

**PROPAGATION IN A FERRITE CIRCULAR
WAVEGUIDE MAGNETIZED THROUGH A ROTARY
FOUR-POLE MAGNETIC FIELD**

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Abstract—The coupled mode approach is applied to the ferrite circular waveguide magnetized through a rotary four-pole transverse bias magnetic fields. The plausible mathematical model of the ferrite waves propagation in the guide is developed which includes gyromagnetic interaction of two orthogonal TE_{11} isotropic modes. The importance of the birefringence effect in determining of phase shift and polarization phenomena are thereby demonstrated. As a result basic design consideration of the circular polarizer applied as a “half-wave plate” in rotary-field phase shifter are provided.

1. INTRODUCTION

Some years ago, Boyd et al. published many papers concerning the nonreciprocal polarizer or latching ferrite phase shifters which used the sections of ferrite circular guide biased through rotary four-pole magnetic field. Although using ferrite transmission line model [1] and basing on the experimental experiences Boyd et al. reviewed the principle of operation [2, 3] and design relationship and construction of these devices [4, 5], a satisfactory physical model describing clearly their operation properties has not been achieved yet. The purpose of this paper is an alternative approach using the coupled mode method [6] to the explanation of the operation principles of such ferrite structures. It extends the coupled mode approach formulated in [7] where such problem was treated for saturated magnetized ferrite only. Our attention is focused on the novel configuration reported in [3–5] where the rotation of biased magnetic field pattern is controlled electronically. The bias yoke consists of two sets of coils, each generating four-pole magnetic field. These two windings are designated as the “sin” and “cos” because of their magnetic field orientations. The rotation angle of the resulting magnetic field pattern depends on the ratio of the control currents. Thus a close relationship between the rotation angle and the transmission phase shift enables to use this structure as “half-wave plate” in Fox variable phase shifter [8] accordingly that for input circularly polarized wave the influence of the rotation angle on the polarization state of the output circularly polarized wave is not observed. It is worth to note that for this configuration the only approximate solutions have been reported [3, 5]. Our procedure is to find simple and plausible coupled mode model of this ferrite guide. The model presented below allows to explain exactly the wave phenomena and operation principles of the considered ferrite circular guide and provides simple relationships that could be used as design of the rotary field ferrite phase shifter and circular polarizers.

2. BASIC CONSIDERATION

2.1. Permeability Tensor

Consider the circular waveguide filled with ferrite magnetized through a yoke producing two transverse magnetic field arrangements, shown in Fig. 1. Bias fields are produced by two interlaced windings that resembles stator located on the wall of circular waveguide.

Resulting flux density of magnetic field around the ferrite rod can be approximately defined as the superposition of the fields for the two

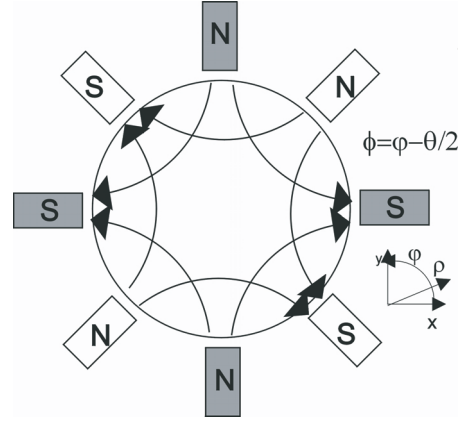


Figure 1. Four-pole magnetic bias field system.

windings

$$\begin{aligned} \mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = & B_{o1} \cos(2\varphi) \mathbf{i}_\rho - B_{o1} \sin(2\varphi) \mathbf{i}_\varphi \\ & + B_{o2} \sin(2\varphi) \mathbf{i}_\rho + B_{o2} \cos(2\varphi) \mathbf{i}_\varphi \end{aligned} \quad (1)$$

Assume that the magnitudes of magnetic fields B_{o1} and B_{o2} of the windings control currents vary as $B_o \cos(\theta)$ and $B_o \sin(\theta)$, respectively, where θ is defined as an electrical angle and B_o define magnitude of the transverse magnetic field in the ferrite cross-section. Applying B_{o1} and B_{o2} in (1) we obtain

$$\mathbf{B} = B_o \cos(2\varphi - \theta) \mathbf{i}_\rho - B_o \sin(2\varphi - \theta) \mathbf{i}_\varphi \quad (2)$$

