) &= \frac{jk}{\eta} \int_{\Omega^{(q)}} G(\mathbf{r}, \mathbf{r}') \mathbf{u}_{z^{(q)}} \times \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) d\Omega\,(\mathbf{r}') \\ &\quad + \frac{1}{j\eta k} \int_{\Omega^{(q)}} \nabla'_s \cdot \left[\mathbf{u}_{z^{(q)}} \times \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) \right] \nabla' G(\mathbf{r}, \mathbf{r}') d\Omega\,(\mathbf{r}') \end{aligned}$$

From (1) and (5), one finally obtains the following modified MFIE

$$\begin{aligned} &-\frac{1}{2}\mathbf{J}_s(\mathbf{r}) + \mathbf{u}_n(\mathbf{r}) \times \int_{\partial V_0} \mathbf{J}_s(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}') \\ &+ \sum_{q=1}^{N_A} \left[\sum_{n=1}^{\infty} Z_{wn}^{(q)} \mathbf{u}_n(\mathbf{r}) \times \mathbf{G}_n^{(q)}(\mathbf{r}) \int_{\Omega^{(q)}} \mathbf{J}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) d\Omega(\mathbf{r}^{(q)}) \right] \\ &= \sum_{q=1}^{N_A} \left[-2\delta^{(q)} \mathbf{u}_n(\mathbf{r}) \times \mathbf{G}_1^{(q)}(\mathbf{r}) \right] - \mathbf{u}_n(\mathbf{r}) \times \mathbf{H}_{ext}^{in}(\mathbf{r}), \quad \mathbf{r} \in \partial V_0 \end{aligned} \tag{6}$$

3. UNIQUENESS OF NEW MFIE

The non-uniqueness problem occurs in integral equation formulations when it is used to describe an isolated scatter or an antenna without introducing the input reference plane T. When an antenna input terminal T exists, the electromagnetic energy exchanges between the source region (enclosed by ∂V_0) and the exterior region (outside of ∂V_0) of the antenna. Thus the physical conditions for interior resonance disappear and one may expect that non-uniqueness problem should also disappear. In what follows, the Fredholm alternative theorem will be used to demonstrate that this is true.

Suppose that (6) has more than one solution and let $\mathbf{J}_{s1}, \mathbf{J}_{s2}$ be two different solutions of (6). Then the difference $\tilde{\mathbf{j}}_s = \mathbf{J}_{s1} - \mathbf{J}_{s2}$ is non-trivial and satisfies the following homogeneous equations

$$-\frac{1}{2}\tilde{\mathbf{j}}_s(\mathbf{r}) + \overleftrightarrow{\mathbf{A}}\left[\tilde{\mathbf{j}}_s(\mathbf{r}')\right] = -\frac{1}{2}\tilde{\mathbf{j}}_s(\mathbf{r}) + \mathbf{u}_n(\mathbf{r}) \times \int\limits_{\partial V_0} \tilde{\mathbf{j}}_s(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}')$$
$$= \mathbf{f}(\mathbf{r}), \quad \mathbf{r} \in \partial V_0 \tag{7}$$

where

$$\mathbf{f}(\mathbf{r}) = \sum_{q=1}^{N_A} \left[\sum_{n=1}^{\infty} Z_{wn}^{(q)} \mathbf{u}_n(\mathbf{r}) \times \mathbf{G}_n^{(q)}(\mathbf{r}) \int\limits_{\Omega^{(q)}} \tilde{\mathbf{j}}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_n^{(q)}(\mathbf{r}^{(q)}) d\Omega\left(\mathbf{r}^{(q)}\right) \right]$$

and $\overleftrightarrow{\mathbf{A}}$ is the integral operator defined by

$$\overleftrightarrow{\mathbf{A}}\left[\tilde{\mathbf{j}}_s\right](\mathbf{r}) = \mathbf{u}_n(\mathbf{r}) \times \int\limits_{\partial V_0} \tilde{\mathbf{j}}_s(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}'), \quad \mathbf{r} \in \partial V_0$$

