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OBLIQUE INCIDENCE PLANE WAVE SCATTERING FROM AN ARRAY OF CIRCULAR DIELECTRIC CYLINDERS

B. H. Henin, A. Z. Elsherbeni, and M. Al Sharkawy

The Center of Applied Electromagnetic Systems Research (CASER)
Electrical Engineering Department
The University of Mississippi
University, MS 38677, USA

Abstract—A rigorous semi-analytical solution is presented for electromagnetic scattering from an array of circular cylinders due to an obliquely incident plane wave. The cylinders are illuminated by either TM_z or TE_z incident plane wave. The solution is based on the application of the boundary conditions on the surface of each cylinder in terms of the local coordinate system of each individual cylinder. The principle of equal volume model is used to represent cylindrical cross-sections by an array of circular cylinders for both dielectric and conductor cases in order to proof the validity of the presented technique.

1. INTRODUCTION

The scattering of electromagnetic plane wave from a single circular dielectric or conductor cylinder for both normal and oblique incidence had been studied for many years [1, 2]. The analysis of scattering by multiple circular cylinders of arbitrary radii and positions makes it possible to simulate the scattering from two-dimensional scatterers of arbitrary cross-section material decomposition. Different techniques were used to solve this scattering problem due to normally incident plane wave. Among those techniques are the integral equation formulation, partial differential equation formulation, and hybrid techniques [3–7]. Furthermore, a rigorous solution had been introduced to solve the scattering problem from an array of dielectric or conductor cylinders for a plane wave excitation of normal incidence [8–11]. In a previous work [12], the principle of equal volume model was used to model any two-dimensional cylindrical object of arbitrary cross section by an array of circular cylinders. The authors provided a detailed derivation for the problem of scattering of a plane wave from an array of parallel circular cylinders in case of normal incidence. The solution is based on applying the continuity of the tangential electric and magnetic fields on the surface of each cylinder. In [13, 14], the same technique was used for an array of circular bianisotropic cylinders in the case of normal incidence.

For many practical applications, the excitation of plane wave is usually not at normal incident with respect to the scatterer. Therefore, in order to provide accurate assessment of the scattering properties of objects, one has to consider oblique incident conditions. The obliquely incident plane wave on the surface of one or two bianisotropic cylinders was considered before [15–17]. In [18], a linear array of bianisotropic cylinders was considered with constant distances between the cylinders. Previous investigations with oblique incident plane wave on multiple cylinders exhibit many limitations. Either the cylinders configurations are confined to linear array, or the solution technique is not proven to tackle practical configurations.

In this paper, the scattering of an obliquely incident plane wave on an array of parallel dielectric circular cylinders of arbitrary radii and positions is considered. The analysis begins by representing each field component by an infinite series of cylindrical harmonic functions with unknown coefficients. Then equations based on the boundary conditions applied on the surface of each cylinder are used to deduce the values of the unknown coefficients. The detailed derivation for the TM_z excitation is given, and the extension to TE_z is outlined.

To prove the validity of the results, some numerical examples are given to illustrate the principle of equal volume model in the case of an obliquely incident plane wave. Both cases of TM_z and TE_z are considered for conductor, dielectric cylinders, or combinations. Furthermore, the near field is calculated to prove the validity of the boundary conditions on the surface of any cylinder.

2. FORMULATION

Without losing generality, the scattering from an obliquely incident E-polarized TM_z plane wave from an array of M cylinders parallel to each other and to the z-axis of a global coordinate system (ρ, ϕ, z) is considered. The incident electric field of a plane wave on cylinder "i"

is expressed in the $(\rho_i,\ \phi_i,\ z)$, cylindrical coordinate system for $e^{j\omega t}$ as

$$\begin{aligned} E_{z_i}^{inc}(\rho_i,\phi_i,z) &= E_0' e^{jk_0 z\cos\theta_0} e^{jk_0\rho_i\sin\theta_0\cos(\phi_i-\phi_0)} e^{jk_0\rho_i'\sin\theta_0\cos\left(\phi_i'-\phi_0\right)} \\ &= E_0' e^{jk_0 z\cos\theta_0} e^{jk_0\rho_i'\sin\theta_0\cos\left(\phi_i'-\phi_0\right)} \\ &\quad \times \sum_{-\infty}^{\infty} j^n J_n\left(k_0\rho_i\sin\theta_0\right) e^{jn(\phi_i-\phi_0)}, \end{aligned} \tag{1}$$

where $E_0' = E_0\sin\theta_0$, θ_0 is the oblique incident angle as shown in Fig. 1 and E_0 is the amplitude of the incident electric field component. The parameter k_0 is the free space wave number, ϕ_0 is the angle of incidence of the plane wave in the x-y plane with respect to the positive x-axis, and the $J_n(\xi)$ is the Bessel function of order n and argument ξ. The second expression of the incident field component is in terms of the cylindrical coordinate of the ith cylinder, whose center is located at (ρ_i', ϕ_i', z) of the global coordinate (ρ, ϕ, z).

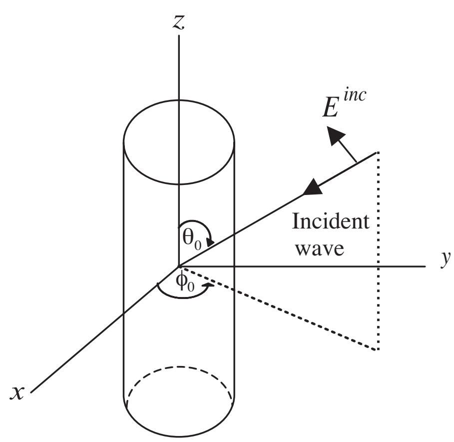


Figure 1. The parameters describing the obliquely incident E-polarized plane wave on a simple cylinder.

