Progress In Electromagnetics Research, PIER 66, 301–315, 2006

2-L-SHAPE TWO-DIMENSIONAL ARRIVAL ANGLE ESTIMATION WITH A CLASSICAL SUBSPACE ALGORITHM

H. Changuel, F. Harabi, and A. Gharsallah

Groupe d'Electronique
Laboratoire de Physique de la matière molle
Faculté des Sciences de Tunis
El-Manar 2092 Tunisie

Abstract—This paper proposes a computationally efficient method for a two-dimensional direction of arrival estimation of multiple narrowband sources. We apply the MUSIC method which requires eigenvalues decomposition to the cross spectral matrix. This paper will employ two L-shape arrays that showed better performances than the one L-shape and the parallel shape arrays. In spite of its computational complexity, simulation results verify that the proposed subspace technique gives much better performance than the propagator method.

1. INTRODUCTION

The direction of arrival (DOA) estimation is very important in the fields of radar, sonar, and high-resolution spectral analysis. The problem of estimating the two-dimensional (2-D) directions of arrival (DOAs), namely, the azimuth and elevation angles, of multiple sources has received considerable attention in the field of array processing [1–3, 6, 7]. Different geometry of the problem schemes have been proposed, in these algorithms, to significantly simplify, and thereby to reduce the costs of, uncertainty estimation. They employ inhomogeneous cylinders [12] or non-uniform linear antenna array configuration [13–15], to solve the problem of estimating the two-dimensional (2-D) directions of arrival (DOAs), of multiple sources.

Sjöberg [16] has proposed a non-uniform array antenna, consisting of 7 identical elements; his paper treats essentially two applications: single scattering against randomly distributed particles and random errors in antenna technology, in great generality. The estimates are given in terms of the deterministic design values and the errors in phase, amplitude and position of the antenna elements. Although the maximum likelihood estimator [2] provides optimum parameter estimation, its computational complexity is extremely demanding.

MUSIC is the most well-known for its super-resolution capability and simplicity and it has less computational complexity than the maximum likelihood methods.

Tayem and Kwon [8] have proposed amelioration in the estimation of angle elevation between 70° and 90° than the algorithm proposed in [5], which is very important in mobile communication. However, the 2-D angle estimation error in [8] is large and it has performance degradation at low SNR. Furthermore, many of the above methods [2, 6, 7] require reasonably accurate initial DOA estimates, 2-D search and/or complex pair matching of the azimuth and elevation angles. Besides, Li et al. [4] have proposed a PM-based DOA estimation method but unfortunately, a 2-D peak search is needed.

The objectives of our paper are as follow

(1) Reduce the estimation error of both the azimuth and elevation angles.
(2) Improve the performance of our method at low SNR.
(3) Reduce the estimation failure problems when elevation angles are between 70° and 90°.

To achieve these objectives, this paper proposes the two L-shape antenna array configuration shown in Fig. 1 and employs the MUSIC method.

The rest of the paper is organized as follows. The data model is presented in Section 2. Our 2-D DOA estimation algorithm is developed in Section 3. Section 4 shows simulation results and Section 5 makes conclusions.

2. DATA MODEL

Consider the two L-shape uniform linear arrays (ULAs) in the x-z and the y-z planes shown in Fig. 1 with inter-element equals d, using three array elements placed on the x, y and z-axes. Each linear array consists of N elements. The element placed at the origin is common for referencing purpose.

Suppose that there are K narrow band sources, $S(t)$, with same wavelength λ impinging on the array, such that kth source has an elevation angle θ_k and an azimuth angle ϕ_k, $k = 1, \ldots, K$.