It is easy to show that the integral operator $\overleftrightarrow{\mathbf{A}}$ is compact [8]. To ensure the uniqueness of (6), one must show that $\tilde{\mathbf{j}}_s$ is zero. Apparently the non-trivial solutions of (7) are not unique. By Fredholm alternative, the following equations must have non-trivial solutions

$$-\frac{1}{2}\mathbf{j}_s(\mathbf{r}) + \mathbf{u}_n(\mathbf{r}) \times \int\limits_{\partial V_0} \mathbf{j}_s(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}') = 0, \quad \mathbf{r} \in \partial V_0$$

and furthermore $\mathbf{f}$ must be orthogonal to $\mathbf{j}_s$, i.e., $(\mathbf{f}, \mathbf{j}_s) = 0$, where the inner product $(\cdot, \cdot)$ for tangential vectors $\mathbf{j}_{s1}$ and $\mathbf{j}_{s2}$ are defined by $(\mathbf{j}_{s1}, \mathbf{j}_{s2}) = \int\limits_{\partial V_0} \mathbf{j}_{s1} \cdot \bar{\mathbf{j}}_{s2} ds$. As a result,

$$(\mathbf{f}, \mathbf{j}_s) = \sum_{q=1}^{N_A} \left[\sum_{n=1}^{\infty} Z_{wn}^{(q)} \int\limits_{\partial V_0} \bar{\mathbf{j}}_s(\mathbf{r}) \cdot \mathbf{u}_n(\mathbf{r}) \times \mathbf{G}_n^{(q)}(\mathbf{r}) ds(\mathbf{r}) \right.$$
$$\left. \times \int\limits_{\Omega^{(q)}} \tilde{\mathbf{j}}_s(\mathbf{r}^{(q)}) \cdot \mathbf{e}_n^{(q)}\left(\mathbf{r}^{(q)}\right) d\Omega\left(\mathbf{r}^{(q)}\right) \right] = 0$$

By the completeness of the transverse vector modal functions $\{\mathbf{e}_n^{(q)}\}$ [36], the above equation implies $\tilde{\mathbf{j}}_s(\mathbf{r}) = 0, \mathbf{r} \in \Omega^{(q)}$. Therefore we must have $\mathbf{J}_{s1} = \mathbf{J}_{s2}, \mathbf{r} \in \Omega^{(q)}$. In other words, the non-trivial solution $\tilde{\mathbf{j}}_s(\mathbf{r})$

must satisfy

$$-\frac{1}{2}\tilde{\mathbf{j}}_s(\mathbf{r}) + \mathbf{u}_n(\mathbf{r}) \times \int\limits_{\partial V_0} \tilde{\mathbf{j}}_s(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}') = 0, \quad \mathbf{r} \in \partial V_0 \tag{8}$$

from (7). By the Fredholm alternative, the above equation has a non-trivial solution if and only if its adjoint

$$\frac{1}{2}\mathbf{a}_s(\mathbf{r}) + \int\limits_{\partial V_0} [\mathbf{u}_n(\mathbf{r}')] \times \mathbf{a}_s(\mathbf{r}')] \times \nabla' \overline{G}(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}') = 0, \quad \mathbf{r} \in \partial V_0 \tag{9}$$

has a non-trivial solution [8]. Now let $\tilde{\mathbf{j}}_{sm}(\mathbf{r}) = \mathbf{u}_n(\mathbf{r}) \times \mathbf{a}_s(\mathbf{r})$ and one may rewrite (9) as

$$\frac{1}{2}\tilde{\mathbf{j}}_{ms}(\mathbf{r}) + \mathbf{u}_n(\mathbf{r}) \times \int\limits_{\partial V_0} \tilde{\mathbf{j}}_{ms}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}') = 0, \quad \mathbf{r} \in \partial V_0 \tag{10}$$

Now the following electromagnetic fields can be constructed

$$\mathbf{E}_m(\mathbf{r}) = -jk\eta \int\limits_{\partial V_0} G(\mathbf{r}, \mathbf{r}') \tilde{\mathbf{j}}_{sm}(\mathbf{r}') ds(\mathbf{r}') - \frac{\eta}{jk} \int\limits_{\partial V_0} \nabla_s \cdot \tilde{\mathbf{j}}_{sm}(\mathbf{r}') \nabla' G(R) ds(\mathbf{r}')$$

$$\mathbf{H}_m(\mathbf{r}) = \int\limits_{\partial V_0} \tilde{\mathbf{j}}_{sm}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}')$$

