

FLAT MULTILAYER DIELECTRIC REFLECTOR ANTENNAS

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Abstract—In this paper, a flat multilayer dielectric reflector antenna has been designed and analyzed. The reflector is made of several sandwiched dielectric layers. The structure is hence an array made of these layers, where the sum of the reflection coefficient of each layer acts as the array factor.

By optimizing the permittivity and thickness of each layer, any desired reflection coefficient to shape the reflected pattern can be obtained.

It is also possible to synthesize a zero or a maximum in any arbitrary direction. The significance of the reflector is ability to radiate high power and at the same time having small dimensions.

1. INTRODUCTION

Multilayer dielectric structures are widely used in several fields of electromagnetic applications especially in filter designing. Lots of papers have concentrated on designing and optimizing filters' characteristics for especial applications [1–3]. Also multilayer structures are widely used as the radome of radiating systems [4, 5]. In this article a flat multilayer dielectric structure is used instead of ordinary metallic reflectors in front of radiating antennas. For this purpose at first, in Section 2, the image theory for nonmetallic flat surfaces has been generalized. In Section 3 an especial structure is proposed which is assumed to produce any desired reflection coefficient. The Sections 4 and 5 include an analytically method to synthesize zero or maximum in any desired angle which a method to optimize their bandwidth and beamwidth is followed in Section 6. An example to

In the standard spherical coordinate system (when the surface of the reflector is parallel to yz plane and is placed at the distance of x_0 from the origin) it can be written as

$$E^{total} = E_0(1 + \Gamma(\psi, p, f)e^{-2\beta_0 x_0 \sin \theta \cos \varphi}) \quad (1)$$

By setting $\Gamma = -1$ for perfect conductors, the results will be reduced to ordinary image theory.

This equation shows although metallic flat reflectors cannot have angular sensitive reflection patterns, nonmetallic reflectors can give various shapes to the reflected field. In another word the desired reflected pattern can be obtained if desirable Γ for a surface can be designed. In Section 3 a method to obtain any desirable Γ , using multilayer dielectric structures is developed.

3. MULTILAYER DIELECTRIC ARRAYS (MDA)

Consider the reflector shown in Fig. 2. This reflector is constructed of several lossless dielectric layers placing on top of one another and has infinite area. The dielectric coefficients of layers and their thicknesses are shown by $(\epsilon_{r1}, x_1), (\epsilon_{r2}, x_2), \dots$ and (ϵ_{rn}, x_n) . Primarily, it's assumed that the layers are fixed on each other but no adhesive is used and we don't need to consider its effect. All the substances are nonmagnetic i.e., $\mu_{ri} = 1$. Using small reflection theory we can analyze this problem to calculate the reflection coefficient. This theory allows

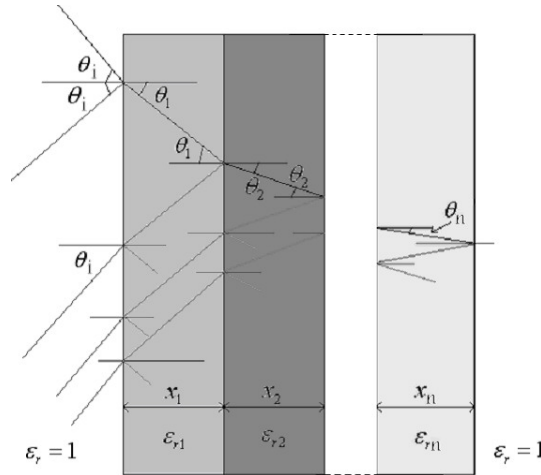


Figure 2. Multilayer reflector prototype. θ_i represents the angle of incident wave.

us to equal the total reflected field to the phasor summation of the first reflection of each layer by assuming small values for reflection coefficients between each layer [7]. By using this theory the obtained expression is much more convenient to use in the design procedure. But if the dielectric coefficients of two adjacent layers differ largely, this approximation will no longer be valid.

The reflection coefficient from i th surface is shown by Γ_i and the total reflection coefficient is

$$\begin{aligned}\Gamma &= \Gamma_1 + \Gamma_2 e^{-2j\beta_1 x_1 \sec \theta_1} + \Gamma_3 e^{-2j\beta_1 x_1 \sec \theta_1} e^{-2j\beta_2 x_2 \sec \theta_2} \\ &\quad + \dots + \Gamma_{N+1} e^{-2j\beta_1 x_1 \sec \theta_1} \dots e^{-2j\beta_N x_N \sec \theta_N} \\ &= \Gamma_1 + \sum_{n=2}^{N+1} \Gamma_n \exp \left(-2j \sum_{i=1}^{n-1} \beta_i x_i \sec \theta_i \right)\end{aligned}\quad (2)$$

θ_i and β_i represent respectively the angle of wave and the propagation constant in i th layer. The waves travel the distance $x_i \sec \theta_i$ between i th and $i+1$ th surfaces. The number of layers is chosen to be odd and also the layers are symmetrically arrayed from the center of the structure. So we have

$$\begin{cases} \varepsilon_r(i) = \varepsilon_r(N+1-i) \\ x_i = x_{N+1-i} \end{cases} \quad 1 \leq i \leq N \quad (3a)$$

This leads to

$$\Gamma_i = -\Gamma_{N+2-i} \quad 1 \leq i \leq N \quad (3b)$$

The wave travels the same distances in symmetric layers. So

$$\theta_i = \theta_{N+1-i} \Rightarrow \beta_i x_i \sec \theta_i = \beta_{N+1-i} x_{N+1-i} \sec \theta_{N+1-i} \quad 1 \leq i \leq N \quad (3c)$$

The thickness of the central layer is chosen to be $2x_{(N+1)/2}$. This choice will be useful to simplify the final relation.

