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MEASUREMENT OF DIELECTRIC CONSTANT AND LOSS FACTOR OF THE DIELECTRIC MATERIAL AT MICROWAVE FREQUENCIES

A. Kumar and S. Sharma

Department of Electronics and Communication Engineering
S. D. D. Institute of Engineering and Technology
Barwala, India

G. Singh

Department of Electronics and Communication Engineering
Jaypee University of Information Technology
Solan, 173 215, India

Abstract—A new technique to evaluate the dielectric constant and loss factor of a homogeneous dielectric material using rectangular shaped perturb cavity has been developed. The values of S-parameters are measured experimentally by placing the sample in the center of the cavity resonator. Sample under test is fabricated in the form of a cylinder. The real and imaginary part of the permittivity can be then calculated from the shift in the resonance frequency and Q-factor. The results of a Teflon sample are also tabulated.

1. INTRODUCTION

The dielectric constant is an essential property of dielectric materials hence its determination is very important. There are many techniques have been developed to this end [1–14]. The most used technique depends on the measurement of either reflection coefficients or resonant frequencies. In the later case material is characterized to load a resonant cavity [2–11] and the sample permittivity is evaluated from the shift of the resonant frequency value, compared to that of the empty (unload) cavity. This approach is based on the perturbation theory, thus it requires the sample to be small enough so that field distribution inside the empty cavity changes slightly when the cavity is loaded. The cavity perturbation method for the measurement of dielectric constant is different from other methods and very sensitive. It involve approximations in their formulation which lead to acceptable results only under very restricted conditions: (i) The sample must be very small compared with the cavity itself so that a frequency shift which is small compared with the resonant frequency shift of the empty cavity is produced by the insertion of the sample. (ii) The cavity without and with sample must be very much alike. The cavity perturbation method has been extensively used for measuring dielectric parameters of material at microwave frequencies. This measurement method can be highly accurate and particularly advantageous in the determination of small loss tangent or loss factors.

This paper introduces new cavity perturbation technique in which a cavity has been designed with very small slot at the center of broader side of the waveguide in order to insert a sample material. Using cavity perturbation technique rectangular cavity resonator is designed to measure the dielectric parameters of Teflon. Measuring resonance frequency of empty cavity and then measuring the shift in resonance frequency with the sample material placed at its center and then the dielectric constant is calculated from the shift in resonance frequency and the sample volume whereas the loss factor is calculated from quality factor with and without sample. The material under test (Teflon) is fabricated in the form of a cylinder and inserted into the center of the rectangular cavity. This measurement method also describes the application of perturbation method to a microwave cavity resonator with a dielectric perturber. The measurement setup uses rectangular waveguide cavity resonator, HP8510 Network Analyzer and PC. The real and imaginary part of the permittivity can then be calculated from the shift in the resonance frequency and Q-factor. The result shows that the dielectric constant of the material can be measured with existing cavity with good accuracy.

2. THEORETICAL ANALYSIS

Many researchers have reported the theoretical [12–14] and experimental [2–11] results of the cavity perturbation techniques. The measurements of permittivity and permeability of the dielectric materials are performed by inserting a small and appropriately shaped sample into a cavity and determining the properties of the sample from the resultant change in the resonant frequency and loaded quality factor of the cavity. The basic idea of the cavity perturbation is the change in the overall geometric configuration of the electromagnetic fields with the insertion of a small sample must be small. Based on this assumption, a detailed derivation of the perturbation equation for the frequency shift upon the insertion of a sample into a cavity was given by Harrington [1]. When a small sample is inserted in a cavity which has an electric field E_0 and magnetic field H_0 in the unperturbed state and the fields in the interior of the sample is E and H, then for loss less sample, the variation of resonance frequency is given by [1, 6] as

$$\frac{f_s - f_0}{f_s} = -\frac{\int (\Delta\varepsilon E \cdot E_0^* + \Delta\mu H \cdot H_0^*) d\tau}{\int (\varepsilon E \cdot E_0^* + \mu H \cdot H_0^*) d\tau} \tag{1}$$

where ε and μ are the permittivity and permeability of the medium in the unperturbed cavity. $d\tau$ is the elementary volume and $\Delta\varepsilon$ and $\Delta\mu$ are the changes in the permittivity and permeability due to the introduction of the sample in the cavity. Without affecting the generality of Maxwell's equations, the complex frequency shift due to lossy sample in the cavity is given by [2, 3, 6] as

$$\frac{-df^*}{f^*} = \frac{(\varepsilon_r - 1)\varepsilon_0 \int\limits_{v_s} E \cdot E_0^* dv + (\mu_r - 1)\mu_0 \int\limits_{v_s} H \cdot H_0^* dv}{\int\limits_{v_s} (D_0 \cdot E_0^* + B_0 H_0^*) dv} \tag{2}$$

where df^* is the complex frequency shift because the permittivity of practical materials is a complex quantity, so the resonance frequency is also complex. B_0, H_0, D_0 and E_0 are the fields in the unperturbed cavity and E and H is the field in the interior of the sample [4, 12].

