PHASE-ONLY AND AMPLITUDE-PHASE SYNTHESIS OF DUAL-PATTERN LINEAR ANTENNA ARRAYS USING FLOATING-POINT GENETIC ALGORITHMS

G. K. Mahanti and A. Chakrabarty

Department of Electronics and Electrical Communication Engineering
Indian Institute of Technology
Kharagpur-721302, India

S. Das

Advanced Technology Center
Indian Institute of Technology
Kharagpur-721302, India

Abstract—In this paper, we present a comparison study between phase-only and amplitude-phase synthesis of symmetrical dual-pattern linear antenna arrays using floating-point or real-valued genetic algorithms (GA). Examples include a sum pattern and a sector beam pattern. In the former, phase is only optimized with predetermined Gaussian amplitude distribution of fixed dynamic range ratio ($|a_{max}/a_{min}|$) and in the latter, both are optimized with less dynamic range ratio than the former and yet share a common amplitude distribution.

1. INTRODUCTION

Phase-only reconfigurable array antennas capable of radiating multiple radiation patterns with a fixed amplitude distribution are used in many applications such as communication and radar. The generation of multiple radiation patterns by a single antenna array with pre-fixed or common amplitude distribution greatly simplifies the hardware implementation of the feed network, since it is technically easier to design a feed network if the element excitations corresponding to different patterns differ only in phase than if they also differ in amplitude. Several methods of generating phase-only multiple pattern antenna arrays have been described [1–7].

F. Ares et al. [1] reported the synthesis of phase-only multiple radiation patterns with pre-fixed amplitude distributions using modified Woodward-Lawson technique. Bucci et al. [2] proposed the method of projection to synthesize reconfigurable array antennas with asymmetrical pencil and flat-top beam patterns using common amplitude and varying phase distributions. The design of a phase-differentiated reconfigurable array has been described [3] using particle swarm optimization in theta domain. Phase-only beam shaping with pre-fixed amplitude distributions was reported [4] using an analytical technique. Baskar et al. [5] synthesized reconfigurable array antennas with phase-only control of 6-bit discrete phase shifter and continuous amplitude distribution using generalized generation gap model genetic algorithm and better synthesis results were obtained, as compared to continuous phase excitations with subsequent quantization. F. Ares et al. [6] described two-pattern linear array antennas using 8-bit phase shifter, and the tolerance of the radiation patterns to errors in different antenna parameters was also investigated. Design of phase-differentiated multiple pattern antenna arrays [7] has been reported based on simulated annealing optimization technique.

Genetic algorithms [8] on the other hand are a class of global optimization technique, which are especially powerful for problems that have large number of variables and many local optima. Unlike binary GA, floating-point GA avoids coding, decoding and directly deals with array excitation coefficient vectors and thus takes less memory space than binary GA.

In this paper, a phase-only synthesis with prefixed Gaussian amplitude distribution [1] and a jointly optimized amplitude-phase synthesis of a dual-pattern array with less dynamic range ratio (DRR) than the former using floating-point GA are presented and their results are compared. Patterns are optimized in sine space (sine of far-field polar angle) instead of angle space [3, 5]. In case of amplitude-phase synthesis, phases are all set to zero degree to generate a sum pattern and are varied in the range 180 to +180 degree to obtain a sector beam pattern.

