

FREE-SPACE RELATIVISTIC LOW-FREQUENCY SCATTERING BY MOVING OBJECTS

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Abstract—The present study brings together two aspects of electromagnetic theory: the recently discussed low-frequency series expansions based on the concept of Consistent Maxwell Systems, and Einstein’s Relativistic Electrodynamics. Combined, this facilitates the analysis of pertinent low-frequency scattering problems involving objects moving with arbitrary constant velocities in free space.

The low-frequency series expansions start with leading terms that are prescribed by solutions of the vector Laplace equation, thus significantly simplifying the conventional analysis in terms of the Helmholtz wave equation. The method is demonstrated by deriving relativistically exact explicit results leading terms for perfectly conducting circular-cylindrical and spherical scatterers. The results apply to arbitrary reference frames where the objects are observed in motion. For simplicity of notation expressions are given in terms of spatiotemporal coordinates native to the object’s rest-frame. Subsequent substitution of the Lorentz transformation for the coordinates is then a straightforward matter.

Previous exact relativistic results for scattering by moving objects have demonstrated the existence of velocity induced mode coupling. It is shown that the low-frequency expansions used here display the same effects for various orders of the partial fields appearing in the series.

1. INTRODUCTION

Einstein’s Special Relativity theory [1] relates the measurement of electromagnetic fields in relatively moving inertial systems. Thus using the “frame hopping” method (a term coined by Van Bladel [2]), whereby boundary value problems are solved in one reference frame and the fields are then transformed into another one, facilitates the

discussion of scattering by moving objects. This class of problems has been comprehensively reviewed, [2], (also citing early results by the present author), but relativistic electromagnetic scattering is still a wide open area and new investigations are constantly reported.

Another important class of problems in electromagnetic scattering involves low-frequency series representations of the scattered fields. Recently [3] the theory has been based on the Consistent Maxwell Systems approach, as summarized below. The main feature of the low-frequency series is the fact that leading terms involve the solution of the vector Laplace equation, rather than the full blown solutions of the more complicated vector Helmholtz equation.

Combining these two subjects facilitates the analysis of low-frequency scattering by moving objects.

1.1. Relativistic Electrodynamics

Einstein's Relativistic Electrodynamics [1] is based on two main postulates: The first is the kinematical postulate of the constancy of c , the speed of light in free space (vacuum), leading to the Lorentz transformation

$$\begin{aligned} \mathbf{r}' &= \tilde{\mathbf{U}} \cdot (\mathbf{r} - \mathbf{v}t), & t' &= \gamma (t - \mathbf{v} \cdot \mathbf{r}/c^2) \\ \gamma &= (1 - v^2/c^2)^{-1/2}, & \tilde{\mathbf{U}} &= \tilde{\mathbf{I}} + (\gamma - 1)\hat{\mathbf{v}}\hat{\mathbf{v}}, \quad \hat{\mathbf{v}} = \mathbf{v}/v, \quad v = |\mathbf{v}| \end{aligned} \quad (1)$$

relating the spatiotemporal coordinates of two relatively moving inertial reference systems, with \mathbf{v} denoting the constant velocity of the origin of reference system $\mathbf{\Gamma}'$ as observed from $\mathbf{\Gamma}$. The role of $\tilde{\mathbf{U}}$ is to multiply coordinates parallel to \mathbf{v} by γ . The inverse transformation is obtained by solving (1), which yields

$$\begin{aligned} \mathbf{r} &= \tilde{\mathbf{U}}' \cdot (\mathbf{r}' - \mathbf{v}'t'), & t &= \gamma' (t' - \mathbf{v}' \cdot \mathbf{r}'/c^2) \\ \mathbf{v}' &= -\mathbf{v}, & \tilde{\mathbf{U}}' &= \tilde{\mathbf{U}}, \quad \gamma' = \gamma \end{aligned} \quad (2)$$

where upon using the notation $\mathbf{v}' = -\mathbf{v}$, the Lorentz transformations (1), (2), become form-invariant. The second postulate concerns the dynamics, i.e., the model involving physically measurable fields. Einstein postulated “the principle of relativity” as he dubbed it, stating that in both $\mathbf{\Gamma}$, $\mathbf{\Gamma}'$, Maxwell's equations for the electromagnetic field are form-invariant. In source-free regions we have, for $\mathbf{\Gamma}$, $\mathbf{\Gamma}'$, respectively

$$\begin{aligned} \partial_{\mathbf{r}} \times \mathbf{E} &= -\partial_t \mathbf{B}, \quad \partial_{\mathbf{r}} \times \mathbf{H} = \partial_t \mathbf{D}, \quad \partial_{\mathbf{r}} \cdot \mathbf{B} = 0, \quad \partial_{\mathbf{r}} \cdot \mathbf{D} = 0 \\ \partial_{\mathbf{r}'} \times \mathbf{E}' &= -\partial_{t'} \mathbf{B}', \quad \partial_{\mathbf{r}'} \times \mathbf{H}' = \partial_{t'} \mathbf{D}', \quad \partial_{\mathbf{r}'} \cdot \mathbf{B}' = 0, \quad \partial_{\mathbf{r}'} \cdot \mathbf{D}' = 0 \end{aligned} \quad (3)$$

Throughout we consistently use $\partial_{\mathbf{r}}$, $\partial_{\mathbf{r}'}$, instead of the traditional ∇ symbol, in order to keep track of the coordinates involved [4].

In (3) and throughout, except where otherwise indicated, the fields are denoted as functions of the native spatiotemporal coordinates, compacted by the symbols

$$\Gamma = (\mathbf{r}, t), \quad \Gamma' = (\mathbf{r}', t') \quad (4)$$

i.e., $\mathbf{E} = \mathbf{E}(\Gamma) = \mathbf{E}(\mathbf{r}, t)$, etc., and $\mathbf{E}' = \mathbf{E}'(\Gamma') = \mathbf{E}'(\mathbf{r}', t')$, etc.. Thus $\Gamma'[\Gamma]$ and $\Gamma[\Gamma']$ can be used to symbolize (1), (2), respectively.

Using the chain rule of calculus, (1) and (2) yield the form-invariant Lorentz transformations for derivatives

$$\begin{aligned} \partial_{\mathbf{r}'} &= \tilde{\mathbf{U}} \cdot (\partial_{\mathbf{r}} + \mathbf{v} \partial_t / c^2), & \partial_{t'} &= \gamma (\partial_t + \mathbf{v} \cdot \partial_{\mathbf{r}}) \\ \partial_{\mathbf{r}} &= \tilde{\mathbf{U}}' \cdot (\partial_{\mathbf{r}'} + \mathbf{v}' \partial_{t'} / c^2), & \partial_t &= \gamma' (\partial_{t'} + \mathbf{v}' \cdot \partial_{\mathbf{r}'} \end{aligned} \quad (5)$$

Combining (1)–(5) Einstein [1] derived the field transformation formulas

$$\begin{aligned} \mathbf{E}' &= \tilde{\mathbf{V}} \cdot (\mathbf{E} + \mathbf{v} \times \mathbf{B}), & \mathbf{B}' &= \tilde{\mathbf{V}} \cdot (\mathbf{B} - \mathbf{v} \times \mathbf{E} / c^2) \\ \mathbf{D}' &= \tilde{\mathbf{V}} \cdot (\mathbf{D} + \mathbf{v} \times \mathbf{H} / c^2), & \mathbf{H}' &= \tilde{\mathbf{V}} \cdot (\mathbf{H} - \mathbf{v} \times \mathbf{D}) \\ \tilde{\mathbf{V}} &= \gamma \tilde{\mathbf{I}} + (1 - \gamma) \hat{\mathbf{v}} \hat{\mathbf{v}} \end{aligned} \quad (6)$$

where in (6) all the fields are functions of Γ , whether measured in Γ or Γ' , i.e., we derive for example $\mathbf{E}'(\Gamma) = \tilde{\mathbf{V}} \cdot (\mathbf{E}(\Gamma) + \mathbf{v} \times \mathbf{B}(\Gamma))$, but $\mathbf{E}'(\Gamma)$ is the electric field measured in Γ' . The form-invariant inverse of (6) is

$$\begin{aligned} \mathbf{E} &= \tilde{\mathbf{V}}' \cdot (\mathbf{E}' + \mathbf{v}' \times \mathbf{B}'), & \mathbf{B} &= \tilde{\mathbf{V}}' \cdot (\mathbf{B}' - \mathbf{v}' \times \mathbf{E}' / c^2) \\ \mathbf{D} &= \tilde{\mathbf{V}}' \cdot (\mathbf{D}' + \mathbf{v}' \times \mathbf{H}' / c^2), & \mathbf{H} &= \tilde{\mathbf{V}}' \cdot (\mathbf{H}' - \mathbf{v}' \times \mathbf{D}'), & \tilde{\mathbf{V}}' &= \tilde{\mathbf{V}} \end{aligned} \quad (7)$$

with $\mathbf{E}(\Gamma')$ etc..

The present study assumes free space (vacuum) as the ambient propagation medium, therefore the exterior of the scattering objects is characterized by the constitutive relations

$$\mathbf{D} = \varepsilon \mathbf{E}, \mathbf{B} = \mu \mathbf{H}, \mathbf{D}' = \varepsilon \mathbf{E}', \mathbf{B}' = \mu \mathbf{H}', c = (\mu \varepsilon)^{-1/2} \quad (8)$$

considerably simplifying the above formulas (6), (7). In a nutshell, this is the statement of Einstein's Relativistic Electrodynamics.