Using the relations of (3) we rewrite the RL in equation (2) as a normalized series of

$$\Gamma = \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(2 \sum_{i=k}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i \right) \quad (4a)$$

Derivation of (4a) can be found in Appendix A. From Snell's law we have

$$\sin \theta_i = \frac{\beta_0}{\beta_i} \sin \theta \Rightarrow \cos \theta_i = \sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta \right)^2}$$

This gives the final relation to describe the multilayer dielectric reflection coefficient as:

$$\Gamma = \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(2 \sum_{i=k}^{\frac{N+1}{2}} \frac{\beta_i x_i}{\sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta \right)^2}} \right) \quad (4b)$$

θ represents the incident angle over the first layer. The relation of (4b) suggests an RL which is akin to array factor of array antennas. Hence we call these reflectors as “**Multilayer Dielectric Arrays**” (MDA).

For vertical polarization we have:

$$\begin{aligned} \Gamma_k &= \frac{\cos \theta_k - \sqrt{\xi_k - \sin^2 \theta_k}}{\cos \theta_k + \sqrt{\xi_k - \sin^2 \theta_k}} \\ &= \frac{\sqrt{1 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2} - \sqrt{\left(\frac{\beta_k}{\beta_{k-1}} \right)^2 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2}}{\sqrt{1 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2} + \sqrt{\left(\frac{\beta_k}{\beta_{k-1}} \right)^2 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2}} \end{aligned} \quad (5a)$$

and for parallel polarization:

$$\begin{aligned} \Gamma_k &= \frac{-\xi_k \cos \theta_k + \sqrt{\xi_k - \sin^2 \theta_k}}{\xi_k \cos \theta_k + \sqrt{\xi_k - \sin^2 \theta_k}} \\ &= \frac{-\left(\frac{\beta_k}{\beta_{k-1}} \right)^2 \sqrt{1 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2} + \sqrt{\left(\frac{\beta_k}{\beta_{k-1}} \right)^2 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2}}{\left(\frac{\beta_k}{\beta_{k-1}} \right)^2 \sqrt{1 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2} + \sqrt{\left(\frac{\beta_k}{\beta_{k-1}} \right)^2 - \left(\frac{\beta_0}{\beta_k} \sin \theta \right)^2}} \end{aligned} \quad (5b)$$

Where $\xi_k = (\beta_k/\beta_{k-1})^2$ [6]. Now we are capable of designing an MDA with any desired RL using any optimization algorithm. For example a desired RL for vertical polarization may have a maximum (=1) between two minimums (=0) i.e., $\theta_{Min1} < \theta_{Max} < \theta_{Min2}$. So we can choose the sinc function as a goal function and GA as the optimization algorithm.

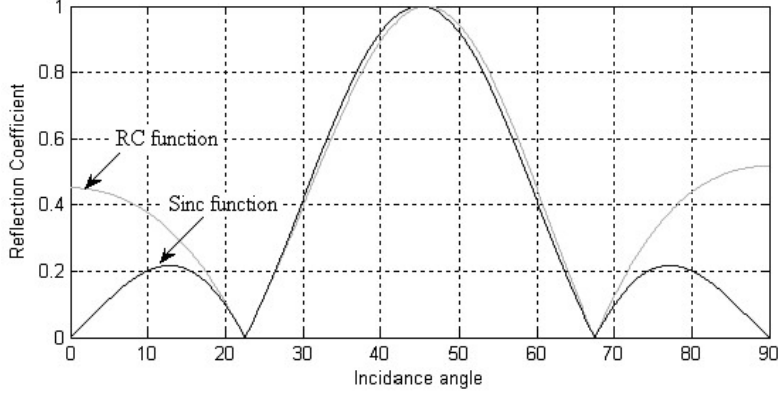


Figure 3. Typical obtained RL using GA algorithm.

For a seven-layer structure we have

$$\Gamma = \sum_{k=1}^4 \frac{\sqrt{1 - \frac{\sin^2 \theta}{\varepsilon_{rk}}} - \sqrt{\frac{\varepsilon_{rk}}{\varepsilon_{rk-1}} - \frac{\sin^2 \theta}{\varepsilon_{rk}}}}{\sqrt{1 - \frac{\sin^2 \theta}{\varepsilon_{rk}}} + \sqrt{\frac{\varepsilon_{rk}}{\varepsilon_{rk-1}} - \frac{\sin^2 \theta}{\varepsilon_{rk}}}} \sin \left(2 \sum_{i=k}^4 \frac{\varepsilon_{ri} \omega \sqrt{\mu_0 \varepsilon_0} x_i}{\sqrt{1 - \frac{\sin^2 \theta}{\varepsilon_{ri}}}} \right)$$

and our goal function for $\theta_{Max} = 45$, $\theta_{Min1} = 22.5$ and $\theta_{Min2} = 67.5$ degrees becomes

$$\Gamma = \text{sinc} \left(\frac{\theta - 45^\circ}{22.5^\circ} \right)$$

A typical RL for an MDA at frequency of 3 GHz is shown in Fig. 3. It can be seen in Fig. 3, the obtained RL is very similar to the sinc function, except at incidence angles less than 15 and greater than 75 degrees. We will show in Section 6.2 that the derivative of RL function with respect to θ , at $\theta = 0$ and $\theta = 90$ is always equal to zero, so at these points it cannot follow the sinc function.

