

MICROWAVE SURFACE IMPEDANCE OF A NEARLY FERROELECTRIC SUPERCONDUCTOR

C.-J. Wu

Department of Applied Physics
National University of Kaohsiung
Kaohsiung 811, Taiwan

C.-M. Fu

Department of Physics
National Taiwan University
Taipei 106, Taiwan

W.-K. Kuo

Department of Electro-Optics Engineering
National Formosa University
Yunlin 632, Taiwan

T.-J. Yang

Department of Electrophysics
National Chiao-Tung University
Hsinchu 300, Taiwan

Abstract—The intrinsic microwave surface impedance for a nearly ferroelectric superconducting film of finite thickness in the dielectric-like response is theoretically investigated. It is based on the electrodynamics of a nearly ferroelectric superconductor that incorporates the Maxwell's equations, the lattice equations for an ionic lattice, and the superconducting London equation as well. It is found that the surface resistance will be enhanced with decreasing the film thickness when the thickness is less than the London penetration depth. However it will begin to resonate as a function of film thickness at the thickness being more than one London penetration depth. The anomalous resonance peaks occur when the thickness equals the even multiple of the London penetration depth. In the frequency-dependent surface resistance, the number of the resonance peaks is strongly dependent on the film thickness, increasing with increasing the thickness. In addition, these

peaks are not regularly spaced at a fixed interval. Discussion on this anomaly in the surface resistance will be given.

1. INTRODUCTION

Microwave measurements have played a useful and important role in the study of fundamental physics of superconductivity for a superconductor. The most common quantity to be measured in the microwave experiments is the surface impedance, $Z_s = R_s + jX_s$, where the real part, R_s , is called the surface resistance, whereas the imaginary part, X_s , is known as the surface reactance. R_s quantifies the power dissipation in a superconductor and X_s is related to the magnetic penetration depth. Both can be, in general, well described on the basis of the two-fluid model of superconductor [1, 2].

There have been lots of reports on the microwave surface impedances since the discovery of high- T_c superconductors. The reports are mainly focused on the cuprate superconductors, especially on the Y-123 and Bi-2212 systems [3–7]. On the other hand, there exists another special class of superconductor called nearly ferroelectric superconductor (NFE-SC). A NFE-SC could be a high- T_c one such as Na-doped WO_3 (Na_xWO_3 , $x \sim 0.05$, $T_c \sim 90$ K) [8]. It could be also a superconductor with very low T_c like the n -doped SrTiO_3 (STO) having a $T_c \sim 1 - 3$ K [9]. A NFE-SC material means that it can be in the superconducting state and in the nearly ferroelectric state as well. Nearly ferroelectric state means that a material is a soft-mode ionic system with relatively high permittivity. For example, the host STO is a familiar nearly ferroelectric with $\epsilon_r \sim 10^4$ in the temperature range at which superconductivity occurs [10]. The electrodynamics of a NFE-SC material has recently been proposed by Birman and Zimbovskaya [11]. It is indicated that the response to an electromagnetic wave can be divided into two types. In some frequency regime one has the so-called Meissner response where the NFE-SC materials behave as the usual superconductors. In some other frequency regime, one will have the dielectric-like response where the electromagnetic field can be allowed to propagate as in a usual dielectric. Using the material parameters of n -STO it is found that the dielectric-like response occurs at frequency ranging from 14.3 GHz to 25.4 GHz which falls in the microwave regime [11]. The study of microwave surface impedance for this material will thus be of great interest and use. However, this has been lacking thus far to our knowledge.

In this paper, we shall give a theoretical study on the microwave

properties for a NFE-SC thin film. We calculate the intrinsic film surface impedance as a function of the film thickness and radiation frequency. We then make some numerical analyses on the surface resistance and then the interesting anomalous resonance behavior can be explored. The results are expected to trigger some related microwave measurements which would be helpful in testing the validity of the electrodynamic theory of NFE-SC.