The resulting z component of the scattered electric field from the ith cylinder and the transmitted z component of the field inside the cylinder material can be expressed, respectively, as

$$E_z^s(\rho_i,\phi_i,z) = E_0' e^{jk_0 z\cos\theta_0} \sum_{-\infty}^{\infty} A_{in} H_n^{(2)}\left(k_0\rho_i\sin\theta_0\right) e^{jn(\phi_i-\phi_0)}, \tag{2}$$

$$E_z^d(\rho_i,\phi_i,z) = E_0' e^{jk_0 z\cos\theta_0} \sum_{-\infty}^{\infty} B_{in} J_n\left(k_0\rho_i\sqrt{\frac{k_d^2}{k_0^2}-\cos^2\theta_0}\right) e^{jn(\phi_i-\phi_0)}, \tag{3}$$

where $H_n^{(2)}(\xi)$ is the Hankel function of the second type of order n and argument ξ and k_d is the wave number inside the cylinder material . The corresponding magnetic field components are obtained as

$$H_z^s(\rho_i, \phi_i, z) = E_0' e^{jk_0 z \cos\theta_0} \sum_{-\infty}^{\infty} C_{in} H_n^{(2)} (k_0 \rho_i \sin\theta_0) \, e^{jn(\phi_i - \phi_0)}, \qquad (4)$$

$$H_z^d(\rho_i, \phi_i, z) = E_0' e^{jk_0 z \cos\theta_0} \sum_{-\infty}^{\infty} D_{in} J_n \left(k_0 \rho_i \sqrt{\frac{k_d^2}{k_0^2} - \cos^2\theta_0} \right) e^{jn(\phi_i - \phi_0)}. \qquad (5)$$

The coefficients A_{in}, B_{in}, C_{in} and D_{in} are unknowns to be determined. Using Maxwell's equations, the ϕ components of the magnetic field can be obtained as follow

$$H_\phi = \frac{-1}{j\omega\mu} \left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} \right),$$

where $E_\rho = \frac{1}{j\omega\varepsilon} \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right)$.

The only dependence on the parameter z is in the exponential term $e^{jk_0 z \cos\theta_0}$, thus the differentiation with respect to z can be expressed in the form of $\frac{\partial}{\partial z} = jk_0 \cos\theta_0$, and therefore H_ϕ can be written as

$$H_\phi(\rho, \phi, z) \left[1 - \frac{k_0^2 \cos^2\theta_0}{\omega^2 \mu\varepsilon} \right] = -\frac{k_0 \cos\theta_0}{j\omega^2 \mu\varepsilon\rho} \frac{\partial H_z}{\partial \phi} + \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial \rho}. \qquad (6)$$

According to equation (6) and based on the fact that the z component of the incident magnetic field is equal to zero for the TM_z case, then the ϕ component of the incident magnetic field on cylinder "i" can be written as

$$\begin{aligned} H_{\phi_i}^{inc}(\rho_i, \phi_i, z) &= \frac{1}{j\omega\mu_0 \sin^2\theta_0} \frac{\partial E_z^{inc}}{\partial \rho_i} \\ &= \frac{E_0' k_0}{j\omega\mu_0 \sin\theta_0} e^{jk_0 z \cos\theta_0} e^{jk_0 \rho_i' \sin\theta_0 \cos\left(\phi_i' - \phi_0\right)} \\ &\quad \sum_{-\infty}^{\infty} j^n J_n' (k_0 \rho_i \sin\theta_0) \, e^{jn(\phi_i - \phi_0)}, \end{aligned} \qquad (7)$$

where the prime represents the derivative of the Bessel function with respect to its full argument.

Based on equation (6), the ϕ components of the scattered magnetic field and the magnetic field transmitted in the dielectric material of cylinder "i" can be expressed, respectively, as

$$\begin{aligned} H^s_{\phi_i}(\rho_i,\phi_i,z) &= E_0' G \frac{k_0}{j\eta_0\lambda_0}\sum_{-\infty}^{\infty} A_{in} H_n^{(2)'}(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)} \\ &\quad - E_0' G \frac{h}{\lambda_0^2\rho_i}\sum_{-\infty}^{\infty} nC_{in} H_n^{(2)}(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)}, \quad (8) \\ H^d_{\phi_i}(\rho_i,\phi_i,z) &= E_0' G \frac{k_d}{j\eta_d\lambda_d}\sum_{-\infty}^{\infty} B_{in} J_n'(\lambda_d\rho_i)\, e^{jn(\phi_i-\phi_0)} \\ &\quad - E_0' G \frac{h}{\lambda_d^2\rho_i}\sum_{-\infty}^{\infty} nD_{in} J_n(\lambda_d\rho_i)\, e^{jn(\phi_i-\phi_0)}, \quad (9) \end{aligned}$$

where $h = k_0\cos\theta_0$, $\lambda_0 = k_0\sin\theta_0$, $G = e^{jk_0 z\cos\theta_0}$, $k_d = k_0\sqrt{\mu_{d_r}\varepsilon_{d_r}}$, $\lambda_d = \sqrt{k_d^2 - k_0^2\cos^2\theta_0}$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, and $\eta_d = \eta_0\sqrt{\mu_{d_r}/\varepsilon_{d_r}}$.

In the same manner, the ϕ components of the electric field can be derived using Maxwell's equations as follow

$$E_\phi = \frac{1}{j\omega\varepsilon}\left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right),$$

where $H_\rho = \frac{-1}{j\omega\mu}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right)$.