Note, that a change of the electrical angle θ is tantamount to a mechanical rotation of the resultant four-pole field by an angle $\theta/2$ as shown in Fig. 2. In this way the magnetic field pattern can be turned electrically. Then in order to define the permeability tensor in the ferrite we exploited an expression given in [8] for ferrite magnetized through four-pole field pattern. In dyadic form it yields

$$\begin{aligned} \vec{\mu} = & \mu_{\rho\rho} \mathbf{i}_\rho \cdot \mathbf{i}_\rho + \mu_{\rho\varphi} \mathbf{i}_\rho \cdot \mathbf{i}_\varphi + \mu_{\rho z} \mathbf{i}_\rho \cdot \mathbf{i}_z \\ & + \mu_{\varphi\rho} \mathbf{i}_\varphi \cdot \mathbf{i}_\rho + \mu_{\varphi\varphi} \mathbf{i}_\varphi \cdot \mathbf{i}_\varphi + \mu_{\varphi z} \mathbf{i}_\varphi \cdot \mathbf{i}_z \\ & + \mu_{z\rho} \mathbf{i}_z \cdot \mathbf{i}_\rho + \mu_{z\varphi} \mathbf{i}_z \cdot \mathbf{i}_\varphi + \mu_{zz} \mathbf{i}_z \cdot \mathbf{i}_z \end{aligned} \quad (3)$$

where:

$$\begin{aligned} \mu_{\rho\rho} &= 1 + (1 - \mu) \sin^2(2\phi) \\ \mu_{\varphi\varphi} &= 1 + (1 - \mu) \cos^2(2\phi) \end{aligned}$$

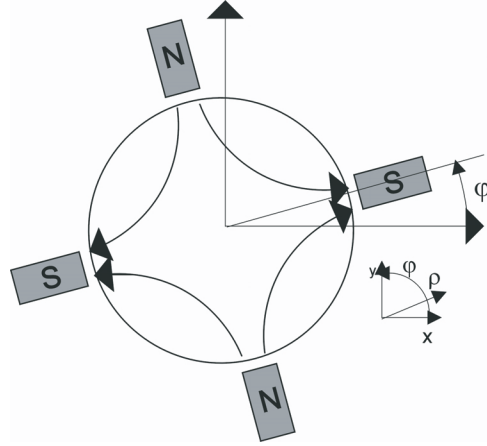


Figure 2. Rotatable magnetic field pattern.

$$\begin{aligned}
 \mu_{\rho\varphi} &= \mu_{\varphi\rho} = (1 - \mu) \sin(2\phi) \cos(2\phi) \\
 \mu_{\rho z} &= -\mu_{z\rho} = -j\mu_a \sin(2\phi) \\
 \mu_{\varphi z} &= -\mu_{z\varphi} = -j\mu_a \cos(2\phi) \\
 \mu_{zz} &= \mu
 \end{aligned}$$

and $\phi = \varphi - \theta/2$. For “weak ” and “strong” bias magnetic field the elements of the permeability tensor μ, μ_a are well determined in [2].

2.2. Coupled Mode Model

In order to obtain an approximate solution for the considered ferrite guide the coupled mode equations as derived by Awai and Itoh [6] were applied. This method describes electromagnetic wave propagation in the ferrite in terms of interaction between the normal modes of the basis waveguide. In our analysis the basis waveguide originates from the investigated guide where ferrite is substituted by dielectric of the permittivity ε_f and permeability $\mu = 1$. Suppose the basis guide is operating in the single mode frequency range. With a pair of orthogonal TE_{11} base modes, the electric \mathbf{E} and magnetic \mathbf{H} fields in the ferrite are written as, $(\mathbf{E} \vee \mathbf{H}) = a_1 \mathbf{F}_1 + a_2 \mathbf{F}_2$ where $\mathbf{F}_i = (\mathbf{e}_i \vee \mathbf{h}_i)$ are basis electric or magnetic fields and a_i ($i = 1, 2$) represent z -dependent scalar wave functions. Using procedure [6] the wave propagation in the ferrite guide can be described by the following

coupled mode equations

$$\begin{aligned}\frac{\partial a_1}{\partial z} + j(\beta - G_2)a_1 &= -jG_1a_2 \\ \frac{\partial a_2}{\partial z} + j(\beta + G_2)a_2 &= -jG_1a_1\end{aligned}\quad (4)$$

