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ANALYSIS OF LOSSY INHOMOGENEOUS PLANAR LAYERS USING EQUIVALENT SOURCES METHOD

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Abstract—A new method is introduced to analyze lossy Inhomogeneous Planar Layers (IPLs). In this method, the equations of IPLs are converted to the equations of homogeneous planar layers, which have been excited by distributed equivalent sources. Then, the electric and magnetic fields are obtained using an iterative approach. The validity of the method is verified using a comprehensive example.

1. INTRODUCTION

Inhomogeneous Planar Layers (IPLs) are widely used in electromagnetics as optimum shields and filters and etc. Also, the IPLs potentially provide less scattering, less stress, larger bandwidth and better coupling effects than homogeneous planar layers [1–8]. The differential equations describing IPLs have non-constant coefficients and so except for a few special cases no analytical solution exists for them. The most straightforward method to analyze IPLs is to subdivide them into many thin homogeneous planar layers [2] and [9]. Of course, analysis of arbitrary IPLs using Taylor's series and the Fourier series expansion of primary parameters or solving an integral equation has been introduced in [10, 11] and [12], respectively. In this paper, a new method is introduced to analyze arbitrary IPLs, also. In this method, the equations of IPLs are converted to the equations of homogeneous planar layers, which have been excited by distributed equivalent electric and magnetic sources. Then, the electric and magnetic fields are obtained using an iterative approach. This method is applicable to all arbitrary lossy and dispersive IPLs. The validity of the method is verified using a comprehensive example.

2. THE EQUATIONS OF IPLS

In this section, the frequency domain equations of the IPLs are reviewed. Figure 1 shows a typical IPL with the thickness d, whose left and right mediums are arbitrary such as the air or conductor. Two different polarizations are possible, one the TM and other the TE. It is assumed that the incident plane wave propagates obliquely towards positive x and z direction with an angle of incidence θ_i and electric filed strength of E^i.

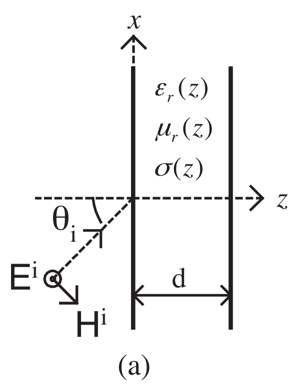

Figure 1. Incident plane wave to IPL structure, (a) TE polarization mode, (b) TM polarization mode.

The transverse electric and magnetic fields on the surface $z =$ const. are defined as follows

$$E_t(z) \triangleq \begin{cases} E_y(z); \text{TE} \\ E_x(z); \text{TM} \end{cases} \tag{1}$$

$$H_t(z) \triangleq \begin{cases} -H_x(z); \text{TE} \\ H_y(z); \text{TM} \end{cases} \tag{2}$$

The differential equations describing lossy IPLs are given by

$$\frac{dE_t(z)}{dz} = -Z(z)H_t(z) \tag{3}$$

$$\frac{dH_t(z)}{dz} = -Y(z)E_t(z) \tag{4}$$

In (3) and (4), two following primary parameters have been defined

$$Z(z) \triangleq \begin{cases} \hat{Z}(z), \mathrm{TE} \\ \hat{Z}(z) + k_x^2 \hat{Y}^{-1}(z), \mathrm{TM} \end{cases} \quad (5)$$

$$Y(z) \triangleq \begin{cases} \hat{Y}(z) + k_x^2 \hat{Z}^{-1}(z), \mathrm{TE} \\ \hat{Y}(z), \mathrm{TM} \end{cases} \quad (6)$$

where

$$\hat{Z}(z) = j\omega\mu_0\mu_r(z) \quad (7)$$

$$\hat{Y}(z) = \sigma(z) + j\omega\varepsilon_0\varepsilon_r(z) \quad (8)$$

In (5) and (6), k_x is the wavenumber for the x direction

$$k_x = \frac{\omega}{c}\sin(\theta_i) \quad (9)$$

where c is the velocity of the light and ω is the angular frequency. Furthermore, there are two boundary conditions as follows

$$E_t(0) + Z_S H_t(0) = E_S \quad (10)$$

$$E_t(d) = Z_L H_t(d) \quad (11)$$

where

$$E_S = \begin{cases} 2E^i, \mathrm{TE} \\ 2E^i \cos(\theta_i), \mathrm{TM} \end{cases} \quad (12)$$

Furthermore,

$$Z_S = \begin{cases} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{1}{\cos(\theta_i)}, \mathrm{TE} \\ \sqrt{\frac{\mu_0}{\varepsilon_0}} \cos(\theta_i), \mathrm{TM} \end{cases} \quad (13)$$

for when the left medium is being the air. Also $Z_L = Z_S$ or $Z_L = 0$ for the air- or conductor-backed IPLs, respectively. The reflection and the transmission coefficients will be determined using the transverse electric fields on the surfaces $z = 0$ and $z = d$ as follows

$$\Gamma_{in} = \begin{cases} \frac{E_t(0)}{E^i} - 1, \mathrm{TE} \\ \frac{E_t(0)}{E^i \cos(\theta_i)} - 1, \mathrm{TM} \end{cases} \quad (14)$$

$$T = \begin{cases} \frac{E_t(d)}{E^i}, \mathrm{TE} \\ \frac{E_t(d)}{E^i \cos(\theta_i)}, \mathrm{TM} \end{cases} \quad (15)$$