It is easy to show that these fields satisfy the Maxwell equations inside ∂V_0 as well as outside ∂V_0. In the following, the subscripts '+' and '−' will signify the values obtained as ∂V_0 is approached from outside and inside respectively. Then one may write

$$\begin{aligned}
\mathbf{E}_{m+}(\mathbf{r}) &= -\frac{\eta}{j2k}\mathbf{u}_n(\mathbf{r}) \nabla_s \cdot \tilde{\mathbf{j}}_{sm}(\mathbf{r}) - jk\eta \int\limits_{\partial V_0} G(\mathbf{r}, \mathbf{r}') \tilde{\mathbf{j}}_{sm}(\mathbf{r}') ds(\mathbf{r}') \\
&\quad - \frac{\eta}{jk} \int\limits_{\partial V_0} \nabla_s \cdot \tilde{\mathbf{j}}_{sm}(\mathbf{r}') \nabla' G(R) ds(\mathbf{r}') \\
\mathbf{H}_{m+}(\mathbf{r}) &= \frac{1}{2}\tilde{\mathbf{j}}_{sm}(\mathbf{r}) \times \mathbf{u}_n(\mathbf{r}) + \int\limits_{\partial V_0} \tilde{\mathbf{j}}_{sm}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}') \\
\mathbf{E}_{m-}(\mathbf{r}) &= \frac{\eta}{j2k}\mathbf{u}_n(\mathbf{r}) \nabla_s \cdot \tilde{\mathbf{j}}_{sm}(\mathbf{r}) - jk\eta \int\limits_{\partial V_0} G(\mathbf{r}, \mathbf{r}') \tilde{\mathbf{j}}_{sm}(\mathbf{r}') ds(\mathbf{r}') \\
&\quad - \frac{\eta}{jk} \int\limits_{\partial V_0} \nabla_s \cdot \tilde{\mathbf{j}}_{sm}(\mathbf{r}') \nabla' G(R) ds(\mathbf{r}')
\end{aligned}$$

$$\mathbf{H}_{m-}(\mathbf{r}) = -\frac{1}{2}\tilde{\mathbf{j}}_{sm}(\mathbf{r}) \times \mathbf{u}_n(\mathbf{r}) + \int_{\partial V_0} \tilde{\mathbf{j}}_{sm}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') ds(\mathbf{r}')$$

Making use of these relations and (10) yields

$$\mathbf{E}_{m+}(\mathbf{r}) - \mathbf{E}_{m-}(\mathbf{r}) = -\frac{\eta}{jk}\mathbf{u}_n(\mathbf{r})\nabla_s \cdot \tilde{\mathbf{j}}_{sm}(\mathbf{r}), \quad \mathbf{r} \in \partial V_0 \quad (11)$$

$$\begin{aligned} \mathbf{H}_{m+}(\mathbf{r}) - \mathbf{H}_{m-}(\mathbf{r}) &= \tilde{\mathbf{j}}_{sm}(\mathbf{r}) \times \mathbf{u}_n(\mathbf{r}), \quad \mathbf{r} \in \partial V_0 \\ \mathbf{u}_n(\mathbf{r}) \times \mathbf{H}_{m+}(\mathbf{r}) &= 0, \quad \mathbf{r} \in \partial V_0 \end{aligned} \quad (12)$$

Since the fields $\mathbf{E}_m, \mathbf{H}_m$ satisfy the Maxwell equations outside ∂V_0 as well as radiation condition in the infinity, it may be concluded that these fields must be identically zero outside ∂V_0 from (12) and the uniqueness theorem for exterior electromagnetic fields [31]. Thus $\mathbf{E}_{m+}(\mathbf{r}) = 0$, which implies

$$\mathbf{u}_n(\mathbf{r}) \times \mathbf{E}_{m+}(\mathbf{r}) = \mathbf{u}_n(\mathbf{r}) \times \mathbf{E}_{m-}(\mathbf{r}) = 0, \quad \mathbf{r} \in \partial V_0 \quad (13)$$

from (11). Since the fields $\mathbf{E}_m, \mathbf{H}_m$ satisfy the Maxwell equations in the infinite domain $R^3 - V_0$ and the radiation condition in the infinity, by the use of uniqueness theorem again, one may conclude that fields $\mathbf{E}_m, \mathbf{H}_m$ are zero everywhere. This implies $\tilde{\mathbf{j}}_{sm} = 0$, contradicting the assumption that $\tilde{\mathbf{j}}_{sm}$ is a non-trivial solutions. The proof is completed.