1.2. Consistent Maxwell Systems and Low-frequency Series

Low-frequency scattering has been recently discussed [3]. It has been shown that the Maxwell equations (3) can equivalently be stated by either of the two sets of so-called Consistent Maxwell

Systems. Substituting (8) into (3) yields in Γ (or Γ' with appropriate apostrophes)

$$\partial_{\mathbf{r}} \times \mathbf{E} = -\mu \partial_t \mathbf{H}, \partial_{\mathbf{r}} \times \mathbf{H} = \varepsilon \partial_t \mathbf{E}, \partial_{\mathbf{r}} \cdot \mathbf{E} = 0, \partial_{\mathbf{r}} \cdot \mathbf{H} = 0 \quad (9)$$

Further substitution within (9) yields the *first* Consistent Maxwell System in the form

$$\left(\partial_{\mathbf{r}}^2 - c^{-2} \partial_t^2\right) \mathbf{E} = 0, \partial_{\mathbf{r}} \times \mathbf{E} = -\mu \partial_t \mathbf{H}, \partial_{\mathbf{r}} \cdot \mathbf{E} = 0, \partial_{\mathbf{r}} \cdot \mathbf{H} = 0 \quad (10)$$

Similarly we obtain the *second* Consistent Maxwell System

$$\left(\partial_{\mathbf{r}}^2 - c^{-2} \partial_t^2\right) \mathbf{H} = 0, \partial_{\mathbf{r}} \times \mathbf{H} = \varepsilon \partial_t \mathbf{E}, \partial_{\mathbf{r}} \cdot \mathbf{E} = 0, \partial_{\mathbf{r}} \cdot \mathbf{H} = 0 \quad (11)$$

For time-harmonic fields with a time factor $e^{-i\omega t}$, we replace in (10) and (11)

$$\partial_t \Leftrightarrow -i\omega \quad (12)$$

yielding the time-domain Fourier transformed fields

$$\begin{aligned} (\partial_{\mathbf{r}}^2 + k^2) \mathbf{E} &= 0, \partial_{\mathbf{r}} \times \mathbf{E} = i\omega\mu\mathbf{H}, \partial_{\mathbf{r}} \cdot \mathbf{E} = 0, \partial_{\mathbf{r}} \cdot \mathbf{H} = 0 \\ (\partial_{\mathbf{r}}^2 + k^2) \mathbf{H} &= 0, \partial_{\mathbf{r}} \times \mathbf{H} = -i\omega\varepsilon\mathbf{E}, \partial_{\mathbf{r}} \cdot \mathbf{E}' = 0, \partial_{\mathbf{r}} \cdot \mathbf{H} = 0, k = \omega/c \end{aligned} \quad (13)$$

for the first and second Consistent Maxwell Systems, respectively.

The Taylor expansion for a plane wave yields a series in terms of ascending powers of the location vector \mathbf{r}

$$\begin{aligned} \mathbf{E}(\Gamma) &= \hat{\mathbf{e}} e_0 e^{i\mathbf{k} \cdot \mathbf{r}}, \mathbf{H}(\Gamma) = \hat{\mathbf{h}} h_0 e^{i\mathbf{k} \cdot \mathbf{r}} \\ e^{i\mathbf{k} \cdot \mathbf{r}} &= \sum_{n=0}^{\infty} (ik)^n \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}} r\right)^n / n!, e_0 = E_0 e^{-i\omega t}, h_0 = H_0 e^{-i\omega t} \\ E_0/H_0 &= e_0/h_0 = Z = (\mu_0/\varepsilon_0)^{1/2}, \hat{\mathbf{e}}_i \cdot \hat{\mathbf{h}}_i = 0, \hat{\mathbf{e}} \times \hat{\mathbf{h}}_i = \hat{\mathbf{k}} \end{aligned} \quad (14)$$

Inasmuch as the Helmholtz Equations (13) are linear, an arbitrary solution, in particular the scattered fields $\mathbf{E}_s, \mathbf{H}_s$, can be represented as superposition or integral of plane waves, generally propagating in complex directions specified by a complex contour C , e.g., see [5–10]. The choice of C is dictated by the pertinent geometry of the scatterers and the associated boundary conditions. Thus we have Taylor series (14) for the plane waves in the integrand, and upon interchanging summation and integration, we find, e.g., see [3], for the electric field

$$\begin{aligned} \mathbf{E}_s(\Gamma) &= e_0 \int_C e^{ik\hat{\mathbf{k}} \cdot \mathbf{r}} \mathbf{g}(\hat{\mathbf{k}}) d\Omega_{\hat{\mathbf{k}}} = e_0 \sum_{n=0}^{\infty} (ik)^n \mathbf{E}_n(\mathbf{r}) / n! \\ \mathbf{E}_n(\mathbf{r}) &= \int_C \mathbf{g}(\hat{\mathbf{k}}) \left(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}} r\right)^n d\Omega_{\hat{\mathbf{k}}} \end{aligned} \quad (15)$$

where the weighting function \mathbf{g} in (15) is usually referred to as the scattering amplitude.

Accordingly the low-frequency series representations for the scattered waves, as solutions of the Helmholtz wave equation, can be represented as series of partial fields in ascending powers of the *constant parameter* $ik = i\omega/c$

$$\mathbf{E}_s(\Gamma) = e_0 \sum_{n=0}^{\infty} (ik)^n \mathbf{E}_n(\mathbf{r})/n!, \quad \mathbf{H}_s(\Gamma) = h_0 \sum_{n=0}^{\infty} (ik)^n \mathbf{H}_n(\mathbf{r})/n! \quad (16)$$

as given for the scalar case by Morse and Feshbach [11, p. 1085].

It is very suggestive to consider (16) as power series in k , as done, e.g., by [12–15]. Some reflection on the structure of (16) reveals that this is a misconception, because the series (16) involve k only to the extent that this is the *constant parameter* appearing in the Helmholtz equation. A power series proper involves powers of a *variable*, not a *constant*. Consequently we cannot substitute (16) in (8) and equate equal powers of k .

Instead, the fields (16) must be substituted in the corresponding Helmholtz equations in (13). The Helmholtz equation is not satisfied term by term by (16), only by pairs of terms of the pertinent series. To bring this out, one can re-adjust indices to derive recurrence equations on the partial fields. Thus from the first line (13) we obtain the first Consistent Maxwell System for the partial fields

$$\begin{aligned} \partial_{\mathbf{r}}^2 \mathbf{E}_n(\mathbf{r}) &= n(n-1) \mathbf{E}_{n-2}(\mathbf{r}), \quad \partial_{\mathbf{r}} \times \mathbf{E}_n(\mathbf{r}) = ik \mathbf{H}_n(\mathbf{r}) \\ \partial_{\mathbf{r}} \cdot \mathbf{E}_n(\mathbf{r}) &= 0, \quad \partial_{\mathbf{r}} \cdot \mathbf{H}_n(\mathbf{r}) = 0 \end{aligned} \quad (17)$$

Similarly, the second Consistent Maxwell System for the partial fields is

$$\begin{aligned} \partial_{\mathbf{r}}^2 \mathbf{H}_n(\mathbf{r}) &= n(n-1) \mathbf{H}_{n-2}(\mathbf{r}), \quad \partial_{\mathbf{r}} \times \mathbf{H}_n(\mathbf{r}) = -ik \mathbf{E}_n(\mathbf{r}) \\ \partial_{\mathbf{r}} \cdot \mathbf{E}_n(\mathbf{r}) &= 0, \quad \partial_{\mathbf{r}} \cdot \mathbf{H}_n(\mathbf{r}) = 0 \end{aligned} \quad (18)$$

The special feature characteristic of the low-frequency series expansions is that for the leading terms $n = 0$, $n = 1$, the Consistent Maxwell Systems (17) and (18) prescribe solutions of the vector Laplace equation, rather than the vector Helmholtz equation.

As a consequence of (17) we have

$$\partial_{\mathbf{r}} \times \partial_{\mathbf{r}} \times \mathbf{E}_n(\mathbf{r}) = \partial_{\mathbf{r}}^2 \mathbf{E}_n(\mathbf{r}) = n(n-1) \mathbf{E}_{n-2}(\mathbf{r}) = ik \partial_{\mathbf{r}} \times \mathbf{H}_n(\mathbf{r}) \quad (19)$$

prescribing for the leading terms in first Consistent Maxwell System $\partial_{\mathbf{r}} \times \mathbf{H}_0(\mathbf{r}) = 0$, $\partial_{\mathbf{r}} \times \mathbf{H}_1(\mathbf{r}) = 0$. Similarly from (18), for the second Consistent Maxwell System we have

$$\partial_{\mathbf{r}} \times \partial_{\mathbf{r}} \times \mathbf{H}_n(\mathbf{r}) = \partial_{\mathbf{r}}^2 \mathbf{H}_n(\mathbf{r}) = n(n-1) \mathbf{H}_{n-2}(\mathbf{r}) = -ik \partial_{\mathbf{r}} \times \mathbf{E}_n(\mathbf{r}) \quad (20)$$

prescribing for the second Consistent Maxwell System $\partial_{\mathbf{r}} \times \mathbf{E}_0(\mathbf{r}) = 0$, $\partial_{\mathbf{r}} \times \mathbf{E}_1(\mathbf{r}) = 0$.