In terms of energy, the numerator of equation (2) represents the energy stored in the sample and the denominator represents the total energy stored in the cavity. The total energy $W = W_e + W_m$ where W_e and W_m are the electric and magnetic energy, respectively. With the aforementioned assumptions applied on equation (2), the fields in the empty part of the cavity are negligible changed by the insertion of the sample. The fields in the sample are uniform over its volume. Both of these assumptions can be considered valid if the sample is sufficiently small relative to the resonant wavelength. The negative sign in equation (2) indicates that by introducing the sample, the resonance frequency is lowered. When a dielectric sample is inserted into the cavity resonator where the maximum perturbation occurs that is at the position of maximum electric field, only the first term in the numerator is significant, since a small change in ε_r at a point of zero electric field or a small change in μ_r at a point of zero magnetic field does not change the resonance frequency. Therefore equation (2) can be reduced to

$$\frac{-df^*}{f} = \frac{(\varepsilon_r - 1)\int\limits_{v_s} E \cdot E^*_{0\,\max} dv}{2\int\limits_{v_s} |E|^2 dv} \tag{3}$$

3. DIELECTRIC CONSTANT ε' AND LOSS FACTOR ε''

A sample of complex permittivity $\varepsilon_r = \varepsilon' - j\varepsilon''$ is kept at the maximum electric field location of the cavity. The sample is taken, as cylinder with uniform cross sectional area 's' and length is greater than narrow dimension 'b' so that it will occupy the entire narrow dimension of the cavity. After the introduction of the sample the empty resonant frequency and Q-factor alter, due to the change in the overall capacitance and conductance of the cavity. If f_0 and Q_0 are the resonance frequency and quality factor of the cavity without sample and f_s and Q_s all the corresponding parameters of the cavity loaded with the sample. The complex resonant frequency shift is related to measurable quantities by [2, 6, 12]

$$\frac{df^*}{f} = \frac{f_s^2 - f_0^2}{f_s^2} + \frac{j}{2}\left(\frac{1}{Q_s} - \frac{1}{Q_0}\right) \tag{4}$$

On equating real and imaginary parts of equation (3) and (4) we have

For real part:

$$\frac{-(f_s - f_0}{f_s} = \frac{(\varepsilon_r' - 1)\int\limits_{V_s} E \cdot E^*_{0\max} dv}{2\int\limits_{V_c} |E_0|^2 dv} \tag{5}$$

We may assume that $E = E_0$ and the valve of E_0 in the TE_{10p} mode is $E_0 = E_{0\max} \sin(p\pi z/l) \sin(p\pi z/l)$ where a is the broader dimension of the wave guide and l is the length of the cavity. Integrating and rearranging the equation (5), we obtain

$$\varepsilon' = \frac{V_c\left(f_0^2 - f_s^2\right)}{4V_s f_s^2} \tag{6}$$

where V_c is volume of the cavity $= a \times b \times l$ (Dimensions of the cavity) and V_s is the volume of the sample $= \pi r^2 h$ (r is the radius and h is the length of the sample).

For Imaginary part:

$$\frac{1}{2}\left(\frac{1}{Q_s}-\frac{1}{Q_0}\right)=\frac{\varepsilon_r''\int\limits_{V_s}E\cdot E_{0\max}^{*}dv}{2\int\limits_{V_s}|E|^2dv} \tag{7}$$

Integrating and rearranging the equation (7), we obtain

$$\left(\frac{1}{Q_s}-\frac{1}{Q_0}\right)\frac{V_c f_0^2}{4V_s f_s^2}=\varepsilon'' \tag{8}$$

where Q_s is the quality factor of cavity with sample and Q_0 is the quality factor without sample. Equation (6) and (8) are the standard form of the expression for dielectric parameters using the perturbation technique.

4. EXPERIMENTAL SET UP

A rectangular X band waveguide cavity is constructed with a brass waveguide of nearly 140 mm length. The cross section dimensions are 22.9 mm in width and 10 mm in height. Two thin conducting sheets are used to form the cavity and to close the two ends of the waveguide. The inductive coupling is provided with two symmetric holes of diameter 4 mm on these end sheets. Fig. 1 and Fig. 2 show the cavity resonator and block diagram of experimental setup for the measurements respectively.