where; $\beta = \beta_o + (\mu - 1)I_0$ is a propagation coefficient of the equivalent dielectric wave. For $\mu = 1$, the β is equal to the propagation coefficient β_o of the base mode and this wave relates to the base mode. The quantities G_1 and G_2 have a dimension of propagation coefficient and are expressed as

$$G_1 = \mu_a \cos(\theta) \cdot I_1 \quad \text{and} \quad G_2 = \mu_a \sin(\theta) \cdot I_1 \quad (5)$$

and quantities I_0 and I_1 given by

$$I_1 = k_0\eta_0 \int_0^a (B - A)C\rho d\rho \quad (5a)$$

$$I_0 = \frac{1}{2}k_0\eta_0 Y_f \left[\frac{1}{2} - Z_f^2 \int_0^a C^2\rho d\rho \right] \quad (5b)$$

where;

$$A = \frac{g}{\rho} J_1(p \cdot \rho), \quad B = pgJ_1'(p \cdot \rho), \quad C = \frac{gp^2}{k_0\eta_0} J_1(p \cdot \rho)$$

and coefficient $g = 0.8871$.

In the above expressions k_0 is wave number and η_0 is wave impedance in vacuum, Y_f is wave admittance of the base TE₁₁ mode, the J_1 is Bessel function, eigenvalue $p = 1.184/a$ and a is a radius of circular guide. Applying solution of the form $a_i = A_i e^{-jkz}$ to the eqs. (4) we found two normal modes in the ferrite whose propagation coefficients are given by:

$$k_1 = \beta + \Delta \quad \text{and} \quad k_2 = \beta - \Delta \quad (6)$$

where;

$$\Delta = \sqrt{G_1^2 + G_2^2} = \mu_a \cdot I_1 \quad (7)$$

The values of k_1 and k_2 are plotted in Fig. 3. The nonreciprocity properties of the guide are seen because for (+z) wave propagation direction the coefficients $k_1^+ = k_1$ and $k_2^+ = k_2$ while for (-z) direction,

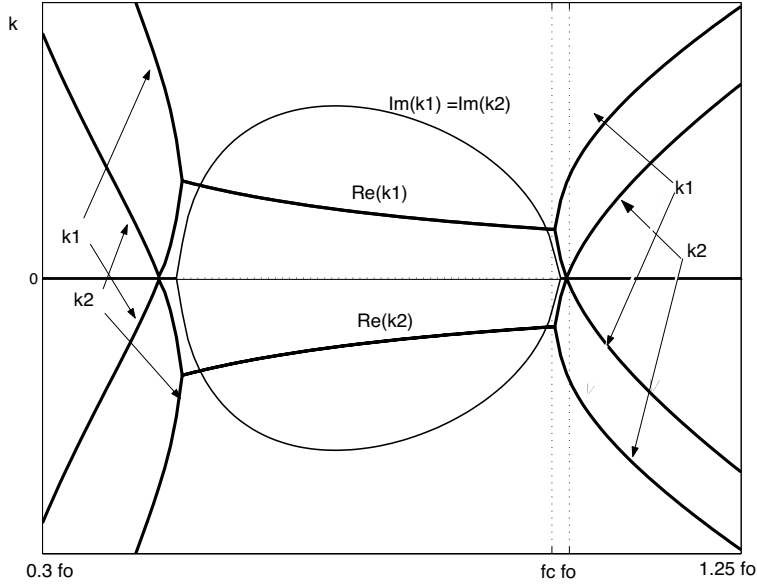


Figure 3. Dispersion characteristics.

$k_1^- = k_2$ and $k_2^- = k_1$. Note that the same relations for $k^{+/-}$ are obtained when magnetization is reversed. Proceeding on this basis note that the phase differences between normal modes is equal.

$$\delta^\pm = k_1^\pm - k_2^\pm = 2 \cdot \Delta^\pm \quad (8)$$

Applying (5) and (6) to (8) we find that phase difference δ is independent of electrical angle θ and ferrite permittivity ε_f . The fact that the $\Delta^+ = -\Delta^- = \Delta$ implies that the value of differential phase shift differs only in sign for opposite direction of propagation or magnetization. For frequency $f = f_0$, the $\beta = \Delta$ and $k_1 = 2\Delta$ and $k_2 = 0$. Now, only one of the normal mode propagates in the ferrite guide. Below the frequency f_0 but above the cutoff frequency f_c the backward wave appears together with the forward mode. Below frequency f_c propagation coefficients β will be purely imaginary ($\beta = j\alpha$) while the value of Δ remains real. Then propagation coefficients $k_{1/2}$ will be both complex values, again equal;