The sensitivity of RL in (4b), to ε_i s and x_i s can be calculated. Due to undesired variations of layers' permittivity (because of nonlinear behavior or inexact implementation) the sensitivity parameter can be defined as

$$S_{\varepsilon}^{\Gamma} = \frac{d\Gamma}{\Gamma} = \frac{1}{\Gamma} \left[\frac{\partial \Gamma}{\partial \varepsilon_1} \Big|_{\forall k \neq 1; d\varepsilon_k = 0} d\varepsilon_1 + \cdots + \frac{\partial \Gamma}{\partial \varepsilon_{\frac{N+1}{2}}} \Big|_{\forall k \neq (N+1)/2; d\varepsilon_k = 0} d\varepsilon_{\frac{N+1}{2}} \right]$$

in which

$$\begin{aligned} \frac{\partial \Gamma}{\partial \varepsilon_s} = & \frac{\omega x_s}{C} \frac{\varepsilon_s - 2 \sin^2 \theta}{(\varepsilon_s - \sin^2 \theta)^{3/2}} \sum_{k=1}^s \Gamma_k(\varepsilon_k, \varepsilon_{k-1}) \cos \left(2 \frac{\omega}{C} \sum_{i=k}^{\frac{N+1}{2}} \frac{\varepsilon_i x_i}{\sqrt{\varepsilon_i - \sin^2 \theta}} \right) \\ & + \sum_{k=s}^{s+1} \frac{\partial \Gamma_k(\varepsilon_k, \varepsilon_{k-1})}{\partial \varepsilon_k} \sin \left(2 \frac{\omega}{C} \sum_{i=k}^{\frac{N+1}{2}} \frac{\varepsilon_i x_i}{\sqrt{\varepsilon_i - \sin^2 \theta}} \right) \end{aligned}$$

If Γ_k variations are negligible, the above relation can be rewritten as

$$\frac{\partial \Gamma}{\partial \varepsilon_s} \approx \frac{\omega x_s}{C} \frac{\varepsilon_s - 2 \sin^2 \theta}{(\varepsilon_s - \sin^2 \theta)^{3/2}} \sum_{k=1}^s \Gamma_k(\varepsilon_k, \varepsilon_{k-1}) \cos \left(2 \frac{\omega}{C} \sum_{i=k}^{\frac{N+1}{2}} \frac{\varepsilon_i x_i}{\sqrt{\varepsilon_i - \sin^2 \theta}} \right)$$

So if the term of $\frac{\omega x_s}{C}$ is great, the reflected pattern will have a high sensitivity to ε_s . The sensitivity to the layers' thickness can be defined as

$$\begin{aligned} S_x^\Gamma &= \frac{d\Gamma}{\Gamma} \\ &= \frac{1}{\Gamma} \left[\frac{\partial \Gamma}{\partial x_1} \Big|_{\forall k \neq 1; dx_k=0} dx_1 + \dots + \frac{\partial \Gamma}{\partial x_{\frac{N+1}{2}}} \Big|_{\forall k \neq (N+1)/2; dx_k=0} dx_{\frac{N+1}{2}} \right] \end{aligned}$$

in which

$$\frac{\partial \Gamma}{\partial x_s} = \frac{\omega \varepsilon_s}{C} \frac{2}{\sqrt{\varepsilon_s - \sin^2 \theta}} \sum_{k=1}^s \Gamma_k(\varepsilon_k, \varepsilon_{k-1}) \cos \left(2 \frac{\omega}{C} \sum_{i=k}^{\frac{N+1}{2}} \frac{\varepsilon_i x_i}{\sqrt{\varepsilon_i - \sin^2 \theta}} \right)$$

And in the same manner if the term of $\frac{\omega \varepsilon_s}{C}$ is great, the reflected pattern will have a high sensitivity to x_s . These equations show that in high frequency applications the implementation must be done as exactly as possible. Also comparing these two equations leads to

$$x_s \frac{\partial \Gamma}{\partial x_s} \approx \varepsilon_s \frac{\partial \Gamma}{\partial \varepsilon_s} \frac{2\varepsilon_s - 2 \sin^2 \theta}{\varepsilon_s - 2 \sin^2 \theta}$$

This shows that the sensitivity of RL to the thickness is greater than the sensitivity to the permittivity.

4. ZERO SYNTHESIZING AT DESIRED ANGLES

Although we are able to design arbitrary RL using GA, analytical syntheses of zero or maximum at desired angles is possible.

To synthesize a zero at $\theta = \theta_0$ we must have $\Gamma|_{\theta=\theta_0} = 0$. This leads to

$$\begin{aligned} 0 &= \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(2 \sum_{i=k}^{\frac{N+1}{2}} \frac{\beta_i x_i}{\sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta \right)^2}} \right) \\ &= \Gamma_{\frac{N+1}{2}} \sin \left(\psi_{0\frac{N+1}{2}} \right) + \Gamma_{\frac{N-1}{2}} \sin \left(\psi_{0\frac{N+1}{2}} + \psi_{0\frac{N-1}{2}} \right) + \\ &\quad \dots + \Gamma_1 \sin \left(\psi_{0\frac{N+1}{2}} + \dots + \psi_{01} \right) \end{aligned}$$

In which

$$\psi_{0i} = \frac{2\beta_i x_i}{\sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta \right)^2}}$$

Here we have two degrees of freedom to determine the values of ψ_{0i} s namely β_i s and x_i s. If we equate all ψ_{0i} s to $n\pi$, the RL at $\theta = \theta_0$ will be equal to zero. Hence

$$\begin{aligned} \psi_{0i} &= \frac{2\beta_i x_i}{\sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta \right)^2}} = n\pi \Rightarrow \\ x_i &= \frac{n\pi}{2\beta_i} \sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta \right)^2} \quad n = 1, 2, \dots \end{aligned} \quad (6)$$

The relation of (6) determines the thickness of each layer in which the value of RL vanishes at $\theta = \theta_0$. As it can be seen these values are independent of the number of layers. Fig. 4 shows the normalized thickness of each layer for an arbitrary multilayer structure versus β_i/β_0 for desired angles.

To separate the curvatures in Fig. 4 further, different values of n can be chosen for each angle. The final form of RL with a zero at θ_0 becomes

$$\Gamma = \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(\pi \frac{\omega_0}{\omega} \sum_{i=k}^{\frac{N+1}{2}} \sqrt{\frac{\varepsilon_{ri} - \sin^2 \theta_0}{\varepsilon_{ri} - \sin^2 \theta}} \right) \quad (7)$$

In which ω_0 is the central frequency and ω is the working frequency.