Thus the ϕ component of the incident electric field on cylinder "i" takes the form

$$\begin{aligned} E^{inc}_{\phi_i}(\rho_i,\phi_i,z) &= \frac{-hE_0'}{\lambda_0^2\rho_i} e^{jk_0 z\cos\theta_0} e^{jk_0\rho_i'\sin\theta_0\cos\left(\phi_i'-\phi_0\right)} \quad (10) \\ &\quad \sum_{-\infty}^{\infty} nj^n J_n(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)}, \end{aligned}$$

while the ϕ components of the scattered electric field and the electric field transmitted inside cylinder "i" can be expressed, respectively, as

$$\begin{aligned} E^s_{\phi_i}(\rho_i,\phi_i,z) &= -E_0' G \frac{h}{\lambda_0^2\rho_i}\sum_{-\infty}^{\infty} nA_{in} H_n^{(2)}(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)} \\ &\quad - E_0' G \frac{\eta_0 k_0}{j\lambda_0}\sum_{-\infty}^{\infty} C_{in} H_n^{(2)'}(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)}, \quad (11) \end{aligned}$$

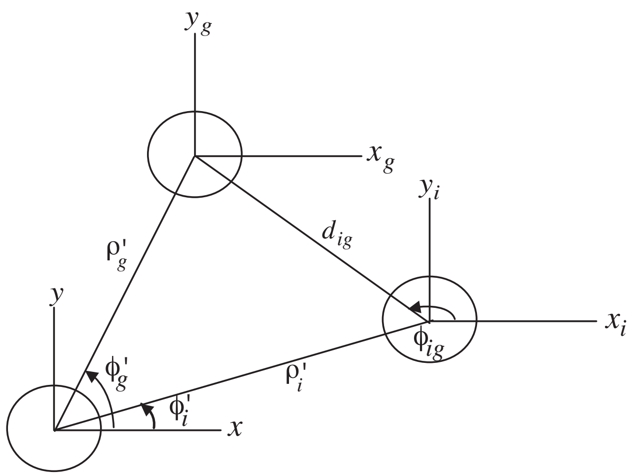


Figure 2. The cross-sectional geometry of the cylinders in the x-y plane.

$$\begin{aligned} E^d_{\phi_i}(\rho_i,\phi_i,z) &= -E'_0 G\frac{h}{\rho_i\lambda_d^2}\sum_{-\infty}^{\infty} nB_{in}J_n(\lambda_d\rho_i)\,e^{jn(\phi_i-\phi_0)} \\ &\quad -E'_0 G\frac{k_d\eta_d}{j\lambda_d}\sum_{-\infty}^{\infty} D_{in}J_n(\lambda_d\rho_i)\,e^{jn(\phi_i-\phi_0)}, \qquad (12) \end{aligned}$$

The expressions in equations (2)–(5), (8)–(9), and (11)–(12) indicate that both the electric and magnetic field components are based on the local coordinates (ρ_i,ϕ_i,z) of cylinder "i". However, the interaction between the M cylinders in terms of multiple scattered fields will require a representation of the scattered field from one cylinder in terms of the local coordinates of another as shown in Fig. 2. Therefore, the addition theorem of Bessel and Hankel functions are used to transfer the scattered field components from one set of coordinates to another. As an example the scattered fields from the gth cylinder in terms of the ith cylinder are presented by [11, 12]

$$\begin{aligned} H^{(2)}(\lambda_0\rho_g)\,e^{jm\phi_g} &= \sum_m J_m(\lambda_0\rho_i)\,H^{(2)}_{m-n}(\lambda_0 d_{ig})\,e^{jm\phi_i}e^{-j(m-n)\phi_{ig}} \qquad (13) \\ d_{ig} &= \rho'^2_i+\rho'^2_g-2\rho'_i\rho'_g\cos\left(\phi'_i-\phi'_g\right), \end{aligned}$$

$$\phi_{ig}=\begin{cases}\cos^{-1}\left(\dfrac{\rho_i'\cos\left(\phi_i'\right)-\rho_g'\cos\left(\phi_g'\right)}{d_{ig}}\right) & \rho_i'\sin(\phi_i')\geq\rho_g'\sin\left(\phi_g'\right)\\ -\cos^{-1}\left(\dfrac{\rho_i'\cos\left(\phi_i'\right)-\rho_g'\cos\left(\phi_g'\right)}{d_{ig}}\right) & \rho_i'\sin(\phi_i')<\rho_g'\sin\left(\phi_g'\right),\end{cases} \tag{14}$$

where (ρ_i', ϕ_i', z) and (ρ_g', ϕ_g', z) are the coordinates of the origins of the ith and gth coordinate system of cylinders i and g respectively in terms of the global coordinate system (x, y), d_{ig} is the distance between the centers of the cylinders, and ϕ_{ig} is the angle between the line joining the centers of the cylinders and the positive x-axis.