1.3. Vector Solutions of the Helmholtz and Laplace Equations

The vector Laplace equation is discussed in [11, p.1784 ff.], and specifically for spherical coordinates, see [11, p.1799 ff.]. In view of the fact that when formally taking $k = 0$ the vector Helmholtz equation reduces to the vector Laplace equation, many properties can be gleaned by mere inspection. Stratton [5, p.392 ff.] derives three independent vector solutions for the vector Helmholtz equation based on the solutions of the scalar Helmholtz equation solution

$$\begin{aligned} (\partial_{\mathbf{r}}^2 + k^2) \mathbf{C}(\mathbf{r}) &= 0, \quad \mathbf{C} = \mathbf{L}, \mathbf{M}, \mathbf{N} \\ \mathbf{L} &= \partial_{\mathbf{r}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{L} = \partial_{\mathbf{r}}^2 \varphi = -k^2 \varphi \neq 0, \partial_{\mathbf{r}} \times \mathbf{L} = 0 \\ \mathbf{M} &= \partial_{\mathbf{r}} \times \hat{\mathbf{a}} \varphi = -\hat{\mathbf{a}} \times \mathbf{L}, \partial_{\mathbf{r}} \cdot \mathbf{M} = 0 \\ k\mathbf{N} &= \partial_{\mathbf{r}} \times \mathbf{M}, \partial_{\mathbf{r}} \cdot \mathbf{N} = 0, k\mathbf{M} = \partial_{\mathbf{r}} \times \mathbf{N} \end{aligned} \quad (21)$$

where in (21) $\hat{\mathbf{a}}$ is an arbitrary *constant* unit vector. We thus have a longitudinal solution \mathbf{L} characterized by nonzero-divergence and zero-curl, and two transverse solutions \mathbf{M} , \mathbf{N} , with zero-divergence and nonzero-curl.

For cylindrical coordinates in particular, we can choose the constant unit vector as $\hat{\mathbf{z}}$, along the cylindrical axis. Moreover, if the functions are independent of the z coordinate, then we can choose

$$\begin{aligned} \mathbf{M} &= \hat{\mathbf{z}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{M} = 0, (\partial_{\mathbf{r}}^2 + k^2) \mathbf{M} = \hat{\mathbf{z}} (\partial_{\mathbf{r}}^2 + k^2) \varphi = 0 \\ \mathbf{L} &= \partial_{\mathbf{r}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{L} = \partial_{\mathbf{r}}^2 \varphi = -k^2 \varphi \neq 0, \partial_{\mathbf{r}} \times \mathbf{L} = 0 \\ k\mathbf{N} &= \partial_{\mathbf{r}} \times \mathbf{M} = \partial_{\mathbf{r}} \times \hat{\mathbf{z}} \varphi \neq 0, \partial_{\mathbf{r}} \cdot \mathbf{N} = 0 \\ (\partial_{\mathbf{r}}^2 + k^2) k\mathbf{N} &= k (\partial_{\mathbf{r}}^2 + k^2) \partial_{\mathbf{r}} \times \hat{\mathbf{z}} \varphi = k \partial_{\mathbf{r}} \times [(\partial_{\mathbf{r}}^2 + k^2) \hat{\mathbf{z}} \varphi] = 0 \\ \partial_{\mathbf{r}} \times \mathbf{N} &= \partial_{\mathbf{r}} \times \partial_{\mathbf{r}} \times \mathbf{M} / k = -\partial_{\mathbf{r}}^2 \mathbf{M} / k = k\mathbf{M} \end{aligned} \quad (22)$$

As shown below, the special case (22) is of interest for the present analysis. This solution is usually not given in general references.

The case of spherical coordinates deserves special attention, see [11, p.1864 ff.], [5, p.414 ff.] the latter also citing early work on the subject. The special solutions involve \mathbf{r} , which is a *non-constant* vector. The proof is outlined by [5], yielding

$$\begin{aligned} (p_{\mathbf{r}}^2 + k^2) \mathbf{C}(\mathbf{r}) &= 0, \quad \mathbf{C} = \mathbf{L}, \mathbf{M}, \mathbf{N} \\ \mathbf{L} &= \partial_{\mathbf{r}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{L} = \partial_{\mathbf{r}}^2 \varphi = -k^2 \varphi \neq 0, \partial_{\mathbf{r}} \times \mathbf{L} = 0 \\ \mathbf{M} &= \partial_{\mathbf{r}} \times \mathbf{r} \varphi = -\mathbf{r} \times \mathbf{L}, \partial_{\mathbf{r}} \cdot \mathbf{M} = 0 \\ k\mathbf{N} &= \partial_{\mathbf{r}} \times \mathbf{M}, \partial_{\mathbf{r}} \cdot \mathbf{N} = 0, k\mathbf{M} = \partial_{\mathbf{r}} \times \mathbf{N} \end{aligned} \quad (23)$$

Consider now the vector Laplace equation, which is relevant for the leading partial wave terms $n = 0$, $n = 1$ in (17)–(20). In order to derive vector solutions in terms of the solutions φ of the scalar Laplace equation, one is tempted to simply assume $k \rightarrow 0$ in (21)–(23). However we run here into inconsistencies resulting from the degeneracy of the Helmholtz system of solutions [11, p. 1784 ff.].

The upshot is that we have only two independent solutions, with \mathbf{L} merging with one of the transversal solutions, say \mathbf{N} . Therefore instead of (21) we end up with

$$\begin{aligned}\partial_{\mathbf{r}}^2 \varphi(\mathbf{r}) &= 0, \partial_{\mathbf{r}}^2 \mathbf{C}(\mathbf{r}) = 0, \mathbf{C} = \mathbf{L}, \mathbf{M}, \mathbf{N} = \mathbf{L} \\ \mathbf{L} &= \partial_{\mathbf{r}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{L} = \partial_{\mathbf{r}}^2 \varphi = 0, \partial_{\mathbf{r}} \times \mathbf{L} = 0 \\ \mathbf{M} &= \partial_{\mathbf{r}} \times \hat{\mathbf{a}} \varphi = -\hat{\mathbf{a}} \times \mathbf{L}, \partial_{\mathbf{r}} \cdot \mathbf{M} = 0\end{aligned}\quad (24)$$

with (24) displaying two zero-divergence solutions, where \mathbf{L} is zero-curl and \mathbf{M} is nonzero-curl.

For the vector Laplace equation (22) becomes

$$\begin{aligned}\mathbf{M} &= \hat{\mathbf{z}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{M} = 0, \partial_{\mathbf{r}} \times \mathbf{M} \neq 0, \partial_{\mathbf{r}}^2 \mathbf{M} = \hat{\mathbf{z}} \partial_{\mathbf{r}}^2 \varphi = 0 \\ \mathbf{L} &= \mathbf{N} = \partial_{\mathbf{r}} \varphi, \partial_{\mathbf{r}} \cdot \mathbf{L} = \partial_{\mathbf{r}}^2 \varphi = 0, \partial_{\mathbf{r}} \times \mathbf{L} = 0\end{aligned}\quad (25)$$

resulting in two zero-divergence solutions, with one nonzero-curl solution \mathbf{M} , and zero-curl solution \mathbf{L} .

As indicated [11, p. 1799 ff.], for spherical coordinates the solution of the vector Laplace equation follows from (23). Similarly to (24) we now have

$$\begin{aligned}\partial_{\mathbf{r}}^2 \varphi_{\nu}^{\mu}(\mathbf{r}) &= 0, \partial_{\mathbf{r}}^2 \mathbf{C}_{\nu}^{\mu}(\mathbf{r}) = 0, \mathbf{C}_{\nu}^{\mu} = \mathbf{L}_{\nu}^{\mu}, \mathbf{M}_{\nu}^{\mu}, \mathbf{N}_{\nu}^{\mu} = \mathbf{L}_{\nu}^{\mu} \\ \mathbf{L}_{\nu}^{\mu}(\mathbf{r}) &= \partial_{\mathbf{r}} \varphi_{\nu}^{\mu}(\mathbf{r}), \partial_{\mathbf{r}} \cdot \mathbf{L}_{\nu}^{\mu}(\mathbf{r}) = \partial_{\mathbf{r}}^2 \varphi_{\nu}^{\mu}(\mathbf{r}) = 0, \partial_{\mathbf{r}} \times \mathbf{L}_{\nu}^{\mu}(\mathbf{r}) = 0 \\ \mathbf{M}_{\nu}^{\mu}(\mathbf{r}) &= \partial_{\mathbf{r}} \times \mathbf{r} \varphi_{\nu}^{\mu}(\mathbf{r}) = -\mathbf{r} \times \mathbf{L}_{\nu}^{\mu}(\mathbf{r}), \partial_{\mathbf{r}} \cdot \mathbf{M}_{\nu}^{\mu}(\mathbf{r}) = 0 \\ \partial_{\mathbf{r}} \times \mathbf{M}_{\nu}^{\mu}(\mathbf{r}) &\neq 0, \varphi_{\nu}^{\mu}(\mathbf{r}) = \varphi_{\nu}^{\mu}(r, \theta, \psi) = r^{-\nu-1} Y_{\nu}^{\mu}(\hat{\mathbf{r}}) \\ Y_{\nu}^{\mu}(\hat{\mathbf{r}}) &= Y_{\nu}^{\mu}(\theta, \psi) = P_{\nu}^{\mu}(C_{\theta}) R_{\mu\psi}, R_{\mu\psi} = C_{\mu\psi}, S_{\mu\psi}\end{aligned}\quad (26)$$

with φ_{ν}^{μ} denoting solutions of the scalar Laplace equation in spherical coordinates, in terms of associated Legendre functions P_{ν}^{μ} , and where $R_{\mu\psi}$ are linear combination of the trigonometric azimuthal functions $C_{\mu\psi}$, $S_{\mu\psi}$. Thusly by operations on the solutions of the scalar Laplace equation, nonzero-curl and zero-curl vector solutions are generated, both types possess zero-divergence.