As a practice these results can be evaluated for the reflector shown in Fig. 5 which is merely composed of one layer. The exact relations

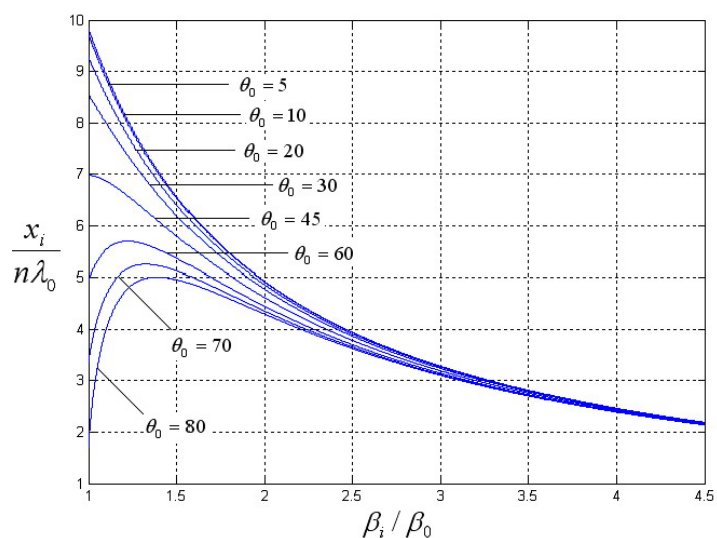


Figure 4. Thickness of each layer relative to wavelength in free space for an arbitrary multilayer structure versus β_i/β_0 for desired angles. n is an integer.

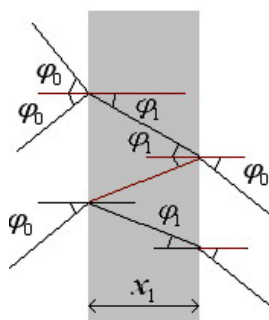


Figure 5. Single layer reflector.

of this reflector for two polarizations are

$$\Gamma_{\perp} = \frac{\frac{\cos \varphi_0 - \sqrt{\varepsilon_r - \sin^2 \varphi_0}}{\cos \varphi_0 + \sqrt{\varepsilon_r - \sin^2 \varphi_0}} \left[1 - \exp \left(-2j\beta_1 x_1 / \sqrt{1 - (\sin^2 \varphi_0)/\varepsilon_r} \right) \right]}{1 - \left(\frac{\cos \varphi_0 - \sqrt{\varepsilon_r - \sin^2 \varphi_0}}{\cos \varphi_0 + \sqrt{\varepsilon_r - \sin^2 \varphi_0}} \right)^2 \exp \left(-2j\beta_1 x_1 / \sqrt{1 - (\sin^2 \varphi_0)/\varepsilon_r} \right)}$$

$$\Gamma_{\parallel} = \frac{\frac{-\varepsilon_r \cos \varphi_0 + \sqrt{\varepsilon_r - \sin^2 \varphi_0}}{\varepsilon_r \cos \theta_0 + \sqrt{\varepsilon_r - \sin^2 \theta_0}} \left[1 - \exp \left(-2j\beta_1 x_1 / \sqrt{1 - (\sin^2 \varphi_0)/\varepsilon_r} \right) \right]}{1 - \left(\frac{-\varepsilon_r \cos \varphi_0 + \sqrt{\varepsilon_r - \sin^2 \varphi_0}}{\varepsilon_r \cos \theta_0 + \sqrt{\varepsilon_r - \sin^2 \theta_0}} \right)^2 \exp \left(-2j\beta_1 x_1 / \sqrt{1 - (\sin^2 \varphi_0)/\varepsilon_r} \right)}$$

From (6) the thickness of the layer is determined as

$$x_1 = \frac{n\pi}{\beta_0} \sqrt{1 - \frac{\sin^2 \varphi_{0z}}{\varepsilon_r}} \quad n = 1, 2, \dots$$

Fig. 6 shows the RL of this reflector for the thicknesses of $x_1 = 0.3\lambda_1$, $x_1 = 2\lambda_1$ and $x_1 = 2.4\lambda_1$ and different dielectric coefficients. The locations of the zeros confirm our formulation. It also shows that the synthesized zeros are exact ones.

5. MAXIMUM SYNTHESIZING AT DESIRED ANGLES

By assuming that all Γ_i s in (4b) are positive, the arguments of Γ in

$$\Gamma_{\max} = \Gamma_{\frac{N+1}{2}} \sin \left(\psi_{M \frac{N+1}{2}} \right) + \Gamma_{\frac{N-1}{2}} \sin \left(\psi_{M \frac{N+1}{2}} + \psi_{M \frac{N-1}{2}} \right) + \dots + \Gamma_1 \sin \left(\psi_{M \frac{N+1}{2}} + \dots + \psi_{M1} \right)$$

can be chosen as

$$\psi_{M \frac{N+1}{2}} = \pi/2 \quad \psi_{Mi} = 2\pi \quad 1 \leq i \leq \frac{N-1}{2}$$

to have a maximum at θ_M . Therefore

$$\psi_{M \frac{N+1}{2}} = \frac{2\beta_{\frac{N+1}{2}} x_{\frac{N+1}{2}}}{\sqrt{1 - \left(\frac{\beta_0}{\beta_{\frac{N+1}{2}}} \sin \theta_M \right)^2}} = \pi/2 \Rightarrow$$