4. APPLICATIONS OF NEW MFIE

Following a similar procedure described in a previous paper [33], where the MFIE has been used to solve the metal cavity problems, the integral Equation (6) can be discretized by subdividing the boundary ∂V_0 into N elements $\Gamma_j (j = 1, 2, \cdots, N)$ which are then approximated by a plane triangles or quadrilaterals. The unknown currents are assumed to be constant over each element. For each element Γ_j, we choose a point $\mathbf{r}^j$ (collocation point) and let the integral equations be satisfied at these points. The above procedure yields a $3N \times 3N$ algebraic equation. It is known that the MFIE is effective for generally shaped large structures but not good for thin wires [9]. In the following, several large antenna structures are used to demonstrate the validity of the new MFIE formulation developed in this paper. The quadrilateral mesh is deployed for all our examples and is generated with HyperMesh.

4.1. A Circular Aperture Antenna with Infinite Flange

Let us consider an aperture antenna fed by a coaxial line consisting of an inner conductor of radius a and an outer conductor of radius b with

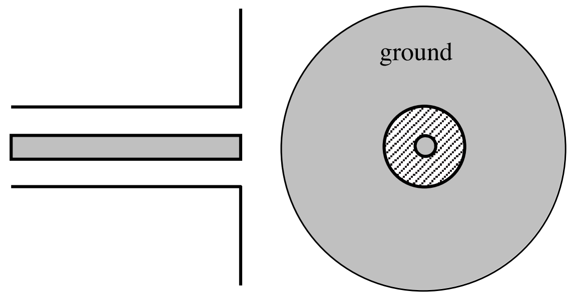

Figure 3. A circular aperture antenna with infinite conducting flange.

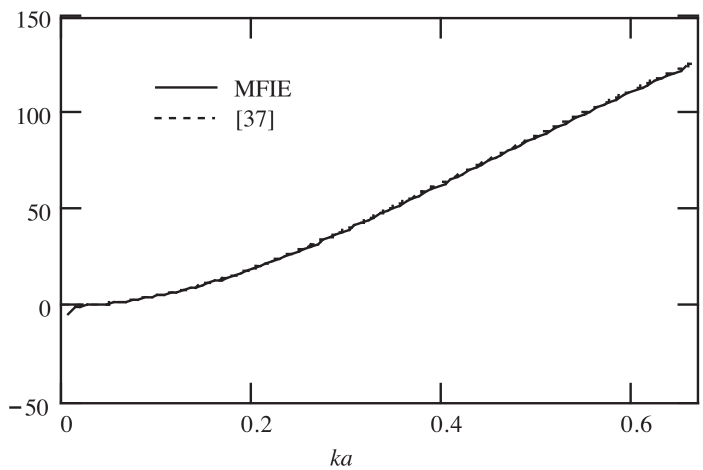

Figure 4. Radiation resistance of the coaxial aperture antenna with infinite flange.

$b = 2a$, as shown in Fig. 3. The infinite flange has been truncated to a finite circular region. The number of the quadrilateral elements used in the calculation is $N = 445$. It is assumed that only dominant mode (TEM mode) is propagating in the coaxial cable and the reference plane is right at the aperture. The radiation resistance and reactance have been calculated by MFIE and compared to the analytical results [37]. A perfect agreement has been obtained as shown in Fig. 4 and Fig. 5. Note that the operating frequency is limited in between the cut-off frequency $k_c a = 0$ of dominant TEM mode and the cut-off frequency $k_c a \approx 0.68$ of the first higher order TE_{11} mode.

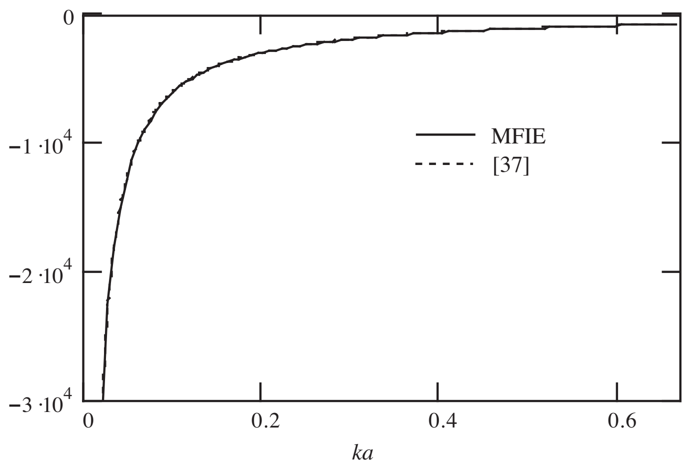

Figure 5. Reactance of the coaxial aperture with infinite flange.