We put the complex base-band representation of the signal received by the nth element of one sub-array as $x_n(t)\,(n = 1, 2, \ldots, N)$,

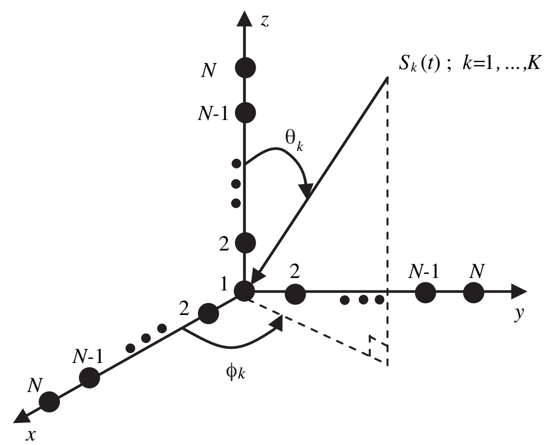

Figure 1. The two L-shape array configuration used for the joint azimuth and elevation (ϕ_k, θ_k) DOA estimation.

the signal sources are far apart from the sub-array. The sub-array output vector at the snapshot t is then given by

$$x(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T = \sum_{k=1}^{K} a(\theta_k, \phi_k)\, s_k(t) + \eta(t) \qquad t = 1, \ldots, P \tag{1}$$

Where $\eta(t)$ is an N-dimensional complex white noise vector with mean zero and covariance $\sigma^2 I$, I is the identity matrix of size N, superscript T denotes transpose of a matrix and $a(\theta_k, \phi_k)$ is the steering vector defined by

$$a(\theta_k, \phi_k) = [1, \exp(-j\varphi_k), \ldots, \exp(-j\varphi_k(N-1))]^T \tag{2}$$

with φ_k depends on the position of the sub-array. The sample correlation matrix of the array output vector is defined by

$$R_t = E\left\{x(t)x(t)^H\right\} = \sum_{k=1}^{K} \sigma_k^2 a(\theta_k, \phi_k)\, a(\theta_k, \phi_k)^H + \sigma^2 I \tag{3}$$

where H is the Hermitian operator.

3. PROPOSED TWO-DIMENSIONAL DIRECTION OF ARRIVAL ESTIMATION ALGORITHM

3.1. Previous Work (MUSIC Algorithm)

Let us review the procedure of the MUSIC algorithm [9]. This classical subspace algorithm relies on the eigen decomposition of the sample correlation matrix R_t calculated in (3).

We denote its eigenvalues (in decreasing order) and their corresponding eigenvectors by λ_k and v_k respectively.

$$R_t = V \Lambda V^H \tag{4}$$

where

$$V = [v_1, \ldots, v_N] \quad \text{and} \quad \Lambda = diag[\lambda_1, \lambda_2, \ldots, \lambda_N] \tag{5}$$

It can be shown [10, 11] that

$$\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_K \geq \lambda_{K+1} = \ldots = \lambda_N = \sigma^2 \tag{6}$$

Here we explain the mechanism of the MUSIC algorithm. If the number K of sources is smaller than the number N of sensors, all the signal components are represented in the signal subspace spanned by the first K eigenvectors $v_1, \ldots, v_K$, and the remaining N-K eigenvectors $v_{K+1}, \ldots, v_N$ represents the noise subspace.

3.2. Elevation and Azimuth Angles Estimation Algorithm

3.2.1. Elevation Angle Estimation Method

Let $Z(t)$ be the $N \times 1$ signal received at the linear sub-array in the z-axes at the snapshot t.

$$Z(t) = (z_1(t), z_2(t), \ldots, z_N(t))^T = \sum_{k=1}^{K} a(\theta_k) s_k(t) + \eta_{kz}(t), \quad t = 1, \ldots, P \tag{7}$$

where

$$a(\theta_k) = [1, \exp(-j\varphi_k), \ldots, \exp(-j\varphi_k(N-1))]^T \tag{8}$$

and

$$\varphi_k = \exp\left(-j\frac{2\pi d \cos\theta_k}{\lambda}\right) \tag{9}$$

θ_k is the elevation angle of the kth source signal. $\eta_{kz}(t)$ is the additive White Gaussian noise of the kth source signal at the snapshot t.