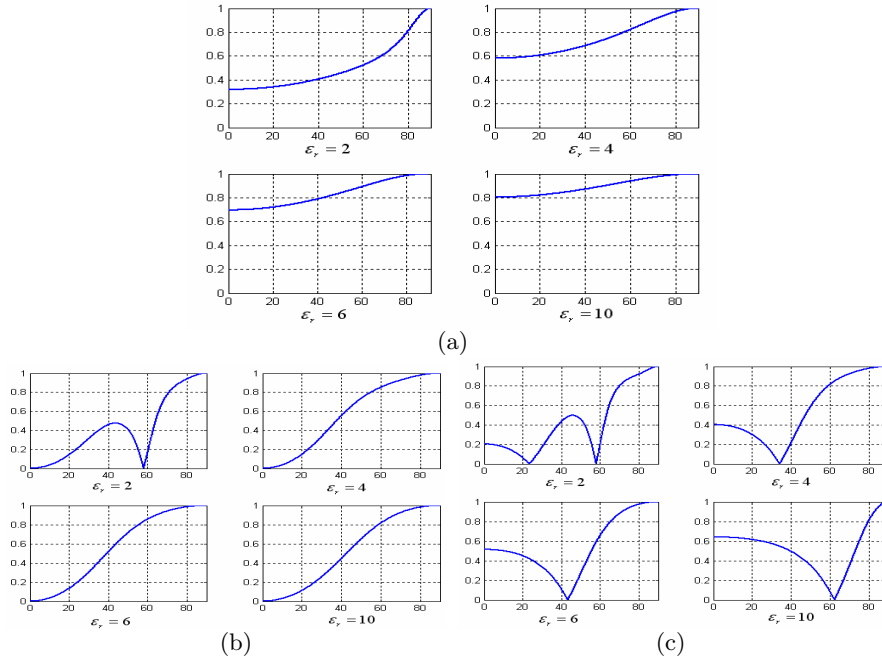


Figure 6. The RLs for single layer reflector. (a) $x_1 = 0.3\lambda_1$ (b) $x_1 = 2\lambda_1$ (c) $x_1 = 2.4\lambda_1$.

$$\begin{aligned}
 x_{\frac{N+1}{2}} &= \frac{\pi}{4\beta_{\frac{N+1}{2}}} \sqrt{1 - \left(\frac{\beta_0}{\beta_{\frac{N+1}{2}}} \sin \theta_M \right)^2} \\
 \psi_{Mi} &= \frac{2\beta_i x_i}{\sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta_M \right)^2}} = 2\pi \Rightarrow \\
 x_i &= \frac{\pi}{\beta_i} \sqrt{1 - \left(\frac{\beta_0}{\beta_i} \sin \theta_M \right)^2} \quad 1 \leq i \leq \frac{N-1}{2} \quad (8)
 \end{aligned}$$

The RL for this maximum will be equal to

$$\Gamma = \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \frac{\omega_0}{\omega} \left(\frac{\pi}{2} \sqrt{\frac{\varepsilon_{r\frac{N+1}{2}} - \sin^2 \theta_M}{\varepsilon_{r\frac{N+1}{2}} - \sin^2 \theta}} + 2\pi \sum_{i=k}^{\frac{N-1}{2}} \sqrt{\frac{\varepsilon_{ri} - \sin^2 \theta_M}{\varepsilon_{ri} - \sin^2 \theta}} \right) \quad (9)$$

In next sections we'll concentrate on zero synthesizing which is more interesting. All the procedure can be easily generalized for maximums.

6. BANDWIDTH AND BEAMWIDTH

To derive exact relations of bandwidth and beamwidth for the given RL equations in (7) and (9) we need the values of Γ_k . This can be done parametrically which may lead to intricate relations. As mentioned before this method is restricted to equation (7).

6.1. Bandwidth

Due to (7) we have

$$\begin{aligned}\Gamma &= \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(\frac{\omega_0}{\omega} \sum_{i=k}^{\frac{N+1}{2}} \gamma_i \right) \\ &= \Gamma_1 \sin \left(\frac{\omega_0}{\omega} \left(\gamma_{\frac{N+1}{2}} + \dots + \gamma_1 \right) \right) + \dots + \Gamma_{\frac{N+1}{2}} \sin \left(\frac{\omega_0}{\omega} \gamma_{\frac{N+1}{2}} \right)\end{aligned}$$

where $\gamma_i = \pi \sqrt{\frac{\varepsilon_{ri} - \sin^2 \theta_0}{\varepsilon_{ri} - \sin^2 \theta}}$.

By rewriting this equation at $\theta = \theta_0$ we get

$$\begin{aligned}\Gamma|_{\theta=\theta_0} &= \Gamma_1 \sin \left(\frac{\omega_0}{\omega} \frac{N+1}{2} \pi \right) + \dots + \Gamma_{\frac{N+1}{2}} \sin \left(\frac{\omega_0}{\omega} \pi \right) \\ &= \Gamma_1 \sin \left(\frac{1}{\bar{\omega}} \frac{N+1}{2} \pi \right) + \dots + \Gamma_{\frac{N+1}{2}} \sin \left(\frac{1}{\bar{\omega}} \pi \right)\end{aligned}\quad (10)$$

Where $\bar{\omega} = \frac{\omega}{\omega_0}$. We can equate (10) to sinusoidal expansion of any desired function of $n/\bar{\omega}$.

Another way to achieve a flatter response is to calculate the derivatives of Γ with respect to $\bar{\omega}$ at $\bar{\omega} = 1$ and try to minimize them. A useful objective function can be

$$OF = \sqrt{\sum_{i=1} W_i \left(\frac{\partial^i}{\partial \bar{\omega}^i} \left\{ \Gamma|_{\theta=\theta_0} \right\}_{\bar{\omega}=1} \right)^2} \quad (11)$$

W_i s are suitable weighting coefficients. The function can be optimized using any optimization algorithm. For a relatively flat response we can set $W_1 = 1$ and the others equal to zero. So we have

$$OF = \frac{\partial}{\partial \bar{\omega}} \left[\Gamma|_{\theta=\theta_0} \right]_{\bar{\omega}=1}$$

$$\begin{aligned}
&= \left[-\Gamma_1 \frac{1}{\bar{\omega}^2} \frac{N+1}{2} \pi \cos \left(\frac{1}{\bar{\omega}} \frac{N+1}{2} \pi \right) - \cdots - \Gamma_{\frac{N+1}{2}} \frac{1}{\bar{\omega}^2} \pi \cos \left(\frac{1}{\bar{\omega}} \pi \right) \right]_{\bar{\omega}=1} \\
&= -\Gamma_1 \frac{N+1}{2} \pi \cos \left(\frac{N+1}{2} \pi \right) - \cdots + \Gamma_{\frac{N+1}{2}} \pi
\end{aligned} \tag{12}$$