The solution of the unknown coefficients A_{in}, B_{in}, C_{in} and D_{in} can be obtained by applying the appropriate boundary conditions on the surface of all cylinders. The boundary conditions on the surface of the ith cylinder are given by

$$E_{z_i}^{inc}+\sum_{g=1}^{M}E_{z_g}^{s}=E_{z_i}^{d}, \tag{15}$$

$$H_{z_i}^{inc}+\sum_{g=1}^{M}H_{z_g}^{s}=H_{z_i}^{d}, \tag{16}$$

$$E_{\phi_i}^{inc}+\sum_{g=1}^{M}E_{\phi_g}^{s}=E_{\phi_i}^{d}, \tag{17}$$

$$H_{\phi_i}^{inc}+\sum_{g=1}^{M}H_{\phi_g}^{s}=H_{\phi_i}^{d}. \tag{18}$$

The solution for the unknown coefficients A_{in}, B_{in}, C_{in} and D_{in}, related to the ith cylinder, is assumed to include the effect of all interactions between the cylinders. After some mathematical manipulations and the application of the boundary conditions on the surface of all M cylinders one obtains

$$V_1=\sum_{g=1}^{M}\sum_{n=-\infty}^{\infty}A_{gn}S_1+C_{gn}R_1, \tag{19}$$

$$V_2=\sum_{g=1}^{M}\sum_{n=-\infty}^{\infty}A_{gn}S_2+C_{gn}R_2, \tag{20}$$

where

$$V_1 = \frac{hP_i}{a_i} l j^l J_l \left(\frac{1}{\lambda_d^2} - \frac{1}{\lambda_0^2} \right), \tag{21}$$

$$V_2 = P_i j^{l-1} \left(\frac{k_d}{\eta_d \lambda_d} J_{ld} J_l - \frac{k_0}{\eta_0 \lambda_0} J_l' \right), \tag{22}$$

$$\begin{aligned} S_1 &= 0 && i = g,\, l \neq n \\ &= \left[\frac{hlH_l}{\lambda_0^2 a_i} - \frac{hlH_l}{\lambda_d^2 a_i} \right] && i = g,\, l = n \\ &= \left[\frac{hlJ_l}{\lambda_0^2 a_g} - \frac{hlJ_l}{\lambda_d^2 a_i} \right] H_{ln} && i \neq g, \end{aligned} \tag{23}$$

$$\begin{aligned} S_2 &= 0 && i = g,\, l \neq n \\ &= \left[\frac{k_0}{j\eta_0 \lambda_0} H_l' - \frac{k_d}{j\eta_d \lambda_d} H_l J_{ld} \right] && i = g,\, l = n \\ &= \left[\frac{k_0}{j\lambda_0} J_l' - \frac{k_d}{j\eta_d \lambda_d} J_l J_{ld} \right] H_{ln} && i \neq g, \end{aligned} \tag{24}$$

and

$$\begin{aligned} R_1 &= 0 && i = g,\, l \neq n \\ &= \left[\frac{k_0 \eta_0}{j\lambda_0} H_l' - \frac{k_d \eta_d}{j\lambda_d} H_l J_{ld} \right] && i = g,\, l = n \\ &= \left[\frac{k_0 \eta_0}{j\lambda_0} J_l' - \frac{k_d \eta_d}{j\lambda_d} J_l J_{ld} \right] H_{ln} && i \neq g, \end{aligned} \tag{25}$$

$$\begin{aligned} R_2 &= 0 && i = g,\, l \neq n \\ &= -\left[\frac{hlH_l}{\lambda_0^2 a_i} - \frac{hlH_l}{\lambda_d^2 a_i} \right] && i = g,\, l = n \\ &= -\left[\frac{hlJ_l}{\lambda_0^2 a_g} - \frac{hlJ_l}{\lambda_d^2 a_i} \right] H_{ln} && i \neq g, \end{aligned} \tag{26}$$

while

$$\begin{aligned} J_l &= \frac{J_l(\lambda_0 a_i)}{J_l(\lambda_d a_i)},\ J_l' = \frac{J_l'(\lambda_0 a_i)}{J_l(\lambda_d a_i)},\ J_{ld} = \frac{J_l'(\lambda_d a_i)}{J_l(\lambda_d a_i)},\ H_l = \frac{H_l^{(2)}(\lambda_0 a_i)}{J_l(\lambda_d a_i)}, \\ H_l' &= \frac{H_l^{(2)'}(\lambda_0 a_i)}{J_l(\lambda_d a_i)},\ H_{ln} = H_{l-n}^{(2)}(\lambda_0 d_{ig})\, e^{-j(l-n)(\phi_{ig}-\phi_0)}, \\ P_i &= e^{jk_0 \rho_i' \cos\left(\phi_i' - \phi_0\right) \sin(\theta_0)} \end{aligned}$$

where the integers n, $l = 0, \pm 1, \pm 2, \dots, \pm N_i$ and i, $g = 0, 1, 2, \dots, M$. Theoretically, N_i is an integer which is equal to infinity; however, it is related to the radius "a_i" of cylinder "i", and type of the ith cylinder by the relation $N_i \approx (1 + 2k_i a_i)$. Equations (19) and (20) are then cast into a matrix form such as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} S_1 & R_1 \\ S_2 & R_2 \end{bmatrix} \begin{bmatrix} A \\ C \end{bmatrix}. \tag{27}$$

The solution of the above truncated matrix equation yields the unknown scattering coefficients A_{in} and C_{in}.

For the case of incident H-polarized TE_z plane wave, the incident magnetic field is expressed in the (ρ_i, ϕ_i, z), cylindrical coordinate system as,

$$\begin{aligned} H_{z_i}^{inc}(\rho_i, \phi_i, z) \;=\; & H_0^* e^{jk_0 z \cos\theta_0} e^{jk_0 \rho_i' \sin\theta_0 \cos\left(\phi_i' - \phi_0\right)} \\ & \sum_{-\infty}^{\infty} j^n J_n\left(k_0 \rho_i \sin\theta_0\right) e^{jn(\phi_i - \phi_0)}, \end{aligned} \tag{28}$$

where $H_0^* = H_0 \sin\theta_0$. One can easily express the field components for a TE_z case, be applying the same procedure used to derive the fields from a TM_z illumination, where the z component of the scattered field and the transmitted z component of the field inside the cylinder material have the same form as in equation (2) to equation (5). The only difference in the final expressions is in equations (21) and (22), where these two equations are to be replaced by,

$$V_1 = P_i j^{l-1} \left(\frac{k_d \eta_d}{\lambda_d} J_{ld} J_l - \frac{k_0 \eta_0}{\lambda_0} J_l' \right), \tag{29}$$

$$V_2 = \frac{h P_i}{a_i} l j^l J_l \left(\frac{1}{\lambda_0^2} - \frac{1}{\lambda_d^2} \right). \tag{30}$$

Therefore the numerical simulation of TE_z polarization can be easily obtained from the TM_z simulation code.