2. PROBLEM FORMULATION

In amplitude-phase synthesis, the design of a reconfigurable dual-pattern antenna array is based on finding a common amplitude distribution that can generate either a sum or a sector beam power pattern, when the phase distribution of the array is modified appropriately. All the excitation phases are set at $0°$ to generate a sum pattern and are varied in the range $-180° \leq \varphi \leq 180°$ to form
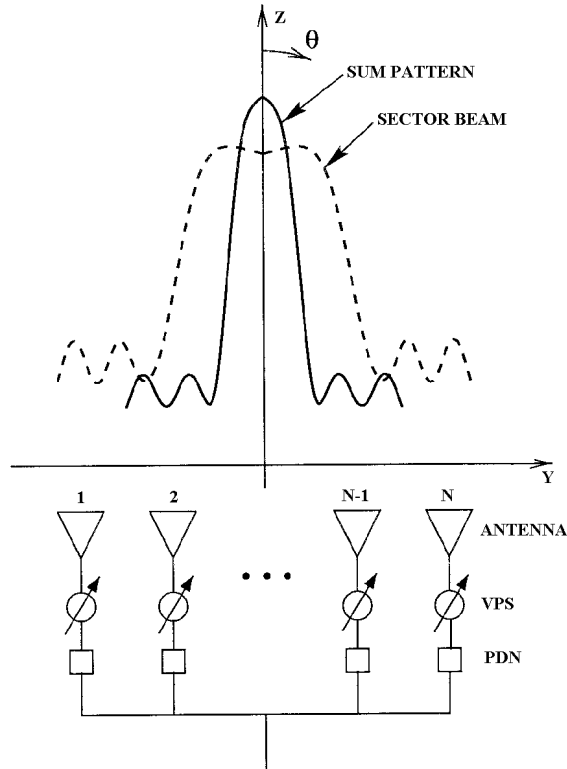
Figure 1. Geometry of a dual-pattern linear antenna array, VPS is variable phase shifter and PDN is power divider network.

a sector beam pattern. However, in phase-only synthesis with pre-fixed Gaussian amplitude distribution, excitation phases in the range $-180° \leq \varphi \leq 180°$ are required to generate sum as well as sector beam pattern.

We consider a linear array of N isotropic antennas [3] that are assumed uncoupled and equally spaced a distance d apart along the Y-axis with its first element at the origin. This is shown in Fig. 1. The free space [9] far-field pattern $F(u)$ in the principal plane (YZ-plane) with symmetric amplitude and phase distributions is given by eqn. (1):

$$F(u) = \sum_{n=1}^{N} a_n e^{i\varphi_n} e^{i(n-1)\frac{2\pi}{\lambda}du} \qquad (1)$$

Where n the element number, λ the wavelength, φ_n the excitation current phases of the elements, a_n the excitation current amplitudes of

the elements, i the imaginary unit, d is the inter-element spacing, and $u = \sin\theta$, θ being the polar angle of far-field measured from broadside $(-90° \text{ to } +90°)$.

$$\text{Normalized absolute far-field, } F_n(u) = \frac{|F(u)|}{|F(u)|_{\max}} \qquad (2)$$

For the dual-pattern array optimization, the objective function must quantify the entire array radiation pattern. The fitness function to be minimized for dual-pattern array optimization problem can be expressed as follows:

$$Fitness = \sum_{s=1}^{3}\left(Q_{s,d}^{(p)} - Q_s^{(p)}\right)^2 + \sum_{s=1}^{4}\left(Q_{s,d}^{(f)} - Q_s^{(f)}\right)^2 + (DRR_d - DRR_o)^2 \qquad (3)$$

where the superscript p is meant for the design specification of sum pattern and the superscript f is meant for the design specification of sector beam pattern.

$Q_{s,d}$ and Q_s represent respectively the applicable desired and calculated value of each design specification, as given in Table1 and Table 2. The fourth term in the second summation in eqn. (3) is a ripple parameter for the sector beam pattern. The third term in eqn. (3) is the difference between desired and obtained value of dynamic range ratio (DRR) of excitation amplitude, defined as:

$$DRR = (a_n)_{\max}/(a_n)_{\min}\forall n \in [1, N] \qquad (4)$$

The last term will not be there in case of design specifications for dual-pattern array with pre-fixed Gaussian amplitude distribution [1] as given in table1. Moreover, the first and second term can be separated out to act as single independent objective function for the design specifications given in table1. The lower the fitness, the more fit the array to the desired specifications. The desired maximum ripple level (RL) in the entire coverage region near zero dB $(-0.2 \leq u \leq 0.2)$ is not to exceed 0.5 dB from the peak value of 0 dB. The difference terms in connection to side lobe level, ripple level and dynamic range ratio in fitness function eqn. (3) are made zero when their respective calculated values are less than their desired values by multiplying appropriate Heaviside step function with these terms whose value is unity when the calculated value is greater than or equal to the desired value and zero elsewhere.
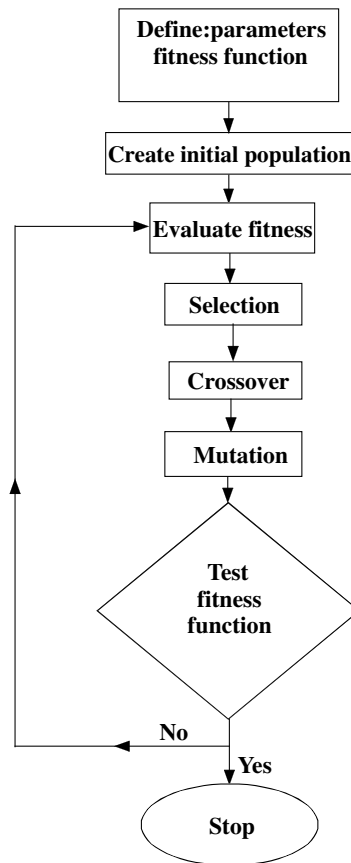
Figure 2. Flow chart of floating-point GA.

3. OPTIMIZATION USING FLOATING-POINT GA

Genetic Algorithm [8] is an iterative stochastic optimizer that works on the concept of survival of the fittest motivated by Darwin, using methods based on the mechanics of natural genetics and natural selection to construct search and optimization procedures that best satisfies a predefined goal. Floating-point GA uses floating-point number representation for the real variables and thus is free from binary encoding and decoding. It takes less memory space and works faster than binary GA. Programs are written in Matlab6.5 code and run on a PentiumIV personal computer. The flow chart of floating-point GA is shown in Fig. 2. The floating-point GA is summarized as follows:

Step 1: Randomly generate an initial population of P individuals within the variable constraint range.

Step 2: Evaluate the fitness of the population from the fitness function.

Step 3: Select the superior individuals using nonlinear ranking [8] and place them in the mating pool. Numbers of individuals in the mating pool are same as P in order to accommodate more and more copies of superior individuals in the new population.

Step 4: Individuals so called parents placed in the mating pool are now allowed to mate followed by mutate using heuristic crossover and non-uniform mutation [8] respectively. In the crossover process, two parents produce two children. Subsequent mutations of the parents add diversity to the population and explore new areas of parameter search space. Select C pairs of parents at random to participate in crossover to produce C pairs of offspring with immediate replacement of the parents. Select M number of parents at random to participate in mutation to produce M number of offspring with replacement.

Step 5: Repeat steps 2–4 until a stopping criterion, such as a sufficiently good solution being discovered or a maximum number of generations being completed, is satisfied. The best scoring individual in the population is taken as the final answer.

3.1. Nonlinear Ranking

Ranking methods only require the evaluation function to map the solutions to a partially ordered set. All individuals in a population are ranked from best to worst based on its fitness value. It assigns the probability of an individual based on its rank (r) and it is expressed as follows:

$$prob(r) = \frac{p(1-p)^{r-1}}{1-(1-p)^P} \tag{5}$$

such that

$$\sum_{r=1}^{P} prob(r) = 1 \tag{6}$$

Where

$p =$ the probability of selecting the best individual $= [0,1]$,

$P =$ the population size,

$r =$ the rank of the individual $= \begin{cases} 1, & \text{for the best individual} \\ P, & \text{for the worst individual} \end{cases}$

3.2. Heuristic Crossover

It produces a linear extrapolation of the two individuals. It uses values of the fitness function in determining the direction of the search.

It produces two children $I^{A(new)}$ and $I^{B(new)}$ from two parents I^A and I^B according to the following provided I^A is better than I^B in terms of fitness:

$$I^{A(new)} = I^A + r(I^A - I^B) \tag{7}$$

$$I^{B(new)} = I^A \tag{8}$$

Where r is a random number between 0 and 1.

$$\text{Feasibility} = 1, \text{ if } a_i \leq I_i^{A(new)} \leq b_i$$
$$= 0, \text{ otherwise} \tag{9}$$

Where a_i and b_i are lower and upper bounds of each variable in population.

If feasibility is equal to zero, then another random value r is generated and another children created. If after t attempts no new solution meeting the constraints is found, the operator gives up and produces no offspring i.e., the children become equal to parents and stop.