6.2. Beamwidth

Again we return to (7) and assume $\omega = \omega_0$.

$$\begin{aligned}
\Gamma &= \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(\sum_{i=k}^{\frac{N+1}{2}} \gamma_i \right) \\
&= \Gamma_1 \sin \left(\gamma_{\frac{N+1}{2}} + \cdots + \gamma_1 \right) + \cdots + \Gamma_{\frac{N+1}{2}} \sin \left(\gamma_{\frac{N+1}{2}} \right)
\end{aligned}$$

We can also assume that the variations of Γ_k s with respect to θ are negligible in contrast with γ_i s' variations. So we calculate the variations of Γ_k with respect to θ at θ_0 as

$$\begin{aligned}
\frac{\partial \Gamma}{\partial \theta} &= \Gamma_1 \left(\sum_{i=1}^{\frac{N+1}{2}} \frac{\partial \sin \left(\gamma_{\frac{N+1}{2}} + \cdots + \gamma_1 \right)}{\partial \gamma_i} \frac{d\gamma_i}{d\theta} \right) + \\
&\quad \cdots + \Gamma_{\frac{N+1}{2}} \frac{\partial \sin \left(\gamma_{\frac{N+1}{2}} \right)}{\partial \gamma_{\frac{N+1}{2}}} \frac{d\gamma_{\frac{N+1}{2}}}{d\theta} \\
&= \Gamma_1 \left(\cos \left(\gamma_{\frac{N+1}{2}} + \cdots + \gamma_1 \right) \sum_{i=1}^{\frac{N+1}{2}} \frac{d\gamma_i}{d\theta} \right) + \\
&\quad \cdots + \Gamma_{\frac{N+1}{2}} \cos \left(\gamma_{\frac{N+1}{2}} \right) \frac{d\gamma_{\frac{N+1}{2}}}{d\theta}
\end{aligned}$$

But we have

$$\frac{d\gamma_i}{d\theta} = \gamma_i \frac{\sin 2\theta}{2\sqrt{\varepsilon_{ri} - \sin^2 \theta}} \Big|_{\theta=\theta_0} = \frac{\pi}{2} \frac{\sin 2\theta_0}{\sqrt{\varepsilon_{ri} - \sin^2 \theta_0}}$$

And finally

$$\begin{aligned}
\frac{d\Gamma}{d\theta} \Big|_{\theta=\theta_0} &= \frac{\pi}{2} \sin 2\theta_0 \left[\Gamma_1 \left(\cos \left(\frac{N+1}{2} \pi \right) \sum_{i=1}^{\frac{N+1}{2}} \frac{1}{\sqrt{\varepsilon_{ri} - \sin^2 \theta_0}} \right) \right. \\
&\quad \left. + \cdots - \frac{\Gamma_{\frac{N+1}{2}}}{\sqrt{\varepsilon_{r\frac{N+1}{2}} - \sin^2 \theta_0}} \right]
\end{aligned} \tag{13}$$

This relation shows that the RL function of any MDA has two relevant extremums at $\theta_0 = 0^\circ$ and $\theta_0 = 90^\circ$. This result is so important and we can check its correctness by referring to Figures of 3 and 6. GA can be used to minimize or maximize (depending on the purpose) the value of $\frac{\partial \Gamma}{\partial \theta} \Big|_{\theta=\theta_0}$.

In case of not achieving an acceptable beamwidth, another objective function can be defined as

$$OF' = \sqrt{\sum_{i=1} W_i' \left(\frac{\partial^i}{\partial \theta^i} [\Gamma|_{\bar{\omega}=1}]_{\theta=\theta_0} \right)^2} \quad (14)$$

Also we can consider variations of Γ_k with respect to θ in it.

In tracking problems, when we attend to determine the angle of target existence exactly, we can synthesize a zero at an angle less than 45 degrees. For this situation and for vertical polarization due to relation (5a), we have

$$\begin{aligned} \Gamma_k &= \frac{\sqrt{1 - \left(\frac{\beta_0}{\beta_k} \sin \theta\right)^2} - \sqrt{\left(\frac{\beta_k}{\beta_{k-1}}\right)^2 - \left(\frac{\beta_0}{\beta_k} \sin \theta\right)^2}}{\sqrt{1 - \left(\frac{\beta_0}{\beta_k} \sin \theta\right)^2} + \sqrt{\left(\frac{\beta_k}{\beta_{k-1}}\right)^2 - \left(\frac{\beta_0}{\beta_k} \sin \theta\right)^2}} \\ &\approx \frac{1 - \frac{\beta_k}{\beta_{k-1}}}{1 + \frac{\beta_k}{\beta_{k-1}}} = \frac{\beta_{k-1} - \beta_k}{\beta_{k-1} + \beta_k} \Rightarrow \Gamma_k \approx \frac{\sqrt{\varepsilon_{r(k-1)}} - \sqrt{\varepsilon_{r(k)}}}{\sqrt{\varepsilon_{r(k-1)}} + \sqrt{\varepsilon_{r(k)}}} \end{aligned}$$

7. A PRACTICAL EXAMPLE OF MDA REFLECTORS

In this section an example is introduced for a nonmetallic reflector. This reflector is for vertical polarized waves which must have a zero at $\theta_0 = 45$ with first order flatness for bandwidth but very low beam width. These properties can be used to exact tracking of mobile targets. Thus

$$\Gamma|_{\theta=45, \bar{\omega}=1} = 0 \quad (15a)$$

$$\frac{\partial}{\partial \bar{\omega}} [\Gamma|_{\theta=45}] \Big|_{\bar{\omega}=1} = 0 \quad (15b)$$

$$\frac{\partial}{\partial \theta} [\Gamma|_{\bar{\omega}=1}] \Big|_{\theta=45} = 0 \quad (15c)$$