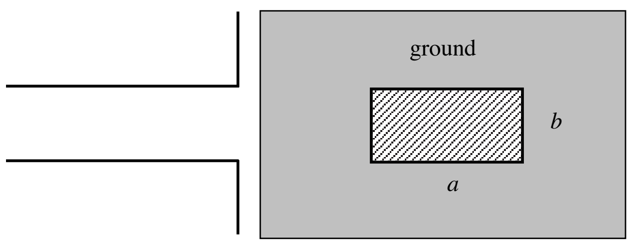

Figure 6. A rectangular aperture in conducting plane.

4.2. A Rectangular Aperture in Infinite Conducting Plane

A rectangular aperture with an infinite flange, whose feeding line is a rectangular waveguide with $a = 2.25$, $b = 1$ is shown in Fig. 6. The infinite flange is truncated to a finite rectangular region. In this case, the total number of quadrilateral element is $N = 1152$. It is assumed that only the dominant TE_{10} mode is propagating in the waveguide and the reference plane is right at the aperture. This structure has been investigated by many authors [38, 39]. The radiation resistance and reactance have been obtained by MFIE and compared to those obtained by correlation matrix method [38]. A good agreement has been obtained as shown in Fig. 7 and Fig. 8. Again the frequency response is limited to the range between the cut-off frequency of the dominant TE_{10} mode $k_c b \approx 1.4$ and the cut-off frequency of the next higher order mode $k_c b \approx 2.8$.

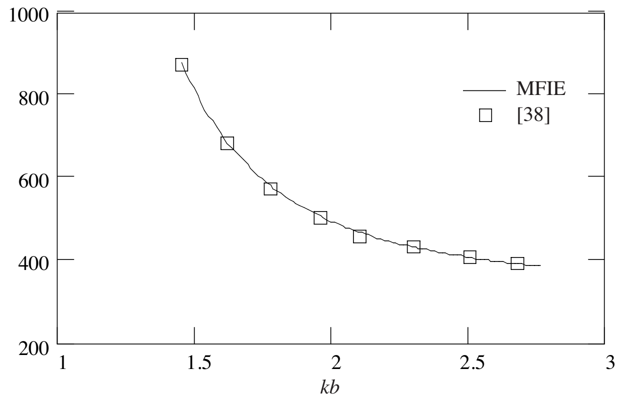

Figure 7. Radiation resistance of the rectangular aperture with infinite flange.

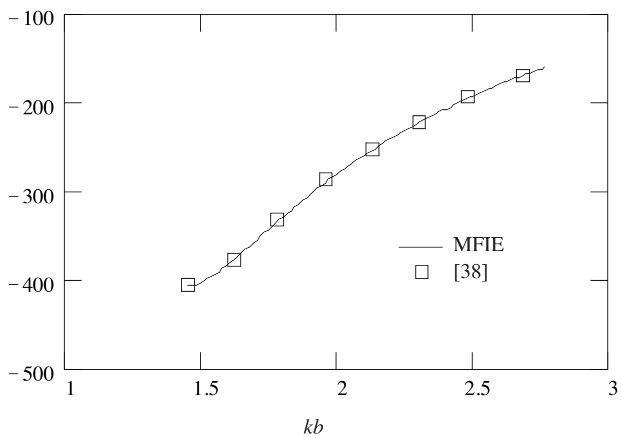

Figure 8. Reactance of the rectangular aperture with infinite flange.

4.3. Coupling between Two Rectangular Apertures

Let us now consider the application of the new MFIE to the antenna coupling problems. Two identical rectangular waveguide apertures are closely placed in a common infinite conducting plane as shown in Fig. 9 with $a = 2.286$, $b = 1.016$ and $d = 0.254$. One of the antennas is in transmitting mode, which is excited by a dominant TE_{10} mode, and the other is in receiving mode. The ground plane has been truncated to a finite rectangular region. The total number of quadrilateral element is $N = 1160$. The coupled power is shown in Fig. 10 and has been

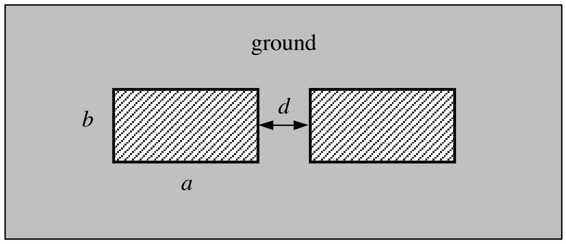

Figure 9. Coupled waveguide geometry.