3. NUMERICAL RESULTS

In this section, sample numerical results are presented to proof the validity of the developed formulation for computing the radar cross-section (RCS) of an array of cylinders excited by an obliquely incident TM_z or TE_z plane wave. For all configurations presented in this paper, the incident wave frequency was set to 300 MHz, and the echo width is defined as

$$\sigma_{2D} = 10 \log \left(\lim_{\rho \to \infty} \left[2\pi\rho \frac{|E_z^s|^2}{|E_z^i|^2} \right] \right). \tag{31}$$

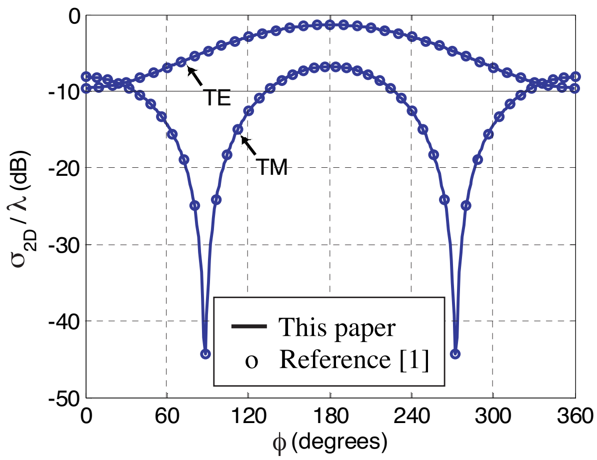


Figure 3. The echo width of a single cylinder of radius $a = 0.1\lambda$, $\varepsilon_r = 4$, $\theta_i = 45$, $\phi_i = 0$.

Figure 3 represents the RCS calculated from a dielectric cylinder of radius $a = 0.1\lambda$, having relative permittivity $\varepsilon_r = 4$, with incident angel $\theta_i = 45$ and $\phi_i = 0$. Both TM_z and TE_z plane wave excitations are considered. The results generated using the presented boundary value solution (BVS) technique are compared to the RCS data calculated from [1]. It is clear from the figure that the two solutions are in complete agreement for both TM_z and TE_z plane wave excitation.

In order to prove the validity of the above calculations for multiple cylinders, the principle of equal volume model can be used to represent one single cylinder by an array of cylinders. According to this principle, the scattered field from one cylinder is expected to be the same as that of an array of cylinders having the total cross section area equal to the cross section area of the cylinder, and having the same outer shape. In Fig. 4, 91 non-intersected cylinders of radius 0.04λ are used to simulate a cylinder of radius 0.4λ. The dielectric constant of the cylinders is chosen to be greater than that of the single cylinder to achieve equal effective area, assuming that the effective area is the product of the cross section area and the relative permittivity of the cylinder.

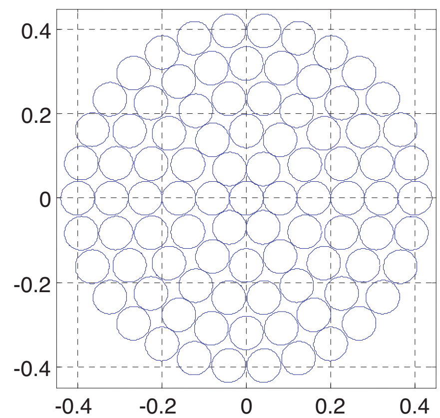


Figure 4. The relative position of an array 91 cylinders.

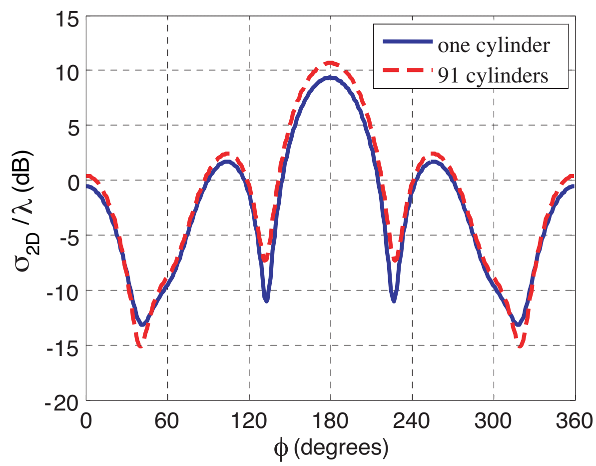


Figure 5. The echo width results of a TM_z plane wave incident on one cylinder of radius 0.4λ, $\varepsilon_r = 4$, $\theta_i = 45$, $\phi_i = 0$, represented by 91 non-intersected cylinders.

Figure 5 shows the echo width calculated from one cylinder of radius 0.4λ, having relative permittivity $\varepsilon_r = 4$, and excited by a TM_z plane wave with incident angel $\theta_i = 45$ and $\phi_i = 0$. The computed echo width is obtained using the expressions provided in [1], and compared with the echo width results generated using an array of cylinders. In Fig. 6, another comparison is presented for the same cylinder definition but at different incident angel $\theta_0 = 75$, $\phi_0 = 0$.