2. STATEMENT OF THE SCATTERING PROBLEM

The scattering problem in the present case involves two main aspects: Firstly we have the problem of relativistically transforming the given

incident wave from the “laboratory” reference frame $\mathbf{\Gamma}$ into the “co-moving” frame $\mathbf{\Gamma}'$ where the scatterer is at rest. Secondly we have to address the scattering problem in $\mathbf{\Gamma}'$, using the low-frequency series representations. Finally we have to implement the “frame hopping” scheme in the reverse direction and transform the scattered fields back into $\mathbf{\Gamma}$.

As already mentioned, the scattered fields *measured* in $\mathbf{\Gamma}$ will be left in terms of $\mathbf{\Gamma}'$ *coordinates*. There is no point in substituting the Lorentz transformations at this stage and deriving everything in terms of $\mathbf{\Gamma}$ coordinates, because the result is cumbersome and totally non transparent. The subject will be better served if the latter step is understood, but only implemented when actual calculations are contemplated.

2.1. Relativistic Considerations

A plane time-harmonic incident wave (14) is assumed in $\mathbf{\Gamma}$. The transformation of plane waves from one inertial system to another is elementary [1, 2, 4, 7]. By exploiting (6), we derive in $\mathbf{\Gamma}'$ an expression given in terms of $\mathbf{\Gamma}$ coordinates

$$\begin{aligned} \mathbf{E}'(\mathbf{\Gamma}) &= \hat{\mathbf{e}}' e'_0 e^{i\mathbf{k} \cdot \mathbf{r}}, \quad \mathbf{H}'(\mathbf{\Gamma}) = \hat{\mathbf{h}}' h'_0 e^{i\mathbf{k} \cdot \mathbf{r}}, \quad e'_0 = E'_0 e^{-i\omega t}, \quad h'_0 = H'_0 e^{-i\omega t} \\ E'_0/H'_0 &= e'_0/h'_0 = Z = (\mu_0/\varepsilon_0)^{1/2}, \quad \hat{\mathbf{e}}'_i \cdot \hat{\mathbf{h}}'_i = 0, \quad \hat{\mathbf{e}}' \times \hat{\mathbf{h}}'_i = \hat{\mathbf{k}}' \end{aligned} \quad (27)$$

The plane wave is unique in that it can be represented also in $\mathbf{\Gamma}'$ as a form invariant expression, i.e., once again in a time-harmonic plane wave of the form (14), with the appropriate apostrophes

$$\begin{aligned} \mathbf{E}'(\mathbf{\Gamma}') &= \hat{\mathbf{e}}' e'_0 e^{i\mathbf{k}' \cdot \mathbf{r}'}, \quad \mathbf{H}'(\mathbf{\Gamma}') = \hat{\mathbf{h}}' h'_0 e^{i\mathbf{k}' \cdot \mathbf{r}'} \\ e^{i\mathbf{k}' \cdot \mathbf{r}'} &= \sum_{n=0}^{\infty} (i k')^n \left(\hat{\mathbf{k}}' \cdot \hat{\mathbf{r}}' r' \right)^n / n!, \quad \hat{\mathbf{k}}' \cdot \hat{\mathbf{r}}' = C_{\psi'}, \quad e'_0 = E'_0 e^{-i\omega' t'}, \\ h'_0 &= H'_0 e^{-i\omega' t'} \end{aligned} \quad (28)$$

The transformation from (27) into (28) is performed by substituting (2) in (27) and defining in $\mathbf{\Gamma}'$ the new wave parameters according to

$$\begin{aligned} \mathbf{k}' &= \tilde{\mathbf{U}} \cdot (\mathbf{k} - \mathbf{v}\omega/c^2), \quad \mathbf{k} = \tilde{\mathbf{U}} \cdot (\mathbf{k}' - \mathbf{v}'\omega'/c^2) \\ \omega' &= \gamma(\omega - \mathbf{v} \cdot \mathbf{k}), \quad \omega = \gamma(\omega' - \mathbf{v}' \cdot \mathbf{k}') \end{aligned} \quad (29)$$

often referred to as the relativistic Fresnel Drag Effect and the relativistic Doppler Effect, respectively. We have thus used the so-called principle of phase invariance $\mathbf{k}' \cdot \mathbf{r}' - \omega' t' = \mathbf{k} \cdot \mathbf{r} - \omega t$.

Retracing the argument (14)–(16), it follows from (28) that in $\mathbf{\Gamma}'$

we now have for the scattered fields

$$\mathbf{E}'_s(\Gamma') = e'_0 \sum_{n=0}^{\infty} (ik')^n \mathbf{E}'_n(\mathbf{r}')/n!, \quad \mathbf{H}_s(\Gamma') = h'_0 \sum_{n=0}^{\infty} (ik')^n \mathbf{H}'_n(\mathbf{r}')/n! \quad (30)$$

As an alternative to (6), one can substitute from the Maxwell equations (3) into (6), (7), deriving transformation differential operators [7, 16, 17]

$$\begin{aligned} \mathbf{E}'(\Gamma) &= \tilde{\mathbf{W}} \cdot \mathbf{E}(\Gamma), \quad \mathbf{H}'(\Gamma) = \tilde{\mathbf{W}} \cdot \mathbf{H}(\Gamma) \\ \mathbf{E}(\Gamma') &= \tilde{\mathbf{W}}' \cdot \mathbf{E}'(\Gamma'), \quad \mathbf{H}(\Gamma') = \tilde{\mathbf{W}}' \cdot \mathbf{H}'(\Gamma') \\ \tilde{\mathbf{W}} &= \tilde{\mathbf{V}} \cdot \left(\tilde{\mathbf{I}} - \mathbf{v} \times \partial_t^{-1} \partial_{\mathbf{r}} \times \tilde{\mathbf{I}} \right) = \tilde{\mathbf{V}} \cdot \left(\tilde{\mathbf{I}} + \beta \hat{\mathbf{v}} \times \partial_{\mathbf{r}} \times \tilde{\mathbf{I}}/(ik) \right) \quad (31) \\ \tilde{\mathbf{W}}' &= \tilde{\mathbf{V}} \cdot \left(\tilde{\mathbf{I}} - \mathbf{v}' \times \partial_{t'}^{-1} \partial_{\mathbf{r}'} \times \tilde{\mathbf{I}} \right) = \tilde{\mathbf{V}} \cdot \left(\tilde{\mathbf{I}} + \beta \hat{\mathbf{v}} \times \partial_{\mathbf{r}'} \times \tilde{\mathbf{I}}/(ik') \right) \\ &= \tilde{\mathbf{V}} \cdot \left(\tilde{\mathbf{I}} - \beta \hat{\mathbf{v}} \times \partial_{\mathbf{r}'} \times \tilde{\mathbf{I}}/(ik') \right), \quad \beta = v/c \end{aligned}$$

where in (31) ∂_t^{-1} indicates the primitive integral with respect to time, or equivalently, upon multiplying by ∂_t we obtain $\partial_t \tilde{\mathbf{W}} = \tilde{\mathbf{V}} \cdot \left(\partial_t \tilde{\mathbf{I}} - \mathbf{v} \times \partial_{\mathbf{r}} \times \tilde{\mathbf{I}} \right)$ etc.. This compact operator representation is convenient for analyzing the scattering problems at hand.

Substitution from (16), (17), into (6), shows that for the partial waves associated with the first Consistent Maxwell System we have

$$\begin{aligned} \mathbf{E}'_s(\Gamma) &= \tilde{\mathbf{V}} \cdot (\mathbf{E}_s(\Gamma) + \mu \mathbf{v} \times \mathbf{H}_s(\Gamma)) = e_0 \sum_{n=0}^{\infty} (ik)^n \tilde{\mathbf{W}} \cdot \mathbf{E}_n(\mathbf{r})/n! \\ \mathbf{H}'_s(\Gamma) &= \tilde{\mathbf{V}} \cdot (\mathbf{H}_s(\Gamma) - \varepsilon \mathbf{v} \times \mathbf{E}_s(\Gamma)) \end{aligned} \quad (32)$$