The sample correlation matrix of the sub-array output vector is defined by

$$R_z(t) = E\left\{z(t)z(t)^H\right\} = \sum_{k=1}^{K} \sigma_k^2 a\left(\theta_k\right) a\left(\theta_k\right)^H + \sigma^2 I \tag{10}$$

We denote its eigenvalues, in decreasing order, and their corresponding eigenvectors by λ_k and v_k respectively. The K points where the function

$$U_z(\theta) = \sum_{k+1}^{N} v_k^H a\left(\theta_k\right) a^H\left(\theta_k\right) v_k \tag{11}$$

approaches zero correspond to the elevation angles $\theta_1, \ldots, \theta_K$ of the signals. Therefore, the elevation angles estimated $\hat{\theta}_k$, $k = 1, \ldots, K$ can easily be found by maximising the following cost function

$$J_z(\theta) = \frac{1}{U_z(\theta)} \tag{12}$$

The exhaustive sweeping operation is usually used to find the local maxima.

3.2.2. The Azimuth Angle Estimation Method

Let $X(t)$ be the $N \times 1$ signal received at the linear sub-array in the x axes at the snapshot t.

$$X(t) = \left(x_1(t), x_2(t), \ldots, x_N(t)\right)^T = \sum_{k=1}^{K} a(\theta_k, \phi_{kx}) s_k(t) + \eta_{kx}(t), \qquad t = 1, \ldots, P \tag{13}$$

where

$$a(\theta_k, \phi_{kx}) = \left[1, \exp(-j\varphi_k), \ldots, \exp\left(-j\varphi_k(N-1)\right)\right]^T \tag{14}$$

and

$$\varphi_k = \exp\left(-j\frac{2\pi d \sin\theta_k \cos\phi_{kx}}{\lambda}\right) \tag{15}$$

ϕ_{kx} is the azimuth angle of the kth source signal. $\eta_{kx}(t)$ is the additive White Gaussian noise of the kth source signal at the snapshot t in the x sub-array with mean zero and covariance $\sigma^2 I$.

The sample correlation matrix of the sub-array output vector is defined by

$$R_x(t) = E\left\{x(t)x(t)^H\right\} = \sum_{k=1}^{K} \sigma_k^2 a\left(\theta_k, \phi_{kx}\right) a\left(\theta_k, \phi_{kx}\right)^H + \sigma^2 I \quad (16)$$

Its eigenvalues, in decreasing order, and their corresponding eigenvectors are denoted by λ_k and v_k respectively.

With the same MUSIC procedure used for estimation of the elevation angle $\hat{\theta}_k$, we can estimate ϕ_{kx} using the sub-array elements in the x axis with the elevation angle $\hat{\theta}_k$ obtained from (12).

The K points where the function

$$U_x\left(\hat{\theta}, \phi_x\right) = \sum_{k+1}^{N} v_k^H a\left(\hat{\theta}_k, \phi_{kx}\right) a^H\left(\hat{\theta}_k, \phi_{kx}\right) v_k \quad (17)$$

$U_x\left(\hat{\theta}, \phi_x\right)$ approaches zero correspond to the azimuth angles $\phi_{1x}, \ldots, \phi_{Kx}$ of the signals. Therefore, the azimuth angles estimated ϕ_{kx}, $k = 1, \ldots, K$ can easily be found by maximising the following cost function

$$J_x(\phi_x) = \frac{1}{U_x\left(\hat{\theta}, \phi_x\right)} \quad (18)$$

In the same way we estimate the azimuth angle $\hat{\phi}_{ky}$ using the y axis sub-array. The $N \times 1$ signal received $Y(t)$ at the linear sub-array in the y-axes at the snapshot t can be rewritten as

$$Y(t) = \left(y_1(t), y_2(t), \ldots, y_N(t)\right)^T = \sum_{k=1}^{K} a(\theta_k, \phi_{ky}) s_k(t) + \eta_{ky}(t), \quad t = 1, \ldots, P \quad (19)$$

where

$$a(\theta_k, \phi_{ky}) = \left[1, \exp(-j\varphi_k), \ldots, \exp\left(-j\varphi_k(N-1)\right)\right]^T \quad (20)$$

and

$$\varphi_k = \exp\left(-j\frac{2\pi d \sin\theta_k \cos\phi_{ky}}{\lambda}\right) \quad (21)$$