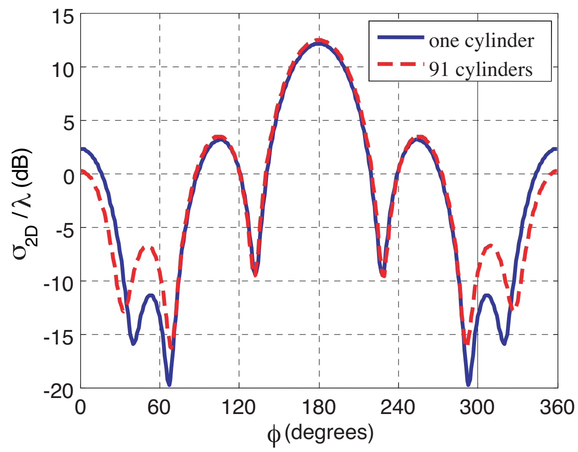


Figure 6. The echo width results of a TM_z plane wave incident on one cylinder of radius 0.4λ, $\varepsilon_r = 4$, $\theta_i = 75$, $\phi_i = 0$, represented by 91 cylinders.

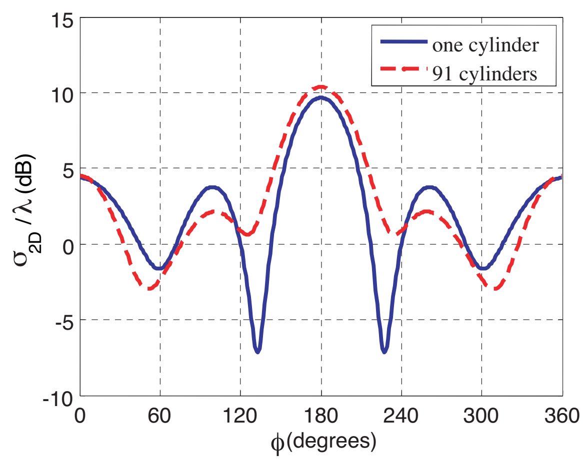


Figure 7. The echo width results of a TE_z plane wave incident on one cylinder of radius 0.4λ, $\varepsilon_r = 4$, $\theta_i = 45$, $\phi_i = 0$, represented by 91 cylinders.

Figure 7 shows the echo width calculated from one cylinder of radius 0.4λ, having relative permittivity $\varepsilon_r = 4$, and excited by a TE_z plane wave with incident angel $\theta_i = 45$ and $\phi_i = 0$. In this case, the results are not completely equivalent due to the presence of the ϕ component of the electric field. The field is affected by the shape of

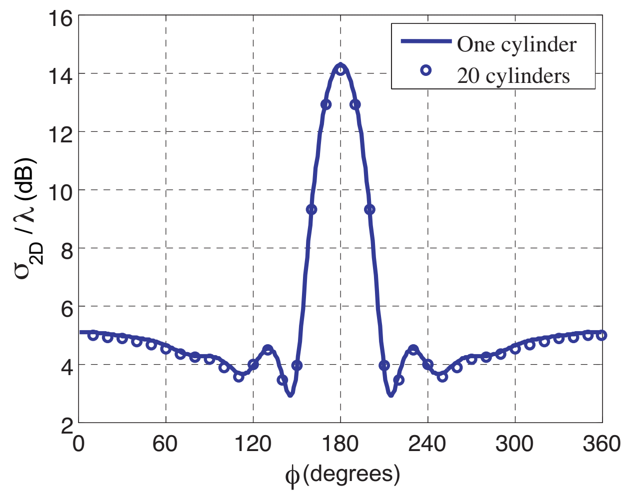


Figure 8. The echo width results of a TM_z plane wave incident on a perfectly conducting cylinder of radius $a = 1\lambda$ for obliquely incident angle $\theta_i = 45$, $\phi_i = 0$.

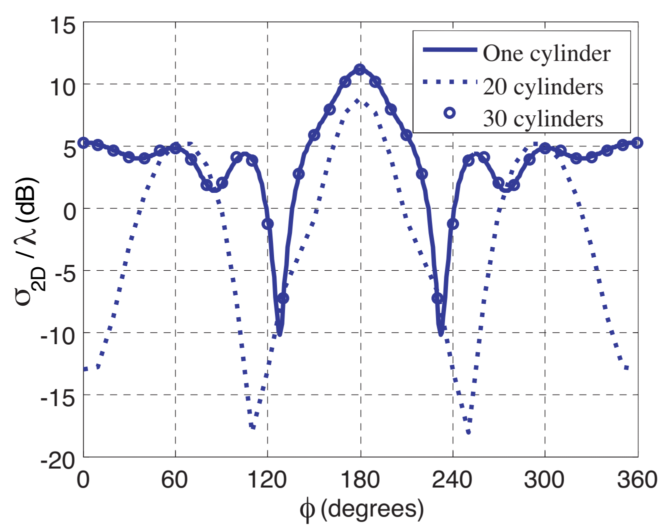


Figure 9. The echo width results of a TE_z plane wave incident on a perfectly conducting cylinder of radius $a = 1\lambda$ for obliquely incident angle $\theta_i = 45$, $\phi_i = 0$.