$k_1^\pm = -j\alpha \pm \Delta$ and $k_2^\pm = -j\alpha \mp \Delta$. Note that the phase difference between forward or backward complex waves is the same as for the propagating waves but their amplitudes decays exponentially at both $\pm z$ directions. Boyd [3] pointed out that periodical guide in which sections of high permittivity dielectric material alternate with ferrite

sections, produces differential phase shift even when ferrite sections are below cutoff frequency. It allowed [4, 5] to reduce diameter of circular waveguide used to build practical ferrite rotary-field phase shifters .

2.3. Ferrite Waves

Generally, the solutions of the coupled mode equations (4) yields two normal modes in the ferrite. For these modes the electric fields are expressed as follow

$$\begin{aligned} \mathbf{E}^1 &= A_1 (\mathbf{e}_1 + K_{21}\mathbf{e}_2) e^{-jk_1z} \\ \mathbf{E}^2 &= A_2 (\mathbf{e}_2 + K_{12}\mathbf{e}_1) e^{-jk_2z} \end{aligned} \quad (9)$$

Then, a mixture of partial waves of the normal modes in the ferrite structure yields two ferrite waves associated with the fields of the base modes. The electric fields of these waves may be expressed as

$$\begin{aligned} \mathbf{E}_1 &= (A_1 e^{-jk_1z} + K_{12}A_2 e^{-jk_2z}) \mathbf{e}_1 \\ \mathbf{E}_2 &= (A_2 e^{-jk_2z} + K_{21}A_1 e^{-jk_1z}) \mathbf{e}_2 \end{aligned} \quad (10)$$

In the above equations the coefficients A_1 and A_2 will be determined by initial conditions. The quantities K_{12} and K_{21} are coupling coefficients describing the field coupling from the base mode 2 to mode 1 and vice versa. The coupling coefficients satisfy condition $K_{21} = -K_{12} = K$ and K is given by

$$K = (G_2 - \Delta)/G_1 \quad (11)$$

The coupling coefficient K evaluated with (5) yields

$$K = \tan(\pi/4 - \theta/2) \quad (12)$$

It is worth to note that coupling coefficient depends on the electrical angle θ . When the ‘‘cos bias’’ winding operates only, then the electrical angle $\theta = 0$ and the coupling between basis modes is observed. The basis modes in the ferrite are uncoupled when the ‘‘sin bias’’ winding is only excited. It occurs when $\theta = \pi/2$ for which coupling coefficient $K = 0$.

2.4. Operation Principles

Let us consider the effect of phase difference on the field distribution along the ferrite guide. Consider the total electric field defined as the

superposition of the fields of two ferrite waves. Then, owing to the relation (10) the total electric field is given by

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_1 + \mathbf{E}_2 \\ &= \left(A_1 e^{-jk_1 z} - K A_2 e^{-jk_2 z} \right) \mathbf{e}_1 + \left(A_2 e^{-jk_2 z} + K A_1 e^{-jk_1 z} \right) \mathbf{e}_2 \end{aligned} \quad (13)$$

Assume that the ferrite guide is excited at the plane $z = 0$ by left hand circularly polarized LHCP wave described as superposition of two linearly polarized base waves. With this input wave the initial conditions at the input plane are given by

$$\mathbf{E}(z = 0) = \mathbf{e}^+ = \mathbf{e}_1 + j\mathbf{e}_2 \quad \text{and} \quad |\mathbf{e}_1| = |\mathbf{e}_2| \quad (14)$$

If initially (13) satisfies (14), coefficients A_1 and A_2 will emerge as

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \frac{1}{1 + K^2} \begin{bmatrix} 1 + jK \\ j(1 + jK) \end{bmatrix} \quad (15)$$

Using (15) in (13), the total electric field distribution $\mathbf{E}(z)$ in the ferrite guide is now formulated with respect of the electric fields $\mathbf{e}^+ = \mathbf{e}_1 + j\mathbf{e}_2$ and $\mathbf{e}^- = \mathbf{e}_1 - j\mathbf{e}_2$ of the base LHCP and RHCP waves, respectively.

