

## **THE MULTIREOLUTION FREQUENCY DOMAIN METHOD FOR GENERAL GUIDED WAVE STRUCTURES**

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**Abstract**—A multiresolution frequency domain (MRFD) analysis similar to the finite difference frequency domain (FDFD) method is presented. This new method is derived by the application of MoM to frequency domain Maxwell's equations while expanding the fields in terms of biorthogonal scaling functions. The dispersion characteristics of waveguiding structures are analyzed in order to demonstrate the advantages of this proposed MRFD method over the traditional FDFD scheme.

### **1. INTRODUCTION**

Over the last decade, multiresolution analysis techniques have successfully been applied to various computational electromagnetic methods yielding significant computational CPU and memory savings, compared to the traditional techniques. Multiresolution analysis

has found application in the improvement of method of moments (MoM) [1–4], finite difference time domain (FDTD) method [5–9] and transmission line matrix (TLM) method [10]. The finite difference frequency domain method, on the other hand, has not yet benefited from the advantages of multiresolution analysis. In this paper, we formulate a 3D frequency domain numerical method based on multiresolution analysis from which a special case leads to the FDFD method.

It was observed in [5] that the FDTD scheme can be derived by applying MoM to Maxwell’s curl equations while using pulse functions as a basis for the expansion of unknown fields. It is also possible to show that the previous statement is true for FDFD scheme. Since pulse functions are also the scaling functions of the Haar wavelet bases, FDFD scheme can also be considered as multiresolution analysis scheme based only on the scaling function and can be improved by adding Haar wavelet functions to the expansion of the fields. One can further improve the method by using the Cohen-Daubechies-Feauveau (CDF) family of wavelets, which is the focus of this work.

## 2. FORMULATION

### 2.1. Wavelet Selection

In order to develop an efficient MRFD formulation, one should choose the appropriate wavelet family from an ever-increasing number of wavelets available. For an effective MRFD algorithm, the appropriate wavelet base should have certain properties; such as compact support, symmetry, interpolation property, regularity (smoothness) and maximum number of vanishing moments.

The support of the wavelet basis is directly related to the number of terms in the field components update equations. Since a great number of terms in each update equation will increase the computational burden and complicate the coding process, it is preferable to have a compactly supported wavelet base. Symmetric wavelet functions will ensure the symmetry of the formulation and in return will simplify the modeling of symmetry and boundary planes [11]. Compact support and symmetry are incompatible aspects of orthogonal wavelet systems, with the Haar wavelets being the exception. A biorthogonal wavelet base can sustain smooth, compact and symmetric wavelets, which is the main reason for the choice of biorthogonal wavelets over orthogonal ones.

In a wavelet expansion, the field value at a certain point is generally reconstructed by a weighted sum of related neighboring basis function coefficients, which requires a complex reconstruction

algorithm. However, the reconstruction algorithm is not essential if the basis function satisfies the interpolation property, as in such cases the basis function coefficients represent the field components at the corresponding position of the grid [12]. Thus, a basis function equipped with the interpolation property will not use the weighted sum and thus yields a more efficient MRFD algorithm.

Two of the most desired properties of a wavelet family are high regularity and maximum number of vanishing moments, owing to their great effect on how well the wavelet expansion approximates a smooth function. Unfortunately, wavelets do not accommodate both high regularity and high number of vanishing moments concurrently and it is unclear which property is more important [13–14]. Biorthogonal wavelets have two dual scaling functions, which can have different regularity properties. It is mentioned in [13] that, it is useful to reserve the high regularity to the synthesis scaling function and maximum vanished moments to the analysis scaling function.

Considering all the requirements mentioned above, Cohen-Daubechies-Feauveau family of wavelets [13], in particular CDF(2, 2) wavelet, is adopted for this work due to its minimal support.