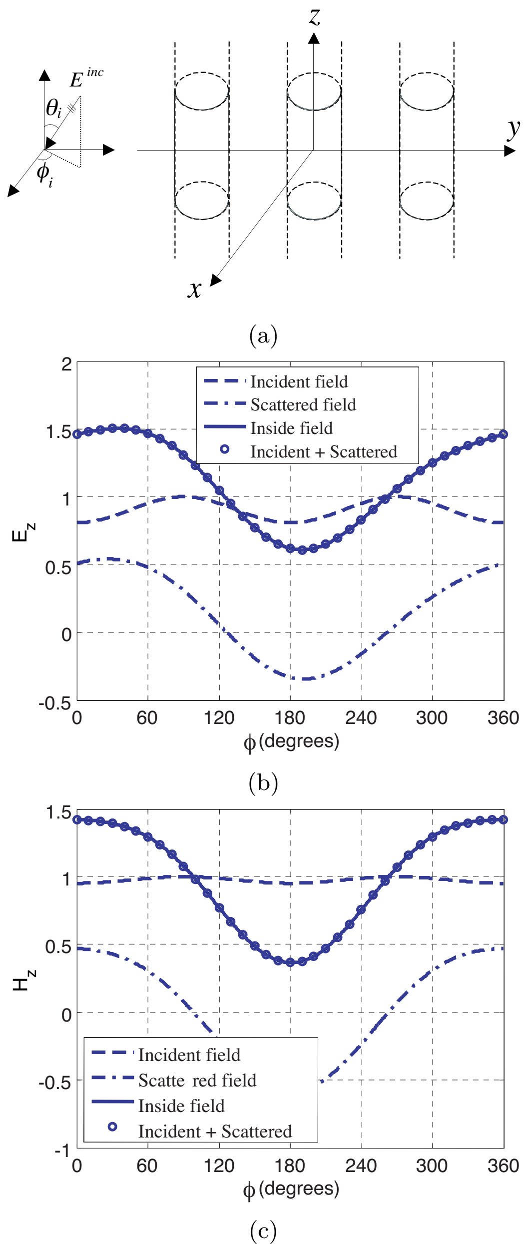
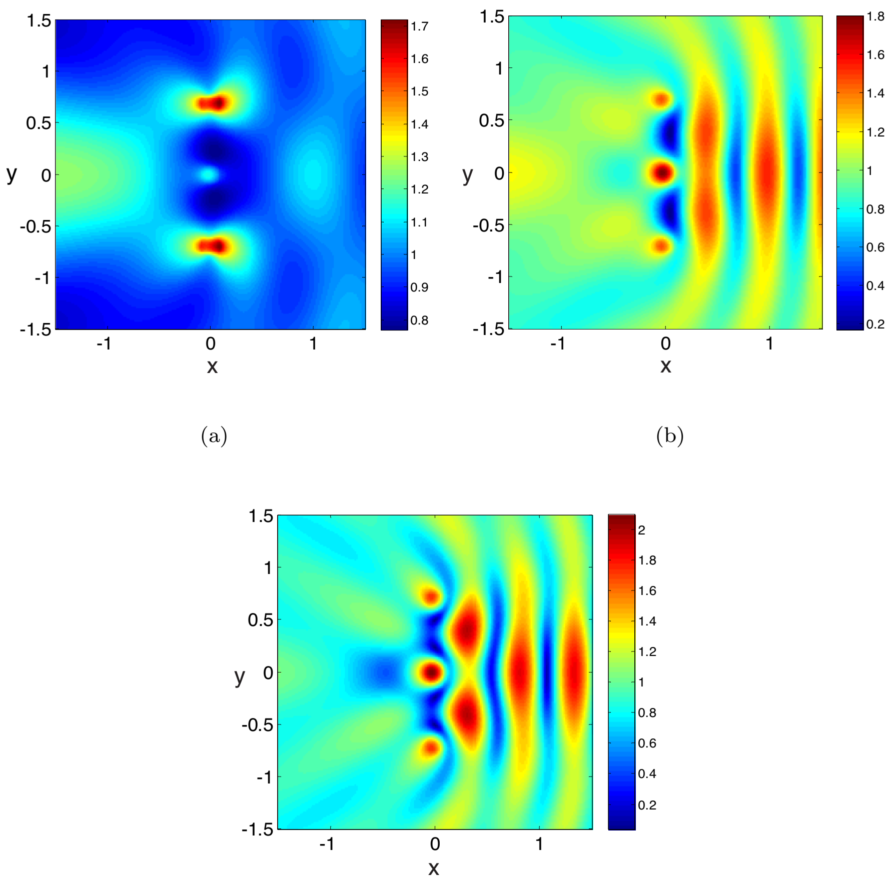


Figure 10. Near field components of three parallel cylinders excited by an incident plane wave at $\theta_i = 30$, $\phi_i = 0$. (a) The arrangement of the cylinders, (b) Continuity of the E_z for TM$_z$ excitation, (c) Continuity of the H_z for TE$_z$ excitation.

the outer surface of the array of cylinders which is not as smooth as it is in the one cylinder case. However, the agreement is acceptable to validate the TE_z developed formulation.

Figure 8 shows the echo width calculated from one conducting cylinder of radius 1λ, where the cylinder is excited by a TM_z plane wave with incident angel $\theta_i = 45$ and $\phi_i = 0$. The computed echo width is compared with that of an array of 20 perfectly conducting cylinders of radius $a = 0.1\lambda$, and all the cylinders are uniformly located on the circumference of a circle of radius $a = 0.9\lambda$. The results show excellent



(a) (b)

(c)

Figure 11. The near field distribution of a TM_z plane wave incident on array of three cylinders at incident angle. (a) $\theta_i = 30$, $\phi_i = 0$, (b) $\theta_i = 60$, $\phi_i = 0$, (c) $\theta_i = 90$, $\phi_i = 0$.