2. BASIC EQUATIONS

The microwave surface impedance can be determined by the relevant theory of a superconductor. For a NFE-SC material, the electrodynamics can be described by the combination of Maxwell's equations, London equations of superconductors, and lattice equations for the host ionic lattice of a ferroelectric. The governing field equation for a NFE-SC can be expressed as [11]

$$\nabla \times \nabla \times \mathbf{E} = -\mu_0 \varepsilon(\omega) \frac{\partial^2 \mathbf{E}}{\partial t^2} - \frac{1}{\lambda_L^2} \mathbf{E}, \quad (1)$$

where the London penetration length is given by

$$\lambda_L = \left(\frac{m^*}{\mu_0 n_s e^2} \right)^{1/2}, \quad (2)$$

where m^* is the electron effective mass, e the electronic charge, and n_s the carrier density of the superelectrons. The dielectric function in Eq. (1) is given by

$$\varepsilon(\omega) = \varepsilon'_\infty \frac{\omega_{LO}^2 - \omega^2}{\omega_{TO}^2 - \omega^2}, \quad (3)$$

where ω_{TO} and ω_{LO} are the transverse and longitudinal soft-mode lattice frequencies, respectively. For a time-harmonic electromagnetic wave propagation in the z -direction, Eq. (1) reduces to Helmholtz equation from which the wavenumber can be defined by

$$k = \tilde{\lambda}^{-1} = \left(\omega \mu_0 \varepsilon(\omega) - \lambda_L^{-2} \right)^{1/2}. \quad (4)$$

It follows from Eq. (4) that the electromagnetic response depends on the value of k . If $k > 0$, an electromagnetic wave is allowed to propagate in the material and it is referred to as the dielectric-like response. If k is an imaginary, then the wave cannot propagate and only exponentially decay into the material characterized by the field

penetration depth. In this case, the NFE-SC would behave like a simple superconductor and we thus have a Meissner response. If $k = 0$, the corresponding frequencies calculated from Eq. (4) are called the cutoff frequencies which are determined by the following expression

$$\omega_{c1,c2} = \omega_{TO} \left(\frac{\omega_{LO}^2}{2\omega_{TO}^2} \left(1 + \frac{1}{a^2} \right) \pm \frac{\omega_{LO}}{2\omega_{TO}} \left(\left(1 + \frac{1}{a^2} \right)^2 \frac{\omega_{LO}^2}{\omega_{TO}^2} - \frac{4}{a^2} \right)^{1/2} \right)^{1/2}, \quad (5)$$

where the smaller one is the lower cutoff frequency ω_{c1} and the larger the upper cutoff frequency ω_{c2} and $a^2 = \mu_0 \varepsilon'_\infty \omega_{LO}^2 \lambda_L^2$. For n -STO, $\omega_{TO} = 1.6 \times 10^{11}$ rad/s, $\omega_{LO} = 5.2 \times 10^{12}$ rad/s, and $a^2 = 2.1$ [11], one has $\omega_{c1} = 9 \times 10^{10}$ rad/s, $\omega_{c2} = 6.32 \times 10^{12}$ rad/s. Thus, the low frequency region of dielectric-like response is $\omega_{c1}/2\pi = 14.3$ GHz $\sim \omega_{TO}/2\pi = 25.4$ GHz, falling in the microwave regime.

For a superconductor occupying the half space $z \geq 0$, surface impedance at $z = 0$ is given by

$$Z_s = \frac{\omega \mu_0}{k} = \frac{\omega \mu_0}{\sqrt{\omega^2 \mu_0 \varepsilon(\omega) - \lambda_L^{-2}}}, \quad (6)$$

This one is called the intrinsic surface impedance (intrinsic to a bulk material). In the dielectric-like response, surface impedance is real, i.e., $Z_s = R_s$. As for a superconducting thin film of thickness d ($0 \leq z \leq d$), the intrinsic film surface impedance $Z_{s,int} = R_{s,int} + jX_{s,int}$ can be calculated by the impedance transformation, with the result

$$R_{s,int} = \frac{R_s^2}{Z_0} \frac{1}{\sin^2(d/\tilde{\lambda}) + (R_s^2/Z_0^2) \cos^2(d/\tilde{\lambda})}, \quad (7)$$

$$X_{s,int} = \frac{1}{2} \left(\frac{R_s^3}{Z_0^2} - R_s \right) \frac{\sin(2d/\tilde{\lambda})}{\sin^2(d/\tilde{\lambda}) + (R_s^2/Z_0^2) \cos^2(d/\tilde{\lambda})}. \quad (8)$$

where $Z_0 = 120\pi \Omega$ is the intrinsic impedance of free space. In the limit of $d \rightarrow 0$, Eqs. (7) and (8) then reduce to the intrinsic bulk surface impedance given in Eq. (6). In the case where the superconducting film is deposited on the dielectric substrate with relative permittivity ε_d . Eqs. (7) and (8) will still hold with the replacement of $Z_0 \rightarrow Z_0/\sqrt{\varepsilon_d}$. In this case the impedance is generally referred to as the effective microwave surface impedance that is related to the material parameters and the thicknesses of both superconductor and dielectric substrate.