A 5-layer MDA is chosen to satisfy desired requirements. The RL of this structure is

$$\Gamma = \sum_{k=1}^3 \frac{\sqrt{\varepsilon_{r(k-1)}} - \sqrt{\varepsilon_{r(k)}}}{\sqrt{\varepsilon_{r(k-1)}} + \sqrt{\varepsilon_{r(k)}}} \sin \left(\pi \frac{\omega_0}{\omega} \sum_{i=k}^3 \sqrt{\frac{\varepsilon_{ri} - 0.5}{\varepsilon_{ri} - \sin^2 \theta}} \right)$$

By referring to (11) and (12) we must have

$$\begin{aligned} \frac{\partial}{\partial \omega} [\Gamma|_{\theta=45}] \Big|_{\omega=1} &= \pi(3\Gamma_1 - 2\Gamma_2 + \Gamma_3) \rightarrow 0 \\ \Rightarrow F_1 &= 3 \frac{1 - \sqrt{\varepsilon_{r1}}}{1 + \sqrt{\varepsilon_{r1}}} - 2 \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} + \frac{\sqrt{\varepsilon_{r2}} - \sqrt{\varepsilon_{r3}}}{\sqrt{\varepsilon_{r2}} + \sqrt{\varepsilon_{r3}}} \rightarrow 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \Gamma}{\partial \theta} \Big|_{\theta=45} &= \frac{\pi}{2} \left[\Gamma_1 \left(\cos \left(\frac{N+1}{2} \pi \right) \sum_{i=1}^3 \frac{1}{\sqrt{\varepsilon_{ri} - 0.5}} \right) + \dots - \frac{\Gamma_3}{\sqrt{\varepsilon_{r3} - 0.5}} \right] \\ &\quad \xrightarrow{\rightarrow \infty} \\ \Rightarrow F_2 &= \left[\begin{aligned} &-\frac{1 - \sqrt{\varepsilon_{r1}}}{1 + \sqrt{\varepsilon_{r1}}} \left(\frac{1}{\sqrt{\varepsilon_{r1} - 0.5}} + \frac{1}{\sqrt{\varepsilon_{r2} - 0.5}} + \frac{1}{\sqrt{\varepsilon_{r3} - 0.5}} \right) \\ &+ \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} \left(\frac{1}{\sqrt{\varepsilon_{r2} - 0.5}} + \frac{1}{\sqrt{\varepsilon_{r3} - 0.5}} \right) \\ &- \frac{\sqrt{\varepsilon_{r2}} - \sqrt{\varepsilon_{r3}}}{\sqrt{\varepsilon_{r2}} + \sqrt{\varepsilon_{r3}}} \frac{1}{\sqrt{\varepsilon_{r3} - 0.5}} \end{aligned} \right] \xrightarrow{\rightarrow \infty} \end{aligned}$$

Our objective function can be set to be $OF = F_1/F_2$. Using the genetic algorithm we can obtain the following results.

$$\varepsilon_1 = 2.25 \quad \varepsilon_2 = 6 \quad \varepsilon_3 = 3$$

The thicknesses of each layer can be calculated using relation of (6). The Obtained RL is shown in Fig. 7. The central frequency is 3 GHz and the bandwidth is about 6 MHz. it's evident that a better bandwidth can be obtained by reducing the upper order derivatives of RL to zero.

For example if we put this reflector in front of a very long wire carrying a constant electric current, we can calculate the reflected and total electric field patterns. The generated fields of this source for the first mode are [6]

$$\begin{cases} E_z^i = -I_e \frac{\beta^2}{4\omega\varepsilon_0} H_0^{(2)}(\beta\rho) a_z \\ H_\varphi^i = -jI_e H_1^{(2)}(\beta\rho) a_\varphi \end{cases}$$

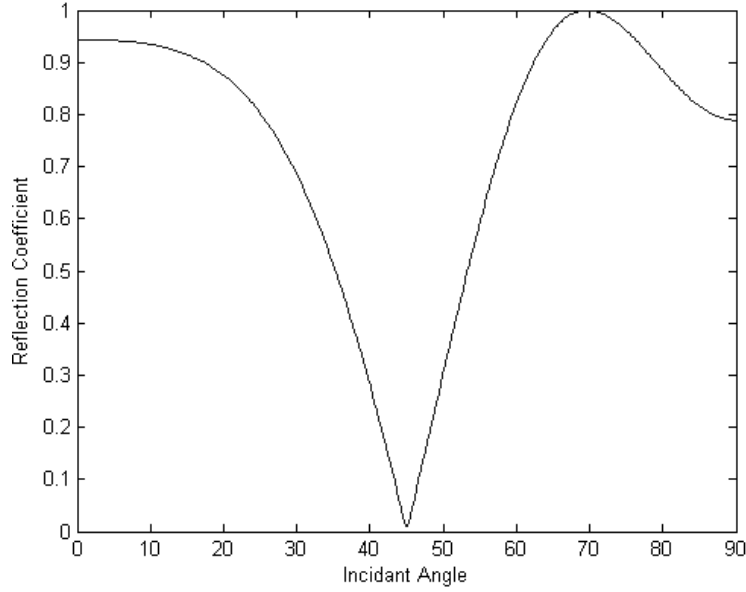


Figure 7. Obtained RL with acceptable beamwidth and bandwidth.

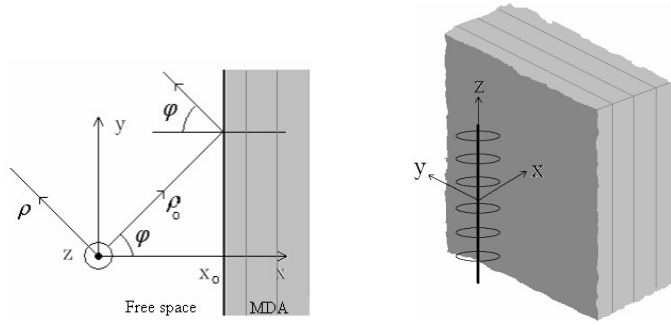


Figure 8. Geometry of an infinite electric line in front of designed MDA.