Finally, the azimuth angle estimation $\hat{\phi}_k$ can be written as

$$\hat{\phi}_k = \begin{cases} \frac{1}{2}\left(\hat{\phi}_{kx} + \hat{\phi}_{ky}\right) & \text{if both } \hat{\phi}_{kx} \text{ and } \hat{\phi}_{ky} \text{ are real} \\ \hat{\phi}_{kx} & \text{if } \hat{\phi}_{ky} \text{ is complex} \\ \hat{\phi}_{ky} & \text{if } \hat{\phi}_{kx} \text{ is complex} \end{cases} \quad (22)$$

3.2.3. Computational complexity Algorithm

Regarding major computational complexity, the MUSIC algorithm requires $O(N^3 + 3N^2L)$ multiplication in calculating the eigen decomposition for a sample correlation matrix with an N-element array and L snapshots. The computational load of the PM algorithm is in $O(3NLK)$, where K is the number of incident sources. Using MUSIC techniques, the complexity is costly and high especially when the number of sources and antenna elements are large.

However, the PM requires pair matching between the 2-D azimuth and elevation angle estimation (θ, ϕ) and can have an estimation failure problem when elevation angles are between 70° and 90°. The elevation angle in typical mobile communications is in the range of 70° and 90°. Thus, the application of the MUSIC method of the two L-shape arrays to mobile communications should be considered.

The cost and time requirement of rigorous uncertainty analysis is quite high, which in general means that methods that employ such rigorous analysis are normally implemented only by National Measurement Institutes and are rarely applied to routine testing or calibration [17]. The purpose of this paper is to show how to remove these problems in the MUSIC method without additional computational loads. This paper employ a configuration of two L-shape arrays, which allow no pair matching between the azimuth angle estimate and the elevation angle estimate of the source k. In addition, with the proposed two L-shape arrays in the x-z and y-z planes, we can completely remove the failure problem. Comparing with the PM, MUSIC algorithm reduces the means, variances and standard deviations significantly.

4. SIMULATION RESULT

Computer simulations have been conducted to evaluate the 2-D DOA estimation performance of the proposed method. The performance of the two L-shape array with the MUSIC method is compared to that of [8] using the PM method.

A half wavelength of the incoming signals is used for the spacing between the adjacent elements in each uniform linear array. The total number of elements for the proposed two L-shape algorithm was 16. We assume one single source $K = 1$, with direction of arrival DOA (θ, ϕ). The additive noise is White Gaussian processes. The $L = 200$ number of snapshots per trial and 1000 independent trials in total are used.

Table 3 lists the standard deviation of the azimuth angle estimation of the four latest values, witch we used for simulation,

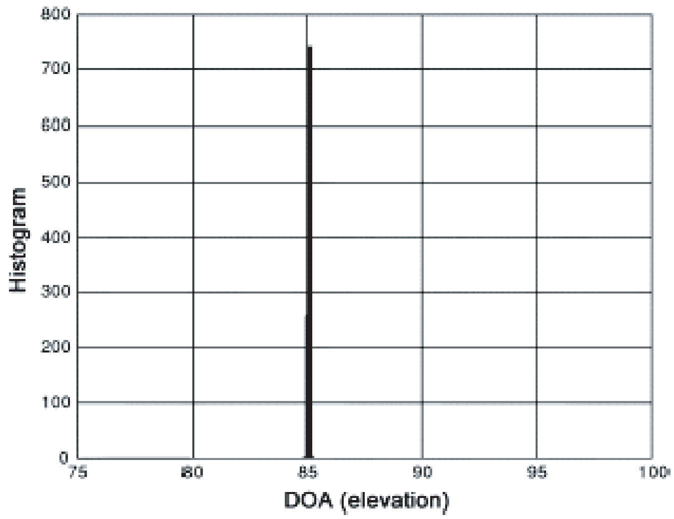

Figure 2. Histogram of elevation DOA estimations for a single source of DOA at (85°, 40°), $SNR = 10\,\mathrm{dB}$, and $N = 5$ elements by using the PM algorithm.