3. NUMERICAL RESULTS AND DISCUSSION

In what follows we will present the numerical results for the intrinsic film surface resistance for the system, n -STO. This is the only NFE-SC with the all related material parameters being available thus far [11]. Fig. 1 shows the calculated $R_{s,int}$ as a function of the film thickness. It is interesting to find that there are two much different regimes for the thickness-dependent $R_{s,int}$. In the first regime, it is seen that the resistance decreases with increasing film thickness for d being small than the London penetration length λ_L . In the second one where $d > \lambda_L$, $R_{s,int}$ will start to resonate as a function of the film thickness. The first one consists with the result of the high- T_c cuprate system, Y-123 [12, 13]. In the cuprate material the microwave surface impedance can be well described only by the two-fluid model. The result here indicates that a NFE-SC in this region behaves like a pure superconductor and the ferroelectric effect is smeared out due to the size effect. The second resonant part shown in Fig. 1 is replotted in Fig. 2, where the horizontal axis is in linear scale for the purpose of observing the resonant positions. The resonant peaks of $R_{s,int}$ are regularly spaced and resonant positions occur at the condition where $d \cong 2\lambda_L, 4\lambda_L, \dots$

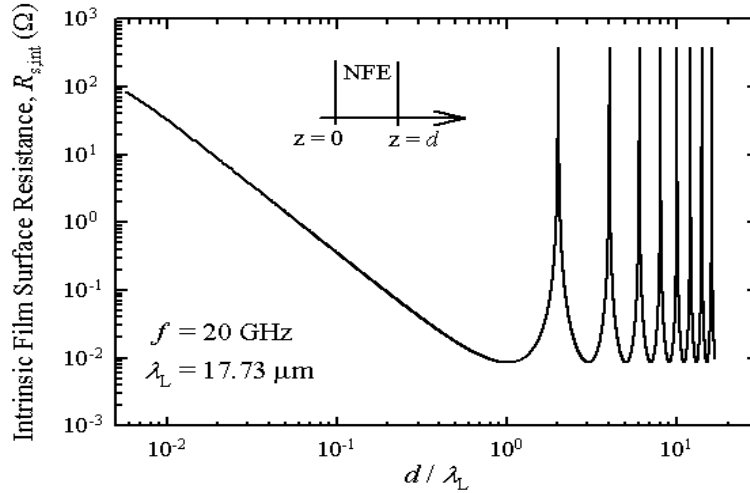


Figure 1. Calculated intrinsic film surface resistance versus the thickness of superconducting film. The radiation microwave frequency is taken to be 20 GHz and the calculated London penetration depth is $17.73 \mu\text{m}$.

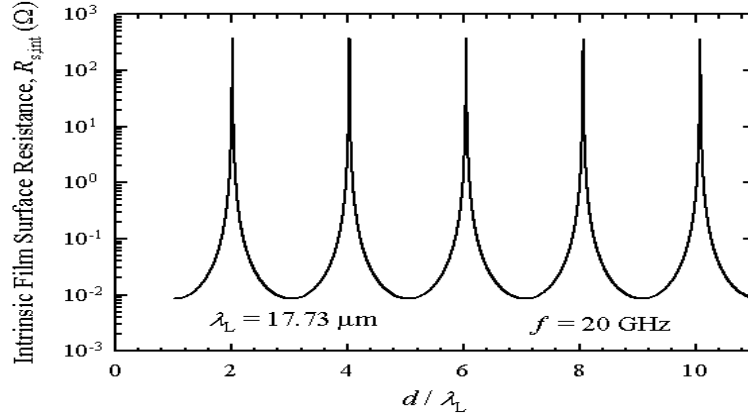


Figure 2. Resonant intrinsic film surface resistance extracted from Fig. 1 for $d > \lambda_L$. The horizontal axis is in the linear scale while in Fig. 1 is in log scale.

