

## GEOMETRICAL ANALYSIS OF HIGH ALTITUDE PLATFORMS CELLULAR FOOTPRINT

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**Abstract**—Cellular communications using high altitude platforms will predominate the existing conventional terrestrial or satellite systems. Radio coverage of HAPs is an important issue that affect the cellular system design, therefore a deep analysis of the HAP cell footprint will be presented in two coverage models and the cell parameters are also deduced for subsequent cellular design. In the design stage, a cell generating algorithm is introduced to define their locations more accurate. When deploying the cellular system with such algorithm, the cells have proper overlap and interspacing.

### 1. INTRODUCTION

High altitude platforms (HAPs) has been developed wherein a large scale airship is positioned at a predetermined altitude approximately 20 km high in the stratosphere where it can be used for telecommunications, broadcasting and environmental measurements. Using HAPs in mobile communications is a promising scheme because it takes the advantages of using satellites but at lower altitudes [1–6]. On the other hand the conventional terrestrial system suffers from coverage problems which are mostly eliminated using HAPs. In the terrestrial cellular structure, the geometry of the formed cells can be determined by considering its hexagonal shape and it is easy to find the other cells locations, but in the case of using HAPs, the cell will be defined by the half power contour on the ground which can be considered as an ellipse. In dealing with the footprint analysis, the cell parameters must be determined such as the major and minor axes and there variations with the utilized antenna beamwidths and beam direction [3–6]. In [3], the coverage analysis is well defined based on an

assumption that the earth surface is flat as the platform height is much smaller than the earth radius, but on the other hand, this assumption will not expect the proper cell parameters for those cells at the coverage edge (i.e., cells of lower elevation angles). Therefore in this paper, two coverage analysis models are presented and compared. The paper is arranged as follows, in Section 2, the flat ground approximation model is presented, while in Section 3 the curved ground model is introduced. Section 4 compares between the two coverage models and finally Section 5 concludes

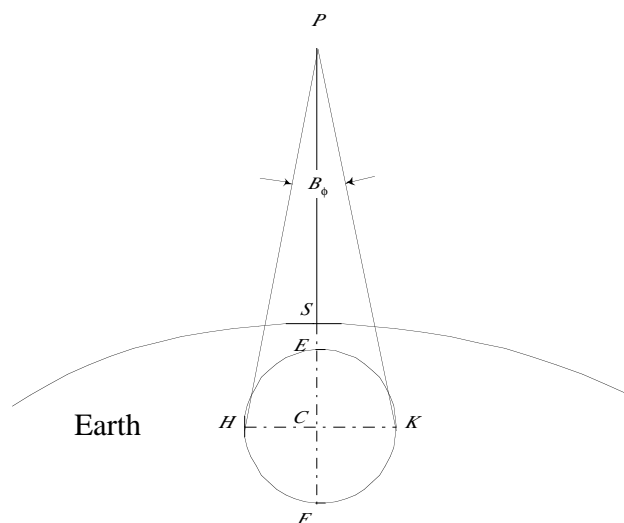
## 2. FLAT EARTH APPROXIMATION FOR HAP RADIO COVERAGE

The HAPs wireless communication system utilizes the directional as well as phased antenna arrays to construct its ground cells. Directional antennas may be in the form of parabolic reflectors, horn antennas, or any other suitable antenna that gives the desired directional pattern. The use of directional antennas has some advantages such as its practical availability and simplicity but on the other hand a failure in one of them results in a coverage hole due to the absence of the beam used in forming its cell. Ground cells also can be formed by directing a beam using phased arrays [7, 8] which has a widespread use. Any of the formed beams is constructed by a number of antenna elements therefore any element failure in the array will slightly distort the beam pattern (the beam will have slightly larger beamwidths) and this can be an advantage compared with the use of directional antennas. As the HAP station is located at an altitude about 20 km high, which is very small compared with the earth's radius, we can approximate the earth as a flat surface as shown in Fig. 1. In this figure, the footprint of a beam formed by any of the mentioned antennas onboard the HAP is shown. The cell as depicted in Fig. 2 is defined by the coverage beam that has a direction of  $\theta_o$  and cross section beamwidth of  $B_\theta$  and  $B_\phi$ , and the projection of the beam on the ground will be an ellipse that has a major axis  $EF$  and minor axis  $HK$ . Denoting the distance  $EF$  as  $b_F$  which is given by