## 2.2. 3D Formulation

In this section, derivation of the new method for lossless simple media is presented. As an example, we can consider one of Maxwell's time-harmonic scalar equations:

$$jw\epsilon E_x(x, y, z) = \frac{\partial H_z(x, y, z)}{\partial y} - \frac{\partial H_y(x, y, z)}{\partial z}. \quad (1)$$

The field components on a Yee cell may be approximated in terms of basis functions as

$$E_x(x, y, z) = \sum_{i,j,k} E_x(i, j, k) \tilde{\phi}_{i+1/2}(x) \tilde{\phi}_j(y) \tilde{\phi}_k(z) \quad (2a)$$

$$H_y(x, y, z) = \sum_{i,j,k} H_y(i, j, k) \tilde{\phi}_{i+1/2}(x) \tilde{\phi}_j(y) \tilde{\phi}_{k+1/2}(z) \quad (2b)$$

$$H_z(x, y, z) = \sum_{i,j,k} H_z(i, j, k) \tilde{\phi}_{i+1/2}(x) \tilde{\phi}_{j+1/2}(y) \tilde{\phi}_k(z) \quad (2c)$$

Here, the indexes  $i, j, k$  indicate the discrete space lattice related to the space grid through  $x = i\Delta x$ ,  $y = j\Delta y$  and  $z = k\Delta z$ . The function  $\tilde{\phi}_n(x)$  is the scaled and shifted CDF dual scaling function ( $\tilde{\phi}(x)$ ) defined as:

$$\tilde{\phi}_n(x) = \tilde{\phi}\left(\frac{x - n\Delta x}{\Delta x}\right) \quad (3)$$

The field expansions are inserted into (1) and both sides are tested with the scaling function according to Galerkin's method. During the sampling process, the following integrals are used:

$$\int_{-\infty}^{\infty} \frac{\partial \tilde{\phi}_{i'+1/2}(y)}{\partial y} \phi_i(y) dy = \sum_{l=1}^{n_a} a(l) (\delta_{i+l-1,i'} - \delta_{i-l,i'}) \quad (4)$$

$$\int_{-\infty}^{\infty} \phi_{i'}(x) \tilde{\phi}_i(x) dx = \Delta x \delta_{i,i'} \quad (5)$$

where  $n_a$  is the stencil size and  $\delta_{i,i'}$  is the kronecker delta given by

$$\delta_{i,i'} = \begin{cases} 1, & i = i' \\ 0, & i \neq i' \end{cases} \quad (6)$$

Stencil sizes and  $a(l)$  coefficients of different CDF wavelets are given in [7]. For CDF(2, 2) wavelet, stencil size is 3 and  $a(l)$  coefficients are listed in Table 1.

**Table 1.** The  $a(l)$  coefficients.

$l$	1	2	3
$a(l)$	1.2291667	-0.0937500	0.0104167

We can now continue the derivation by testing the left-hand side of (1) according to MoM, such that

$$\begin{aligned} & jw\varepsilon \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E_x(x, y, z) \phi_{i+1/2}(x) \phi_j(y) \phi_k(z) dx dy dz \\ &= jw\varepsilon \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \sum_{i,j,k} E_x(i, j, k) \tilde{\phi}_{i+1/2}(x) \tilde{\phi}_j(y) \tilde{\phi}_k(z) \right] \\ & \quad \phi_{i+1/2}(x) \phi_j(y) \phi_k(z) dx dy dz \\ &= jw\varepsilon E_x(i, j, k) \Delta x \Delta y \Delta z \end{aligned} \quad (7)$$

where  $\phi_n(x)$  is the scaled and shifted CDF scaling function ( $\phi(x)$ ) defined similar to (3). Testing the first term of the right-hand side of (1) yields:

$$\begin{aligned}
 & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial H_z(x, y, z)}{\partial y} \phi_{i+1/2}(x) \phi_j(y) \phi_k(z) dx dy dz \\
 &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial}{\partial y} \left[ \sum_{i,j,k} H_z(i, j, k) \tilde{\phi}_{i+1/2}(x) \tilde{\phi}_{j+1/2}(y) \tilde{\phi}_k(z) \right] \\
 & \quad \phi_{i+1/2}(x) \phi_j(y) \phi_k(z) dx dy dz \\
 &= \Delta x \Delta z \sum_{l=1}^{n_a} a(l) [H_z(i, j + l - 1, k) - H_z(i, j - l, k)]. \quad (8)
 \end{aligned}$$

Testing the second term of the right-hand side of (1) similarly yields:

$$\begin{aligned}
 & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\partial H_y(x, y, z)}{\partial z} \phi_{i+1/2}(x) \phi_j(y) \phi_k(z) dx dy dz = \\
 & \Delta x \Delta y \sum_{l=1}^{n_a} a(l) [H_y(i, j, k + l - 1) - H_y(i, j, k - l)]. \quad (9)
 \end{aligned}$$