3.3. Non-uniform Mutation

This is the unary operator responsible for the fine tuning capabilities of the system, so that it can escape from the trap of local minima. It randomly changes one variable of a parent. It is defined as follows: For a parent I^A, if variable I_k^A was selected at random for this mutation, the result is:

$$\overline{I}^A = \left(I_1^A, \ldots, \overline{I}_k^A, \ldots, I_N^A \right)$$

Where

$$\overline{I}_k^A = I_k^A + \left(b_k - I_k^A \right) f(G), \quad \text{if } r_1 < 0.5 \tag{10}$$

$$= I_k^A - \left(I_k^A - a_k \right) f(G), \quad \text{if } r_1 \geq 0.5 \tag{11}$$

Where

$$f(G) = r_2 \left(1 - \frac{G}{G_{\max}} \right)^s \tag{12}$$

Where r_1, r_2 are uniform random number between [0,1], G is the Current generation, $G_{\max}$ is the maximum generation number, s is the

shape parameter determining the degree of non-uniformity, b_k, a_k are upper and lower bounds of I_k^A respectively. The function $f(G)$ returns a value in the range [0,1] such that the probability of $f(G)$ being close to zero increases as G increases (G is the generation number). This property causes this operator to search the space uniformly initially, when G is small, and very locally at later stages.

4. RESULTS

We consider a uniform linear array of 20 isotropic elements spaced 0.5λ apart along Y-axis in order to generate a sector beam and a sum pattern using phase-only control with fixed Gaussian excitation amplitude distribution [1] whose mean is at the origin. This type of distribution gives a dynamic range ratio of 4.98. Phase distribution is assumed symmetric with respect to the center of the array.

Next the dual-pattern is generated by jointly optimizing phase and amplitude distributions with the dynamic range ratio equal to or less than 4.98. Here, both amplitude and phase distributions are assumed symmetric with respect to the center of the array. Because of symmetry, only ten amplitudes and ten phases are to be optimized. All phases are restricted to lie between -180 and 180 degrees, as mentioned before, and amplitudes between 0 and 1.

Table 1. Simulated results for phase-only synthesis.

Design parameters	Sum pattern		Sector beam	
	Desired	Obtained	Desired	Obtained
Side lobe level (SLL, in dB)	-20.00	-24.13	-20.00	-21.17
Half-power beamwidth (HPBW, in u-space)	0.12	0.12	0.50	0.49
Beamwidth at SLL (in u-space)	0.30	0.30	0.70	0.68
Ripple (in dB, $-0.2 \leq u \leq 0.2$)	N/A	N/A	0.50	0.47

For design specifications as given in Table 1 and Table 2, GA is run with an initial population of 200 and nonlinear ranking with probability of 0.15 for selecting the best individual. Crossover and mutation operators are called fourth and sixth times respectively every generation in order to ensure that only four pairs of parents participate in crossover and six number of parents take part in mutation instead of all. This will reduce the overall computational time in optimization

considerably. Number of attempts in heuristic crossover is taken to be three and the shape parameter in non-uniform mutation is taken to be 3.5. Best individual having the lowest fitness value is taken as final.

For design specifications of phase-only method, GA is run for 800 generations. Simulated results are shown in Table 1. When these results are compared with [1], an improvement in the side lobe level of sum and sector beam pattern and ripple of sector beam pattern is noticed. Fig. 3(a) shows the normalized absolute power patterns in dB for dual-pattern array by phase-only synthesis. Fig. 3(b) shows the phase distributions in degree for the dual-pattern array by phase-only synthesis.

Table 2. Simulated results for amplitude-phase synthesis.