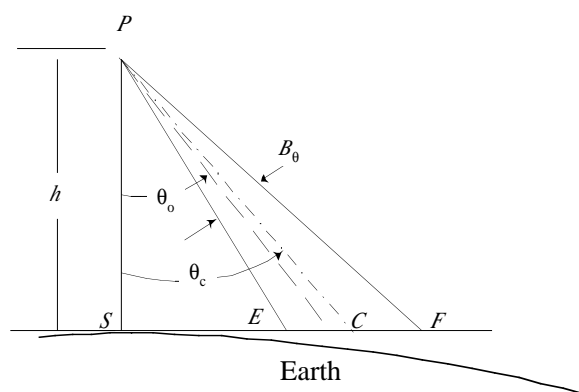
$$b_F = h \left( \tan \left( \theta_o + \frac{B_\theta}{2} \right) - \tan \left( \theta_o - \frac{B_\theta}{2} \right) \right) \quad (1)$$

where the subscript  $F$  stands for flat ground approximation and  $h$  is the platform altitude in km. The cell center point,  $C$ , is located by an angle from the platform given by

$$\theta_c = \tan^{-1} \left( \tan \left( \theta_o - \frac{B_\theta}{2} \right) + \frac{b_F}{2h} \right) \quad (2)$$



**Figure 1.** HAP cell footprint.



**Figure 2.** Flat ground approximation geometry.

and the cell minor axis distance  $HK$  can be denoted as  $a_F$  and can be given by

$$a_F = 2h \sec(\theta_c) \tan\left(\frac{B_\phi}{2}\right) \quad (3)$$

These two quantities (i.e., the minor and major axes) will define the cell shape and this assumption can be used for smaller and moderate coverage areas but when the coverage area increases the approximation error will increase and can't no longer be used.



where the cell center has a ground center angle given by

$$\gamma_o = \frac{1}{2}(\gamma_1 + \gamma_2) \quad (6)$$

and we can get the distance  $PB$  (see Appendix A) as follows:

$$PB = h + R \left( 1 - \frac{1}{2}(\cos(\gamma_1) + \cos(\gamma_2)) \right) \quad (7)$$

and the distance  $BC$  (see Appendix A) will be

$$BC = \frac{1}{2}R(\cos(\gamma_1) + \cos(\gamma_2)) \tan(\gamma_o) \quad (8)$$

therefore the platform-to-cell center slant distance will be

$$PC = \sqrt{PB^2 + BC^2} \quad (9)$$

from the above equations, the cell major axis,  $b_C$ , can be defined as

$$b_C = EF = R(\gamma_2 - \gamma_1) \quad (10)$$

or

$$\begin{aligned} b_C = R & \left( \sin^{-1} \left( \left( 1 + \frac{h}{R} \right) \sin \left( \theta_o + \frac{B_\theta}{2} \right) \right) \right. \\ & \left. - \sin^{-1} \left( \left( 1 + \frac{h}{R} \right) \sin \left( \theta_o - \frac{B_\theta}{2} \right) \right) - B_\theta \right) \end{aligned} \quad (11)$$

and in this case the value of  $\theta_c$  will be

$$\theta_c = \tan^{-1} \left( \frac{BC}{PB} \right) \quad (12)$$

or

$$\theta_c = \tan^{-1} \left\{ \frac{\tan(\gamma_o)}{2 \left( 1 + \frac{h}{R} \right) / (\cos(\gamma_1) + \cos(\gamma_2)) - 1} \right\} \quad (13)$$

therefore the cell minor axis,  $a_c$ , will be

$$a_C = HK = 2PC \tan \left( \frac{B_\phi}{2} \right) \quad (14)$$

or

$$a_C = 2h \sec(\theta_c) \tan \left( \frac{B_\phi}{2} \right) \quad (15)$$

which can be also given by

$$a_C = 2R \tan\left(\frac{B_\phi}{2}\right) \left( \left(1 + \frac{h}{R} - \frac{1}{2}(\cos(\gamma_1) + \cos(\gamma_2))\right)^2 + \frac{1}{4}(\cos(\gamma_1) + \cos(\gamma_2))^2 \tan^2(\gamma_o) \right)^{1/2} \quad (16)$$