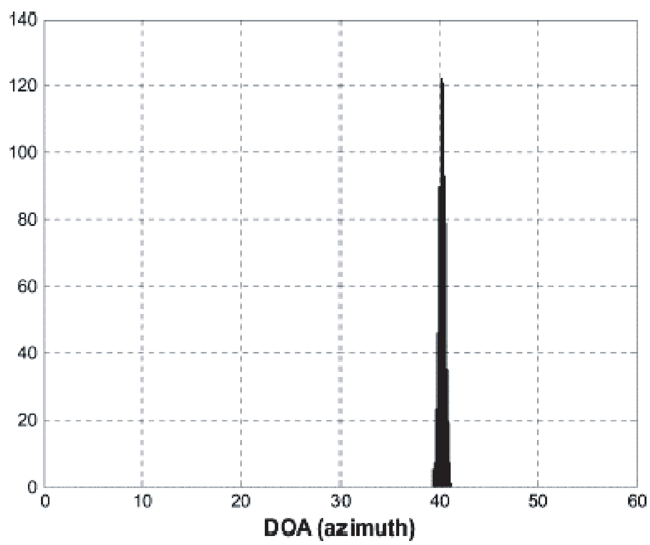

Figure 3. Histogram of azimuth DOA estimations for a single source of DOA at (85°, 40°) $SNR = 10\,\mathrm{dB}$, $N = 5$ elements by using the PM 2-L-shape array configuration.

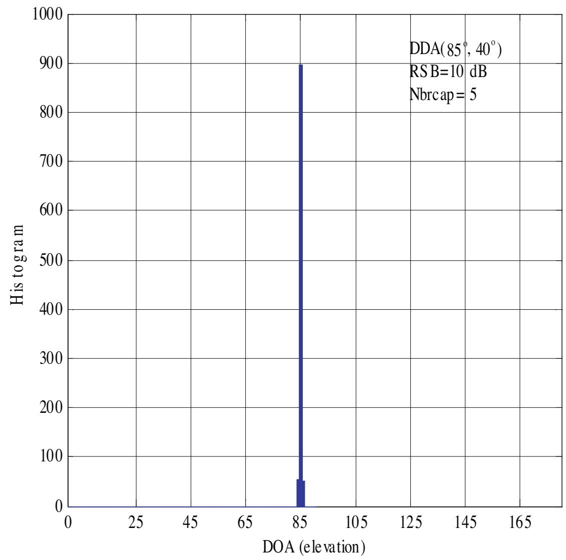

Figure 4. Histogram of elevation DOA estimations for a single source of DOA at (85°, 40°) $SNR = 10\,\text{dB}$, $N = 5$ elements by using the proposed MUSIC of the 2-L-shape arrays configuration.

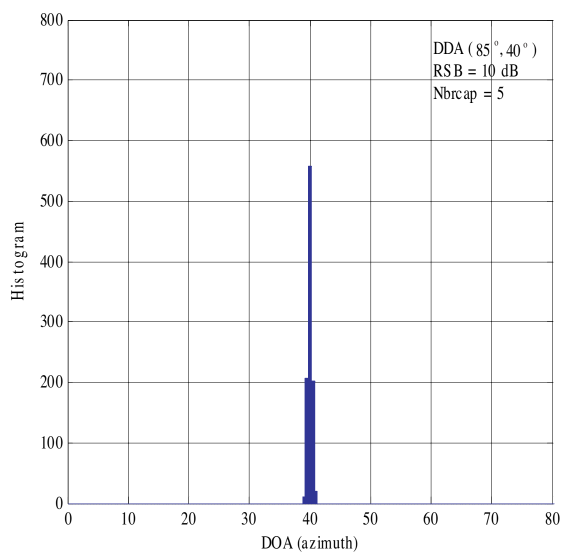

Figure 5. Histogram of azimuth DOA estimations for a single source of DOA at (85°, 40°) $SNR = 10\,\text{dB}$, $N = 5$ elements by using the proposed MUSIC-2-L-shape array configuration.