Equating (1), (7), (8) and (9) results in the MRFD update equation:

$$E_x(i, j, k) = \frac{1}{jw\epsilon} \left( \frac{\sum_{l=1}^{n_a} a(l) [H_z(i, j + l - 1, k) - H_z(i, j - l, k)]}{\Delta y} - \frac{\sum_{l=1}^{n_a} a(l) [H_y(i, j, k + l - 1) - H_y(i, j, k - l)]}{\Delta z} \right). \quad (10)$$

The remaining five update equations, which are not listed here due to space saving considerations, can be derived similarly by applying the same procedure to the rest of the scalar Maxwell's equations.

### 2.3. 2D Formulation

The determination of the dispersion characteristics of waveguiding structures is a fundamental problem in microwave engineering applications. Furthermore, these characteristics (propagation constant, mode patterns, characteristic impedance, etc.) can be used as port data for 3D simulation of microwave devices [15]. Since such problems are well-suited for frequency domain techniques, characterization of guided wave structures is considered for validation of the proposed method.

The eigen-based finite difference frequency domain method was proposed to solve the propagation characteristics of guided wave structures [16]. For the MRFD solution of the problem, a similar approach is adopted. Assuming that the waveguiding structure is uniform along  $z$  axis and the wave propagates in positive  $z$  direction, the electric and magnetic fields inside guided wave structure can be expressed as:

$$\vec{E}(x, y, z) = [E_x(x, y)\hat{x} + E_y(x, y)\hat{y} + E_z(x, y)\hat{z}] e^{-j\beta z} \quad (11a)$$

$$\vec{H}(x, y, z) = [H_x(x, y)\hat{x} + H_y(x, y)\hat{y} + H_z(x, y)\hat{z}] e^{-j\beta z} \quad (11b)$$

where  $\beta$  is the propagation constant. Time harmonic Maxwell's equations for simple dielectric media are:

$$\nabla \times \vec{E} = -j\omega\mu H, \quad \nabla \times \vec{H} = j\omega\varepsilon E \quad (12a)$$

$$\nabla \cdot \vec{D} = 0, \quad \nabla \cdot \vec{B} = 0 \quad (12b)$$

in which the space derivatives with respect to  $z$  can be replaced by  $-j\beta$  (i.e.,  $\partial/\partial z = -j\beta$ ).

Inserting (11) into (12) yields the following scalar equations:

$$\beta E_x = w\mu_y H_y + j \frac{\partial E_z}{\partial x} \quad (13a)$$

$$\beta E_y = -w\mu_x H_x + j \frac{\partial E_z}{\partial y} \quad (13b)$$

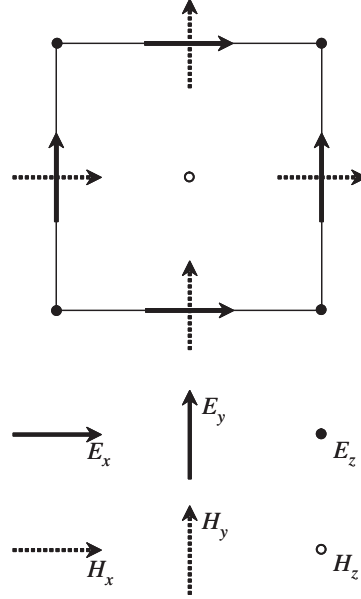
$$\beta H_x = -w\varepsilon_y E_y + j \frac{\partial H_z}{\partial x} \quad (13c)$$

$$\beta H_y = w\varepsilon_x E_x + j \frac{\partial H_z}{\partial y} \quad (13d)$$

$$\beta E_z = -j\varepsilon_x \frac{\partial E_x}{\partial x} - j\varepsilon_y \frac{\partial E_y}{\partial y} \quad (13e)$$

$$\beta \mu_z H_z = -j\mu_x \frac{\partial H_x}{\partial x} - j\mu_y \frac{\partial H_y}{\partial y} \quad (13f)$$

Note that (13a)–(13d) is derived from Maxwell's curl equations (12a) and (13e), (13f) is derived from Maxwell's divergence equations (12b).