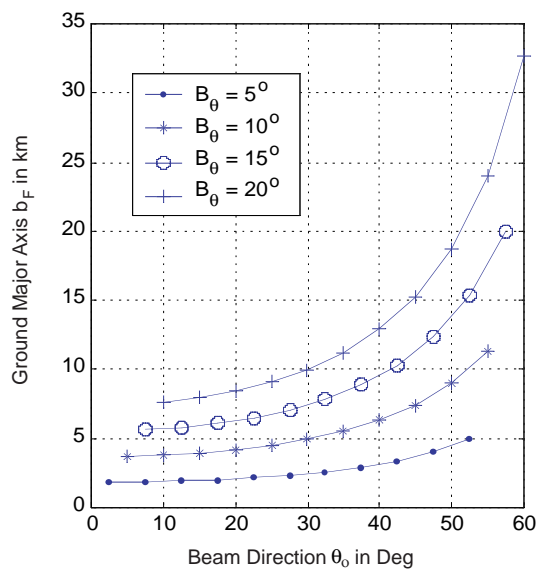
#### 4. ANALYSIS AND COMPARISON FOR THE TWO COVERAGE MODELS

Figs. 4a and b depicts the variations of the cell major axes with the beam direction  $\theta_o$  at different beamwidths  $B_\theta$  for both coverage models using directional antennas. The variations in this figure indicates the increase in the footprint with increasing both the beam direction and the beamwidth. The difference (or the absolute distance error) between the two quantities in km is shown in Fig. 5a while the relative error in the cell major axis between the two models may be defined as:

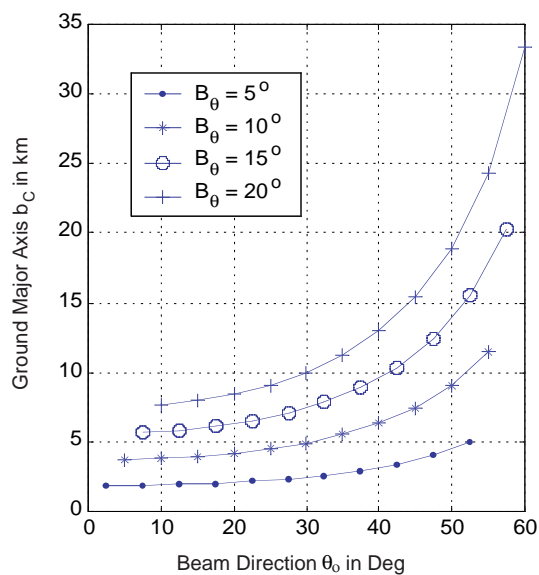
$$\varepsilon_b = \frac{b_C - b_F}{b_C} \times 100\% \quad (17)$$

where its variation with both the beam direction and beamwidth is shown In Fig. 5b. In this figure, the error may approach about 2% of the major axis for beamwidth of  $20^\circ$  at a beam direction of about  $60^\circ$  which corresponds to 700 meter difference. This large difference between the two expected major axis values for the two models will affect the system design especially for the cells at the coverage border or edges. On the other hand, the error is much smaller for the inner coverage cells and for cells of narrower beamwidth. For example a beamwidth of  $5^\circ$  generates cells that have an error not exceeding 12 meters for direction no more than  $40^\circ$  as depicted in Fig. 5(a). The same analysis is done for the cell minor axis as depicted in Figs. 6a and b and the error (absolute and relative) for the two models is shown respectively in Figs. 7a and b.

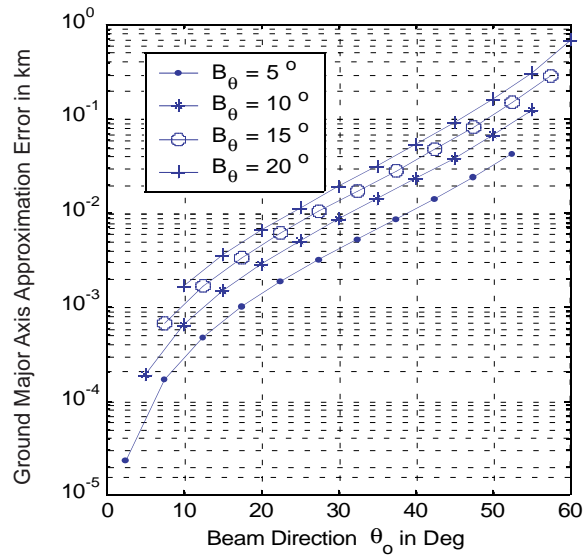
One can therefore utilize the simple equations used in the flat ground model for the range of moderate and acceptable error (such as for cells near the coverage center) while for larger error we can utilize the better curved earth model thus optimizing the use of both models.