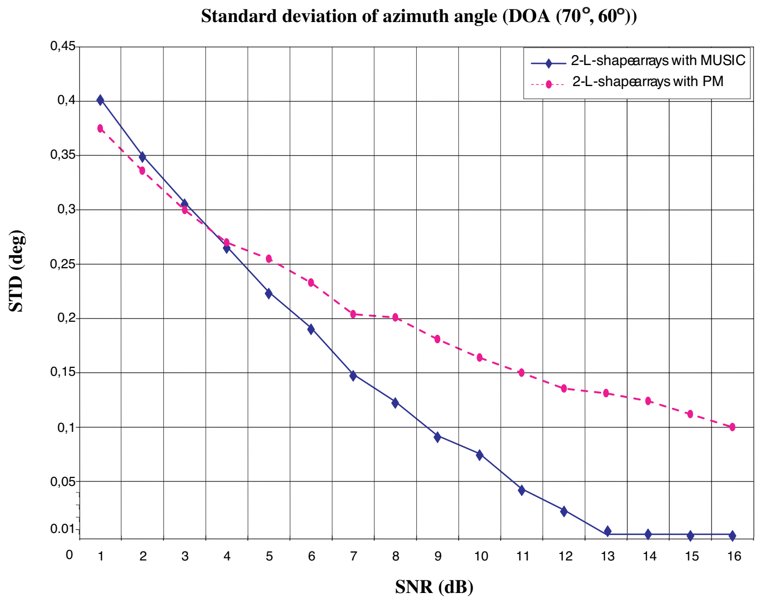

Figure 6. Standard deviation of the azimuth angle estimation versus SNR for a single source at (70°, 60°) using both the PM method in [8] and the proposed MUSIC of the 2-L-shape arrays.

of signal to noise ratio (SNR) cases between 13 dB and 16 dB, using the proposed MUSIC of the 2-L-shape arrays. We observe that the standard deviation tend towards zero when the SNR become high.

Table 4 lists the standard deviation of the elevation angle estimation of the eighth latest values, witch we used for simulation, of signal to noise ratio (SNR) cases between 9 dB and 16 dB. We observe that the standard deviation tend towards zero when the SNR become high.

Figs. 2 and 3 show the histogram plots for the joint elevation and azimuth angles, respectively, for a single source with DOA (85°, 40°) and signal to noise ratio $SNR = 10\,\text{dB}$, by using the PM of the 2-L-shape arrays in [8].

Figs. 4 and 5 show the corresponding histogram plots for the joint elevation and azimuth angles estimation, respectively, by using the proposed MUSIC algorithm of the 2-L-shape arrays

We observe that both of the two methods give a very close joint DOA estimation and the clear peaks appear around (85°, 40°). No failure can be observed in the two cases. However, it is clear that the proposed algorithm improves the performance significantly compared