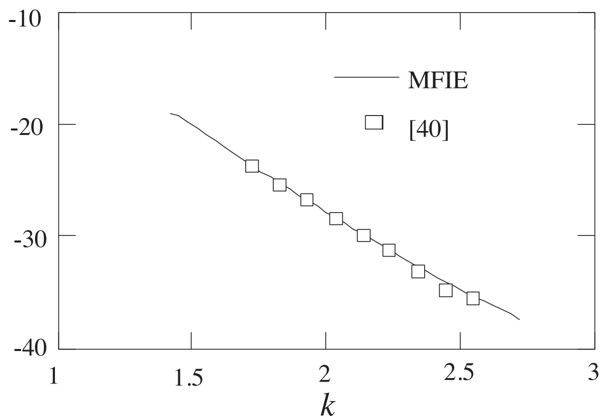

Figure 10. Coupled power $20 \log |S_{12}|$.

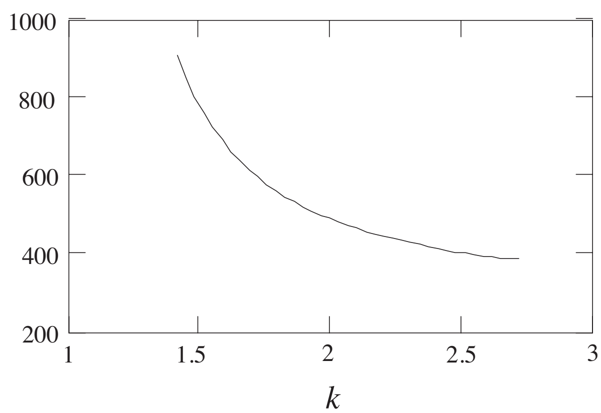

Figure 11. Radiation resistance of coupled waveguide.

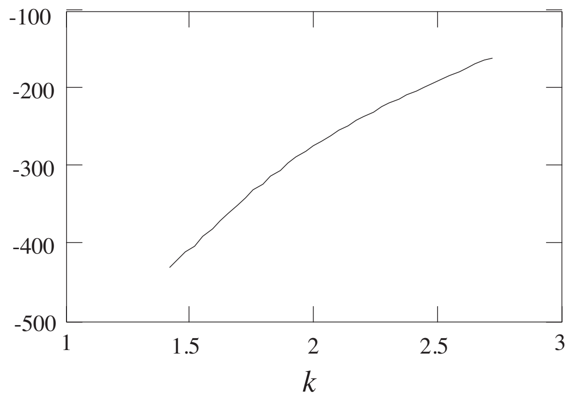

Figure 12. Reactance of coupled waveguide.

compared with that obtained by Mailloux [40], and a good agreement has been obtained. The radiation resistance and input reactance seen from the excited waveguide are also shown in Fig. 11 and Fig. 12. Note that the antenna input reference planes are chosen right at the apertures, and the frequency range is limited in between the cut-off frequency of the dominant TE_{10} mode $k_c \approx 1.37$ and the cut-off frequency of the next higher order mode $k_c \approx 2.75$.

5. CONCLUSION

In this paper the traditional MFIE has been generalized to the analysis of metal antenna systems to which the feeding lines are connected. Since the antenna input terminal has been included, both electric current and magnetic current exist at the antenna terminal. As a result, the integral equation formulation contains both electric current and magnetic current as unknowns. To get rid of one of the unknowns, a relationship between the magnetic current and the electric current at the antenna terminal can be constructed through the use the field expansions in terms of the transverse vector modal functions in the feeding lines.

An integral equation formulation that includes the feeding lines has the advantage of high accuracy in antenna input impedance calculation since it does not rely on the usual approximations made in the antenna source region and thus is more realistic. Another advantage of introducing the feeding lines in the integral equation formulation is that it guarantees a unique solution, thus providing a solid theoretical basis for the analysis of various antennas.

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