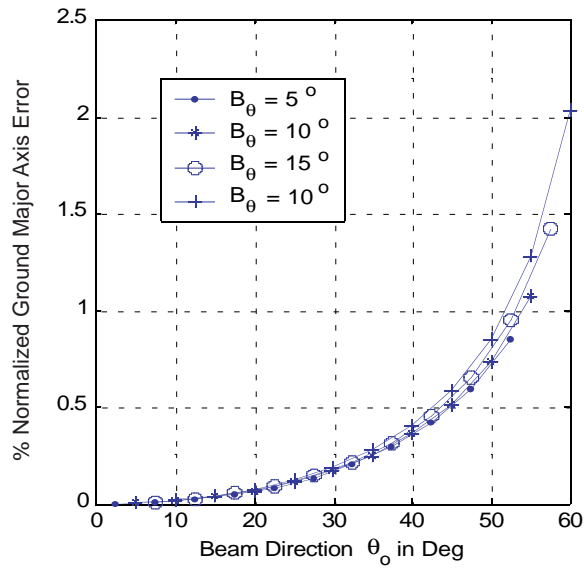
**Figure 4a.**  $b_F$  variation with beam direction at different beamwidths.



**Figure 4b.**  $b_C$  variation with beam direction at different beamwidths.

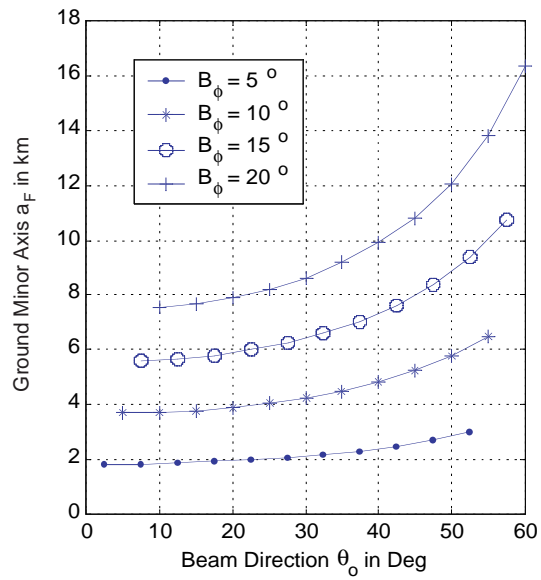


**Figure 5a.** Absolute error variation with beam direction at different beamwidths.

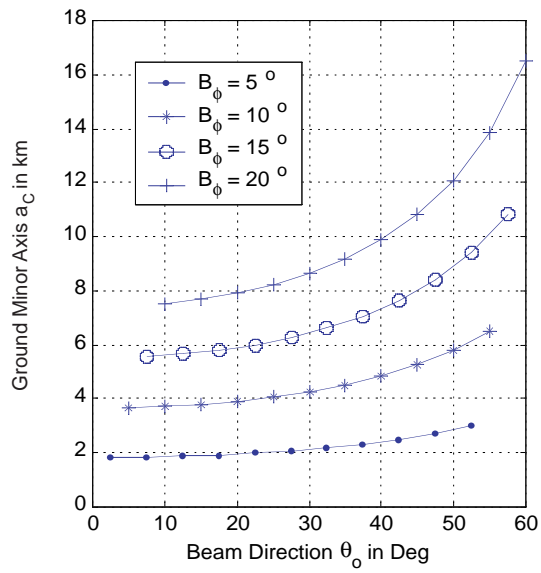


**Figure 5b.** Relative error variation with beam direction at different beamwidths.

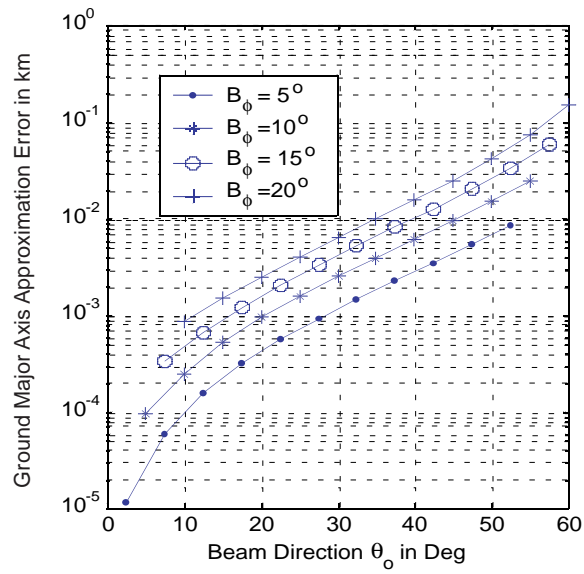




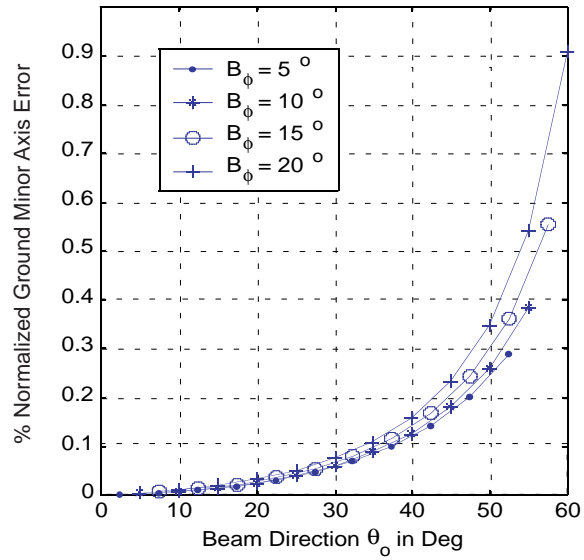
**Figure 6a.**  $a_F$  variation with beam direction at different beamwidth.



**Figure 6b.**  $a_C$  variation with beam direction at different beamwidths.



**Figure 7a.** Absolute error variation with beam direction at different beamwidths.



**Figure 7b.** Relative error variation with beam direction at different beamwidths.

## 5. CONCLUSION

High altitude platforms is an emerging technology for different types of communications especially mobile cellular radio. Two coverage models are presented which explores the cell footprint. The first model approximates the earth as a flat surface which simplifies the equations used for coverage analysis but this approximation will be more erroneous especially for cells at the coverage edges of the HAP. Therefore a composite cell footprint equations may be used to both simplify the analysis for inner cells and for more accurate results for the outermost cells.