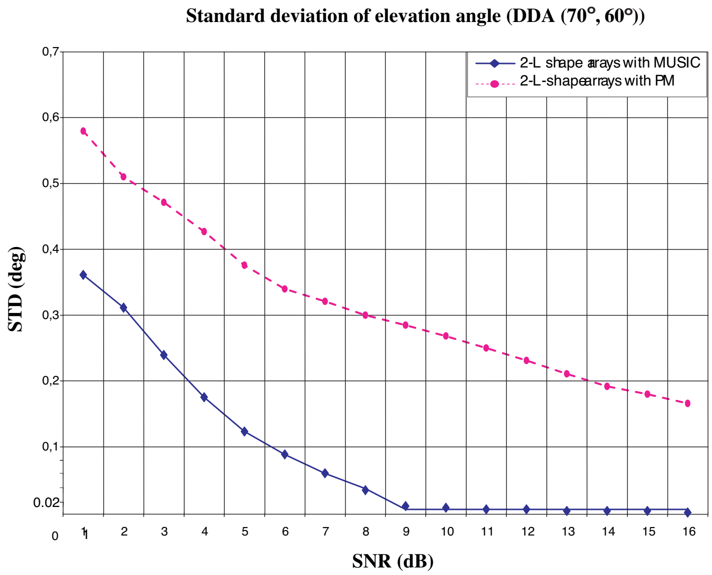

Figure 7. Standard deviation of the elevation angle estimation versus SNR for a single source at (70°, 60°) using both the PM method in [8] and the proposed MUSIC of the 2-L-shape arrays.

to the PM algorithm and it reduces the estimation error of both the azimuth and elevation angles.

Figs. 6 and 7, show the standard deviation of the azimuth and elevation angle estimation versus the SNR in dB for a single source of DOA (70°, 60°), respectively. The total number of elements used is 16 for both methods. It is clear from Fig. 6 that the PM method in [8] is worse than the proposed MUSIC method of the 2-L-shape arrays after 3 dB for the azimuth angle estimation.

In addition, we observe from Fig. 7 that the performance of the proposed algorithm gives much better performance for elevation angle estimation, especially at a low SNR value, than the PM method in [8] and [5].

Tables 1 and 2, list the means, variances and standard deviations of various elevation angle cases between 71° and 89° at a 40° azimuth angle for the PM in [8] and the proposed MUSIC of 2-L-shape arrays, respectively. A single source and 10 dB SNR are considered again. We observe that both of the two methods give good estimation quality with no estimation failure.

As the elevation angle approaches 90°, the DOA estimation quality of the proposed MUSIC method is slightly better than the PM in [8].

Table 1. Means, variances and standard deviation at $SNR = 10\,\text{dB}$ for fixed azimuth angle $\phi = 40°$ and different elevation angle by using the PM method.

in Degrees	Mean of $\hat{\theta}_k$	Variance of $\hat{\theta}_k$	Standard Deviation of $\hat{\theta}_k$
71	71.2528	0.0150	0.1292
74	74.2237	0.0110	0.1047
77	77.1784	0.0068	0.0825
80	80.0998	0.0037	0.0612
83	83.0998	0.0020	0.0442
86	86.0564	0.00063242	0.0251
89	89.0146	0.00037145	0.0061

Table 2. Means, variances and standard deviation at $SNR = 10\,\text{dB}$ for fixed azimuth angle $\phi = 40°$ and different elevation angle by using the Proposed MUSIC method.

θ in Degrees	Mean of θ_k	Variance of θ_k	Standard Deviation of θ_k
71	71.0304	0.0118	0.1049
74	74.0107	0.0103	0.1041
77	77.0068	0.0080	0.0893
80	80.0050	0.0011	0.04048
83	83.0029	0.0008	0.02895
86	86.0010	0.0004	0.00633
89	89.0002	0.0001	0.00096

Table 3. Standard deviation of the azimuth angle estimation at SNR between 13 dB and 16 dB for a single source at $(70°, 60°)$ using the proposed MUSIC of the 2-L-shape arrays.

SNR (dB)	13	14	15	16
STD (deg)	0.0098	0.0064	0.0041	0.0009

Table 4. Standard deviation of the elevation angle estimation at SNR between 9 dB and 16 dB for a single source at $(70°, 60°)$ using the proposed MUSIC of the 2-L-shape arrays.

SNR (dB)	9	10	11	12	13	14	15	16
STD (deg)	0.0019	0.0015	0.0010	0.0008	0.0006	0.0004	0.0002	0.0001

In a typical mobile communication environment, the elevation angle would be between 71° and 89°. Therefore, the proposed algorithm would be more practical than the PM in [8].