**Figure 1.** The compact 2D lattice.

Equation (13) can be discretized by using the compact 2D Yee cell [16–18] shown in Fig. 1. Two of the resulting update equations are presented below as an example:

$$\beta E_x(i, j) = w\mu_y(i, j)H_y(i, j) + \frac{j}{\Delta x} \sum_{l=1}^3 a(l) [E_z(i+l, j) - E_z(i-l+1, j)] \quad (14a)$$

$$\begin{aligned} \beta E_z(i, j) = & \frac{j}{\Delta x \varepsilon_z(i, j)} \sum_{l=1}^3 a(l) \\ & [\varepsilon_x(i-l, j)E_x(i-l, j) - \varepsilon_x(i+l-1, j)E_x(i+l-1, j)] \\ & + \frac{j}{\Delta y \varepsilon_z(i, j)} \sum_{l=1}^3 a(l) \\ & [\varepsilon_y(i, j-l)E_y(i, j-l) - \varepsilon_y(i, j+l-1)E_y(i, j+l-1)] \end{aligned} \quad (14b)$$

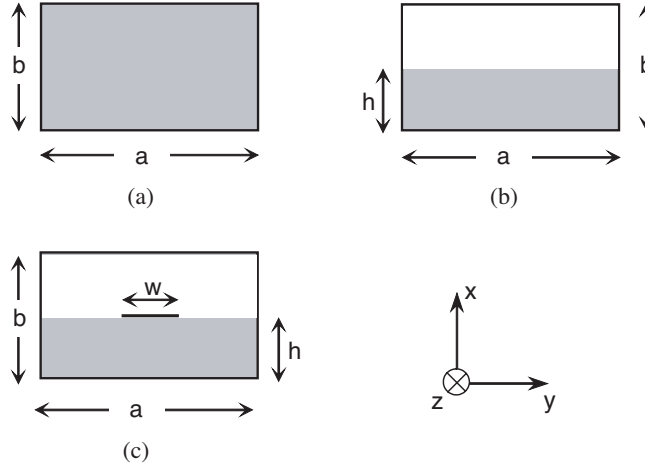
Field update equations can be used to form an eigenvalue problem as:

$$[A] - \beta[I] \cdot x = 0 \quad (15)$$

where  $[A]$  is a sparse coefficient matrix,  $[I]$  is the unit matrix and  $x$  is the unknown field vector. The eigen solution of  $[A]$  delivers the propagation constant and corresponding electromagnetic fields.

### 3. NUMERICAL RESULTS

First, a dielectric filled rectangular waveguide with dimensions  $a = 1.5$  cm,  $b = 0.6$  cm and relative dielectric constant  $\epsilon_r = 2.25$ , as illustrated in Fig. 2(a), is considered. The propagation constants of the first two higher order modes are computed using the FDFD [16] and the MRFD methods. Results are compared in Fig. 3(a) to the analytical calculations [19]. The convergence of the calculated error with respect to cell size for each method is provided in Fig. 4.

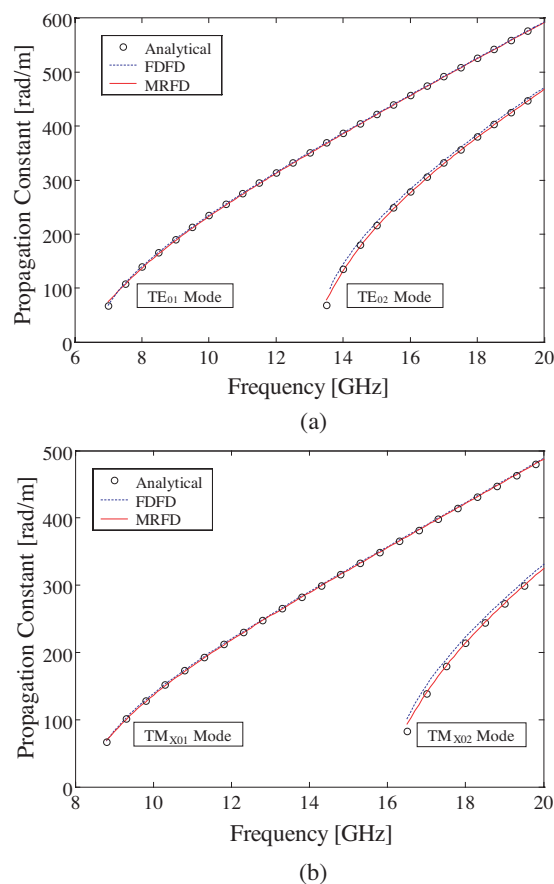


**Figure 2.** Waveguiding structures: (a) dielectric filled waveguide, (b) partially filled waveguide, (c) microstrip line.