$$\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^- = [\cos(\Delta \cdot z)\mathbf{e}^+ + R \sin(\Delta \cdot z)\mathbf{e}^-] e^{-j\beta z} \quad (16)$$

where;

$$R = -j \frac{(1 + jK)^2}{1 + K} \quad (17)$$

Substituting (12) into (17) the coefficient R results in the relation to the electrical angle θ as

$$R = e^{-j\theta} \quad (18)$$

Note from (16) that for different value of z the polarization state of the wave is changed. The polarization state of wave is defined by axial ratio (AR) and rotation angle (ψ). These parameters [9] can be evaluated with (16) as

$$AR = \tan(\pi/4 - \Delta \cdot z) \quad \text{and} \quad \psi = \theta/2, \quad (19)$$

for $\Delta \cdot z$ & $\theta \in (0, \pi/2)$.

The coefficient AR is expressed only in terms of the phase difference angle ($\Delta \cdot z$) while the rotation angle ψ is proportional to the electrical angle θ which simulates mechanical rotation of the bias magnetic field. For $AR = 0$ the wave is linearly polarized (LP) while $AR = +1$ or -1 is designates for (LHCP) and (RHCP) circularly polarized waves. It is significant that for circularly polarized ferrite

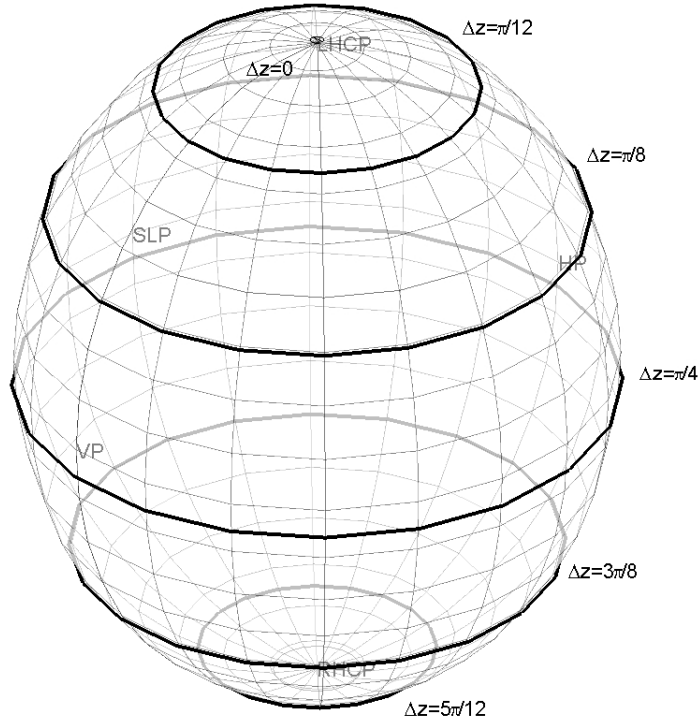


Figure 4. Polarization states of the output wave for input LHCP wave defined by points on circles evaluated versus θ for selected values $\Delta \cdot z$ and performed on Poincare sphere.

wave the rotation angle ψ is not defined so its polarization is not changed with electrical angle θ . This situation is observed on the Poincare sphere shown in Fig. 4 where parallels show the change of axial ratio AR of the wave propagating along the guide. Over the distance, where $\Delta \cdot z = \pi/4$ the input circularly polarized wave is converted into linearly polarized wave whose orientation varies with electrical angle θ . It demonstrates points on equator of Poincare sphere in Fig. 4. Note that for $\theta = 0$ or π , the orthogonal VLP or HLP wave appears, respectively. Therefore for such polarizer called “quarter-wave plate” [8] the principal axis is determined by the transverse four pole magnetic field pattern produced by one winding or defined in our system by $\theta = 0$ or π .

Another important configuration is ferrite section of length $\Delta \cdot z = \pi/2$ where LHCP input wave is transformed to the output RHCP wave and back. It refers to the “rotary half-wave section” where

the phase variation of the output circularly polarized wave is linearly proportional to the electrical angle θ . It bring (16) evaluated with (18), where the quantity of phase variation is equals $\theta + \beta z$ and β is independent of angle θ .