5. CONCLUSION

An antenna array configuration was proposed using the MUSIC method, for the 2-D azimuth and elevation angle estimation problem and compared with the PM algorithm of the 2-L-shape arrays in [8]. MUSIC can be applied to all array configurations. The superiority of the proposed algorithm over PM algorithm is shown through simulations.

(1) The proposed scheme reduces the estimation error of both the azimuth and elevation angles.

(2) It shows better performance at low SNR, than the PM scheme.

(3) The MUSIC algorithm reduces the estimation failure problems when elevation angles are between 70° and 90°. Furthermore, the proposed two L-shape algorithm shows no failure for all pair angles.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for their valuable comments that significantly improved the presentation of this paper.

REFERENCES

1. Krekel, P. and E. Deprettre, "A two dimensional version of matrix pencil method to solve the DOA problem," *European Conference on Circuit Theory and Design,* 435—439, 1989.
2. Hua, Y., T. K. Sarkar, and D. D. Weiner, "An L-shape array for estimating 2-D directions of wave arrival," *IEEE Trans. Antennas Propagation,* Vol. 39, 143–146, 1991.
3. Kedia, V. S. and B. Chandna, "A new algorithm for 2-D DOA estimation," *Signal Process.,* Vol. 60, 325–332, 1997.
4. Li, P., B. Yu, and J. Sun, "A new method for two-dimensional array signal processing in unknown noise environments," *Signal Process.,* Vol. 47, 319–327, December 1995.
5. Wu, Y., G. Liao, and H. C. So, "A fast algorithm for 2-D direction-of-arrival estimation," *Signal Processing,* Vol. 83, 1827–1831, August 2003.
6. Liu, T. and J. Mendel, "Azimuth and elevation direction finding using arbitrary array geometries," *IEEE Trans. Signal Processing,* Vol. 46, 2061–2065, july 1998.
7. Tayem, N. and H. M. Kwon, "Azimuth and elevation angle estimation with no failure and no eigen decomposition," *Signal Process.,* Vol. 86, 8–16, May 2005.
8. Tayem, N. and H. M. Kwon, "L-shape-2-D arrival angle estimation with propagator method," *IEEE Trans. Antennas and Propagation,* Vol. 53. 1622–1630, May 2005.
9. Schmidt, R. O., "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas and Propagation,* Vol. 34, 276–280, March 1986.
10. Horn, R. A. and C. R. Johnson, *Matrix Analysis*, Cambridge, University Press, 1989.
11. Johnson, D. H. and D. E. Dudgeon, *Array Signal Processing*, Prentice Hall, Englewood Cliffs, NJ, 1993.
12. Belkebir, K., A. Baussard, and D. Premel, "Edge-preserving regularization scheme applied to modified gradient method to reconstruct two-dimensional targets from data laboratory-

controlled," *Progress In Electromagnetics Research*, PIER 54, 1–17, 2005.

13. Spencer, N. K., "Nonlinear signal processing and nonuniform linear antenna array design for DOA estimation of coherent sources," *Proc. NSIP-99, Antalya*, 849–853, 1999.
14. Abramovich, Y. I. and N. K. Spencer, "Nonuniform linear antenna array design and signal processing for DOA estimation of Gaussian sources," *Proc. DASP-99, LaSalle, IL*, 1–6, 1999.
15. Lee, K.-C., "Frequency-domain analyses of nonlinearly loaded antenna arrays using simulated annealing algorithms," *Progress In Electromagnetics Research*, PIER 53, 271–281, 2005.
16. Sjöberg, D., "Coherent effects in single scattering and random errors in antenna technology," *Progress In Electromagnetics Research*, PIER 50, 13–39, 2005.
17. Selvan, K. T., "A modified three-antenna gain measurement method to simplify uncertainty estimation," *Progress In Electromagnetics Research*, PIER 57, 197–208, 2006.