Similarly, substituting from (16), (18), into (6), yields in terms of the second Consistent Maxwell System partial waves

$$\begin{aligned} \mathbf{H}'_s(\Gamma) &= \tilde{\mathbf{V}} \cdot (\mathbf{H}_s(\Gamma) - \varepsilon \mathbf{v} \times \mathbf{E}_s(\Gamma)) = h_0 \sum_{n=0}^{\infty} (ik)^n \tilde{\mathbf{W}} \cdot \mathbf{H}_n(\mathbf{r})/n! \\ \mathbf{E}'_s(\Gamma) &= \tilde{\mathbf{V}} \cdot (\mathbf{E}_s(\Gamma) + \mu \mathbf{v} \times \mathbf{H}_s(\Gamma)) \end{aligned} \quad (33)$$

where it is noted that in (32), (33), the use of the operator $\tilde{\mathbf{W}}$ is limited to one of the fields only. The inverse relations follow in an obvious manner, yielding

$$\begin{aligned} \mathbf{E}_s(\Gamma') &= \tilde{\mathbf{V}} \cdot (\mathbf{E}'_s(\Gamma') + \mu \mathbf{v}' \times \mathbf{H}'_s(\Gamma')) = e'_0 \sum_{n=0}^{\infty} (ik')^n \tilde{\mathbf{W}}' \cdot \mathbf{E}'_n(\mathbf{r}')/n! \\ \mathbf{H}_s(\Gamma') &= \tilde{\mathbf{V}} \cdot (\mathbf{H}'_s(\Gamma') - \varepsilon \mathbf{v}' \times \mathbf{E}'_s(\Gamma')) \\ \mathbf{H}_s(\Gamma') &= \tilde{\mathbf{V}} \cdot (\mathbf{H}'_s(\Gamma') - \varepsilon \mathbf{v}' \times \mathbf{E}'_s(\Gamma')) = h'_0 \sum_{n=0}^{\infty} (ik')^n \tilde{\mathbf{W}}' \cdot \mathbf{H}'_n(\mathbf{r}')/n! \quad (34) \\ \mathbf{E}_s(\Gamma') &= \tilde{\mathbf{V}} \cdot (\mathbf{E}'_s(\Gamma') + \mu \mathbf{v}' \times \mathbf{H}'_s(\Gamma')) \end{aligned}$$

where in (34) the first two lines, the last two lines, apply to the first, second, Consistent Maxwell System, respectively.

Some manipulation of indices in (34) yields for the first Consistent Maxwell System

$$\begin{aligned}
\mathbf{E}_s(\Gamma') &= \tilde{\mathbf{V}} \cdot \left(\tilde{\mathbf{I}} - \beta \hat{\mathbf{v}} \times \partial_{\mathbf{r}'} \times \tilde{\mathbf{I}} / (ik') \right) e_0' \sum_{n=0}^{\infty} (ik')^n \mathbf{E}'_n(\mathbf{r}') / n! \\
&= e_0' \sum_{n=0}^{\infty} (ik')^n \tilde{\mathbf{V}} \cdot \mathbf{E}'_n(\mathbf{r}') / n! - \gamma \beta e_0' \sum_{n=0}^{\infty} (ik')^{n-1} \hat{\mathbf{v}} \times \partial_{\mathbf{r}'} \times \mathbf{E}'_n(\mathbf{r}') / n! \\
&= e_0' \sum_{n=0}^{\infty} ((ik')^{n-1} / n!) \left[n \tilde{\mathbf{V}} \cdot \mathbf{E}'_{n-1}(\mathbf{r}') - \gamma \beta \hat{\mathbf{v}} \times \partial_{\mathbf{r}'} \times \mathbf{E}'_n(\mathbf{r}') \right]
\end{aligned} \tag{35}$$

and the analogous expression for $\mathbf{H}_s(\Gamma')$ follows in an obvious manner. It has been shown previously [18] that velocity-dependent scattering is characterized by multipole mode-coupling. In (35) it is seen that for the low-frequency representation the same effect, in terms of the partial fields modes, appears again, i.e., the velocity couples terms of indices n and $n - 1$.

2.2. Scattering by Objects at Rest

We are now in Γ' , where the scatterer is at rest, excited by a time harmonic plane wave (28). Thus the “frame hopping” approach reduced the boundary value problem to the usual one for objects at rest.

The total fields in the exterior domain, denoted by \mathbf{E}'_T , \mathbf{H}'_T , are the sum of the incident fields \mathbf{E}' , \mathbf{H}' , in (28), and the scattered fields \mathbf{E}'_s , \mathbf{H}'_s . Together with \mathbf{E}'_{in} , \mathbf{H}'_{in} , in the interior domain, the boundary conditions for the tangential components of the fields prescribe

$$\begin{aligned}
\hat{\mathbf{n}}' \times (\mathbf{E}'_T - \mathbf{E}'_{in}) &= 0|_{S'}, \quad \hat{\mathbf{n}}' \times (\mathbf{H}'_T - \mathbf{H}'_{in}) = 0|_{S'} \\
\mathbf{E}'_T &= \mathbf{E}' + \mathbf{E}'_s, \quad \mathbf{H}'_T = \mathbf{H}' + \mathbf{H}'_s
\end{aligned} \tag{36}$$

on the surface S' , which together with its associated outwardly directed unit normal vector $\hat{\mathbf{n}}'$ defines the scatterer's geometry in Γ' . It is important to note that in view of the linearity of the problem (36) applies to individual terms of the low-frequency series as well.

For simplicity, the present discussion is limited to perfectly conducting objects hence the internal fields vanish, leaving us with

$$\hat{\mathbf{n}}' \times \mathbf{E}'_T = 0|_{S'} \tag{37}$$

and for circular-cylinders and spheres of radius a , the surface S' is defined by $r' = a$, and $\hat{\mathbf{n}}' = \hat{\mathbf{r}}'$. As far as low-frequency series and relativistic scattering considerations are involved, these simplifications do not affect the generality of the present discussion.

3. SCATTERING BY MOVING CYLINDERS AND SPHERES

3.1. Scattering by a Moving Cylinder, Leading Terms

Scattering by a perfectly conducting moving circular cylinder, at rest in Γ' , has been investigated before [2–4, 16, 18]. Here the emphasis is placed on the fields expressed in terms of the low-frequency series expansions. In Γ , the incident time harmonic plane wave (14) is now specialized to $\hat{\mathbf{e}}$ polarized along the cylindrical axis $\hat{\mathbf{z}}$ and propagating in the $\mathbf{k} = \hat{\mathbf{x}}$ direction.

In order to have a simple example of velocity-dependent scattering, we choose

$$\hat{\mathbf{v}} = \hat{\mathbf{x}} = \hat{\mathbf{x}}' \quad (38)$$

It follows that the transformed wave (28) observed in Γ' is given by

$$\begin{aligned} \mathbf{E}'(\Gamma') &= \hat{\mathbf{z}}' e'_0 e^{ik'\hat{\mathbf{x}}'\cdot\mathbf{r}'}, \quad \mathbf{H}'(\Gamma') = -\hat{\mathbf{y}}' h'_0 e^{ik'\hat{\mathbf{x}}'\cdot\mathbf{r}'}, \quad \hat{\mathbf{x}}' = \hat{\mathbf{x}}, \quad \hat{\mathbf{y}}' = \hat{\mathbf{y}}, \quad \hat{\mathbf{z}}' = \hat{\mathbf{z}} \\ e^{ik'\hat{\mathbf{x}}'\cdot\mathbf{r}'} &= \sum_{n=0}^{\infty} (ik')^n (\hat{\mathbf{x}}' \cdot \hat{\mathbf{r}}' r')^n / n! = \sum_{n=0}^{\infty} (ik')^n (r' C_{\psi'})^n / n!, \quad C_{\psi'} = \cos \psi' \end{aligned} \quad (39)$$

with ψ' in (39) denoting the azimuthal angle, and $\hat{\mathbf{r}}'$ pointing away from the cylindrical axis.

For perfectly conducting circular cylinders and \mathbf{E}' polarization as in (39), we choose nonzero-curl solutions \mathbf{M} for the vector Laplace equation in two-dimensional geometries as in (25), with the appropriate apostrophes. This solution is required according to (17), (18), for the terms $n = 0$, $n = 1$.

The term $n = 0$ in (39) prescribes a solution of the Laplace equation in cylindrical coordinates which is independent of ψ' . Therefore the nonzero-curl solution is

$$\begin{aligned} \mathbf{E}'_{s,0}(\Gamma') &= e'_0 \mathbf{E}'_0(\mathbf{r}'), \quad \mathbf{E}'_0(\mathbf{r}') = \hat{\mathbf{z}}' \varphi'_0(\mathbf{r}') = \hat{\mathbf{z}}' A_0 \ln(k'r') \\ \partial_{\mathbf{r}'}^2 \mathbf{E}'_0(\mathbf{r}') &= 0, \quad \partial_{\mathbf{r}'} \cdot \mathbf{E}'_0(\mathbf{r}') = 0, \quad \partial_{\mathbf{r}'} \times \mathbf{E}'_0(\mathbf{r}') \neq 0 \end{aligned} \quad (40)$$

On the surface $r' = a$, (37) prescribes

$$A_0 = -1 / \ln(k'a), \quad \mathbf{E}'_0(\mathbf{r}') = -\hat{\mathbf{z}}' \ln(k'r') / \ln(k'a) \quad (41)$$

In accordance with (17) (with apostrophes), the associated partial magnetic field is given by

$$\begin{aligned} \mathbf{H}'_0(\mathbf{r}') &= \partial_{\mathbf{r}'} \times \mathbf{E}'_0(\mathbf{r}') / (ik') = \hat{\mathbf{z}}' \times \partial_{\mathbf{r}'} \ln(k'r') / (ik' \ln(k'a)) \\ &= \hat{\boldsymbol{\psi}}' / (ik' r' \ln(k'a)), \\ \mathbf{H}'_{s,0}(\Gamma') &= \partial_{\mathbf{r}'} \times \mathbf{E}'_{s,0}(\Gamma') / (Zik') = h'_0 \mathbf{H}'_0(\mathbf{r}') \end{aligned} \quad (42)$$

with the usual notation of $\hat{\psi}'$ pointing in the direction of increasing ψ' .