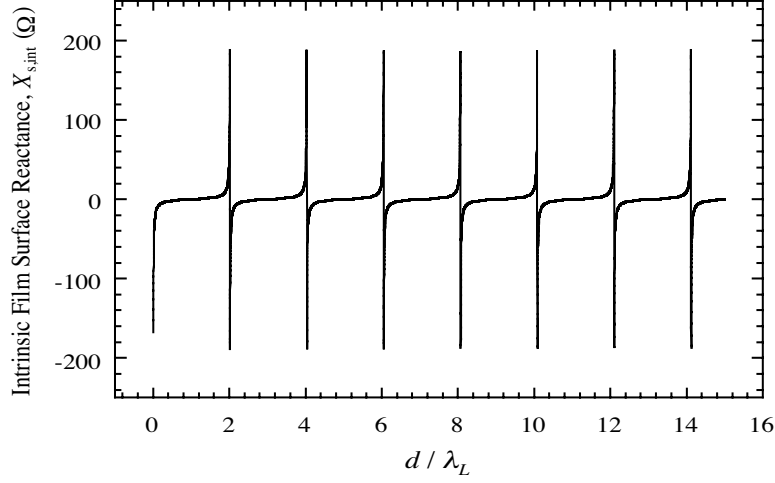


Figure 3. Calculated intrinsic film surface reactance versus the thickness of superconducting film. The radiation microwave frequency is taken to be 20 GHz and the calculated London penetration depth is $17.73 \mu\text{mm}$.

The intrinsic film surface reactance is plotted in Fig. 3, in which it switches quickly from a positive value (inductive) to a negative one (capacitance) around the resonant point. It can be seen from Figs. 1 and 3 that at resonance, surface resistance is equal to 377Ω

whereas surface reactance vanishes, indicating the film is equivalent to a free-space slab. Near the resonance point, the film is equivalent to an inductance as the thickness is smaller than resonant thickness and it then becomes a capacitance when the thickness larger than the resonant thickness.

The resonance position can be further extracted from Eqs. (7) and (8) where it can be seen that the resonance occurs at $\sin(d/\tilde{\lambda}) = 0$, namely $d = n\tilde{\lambda}$, $n = 1, 2, 3, \dots$. With Eq. (4), we can find the resonant thickness of the film, with the result

$$d_{res} = n \frac{1}{\sqrt{\omega\mu_0\varepsilon(\omega) - \lambda_L^{-2}}}. \quad (9)$$

From Eq. (9), a further calculation reveals that the resonance points are $d_{res} = 2.0169\lambda_L$, $4.0338\lambda_L$, and $6.0507\lambda_L$ for $n = 1, 2$, and 3 , respectively. These three points are evidently illustrated in Figs. 2 and 3.

In Fig. 4 we plot the frequency dependence of the intrinsic film surface resistance at film thickness, $d = \lambda_L$, $2\lambda_L$, and $8\lambda_L$, respectively. It can be seen that $R_{s,int}$ exhibits anomalous sharp peaks and the number of peaks increases with increasing film thickness. A comb-like distribution is seen for $d = 8\lambda_L$ and these peaks are not regularly

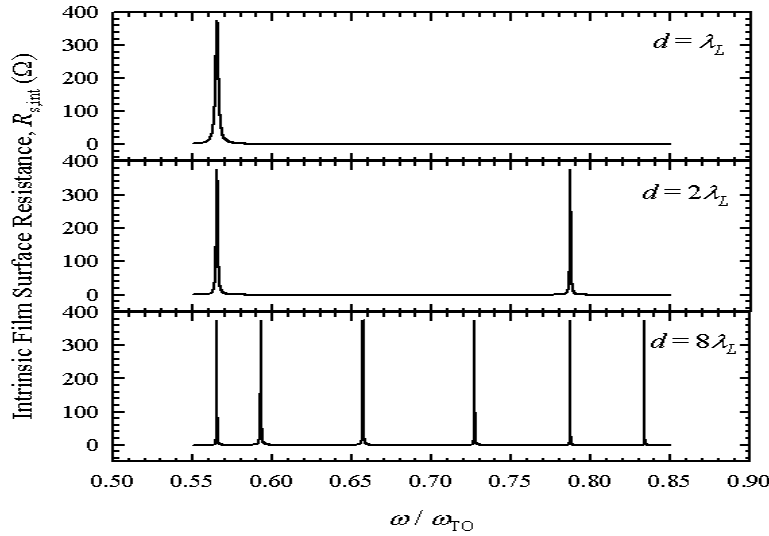


Figure 4. Calculated intrinsic film surface resistance as a function of the frequency at different thicknesses of film.