The second example is a partially filled waveguide as shown in Fig. 2(b). Waveguide dimensions are  $a = 1.5$  cm,  $b = 0.6$  cm and  $h = 0.3$  cm and the relative dielectric constant of the substrate is  $\epsilon_r = 2.25$ . Computed results are given in Fig. 3(b).

Finally, the propagation characteristics of a boxed microstrip line (Fig. 2(c)) are considered. The dimensions are  $a = 1.5$  cm,  $b = 1.5$  cm,  $h = 0.3$  cm and  $w = 0.3$  cm and  $\epsilon_r$  of the substrate is 30. The computed effective dielectric constant of this structure is shown in Fig. 5.



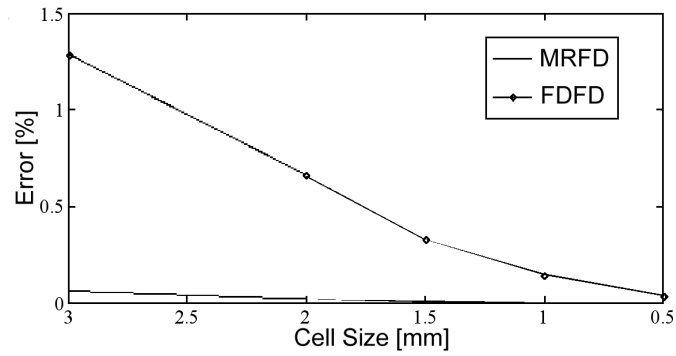
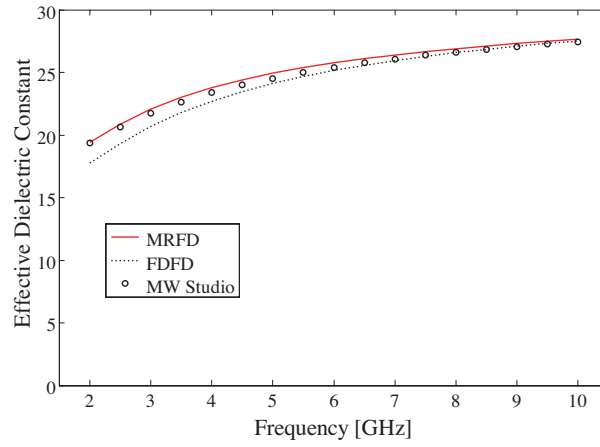


**Figure 3.** Propagation constant of the first two propagating modes of a waveguide: (a) dielectric filled, (b) partially filled.

For all three cases, the MRFD grid was chosen to be coarser than the FDFD grid, yet the accuracy of both methods remained identical. Coarser grid results smaller matrix sizes, reduced computation time and memory requirements. The efficiency of the new method is compared to FDFD in Table 2. Results show that the accuracy of the MRFD method matches that of FDFD method despite savings in terms of memory and in execution time. The memory requirements and execution time of both methods for each example is summarized Table 2. These calculations are performed by MATLAB codes and run on a Windows based personal computer with 3 GB memory and two 600-MHZ Pentium III CPUs.

**Table 2.** Computer resources consumed by the two methods.

		Mesh size	Matrix Size [byte]	Time [sec]
Uniform Waveguide	FDTD	$15 \times 6$	32336	1.549
	MRFD	$5 \times 2$	4384	0.268
Partially Filled Waveguide	FDTD	$15 \times 6$	32336	3.001
	MRFD	$5 \times 4$	11600	0.7433
Microstrip Line	FDTD	$15 \times 6$	32336	1.424
	MRFD	$10 \times 4$	26620	0.925

**Figure 4.** Error convergence of the propagation constant.**Figure 5.** Effective dielectric constant of the boxed microstrip line.

#### 4. CONCLUSION

The multiresolution frequency domain scheme based on the biorthogonal CDF wavelet family is developed. The newly developed method together with traditional FDFD method is used to analyze the propagation characteristics of general guided wave structures. Results indicate substantial savings in terms of execution time and memory requirements. It is expected that the savings will be more significant in three dimensional problems.

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