The corresponding zero-curl solutions (with apostrophes) in (25) that is independent of ψ' is given by the radial field proportional to

$$\partial_{\mathbf{r}'} \varphi'_0(\mathbf{r}') = \partial_{\mathbf{r}'} \ln(k'r') = \hat{\mathbf{r}}'/r' \quad (43)$$

Obviously the $\hat{\mathbf{r}}'$ component in (43) is perpendicular to the tangential boundary conditions (36), (37), and cannot be compensated by the $\hat{\mathbf{z}}'$ -direction components of the incident wave (39). It must be born in mind that equations satisfying the boundary conditions are the mechanism creating the scattered fields, therefore the solution (43) is extraneous to our problem and should be ignored.

From (34), and substituting (40), (42), the scattered field in $\mathbf{\Gamma}$ is obtained as

$$\begin{aligned} \mathbf{E}_{s,0}(\Gamma') &= -\hat{\mathbf{z}}' e'_0 \gamma [\ln(k'r') + \beta C_{\psi'} / (ik'r')] / \ln(k'a) \\ \partial_{\mathbf{r}'}^2 \mathbf{E}_{s,0}(\Gamma') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{E}_{s,0}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{E}_{s,0}(\Gamma') \neq 0 \end{aligned} \quad (44)$$

also a solution of the vector Laplace equation with vanishing divergence and nonzerocurl. Similarly to (35), in (43) the velocity effect term with factor β is a dipole term, demonstrating the mode-coupling effect. Once again it is emphasized that $\mathbf{E}_{s,0}(\Gamma')$ is the field measured in $\mathbf{\Gamma}$, but expressed in terms of $\mathbf{\Gamma}'$ native coordinates.

The associated magnetic field in $\mathbf{\Gamma}$, is derived from (34), (40), (42) in the form

$$\begin{aligned} \mathbf{H}_{s,0}(\Gamma') &= \tilde{\mathbf{V}} \cdot (\mathbf{H}'_{s,0}(\Gamma') - \varepsilon \mathbf{v}' \times \mathbf{E}'_{s,0}(\Gamma')) \\ &= h'_0 \tilde{\mathbf{V}} \cdot (\hat{\psi}' / (ik'r') + \beta \hat{\mathbf{y}}' \ln(k'r')) / \ln(k'a) \\ \hat{\mathbf{y}}' &= \hat{\mathbf{r}}' S_{\psi'} + \hat{\psi}' C_{\psi'}, S_{\psi'} = \sin \psi' \end{aligned} \quad (45)$$

The term $n = 1$ in (28) involves $r' C_{\psi'}$, hence $C_{\psi'}$ must appear in the scattered wave in order for the boundary condition (37) to be satisfied. Similarly to (43), the zero-curl solution derived from (25) having $C_{\psi'}$ as a factor is proportional to

$$\partial_{\mathbf{r}'} \varphi'_1(\mathbf{r}') = \partial_{\mathbf{r}'} (C_{\psi'} / r') = -(\hat{\mathbf{r}}' C_{\psi'} + \hat{\psi}' S_{\psi'}) / r'^2 \quad (46)$$

A similar solution is obtained for $\varphi'_1 = S_{\psi'} / r'$. Both solutions are inadequate for compensating the incident field (39) in the $\hat{\mathbf{z}}'$ direction according to (37).

On the other hand the nonzero-curl solution prescribed by (25) is

$$\begin{aligned} \mathbf{E}'_{s,1}(\Gamma') &= \hat{\mathbf{z}}' e'_0 ik' \varphi'_1(\mathbf{r}') = \hat{\mathbf{z}}' A_1 e'_0 ik' C_{\psi'} / r', A_1 = -a^2 \\ \partial_{\mathbf{r}'}^2 \mathbf{E}'_{s,1}(\Gamma') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{E}'_{s,1}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{E}'_{s,1}(\Gamma') \neq 0 \end{aligned} \quad (47)$$

with the value of A_1 in (47) prescribed by (37).

The associated magnetic field is derived from (17)

$$\begin{aligned}\mathbf{H}'_{s,1}(\Gamma') &= \partial_{\mathbf{r}'} \times \mathbf{E}'_{s,1}(\Gamma') / (Zik') \\ &= -h'_0 a^2 \partial_{\mathbf{r}'} \times (\hat{\mathbf{z}} C_{\psi'} / r') = h'_0 (a/r')^2 (\hat{\mathbf{r}}' S_{\psi'} - \hat{\boldsymbol{\psi}}' C_{\psi'}) \quad (48) \\ \partial_{\mathbf{r}'}^2 \mathbf{H}'_{s,1}(\Gamma') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{H}'_{s,1}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{H}'_{s,1}(\Gamma') = 0\end{aligned}$$

The analog of (44) for the present case is

$$\begin{aligned}\mathbf{E}_{s,1}(\Gamma') &= -\hat{\mathbf{z}} e'_0 \gamma a^2 (ik' C_{\psi'} / r' + \beta C_{2\psi'} / r'^2) \\ \partial_{\mathbf{r}'}^2 \mathbf{E}_{s,1}(\Gamma') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{E}_{s,1}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{E}_{s,1}(\Gamma') \neq 0\end{aligned} \quad (49)$$

where the velocity effects adds a quadrupole term to the velocity independent dipole.

The associated magnetic field, the analog of (45), is now derived according to (34)

$$\begin{aligned}\mathbf{H}_{s,1}(\Gamma') &= \tilde{\mathbf{V}} \cdot (\mathbf{H}'_{s,1}(\Gamma') - \varepsilon \mathbf{v}' \times \mathbf{E}'_{s,1}(\Gamma')) \\ &= h'_0 \tilde{\mathbf{V}} \cdot ((a/r')^2 (\hat{\mathbf{r}}' S_{\psi'} - \hat{\boldsymbol{\psi}}' C_{\psi'}) + \beta \hat{\mathbf{y}}' a^2 ik' C_{\psi'} / r')\end{aligned} \quad (50)$$

For an excitation wave with the magnetic field polarized along $\hat{\mathbf{z}}$, (39) is replaced by

$$\begin{aligned}\mathbf{H}'(\Gamma') &= \hat{\mathbf{z}} h'_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'}, \mathbf{E}'(\Gamma') = \hat{\mathbf{y}}' e'_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} = (\hat{\mathbf{r}}' S_{\psi'} + \hat{\boldsymbol{\psi}}' C_{\psi'}) e'_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} \\ e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} &= \sum_{n=0}^{\infty} (ik')^n (\hat{\mathbf{x}}' \cdot \hat{\mathbf{r}}')^n / n! = \sum_{n=0}^{\infty} (ik')^n (r' C_{\psi'})^n / n!\end{aligned} \quad (51)$$

The analog of (40) for the second Consistent Maxwell System (18) for $n = 0$ and nonzero-curl solutions of the type (25) is

$$\begin{aligned}\mathbf{H}'_{s,0}(\Gamma') &= h'_0 \mathbf{H}'_0(\mathbf{r}'), \mathbf{H}'_0(\mathbf{r}') = \hat{\mathbf{z}}' \varphi'_0(\mathbf{r}') = \hat{\mathbf{z}}' B_0 \ln(k' r') \\ \partial_{\mathbf{r}'}^2 \mathbf{H}'_0(\mathbf{r}') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{H}'_0(\mathbf{r}') = 0, \partial_{\mathbf{r}'} \times \mathbf{H}'_0(\mathbf{r}') \neq 0\end{aligned} \quad (52)$$

The associated electric field is the analog of (42)

$$\begin{aligned}\mathbf{E}'_{s,0}(\Gamma') &= - (e'_0 / k'_0) \partial_{\mathbf{r}'} \times \mathbf{H}'_{s,0}(\Gamma') / (ik') = -e'_0 \partial_{\mathbf{r}'} \times \hat{\mathbf{z}}' B_0 \ln(k' r') / (ik') \\ &= e'_0 B_0 \hat{\mathbf{z}}' \times \partial_{\mathbf{r}'} \ln(k' r') / (ik') = e'_0 B_0 \hat{\boldsymbol{\psi}}' / (ik' r') \quad (53) \\ \partial_{\mathbf{r}'}^2 \mathbf{E}'_{s,0}(\Gamma') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{E}'_{s,0}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{E}'_{s,0}(\Gamma') = 0\end{aligned}$$

Presently the boundary condition (37) for the perfectly conducting cylinder, is specified on the tangential component of the associated

electric field in (51). Inasmuch as (51) for $n = 0$ possesses a factor $\hat{\psi}' C_{\psi'}$ which is missing in (53), we conclude that $B_0 = 0$ must be assumed, i.e., the monopole term solution does not exist in this problem.