The geometry of problem is shown in Fig. 8. From (1) we can calculate the total electric field

$$\begin{aligned}
 E^{total} &= E_0 \left(1 + \Gamma e^{-2x_0 \beta_0 \sin \theta \cos \varphi} \right) \\
 &= -a_z I_e \frac{\beta^2}{4\omega \varepsilon_0} H_0^{(2)}(\beta \rho)
 \end{aligned}$$

$$\times \left[1 + \sum_{k=1}^3 \frac{\sqrt{\varepsilon_r(k-1)} - \sqrt{\varepsilon_r(k)}}{\sqrt{\varepsilon_r(k-1)} + \sqrt{\varepsilon_r(k)}} \sin \left(\pi \frac{\omega_0}{\omega} \sum_{i=k}^3 \sqrt{\frac{\varepsilon_{ri} - 0.5}{\varepsilon_{ri} - \sin^2 \theta}} \right) \times e^{-2x_0 \beta_0 \sin \theta \cos \varphi} \right]$$

With normalized form of

$$E^{total} = 1 + \exp(-2x_0 \beta_0 \sin \theta \cos \varphi) \times \sum_{k=1}^3 \frac{\sqrt{\varepsilon_r(k-1)} - \sqrt{\varepsilon_r(k)}}{\sqrt{\varepsilon_r(k-1)} + \sqrt{\varepsilon_r(k)}} \sin \left(\pi \frac{\omega_0}{\omega} \sum_{i=k}^3 \sqrt{\frac{\varepsilon_{ri} - 0.5}{\varepsilon_{ri} - \sin^2 \theta}} \right)$$

The reflected pattern is shown in Fig. 9

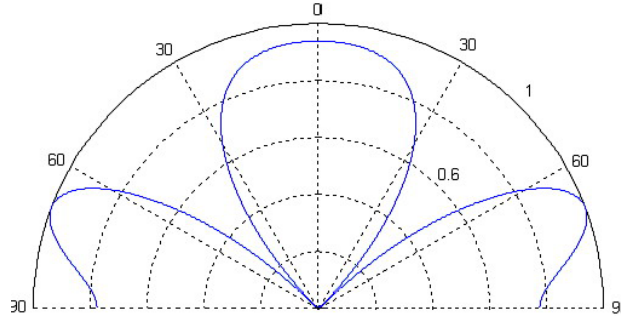


Figure 9. Reflected pattern versus incident angle.

8. CONCLUSION

In this paper, multilayer dielectric structures are proposed which can be used as reflectors in front of any radiating source. These structures are able to give any desired pattern to reflected waves. The power of source is restricted just by dielectric strength which can be very high. Also the ability of synthesizing a zero or a maximum at desired angles is proved analytically using equations of (6) and (8) and then its bandwidth and beamwidth are improved by optimizing objective functions of (11) and (14). Since flat multilayer dielectric reflectors can produce a maximum at a desired angle and minimize the side lobes around it, they can be similar to the curved metallic reflectors.

The Design procedure may lead to use dielectrics with no realizable permittivity where their implementation is difficult. This and non optimum usage of the source power are two disadvantages of these reflectors.

APPENDIX A.

In this appendix (4a) is derived. From (2) we have

$$\begin{aligned}
\Gamma &= \Gamma_1 \left(1 - e^{-2j \sum_{i=1}^N \beta_i x_i \sec \theta_i} \right) + \Gamma_2 e^{-2j \beta_1 x_1 \sec \theta_1} \left(1 - e^{-2j \sum_{i=2}^{N-1} \beta_i x_i \sec \theta_i} \right) \\
&\quad + \dots + \Gamma_{\frac{N+1}{2}} e^{-2j \sum_{i=1}^{\frac{N-1}{2}} \beta_i x_i \sec \theta_i} \left(1 - e^{-2 \times 2j \beta_{\frac{N+1}{2}} x_{\frac{N+1}{2}} \sec \theta_{\frac{N+1}{2}}} \right) \\
&= 2j \Gamma_1 e^{-j2 \sum_{i=1}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i} \sin \left(2 \sum_{i=1}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i \right) \\
&\quad + 2j \Gamma_2 e^{-2j \beta_1 x_1 \sec \theta_1} e^{-j2 \sum_{i=2}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i} \\
&\quad \times \sin \left(2 \sum_{i=2}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i \right) + \dots \\
&\quad + 2j \Gamma_{\frac{N+1}{2}} e^{-2j \sum_{i=1}^{\frac{N-1}{2}} \beta_i x_i \sec \theta_i} e^{-j2 \beta_{\frac{N+1}{2}} x_{\frac{N+1}{2}} \sec \theta_{\frac{N+1}{2}}} \\
&\quad \times \sin \left(2 \beta_{\frac{N+1}{2}} x_{\frac{N+1}{2}} \sec \theta_{\frac{N+1}{2}} \right) \\
\Rightarrow \Gamma &= 2j e^{-j2 \sum_{i=1}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i} \left\{ \Gamma_1 \sin \left(2 \sum_{i=1}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i \right) \right. \\
&\quad + \Gamma_2 \sin \left(2 \sum_{i=1}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i \right) + \dots \\
&\quad \left. + \Gamma_{\frac{N+1}{2}} \sin \left(2 \beta_{\frac{N+1}{2}} x_{\frac{N+1}{2}} \sec \theta_{\frac{N+1}{2}} \right) \right\} \\
\Gamma &= \sum_{k=1}^{\frac{N+1}{2}} \Gamma_k \sin \left(2 \sum_{i=k}^{\frac{N+1}{2}} \beta_i x_i \sec \theta_i \right)
\end{aligned}$$

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