Design parameters	Sum pattern		Sector beam	
	Desired	**Obtained**	**Desired**	**Obtained**
Side lobe level (SLL, in dB)	-25.00	-25.05	-25.00	-25.56
Half-power beamwidth (HPBW, in u-space)	0.12	0.13	0.50	0.51
Beamwidth at SLL (in u-space)	0.30	0.31	0.70	0.72
Ripple (in dB, $-0.2 \leq u \leq 0.2$)	N/A	N/A	0.50	0.87

For design specifications of amplitude-phase synthesis, GA is run for 1200 generations. Dynamic range ratio of excitation amplitude distribution is found to be 4.51, that is again lower than the DRR value of phase-only synthesis. Side lobe level obtained for sum pattern is $-25.05\,$dB and that of sector beam is $-25.56\,$dB with a ripple of $0.87\,$dB. An improvement of about $4.39\,$dB and $0.92\,$dB in the side lobe level of sector beam and sum pattern is obtained over phase-only synthesis even with less value of DRR. Simulated results are shown in Table 2. There is a very good agreement between the desired and the synthesized results using GA. Coverage region near zero dB for calculating ripple of sector beam is not mentioned in [3, 5]. In our case, they are all clearly mentioned. Our method of calculating ripple of flat-top beam is quite different from [3, 5]. Equally spacing pattern points in sine space provide a more uniform sampling and less number of sampling points than angle space [3, 5], which in turn reduces the complexity of optimization.

Fig. 4(a) shows normalized absolute power patterns in dB for dual-pattern array by amplitude-phase synthesis. Fig. 4(b) shows
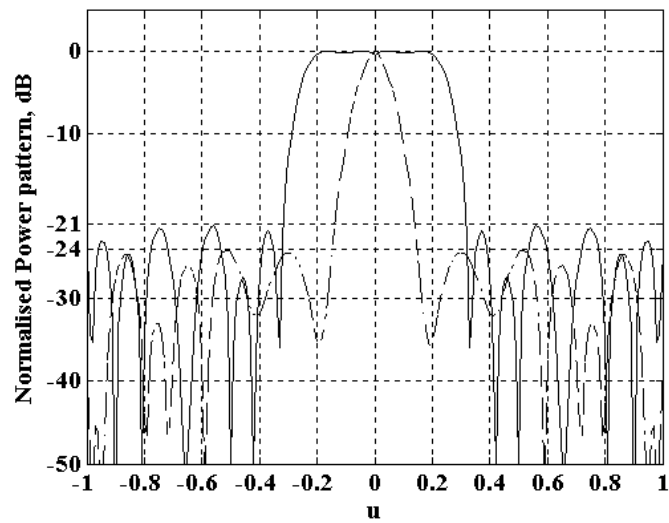
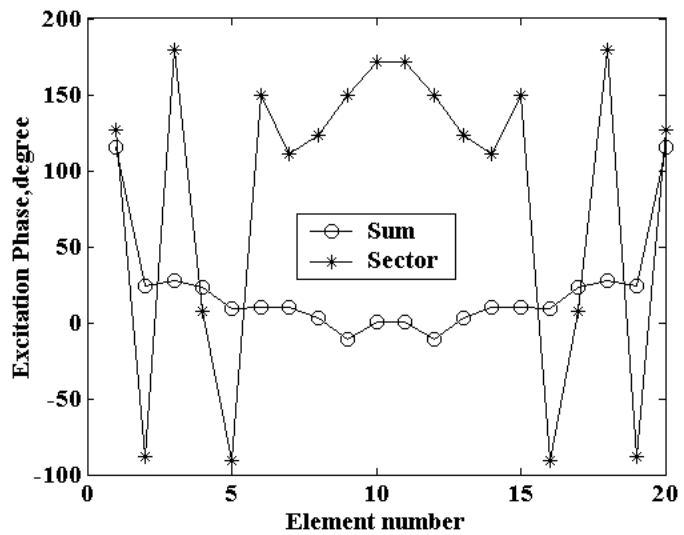**Figure 3a.** Normalized absolute power patterns in dB for dual-pattern array by phase-only synthesis.