## APPENDIX A.

From Fig. 3, if the earth's radius is  $R$ , then the distance from the earth's center to point  $A$  will be

$$GA = R \cos(\gamma_1) \quad (\text{A1})$$

and the distance connecting the earth's center to point  $D$  will be

$$GD = R \cos(\gamma_2) \quad (\text{A2})$$

and from the same figure we have

$$GB = \frac{1}{2}(GA + GD) \quad (\text{A3})$$

therefore we can get the distance  $PB$  as

$$PB = h + R - GB \quad (\text{A4})$$

substituting  $GB$  with Eq. (A3), therefore Eq. (A4) can be rewritten as

$$PB = h + R - \frac{1}{2}(GA + GD) \quad (\text{A5})$$

or

$$PB = h + R - \frac{1}{2}(R \cos(\gamma_1) + R \cos(\gamma_2)) \quad (\text{A6})$$

or

$$PB = h + R \left( 1 - \frac{1}{2}(\cos(\gamma_1) + \cos(\gamma_2)) \right) \quad (\text{A7})$$

and  $BC$  will be

$$BC = GB \tan(\gamma_o) \quad (\text{A8})$$

substituting with Eq. (A3) in Eq. (A8) we have

$$BC = \frac{1}{2}(GA + GD) \tan(\gamma_o) \quad (\text{A9})$$

and finally substituting Eq. (A1) and Eq. (A2) in Eq. (A9) we have

$$BC = \frac{1}{2}R(\cos(\gamma_1) + \cos(\gamma_2)) \tan(\gamma_o) \quad (\text{A10})$$

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