We therefore consider the next multipole term, namely the dipole-type field $\mathbf{H}'_{s,1}(\Gamma')$, analogous to (47)

$$\begin{aligned} \mathbf{H}'_{s,1}(\Gamma') &= \hat{\mathbf{z}}' h'_0 \varphi'_1(\mathbf{r}') = \hat{\mathbf{z}}' h'_0 B_1 C_{\psi'}/r' \\ \partial_{\mathbf{r}'}^2 \mathbf{H}'_{s,1}(\Gamma') &= 0, \partial_{\mathbf{r}'} \cdot \mathbf{H}'_{s,1}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{H}'_{s,1}(\Gamma') \neq 0 \end{aligned} \quad (54)$$

and its associated electric field $\mathbf{E}'_{s,1}(\Gamma')$, the analog of (48)

$$\begin{aligned} \mathbf{E}'_{s,1}(\Gamma') &= -(e'_0/h'_0) \partial_{\mathbf{r}'} \times \mathbf{H}'_{s,1}(\Gamma')/ik' = -e'_0 B_1 \partial_{\mathbf{r}'} \times (\hat{\mathbf{z}}' C_{\psi'}/r') \\ &= e'_0 B_1 \hat{\mathbf{z}}' \times \partial_{\mathbf{r}'} (C_{\psi'}/r') = -e'_0 B_1 \hat{\mathbf{z}}' \times (\hat{\mathbf{r}}' C_{\psi'} + \hat{\psi}' S_{\psi'})/r'^2 \\ &= e'_0 B_1 (\hat{\mathbf{r}}' S_{\psi'} - \hat{\psi}' C_{\psi'})/r'^2 \end{aligned} \quad (55)$$

$$\partial_{\mathbf{r}'}^2 \mathbf{E}'_{s,1}(\Gamma') = 0, \partial_{\mathbf{r}'} \cdot \mathbf{E}'_{s,1}(\Gamma') = 0, \partial_{\mathbf{r}'} \times \mathbf{E}'_{s,1}(\Gamma') = 0$$

Subject to (37), (51), for $n = 0$, and (55) we find

$$B_1 = a^2 \quad (56)$$

We also make the observation that the zero-curl solution according to (25), proportional to

$$\partial_{\mathbf{r}'} \varphi'_1(\mathbf{r}') = \partial_{\mathbf{r}'} (C_{\psi'}/r') = -(\hat{\mathbf{r}}' C_{\psi'} + \hat{\psi}' S_{\psi'})/r'^2 \quad (57)$$

is inadequate for compensating the $n = 0$ or $n = 1$ electric field terms in (39), according to (37). Similarly the magnetic zero-curl solution is inadequate for compensating the $n = 0$ or $n = 1$ electric field terms in (51), according to (37), because the zero-curl property means in (57) that we do not have an associated electric field.

The transformation of (54) back to $\mathbf{\Gamma}$ is once again prescribed by (34). Accordingly we obtain for the scattered electric field

$$\begin{aligned} \mathbf{H}_{s,1}(\Gamma') &= \tilde{\mathbf{W}}' \cdot \mathbf{H}'_{s,1}(\Gamma') = \tilde{\mathbf{V}} \cdot (\tilde{\mathbf{I}} - \beta \hat{\mathbf{v}} \times \partial_{\mathbf{r}'} \times \tilde{\mathbf{I}}/(ik')) \cdot \mathbf{H}'_{s,1}(\Gamma') \\ &= \gamma h'_0 a^2 (\hat{\mathbf{z}}' ik' C_{\psi'}/r' + \beta \hat{\mathbf{v}} \times (\hat{\psi}' C_{\psi'} - \hat{\mathbf{r}}' S_{\psi'})/ik' r'^2) \\ &= \hat{\mathbf{z}}' \gamma h'_0 a^2 (ik' C_{\psi'}/r' + \beta C_{2\psi'}/(ik' r'^2)) \end{aligned} \quad (58)$$

$$\partial_{\mathbf{r}'}^2 \mathbf{H}_{s,1} = 0, \partial_{\mathbf{r}'} \cdot \mathbf{H}_{s,1} = 0, \partial_{\mathbf{r}'} \times \mathbf{H}_{s,1} \neq 0$$

Once again (58) displays mode coupling, in terms of Γ' coordinates, where in the first line the first and second terms in parentheses correspond to a dipole and a quadrupole, respectively. Transformation of the associated electric field (55) is prescribed by (34), and constitutes the analog of (45) in the form

$$\begin{aligned}\mathbf{E}_{s,1}(\Gamma') &= \tilde{\mathbf{V}} \cdot \left(\mathbf{E}'_{s,1}(\Gamma') + \mu \mathbf{v}' \times \mathbf{H}'_{s,1}(\Gamma') \right) \\ &= \tilde{\mathbf{V}} \cdot a^2 e'_0 \left((\hat{\mathbf{r}}' S_{\psi'} - \psi' C_{\psi'}) / r'^2 - \mu \mathbf{v}' \times \hat{\mathbf{z}} h'_0 / e'_0 C_{\psi'} / r' \right) \\ &= \tilde{\mathbf{V}} \cdot a^2 e'_0 \left((\hat{\mathbf{r}}' S_{\psi'} - \hat{\psi}' C_{\psi'}) / r'^2 + \beta \hat{\mathbf{y}}' C_{\psi'} / r' \right)\end{aligned}\quad (59)$$

With this we end the discussion on scattering by a cylinder. As commented above, for brevity the last step of substituting (1) in order to express results explicitly in terms of Γ native coordinates Γ is left for mathematical simulations.

3.2. Scattering by a Moving Sphere, Leading Terms

The present analysis, in terms of spherical solutions of the Laplace equation, is similar to the cylindrical case. As in (39) the incident plane wave is propagating in the $\mathbf{k}' = \hat{\mathbf{x}}'$ direction, and is polarized along the spherical polar axis $\hat{\mathbf{z}}'$, with $\hat{\mathbf{r}}'$ denoting now the spherical radius unit vector. Hence we have

$$\begin{aligned}\mathbf{E}'(\Gamma') &= \hat{\mathbf{z}}' e'_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} = (\hat{\mathbf{r}}' C_{\theta'} - \hat{\theta}' S_{\theta'}) e'_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'}, \quad \mathbf{H}(\Gamma') = -\hat{\mathbf{y}}' h'_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} \\ e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} &= \sum_{n=0}^{\infty} (ik')^n (\hat{\mathbf{x}}' \cdot \hat{\mathbf{r}}' r')^n / n! = \sum_{n=0}^{\infty} (ik')^n (r' S_{\theta'} C_{\psi'})^n / n! \quad (60) \\ \hat{\mathbf{r}}' &= \hat{\mathbf{x}}' S_{\theta'} C_{\psi'} + \hat{\mathbf{y}}' S_{\theta'} S_{\psi'} + \hat{\mathbf{z}}' C_{\theta'}\end{aligned}$$

with θ' in (60) denoting the polar angle, subtended by $\hat{\mathbf{z}}'$ and $\hat{\mathbf{r}}'$, with $\hat{\theta}'$ pointing in the direction of increasing θ' , and with ψ' standing for the azimuthal angle, as commonly denoted.

Here too the scatterer is chosen as a perfectly conducting sphere, hence similarly to the case of the circular cylinder, the boundary condition (37) prescribes that at the surface $r' = a$ the total tangential electric field vanishes.

Consider the solutions $\nu = 0$, $\mu = 0$ of (26). In analogy with (43), the radial lowest order zero-curl solution, proportional to

$$\partial_{\mathbf{r}'} \varphi_0^0 = \partial_{\mathbf{r}'} (1/r') = -\hat{\mathbf{r}}' / r'^2 \quad (61)$$

cannot be involved with the boundary condition (37) regarding tangential fields. The associated \mathbf{M}_0^0 is identically zero.

In (26), the next term in the hierarchy is $\nu = 1$, $\mu = 0$, yielding the nonzero-curl solution of the vector Laplace equation

$$\begin{aligned}\mathbf{M}_1^0(\mathbf{r}') &= -\mathbf{r}' \times \partial_{\mathbf{r}'} \varphi_1^0(\mathbf{r}') = -\mathbf{r}' \times \partial_{\mathbf{r}'} (C_{\theta'}/r'^2) \\ &= \mathbf{r}' \times (\hat{\mathbf{r}}' 2C_{\theta'} + \hat{\boldsymbol{\theta}}' S_{\theta'}) / r'^3 = \hat{\boldsymbol{\psi}}' S_{\theta'} / r'^2, \quad \partial_{\mathbf{r}'} \times \mathbf{M}_1^0(\mathbf{r}') \neq 0\end{aligned}\quad (62)$$

Obviously we cannot choose (62) to describe the scattered electric field excited by (60), because it is pointing in the azimuthal direction $\hat{\boldsymbol{\psi}}'$, and hence cannot compensate for the electric field, as prescribed by (37). On the other hand, using the second Consistent Maxwell System (18) with (30), and the magnetic field prescribed by (62), we derive for the magnetic field and the associated electric field

$$\begin{aligned}\mathbf{H}'_{s,1}(\Gamma') &= B_1 h'_0 i k' \mathbf{H}'_1(\mathbf{r}') = B_1 h'_0 i k' \mathbf{M}_1^0(\mathbf{r}') = \hat{\boldsymbol{\psi}}' B_1 h'_0 i k' S_{\theta'} / r'^2 \\ \mathbf{E}'_1(\mathbf{r}') &= -\partial_{\mathbf{r}'} \times \mathbf{H}'_1(\mathbf{r}') / i k' = -(\hat{\mathbf{r}}' 2S_{\theta'} + \hat{\boldsymbol{\theta}}' S_{\theta'}) / (i k' r'^3) \\ \mathbf{E}'_{s,1}(\Gamma') &= -e'_0 B_1 (\hat{\mathbf{r}}' 2C_{\theta'} + \hat{\boldsymbol{\theta}}' S_{\theta'}) / r'^3 \\ \partial_{\mathbf{r}'}^2 \mathbf{H}'_{s,1}(\Gamma') &= 0, \quad \partial_{\mathbf{r}'} \cdot \mathbf{H}'_{s,1}(\Gamma') = 0, \quad \partial_{\mathbf{r}'} \times \mathbf{H}'_{s,1}(\Gamma') \neq 0 \\ \partial_{\mathbf{r}'}^2 \mathbf{E}'_{s,1}(\Gamma') &= 0, \quad \partial_{\mathbf{r}'} \cdot \mathbf{E}'_{s,1}(\Gamma') = 0, \quad \partial_{\mathbf{r}'} \times \mathbf{E}'_{s,1}(\Gamma') = 0\end{aligned}\quad (63)$$