The similar results are presented in Fig. 5 where output wave polarization state parameters are calculated and measured as a function of magnetization M/M_s that varies with magnetizing current I_m . From equation (7) and relations defined for elements of permeability tensor in [2] is seen that Δ is proportional to M/M_s . Therefore the change of M/M_s as well as the ferrite section length $\Delta \cdot z$ define similar response for the wave polarization state. One may notice that experimental results correspond well to the parallels calculated for values of $M/M_s = 0.5$ and $M/M_s = 0.7$ that relate to the magnetizing current $I_m = 0.2A$ and $I_m = 0.4A$ respectively.

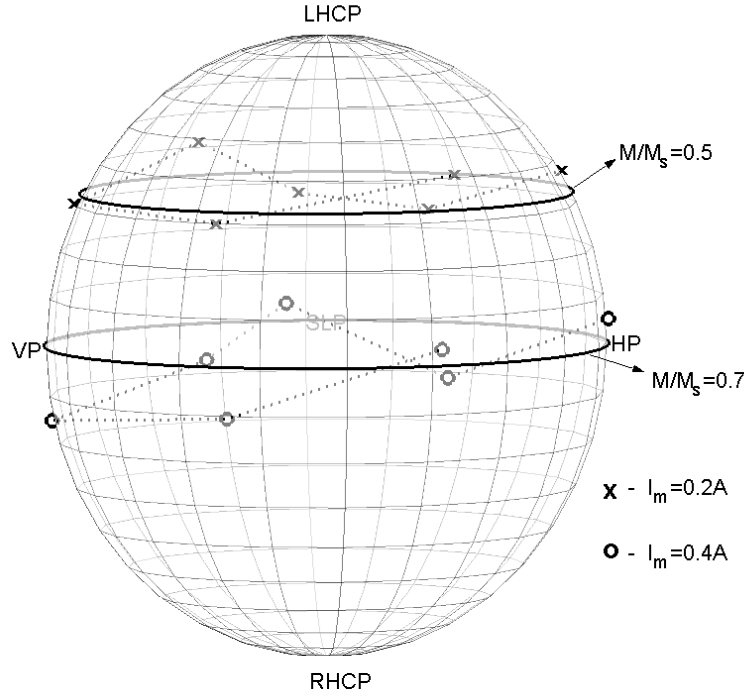


Figure 5. Experimental and simulated polarization state parameters of the wave at the output of the ferrite section for LHCP input, evaluated versus θ and selected two values of magnetization currents I_m (M/M_s) (ferrite rod parameters, radius $a = 18.59$ mm, length $z = 98$ mm, $M_s = 250$ Gauss, $\epsilon_r = 13.5$).

2.5. Matching Properties

When normal TE_{11} wave propagates from a dielectric circular waveguide into a ferrite circular guide, a two reflected linearly polarized orthogonal TE_{11} waves will occur mainly at the interface. In this arrangement the reflection results from differences between the wave impedance Z_d of the TE_{11} mode in dielectric guide and wave impedances of the ferrite modes Z_1 and Z_2 . The ferrite waves are revealed as perturbed linearly polarized dielectric wave characterized by propagation coefficients β and wave impedance $Z = k_0\eta_0/\beta$. Hence, ferrite waves impedances normalized to the dielectric wave impedance Z are approximately defined as

$$\tilde{Z}_1 = \beta/k_1 \quad \text{and} \quad \tilde{Z}_2 = \beta/k_2 \quad (20)$$

Applying (6) and (8) in (20), the ferrite wave impedances are expressed in terms of normalized differential phase shift $\chi = \delta/2\beta$, as follow

$$\tilde{Z}_1 = 1/(1 + \chi) \quad \text{and} \quad \tilde{Z}_2 = 1/(1 - \chi) \quad (21)$$

Now assume the following relationship between normalized dielectric $Z_d/Z = r$ and ferrite waves impedances

$$\sqrt{\tilde{Z}_1\tilde{Z}_2} = r \quad \text{where} \quad r = 1/\sqrt{1 - \chi^2} \quad (22)$$

The reflection coefficients Γ_1 and Γ_2 for input LP dielectric modes will be

$$\Gamma_1 = -\Gamma_2 = \Gamma = \frac{\tau - 1}{\tau + 1} \quad (23)$$

with; $\tau^2 = \tilde{Z}_1/\tilde{Z}_2 = (1 - \chi)/(1 + \chi)$.