spaced as in Fig. 2. At resonance, $R_{s,int} = Z_0$, indicating the film is totally transparent to the incident microwave radiation. In addition, the resonant position can be determined as follows: Based on Eq. (7), the resonance condition is $\sin(d/\tilde{\lambda}) = 0 \rightarrow \tilde{\lambda} = d/(n\pi)$, $n = 1, 2, 3, \dots$. This together with Eq. (4) then explicitly determines the resonance frequency as shown in Fig. 3. In addition, it should be noted that the left most peak for each film thickness is not a real resonant peak. It is the one that occurs at $\omega = \omega_{c1}$ which in turn leads to $\lambda_L \rightarrow \infty$ and thus $\sin(d/\tilde{\lambda}) = 0$ is automatically satisfied.

4. SUMMARY

In summary, the intrinsic microwave surface impedance of a NFE-SC thin film has been theoretically investigated. It is found that the surface resistance will resonant as a function of film thickness as the thickness is larger than the London penetration depth. The resonant peaks are regularly spaced and occur at the even multiple of the London penetration length. The frequency-dependent surface resistance shows an anomalous comb-like resonant peak. The peak number is strongly dependent on the film thickness. These peaks however are not regularly spaced in the frequency domain.

ACKNOWLEDGMENT

The author, C.-J. Wu is indebted to the financial support by the National Science Council of the Republic of China under grant No. NSC-95-2112-M-390-003-MY2.

REFERENCES

1. Shen, Z.-Y., *High-Temperature Superconducting Microwave Circuits*, Artech House, Norwood, 1994.
2. Wu, C. J., "Transmission and reflection in a periodic superconductor/dielectric film multilayer structure," *J. Electromagnetic Waves and Applications*, Vol. 19, 1991–1996, 2005.
3. Pan, V. M., et al., "Microwave impedance of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ high-temperature superconductor films in a magnetic field," *Low Temp. Phys.*, Vol. 31, 254–262, 2005.
4. Trunin, M. R., "Temperature dependence of microwave surface impedance in high-Tc single crystals: Experimental and theoretical aspects," *J. Supercond.*, Vol. 11, 381–408, 1998.

5. Liu, S. F. and S. D. Liu, "A novel wideband high-temperature superconducting microstrip antenna," *J. Electromagnetic Waves and Applications*, Vol. 19, 2073–2079, 2005.
6. Tchernyi, V. V. and E. V. Chensky, "Electromagnetic background for possible magnetic levitation of the superconducting rings of Saturn," *J. Electromagnetic Waves and Applications*, Vol. 19, 987–995, 2005.
7. Tchernyi, V. V. and A. Yu Pospelov, "Possible electromagnetic nature of the saturns rings: superconductivity and magnetic levitation," *Progress In Electromagnetics Research*, PIER 52, 277–299, 2005.
8. Levi, Y., et al., "Evidence for localized high-Tc superconducting regimes on the surface of Na-doped WO_3 ," *Europhys. Lett.*, Vol. 51, 564–570, 2000.
9. Koonce, C. S., et al., "Superconducting transition temperature of semiconducting SrTiO_3 ," *Phys. Rev.*, Vol. 163, 380–390, 1968.
10. Savaguchi, E., et al., "Dielectric constant of Strontium Titanate at low temperature," *J. Phys. Soc. Jpn.*, Vol. 17, 1666–1667, 1962.
11. Birman, J. L. and N. A. Zimbovskaya, "Electrodynamics of nearly ferroelectric superconductors," *Phys. Rev. B*, Vol. 64, 144506, 2001.
12. Klein, N., et al., "Effective microwave surface impedance of high-Tc thin films," *J. Appl. Phys.*, Vol. 67, 6940–6945, 1990.
13. Wu, C.-J. and T.-Y. Tse, "Effective microwave surface impedance of superconducting films in the mixed state," *IEEE Trans. Magnetics*, Vol. 33, 2348–2355, 1997.