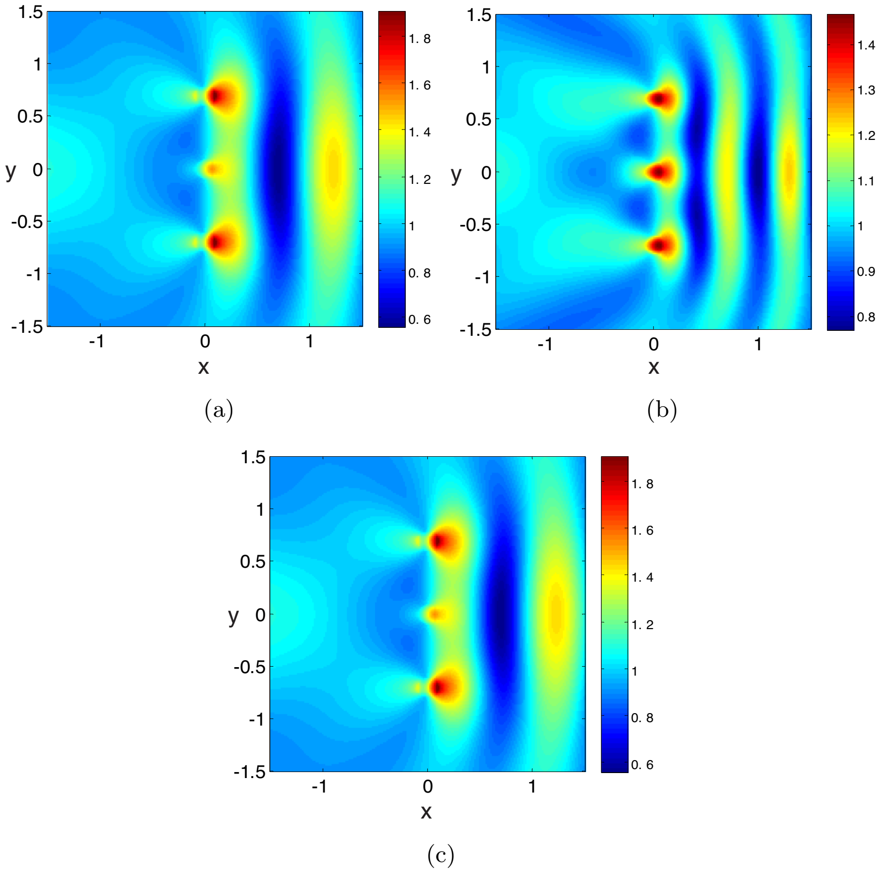


Figure 12. The near field distribution of a TE_z plane wave incident on array of three cylinders at incident angle. (a) $\theta_i = 30$, $\phi_i = 0$, (b) $\theta_i = 60$, $\phi_i = 0$, (c) $\theta_i = 90$, $\phi_i = 0$.

agreement between both curves, which indicates the validity of using this method to model any perfectly conducting object by an array of circular cylinders. For the case of TE_z excitation, Fig. 9 shows that the result is not in complete agreement for an array of 20 cylinders due to the effect of the ϕ component of the electric field as explained in the previous paragraph. The results can be improved by using an array of 30 cylinders as shown in the same figure.

As another method to check the validity of this technique and its accuracy, specially in near field region, the scattering coefficients A_{in} and C_{in} can be used to deduce the coefficients values of the transmitted field inside the cylinders material B_{in} and D_{in}. Then, the numerical values of these four coefficients are used to check the validity of boundary equations from equation (15) to equation (18). For simplicity, three cylinders of radius 0.1λ are used. The cylinders are placed symmetry around the x axis with the center of all cylinders located on the y axis, and the distance between the centers is 0.7λ. Fig. 10 shows the numerical value of the transmitted field inside the first cylinder compared with the summation of the incident field and the scattered fields form all cylinders on the surface of this cylinder. In Fig. 10(b), the continuity of the H_z components in the case of TE_z excitation are considered, and in Fig. 10(c) the continuity of the E_z component in the case of TM_z excitation are considered. Both figures are plotted for incident angle $\theta_i = 30$, $\phi_i = 0$.

Figure 11 shows the near field distribution resulting from the incidence of a TM_z polarized plane wave on an array of three cylinders. The cylinders are placed symmetry around the x axis with the center of the three cylinders located on the y axis, the radius of each cylinder is 0.1λ and the distance between the centers is 0.7λ. The graph shows the numerical value of the transmitted field inside the cylinders and the total field outside the cylinders for three different incident angles $\theta_i = 30$, 60, and 90. In Fig. 12, the near field distribution is plotted for the case of TE_z excitation for the same configuration.

4. CONCLUSIONS

The analyses of an obliquely incident plane wave scattering from an array of parallel circular cylinders is derived for both TM_z, and TE_z polarizations. The derivation is based on the application of the boundary conditions on the surface of each cylinder. This solution is valid for both dielectric and conductor cylinders, and can be used to study of electromagnetic interaction with any two-dimensional scattering object that can be constructed from an array of parallel circular cylinders. Two different methods are presented to check the validity of this technique. First, the principle of equal volume model is used to represent one cylinder by an array of circular cylinders and sample numerical results are given to compare the known echo width of the scattered field from one cylinder to that of an array of circular cylinders. Second, the near field is calculated to prove the validity of the boundary conditions on the surface of each cylinder. The results in both cases show excellent agreement between the calculated field and the known results from pervious work for special cases. The presented scattering technique can be extended for other types of cylinders.

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