Let linearly polarized wave is incident with angle γ on the dielectric ferrite interface. With conditions (23) the magnitudes of co- and cross-polarization reflection depends on incident angle γ . There are $\Gamma^{(co)} = \Gamma \cos(2\gamma)$ and $\Gamma^{(cross)} = -\Gamma \sin(2\gamma)$, where $\Gamma \sim \chi/2$ and $\gamma = (\pi - \theta)/2$. Because the cross-polarization reflection can be easily canceled using loss plate the only ‘‘parasitic’’ is co-polarized reflected wave appearing at the plane of incident wave. Note that this reflection is vanished when $\gamma = \pi/4$ because the polarization planes of reflected and input linearly polarized waves are orthogonal. The situation for incidence of circularly polarized wave is similar. For LHCP input wave the reflected wave is RHCP and vice versa. The maximal magnitude of the reflection is equal Γ and it is less than -20 dB for value of $\chi < 0.2$. Consider now the problem when condition (22) is not satisfied. Then $\Gamma_1 \neq \Gamma_2$ and $\Gamma^{(co)}$ and $\Gamma^{(cross)}$ are given by

$$\Gamma^{(co)} = \Gamma \cos(\xi) \quad \text{and} \quad \Gamma^{(cross)} = \Gamma \sin(\xi) \quad (24)$$

where; $\Gamma = \sqrt{\Gamma_1^2 \cos^2(\gamma) + \Gamma_2^2 \sin^2(\gamma)}$ and $\xi = [\gamma + \arctan[(\Gamma_1/\Gamma_2) \tan(\gamma)]]$.

For $r = \tilde{Z}_1$ the reflection coefficients $\Gamma_1 = 0$ and $\Gamma_2 = -\chi$. With these values the co-polarized reflection is $\Gamma^{(co)} = -\chi \sin \gamma$. For $r = \tilde{Z}_2$ we obtain $\Gamma_2 = 0$ and $\Gamma_1 = \chi$ and hence $\Gamma^{(co)} = \chi \cos(\gamma)$. The plots relating return loss Γ to the normalized phase difference χ and dielectric impedance r for angle $\gamma = 45^\circ$ are shown in Fig. 6. Note that this reflection is less than -15 dB when r is chosen from 0.8 to 1.2 and $\chi < 0.2$. The magnitudes of reflected coefficient achieve minimum when $r \approx 1$ i.e., when the condition (22) is satisfied.

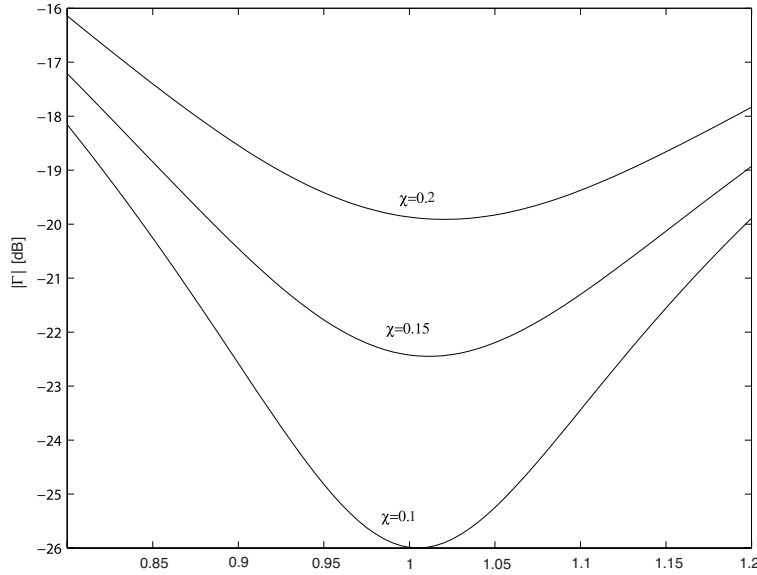


Figure 6. Reflection coefficient Γ . versus normalized impedance r for selected values of χ , angle.

3. CONCLUSION

Coupled mode description of the propagation phenomena in the ferrite circular waveguide magnetized through a rotary four-pole bias magnetic field has been developed. This approach has given useful insight into the effect of bias field rotation on the change of the phase and polarization state of the wave propagated along the guide. The use of coupled waves provides simple design relationships useful to model latching ferrite rotary field phase shifters and polarizer.

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