Figure 3b. Excitation phases in degree for dual-pattern array by phase-only synthesis.
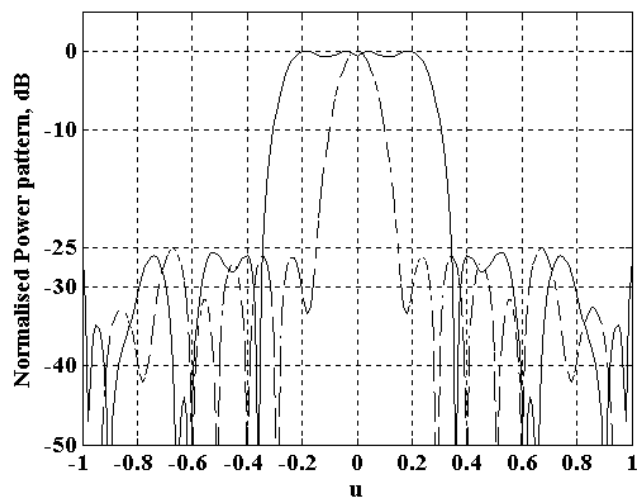
Figure 4a. Normalized absolute power patterns in dB for dual-pattern array by amplitude-phase synthesis.
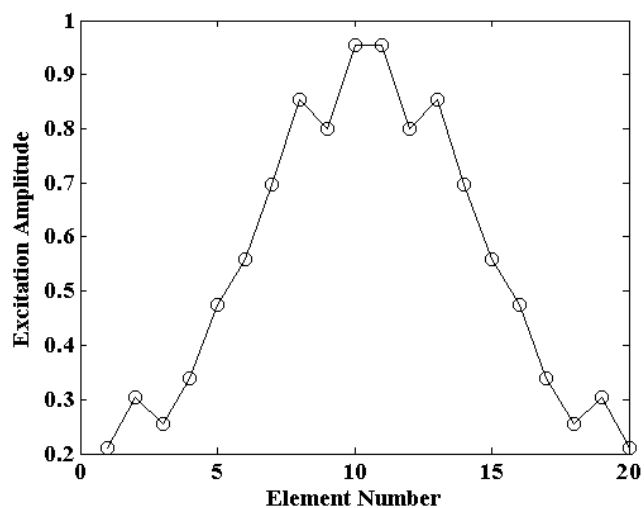

Figure 4b. Excitation amplitude distributions for dual-pattern array by amplitude-phase synthesis.
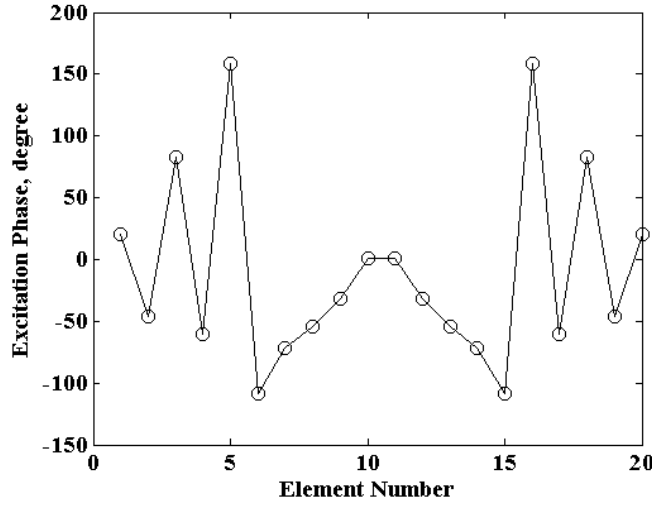
Figure 4c. Excitation phases in degree for sector beam pattern by amplitude-phase synthesis.

common amplitude distribution and Fig. 4(c) shows phase distributions in degree for sector beam pattern.

5. CONCLUSIONS

A comparison study between phase-only and amplitude-phase synthesis of generating dual-pattern is presented in this paper. The results show that amplitude-phase synthesis is better than phase-only synthesis in terms of side lobe level and dynamic range ratio (DRR) of excitation amplitude, but its amplitude distribution is very irregular. By fixing the DRR to a lower value, the differences between the successive excitation amplitudes are minimized and hence the effects of coupling between the neighboring array elements are reduced. Amplitude-phase synthesis requires constant phases of zero degree to generate a sum pattern, where as phase-only method requires phases to generate a sum pattern with desired side lobe level. Amplitude-phase synthesis rather requires less complicated feed network than phase-only synthesis due to constant phase feed network for generating sum pattern and therefore cost of designing the complete feed network is substantially reduced. Results for a linear isotropic antenna array have illustrated the performance of this proposed technique. These methods can be extended to planar array antenna synthesis also.

ACKNOWLEDGMENT

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