In order to insert a sample material into the resonator, a slot is constructed at the center of the broader side of the waveguide. The

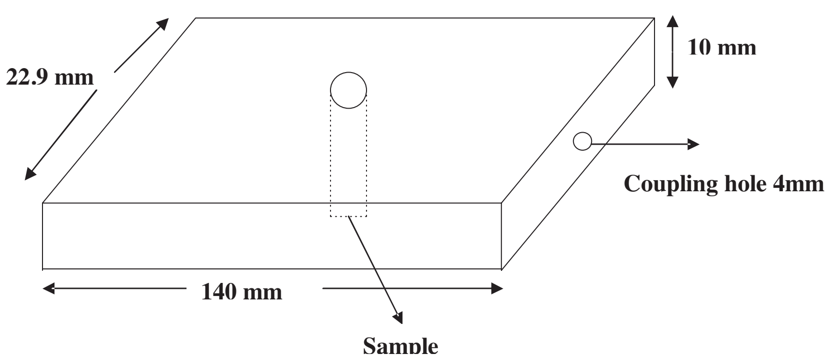

Figure 1. The cavity resonator and its dimensions.

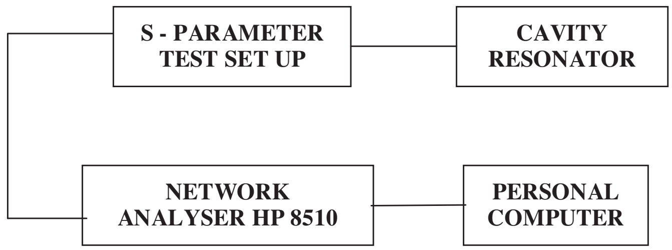

Figure 2. Block diagram of the experimental setup.

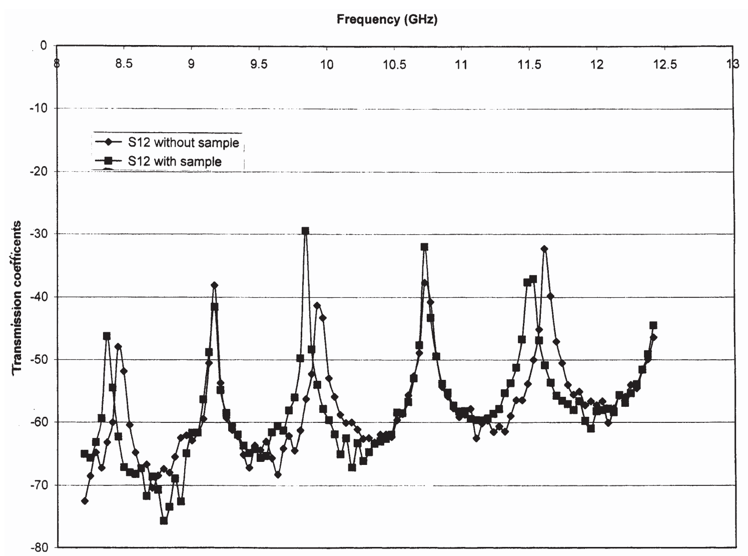

Figure 3. Transmission coefficients of the Cavity measurements in X-band.

Table 1. Dielectric constant of Teflon.

Mode	Radius of Sample	F_0 (GHz)	F_S (GHz)	ε'
TE_{105}	1.98mm	11.6	11.43	1.96
TE_{105}	2.4mm	11.66	11.4	2.05

width of the sample hole is equal to the diameter of the cylindrical sample. This rectangular waveguide cavity resonator is connected to the two ports of the HP 8510 s-parameter test set of the measuring system. It is also operated in the TE_{10P} modes.

5. EXPERIMENTAL RESULTS

An example of the cavity measurement in X-band is shown in Fig. 3. The left peaks of the Fig. 3 are the resonance peaks of the cavity containing sample (Teflon) material and the right peaks are the empty cavity resonance. Fig. 3 shows the expected shift in resonance frequency and Q-factor. The obtained resonance parameters and dielectric calculation are tabulated for TE_{105} mode. Table 1 shows the calculated values of dielectric constant of cylindrical Teflon sample of different radius.

6. CONCLUSION

A new perturbation technique has been developed and discussed for the evaluation of dielectric parameters of dielectric material at microwave frequency. In this technique, a cavity has been designed with very small slot at the center of the broader side of the wave-guide in order to insert a sample material. The existing cavity resonator is constructed with a line slot on the broader side of the wave-guide with moving sample holder. The analysis of the expressions for dielectric constant and loss factor has been discussed. It has been observed from the results that the dielectric parameters of the material can be measured with existing cavity with good accuracy.

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