Application of the boundary condition (37) to the sum of the tangential electric fields of (60) for $n = 0$ and (63) yields

$$B_1 = -a^3 \quad (64)$$

The transformation of $\mathbf{H}'_{s,1}$ in (63), (64), back into Γ is the analog of (58), once again prescribed by (34). Accordingly we obtain for the scattered electric field

$$\begin{aligned}\mathbf{H}_{s,1}(\Gamma') &= h'_0 a^3 \tilde{\mathbf{V}} \cdot \left(-\hat{\boldsymbol{\psi}}' i k' S_{\theta'} / r'^2 + \beta \hat{\mathbf{v}} \times (\hat{\mathbf{r}}' 2C_{\theta'} / r'^3 + \hat{\boldsymbol{\theta}}' S_{\theta'} / r'^3) \right) \\ \hat{\mathbf{v}} &= \hat{\mathbf{x}} = \hat{\mathbf{r}}' S_{\theta'} C_{\psi'} + \hat{\boldsymbol{\theta}}' C_{\theta'} C_{\psi'} - \hat{\boldsymbol{\psi}}' S_{\psi'}\end{aligned}\quad (65)$$

Using (63), (64), and the transformation prescribed by (34), the analog of (59) is derived

$$\begin{aligned}\mathbf{E}_{s,1}(\Gamma') &= \tilde{\mathbf{V}} \cdot \left(\mathbf{E}'_{s,1}(\Gamma') + \mu \mathbf{v}' \times \mathbf{H}'_{s,1}(\Gamma') \right) \\ &= \tilde{\mathbf{V}} \cdot e'_0 a^3 \left((\hat{\mathbf{r}}' 2C_{\theta'} + \hat{\boldsymbol{\theta}}' S_{\theta'}) / r'^3 + \beta \hat{\mathbf{v}} \times \hat{\boldsymbol{\psi}}' i k' S_{\theta'} / r'^2 \right) \\ \hat{\mathbf{v}} \times \hat{\boldsymbol{\psi}}' &= \hat{\mathbf{x}} \times \hat{\boldsymbol{\psi}}' = -\hat{\boldsymbol{\theta}}' S_{\theta'} C_{\psi'} + \hat{\mathbf{r}}' C_{\theta'} C_{\psi'}\end{aligned}\quad (66)$$

For completeness consider the incident wave with the magnetic field polarized along $\hat{\mathbf{z}}'$. Instead of (60) we now have

$$\begin{aligned}\mathbf{H}'(\Gamma') &= \hat{\mathbf{z}}' h_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} = \left(\hat{\mathbf{r}}' C_{\theta'} - \hat{\boldsymbol{\theta}}' S_{\theta'} \right) h_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} \\ \mathbf{E}'(\Gamma') &= \hat{\mathbf{y}}' e_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} = \left(\hat{\mathbf{r}}' S_{\theta'} S_{\psi'} + \hat{\boldsymbol{\theta}}' C_{\theta'} S_{\psi'} + \hat{\boldsymbol{\psi}}' C_{\psi'} \right) e_0 e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} \\ e^{ik' \hat{\mathbf{x}}' \cdot \mathbf{r}'} &= \sum_{n=0}^{\infty} (ik')^n (\hat{\mathbf{x}}' \cdot \hat{\mathbf{r}}' r')^n / n! = \sum_{n=0}^{\infty} (ik')^n (r' S_{\theta'} C_{\psi'})^n / n!\end{aligned}\quad (67)$$

To satisfy the boundary condition (37) we need in (26) the solution $\nu = 1$, $\mu = 1$

$$\begin{aligned}\mathbf{M}_1^1(\mathbf{r}') &= -\mathbf{r}' \times \mathbf{L}_1^1(\mathbf{r}') = -\mathbf{r}' \times \partial_{\mathbf{r}'} (\varphi_1^1) = -\mathbf{r}' \times \partial_{\mathbf{r}'} (S_{\theta'} S_{\psi'} / r'^2) \\ &= -\mathbf{r}' \times \left(-\hat{\mathbf{r}}' 2 S_{\theta'} S_{\psi'} + \hat{\boldsymbol{\theta}}' C_{\theta'} S_{\psi'} + \hat{\boldsymbol{\psi}}' C_{\psi'} \right) / r'^3 \\ &= \left(-\hat{\boldsymbol{\psi}}' C_{\theta'} S_{\psi'} + \hat{\boldsymbol{\theta}}' C_{\psi'} \right) / r'^2, \quad \partial_{\mathbf{r}'} \times \mathbf{M}_1^1(\mathbf{r}') \neq 0\end{aligned}\quad (68)$$

which is assigned to the magnetic field in the form

$$\begin{aligned}\mathbf{H}_{s,1}'(\Gamma') &= C_1 h_0 ik' \mathbf{H}_1'(\mathbf{r}') = C_1 h_0 ik' \left(-\hat{\boldsymbol{\psi}}' C_{\theta'} S_{\psi'} + \hat{\boldsymbol{\theta}}' C_{\psi'} \right) / r'^2, \\ \partial_{\mathbf{r}'} \times \mathbf{H}_1'(\mathbf{r}') &\neq 0\end{aligned}\quad (69)$$

involving the yet undetermined constant C_1 . Inasmuch as (69) involves a nonzero-curl magnetic field, we are working within the second Consistent Maxwell System, (18).

The electric field associated with (69), is now derived as

$$\begin{aligned}ik' \mathbf{E}_1'(\mathbf{r}') &= -\partial_{\mathbf{r}'} \times \mathbf{H}_1'(\mathbf{r}') = \partial_{\mathbf{r}'} \times \left(\left(\hat{\boldsymbol{\psi}}' C_{\theta'} S_{\psi'} - \hat{\boldsymbol{\theta}}' C_{\psi'} \right) / r'^2 \right) \\ &= \hat{\mathbf{r}}' S_{\psi'} (C_{2\theta'} + S_{\psi'}) / \left(r'^3 S_{\theta'} \right) + \left(\hat{\boldsymbol{\theta}}' C_{\theta'} S_{\psi'} + \hat{\boldsymbol{\psi}}' C_{\psi'} \right) / r'^3 \\ \mathbf{E}_{s,1}'(\Gamma') &= C_1 e_0 ik' \mathbf{E}_1'(\mathbf{r}')\end{aligned}\quad (70)$$

Application of the boundary condition (37) to the tangential electric fields in (67), (70), yields

$$C_1 = -a^4 \quad (71)$$

The final step of transforming the Γ' scattered fields (69)–(71) into the scattered fields measured for a moving scatterer in Γ follows once again from (34), and is the analog of (66). Inasmuch as the technique is identical, this final step will be left to the interested reader. This ends the analysis for the leading terms of the low-frequency expansion in spherical coordinates.

4. CONCLUDING REMARKS

The Consistent Maxwell Systems and their application to low-frequency theory has been recently discussed [3]. Presently this method and Einstein's special relativity theory are incorporated in order to investigate scattering by moving objects in the low-frequency regime.

The leading terms of the low-frequency series are solutions of the vector Laplace equation. This is of special interest because solutions of the Laplace equation are simpler than wave solutions of the Helmholtz equation. Furthermore, the Laplace equation is separable in more coordinate systems than the Helmholtz equation, thus offering more canonical solutions.

The pertinent theoretical aspects are revisited and adapted to the present class of problems. In order to facilitate future simulations, explicit solutions have been derived. The resulting formulas are relativistically exact, but must be kept in sufficiently simple form to avoid unnecessary encumbering of the presentation. To this end we employ a few strategies. Firstly, the arguments of the relativistically exact formulas describing the fields in the reference system $\mathbf{\Gamma}$, where the objects are observed in motion, are expressed in terms of the space-time coordinates of the $\mathbf{\Gamma}'$ system where the objects are at rest. Another simplifying aspect is achieved by assuming the ambient propagation medium to be free space (vacuum). Finally, simple geometries of circular-cylinders and spheres are considered, and the boundary relations are limited to perfectly conducting surfaces.

The results show how velocity dependent mode coupling enters in the formulas. This effect has been noted previously for the exact scattering solutions. Essentially low-frequency solutions are derived in the near field, and the extension to large distances from the scatterer involves the corresponding scalar and vector Kirchhoff diffraction integrals, explained elsewhere [3].

The present study should be considered as a general introduction to the subject area. Different scattering geometries, velocity directions, and higher order terms will be derived for this class of problems enhancing our understanding of the subject.

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