3. IPLS EXCITED BY EQUIVALENT SOURCES

In this section, the analysis of arbitrary lossy IPLs using the method of equivalent sources is introduced. First, the average of the primary parameters are defined as follows

$$\bar{Z} = \frac{1}{d}\int_0^d Z(z)dz \tag{16}$$

$$\bar{Y} = \frac{1}{d}\int_0^d Y(z)dz \tag{17}$$

The differential Equations (3) and (4) can be converted to the following equations

$$\frac{dE_t(z)}{dz} = -\bar{Z}H_t(z) + M(z) \tag{18}$$

$$\frac{dH_t(z)}{dz} = -\bar{Y}E_t(z) + J(z) \tag{19}$$

The Equations (18) and (19) are related to homogeneous planar layers, which have been excited by distributed equivalent sources (magnetic and electric types) defined by

$$M(z) \triangleq (\bar{Z} - Z(z))H_t(z) \tag{20}$$

$$J(z) \triangleq (\bar{Y} - Y(z))E_t(z) \tag{21}$$

Combining (18) and (19) with each other, gives the following differential equations

$$\frac{d^2E_t(z)}{dz^2} - \bar{\gamma}^2 E_t(z) = \frac{dM(z)}{dz} - \bar{Z}J(z) \tag{22}$$

$$H_t(z) = \frac{1}{\bar{Z}}\left(M(z) - \frac{dE_t(z)}{dz}\right) \tag{23}$$

where

$$\bar{\gamma} = \sqrt{\bar{Z}\bar{Y}} \tag{24}$$

As it has been shown in [13] for similar equations, the electric and magnetic field distributions are obtained from (22) and (23) as follows

$$\begin{aligned} E_t(z) &= E^+\exp(-\bar{\gamma}z) + E^-\exp(\bar{\gamma}z) \\ &\quad + \frac{1}{2\bar{\gamma}}\exp(\bar{\gamma}z)\int_0^z \exp(-\bar{\gamma}z')\left(\frac{dM(z')}{dz'} - \bar{Z}J(z')\right)dz' \\ &\quad - \frac{1}{2\bar{\gamma}}\exp(-\bar{\gamma}z)\int_0^z \exp(\bar{\gamma}z')\left(\frac{dM(z')}{dz'} - \bar{Z}J(z')\right)dz' \qquad (25) \end{aligned}$$

$$\begin{aligned} H_t(z) &= \frac{1}{\bar{Z}}M(z) + \frac{E^+}{\bar{Z}_c}\exp(-\bar{\gamma}z) - \frac{E^-}{\bar{Z}_c}\exp(\bar{\gamma}z) \\ &\quad - \frac{1}{2\bar{Z}}\exp(\bar{\gamma}z)\int_0^z \exp(-\bar{\gamma}z')\left(\frac{dM(z')}{dz'} - \bar{Z}J(z')\right)dz' \\ &\quad - \frac{1}{2\bar{Z}}\exp(-\bar{\gamma}z)\int_0^z \exp(\bar{\gamma}z')\left(\frac{dM(z')}{dz'} - \bar{Z}J(z')\right)dz' \qquad (26) \end{aligned}$$

The constants E^+ and E^- in (25) and (26) are obtained from the terminal conditions (10) and (11) as follows

$$\begin{aligned} E^+ &= \frac{1}{1 - \bar{\Gamma}_S\bar{\Gamma}_L\exp(-2\bar{\gamma}d)} \times \left[\frac{\bar{Z}_c}{Z_S + \bar{Z}_c}E_S - \frac{1}{\bar{\gamma}}\frac{Z_S}{Z_S + \bar{Z}_c}M(0)\right. \\ &\quad + \bar{\Gamma}_S\frac{\exp(-\bar{\gamma}d)}{\bar{\gamma}}\frac{Z_L}{Z_L + \bar{Z}_c}M(d) \\ &\quad - \bar{\Gamma}_S\frac{1}{2\bar{\gamma}}\int_0^d \exp(-\bar{\gamma}z')\left(\frac{dM(z')}{dz'} - \bar{Z}J(z')\right)dz' \\ &\quad \left. - \bar{\Gamma}_S\bar{\Gamma}_L\frac{\exp(-2\bar{\gamma}d)}{2\bar{\gamma}}\int_0^d \exp(\bar{\gamma}z')\left(\frac{dM(z')}{dz'} - \bar{Z}J(z')\right)dz'\right] \qquad (27) \end{aligned}$$

$$
\begin{aligned}
E^{-} \;=\; & \frac{1}{1-\bar{\Gamma}_S\bar{\Gamma}_L\exp(-2\bar{\gamma}d)} \times \Bigg[\bar{\Gamma}_L\exp(-2\bar{\gamma}d)\frac{\bar{Z}_c}{Z_S+\bar{Z}_c}E_S \\
& -\bar{\Gamma}_L\frac{\exp(-2\bar{\gamma}d)}{\bar{\gamma}}\frac{Z_S}{Z_S+\bar{Z}_c}M(0)+\frac{\exp(-\bar{\gamma}d)}{\bar{\gamma}}\frac{Z_L}{Z_L+\bar{Z}_c}M(d) \\
& -\frac{1}{2\bar{\gamma}}\int_0^d \exp(-\bar{\gamma}z')\left(\frac{dM(z')}{dz'}-\bar{Z}J(z')\right)dz' \\
& -\bar{\Gamma}_L\frac{\exp(-2\bar{\gamma}d)}{2\bar{\gamma}}\int_0^d \exp(\bar{\gamma}z')\left(\frac{dM(z')}{dz'}-\bar{Z}J(z')\right)dz'\Bigg] \qquad (28)
\end{aligned}
$$

where

$$\bar{\Gamma}_S \;=\; \frac{Z_S-\bar{Z}_c}{Z_S+\bar{Z}_c} \qquad (29)$$

$$\bar{\Gamma}_L \;=\; \frac{Z_L-\bar{Z}_c}{Z_L+\bar{Z}_c} \qquad (30)$$

in which

$$\bar{Z}_c=\sqrt{\frac{\bar{Z}}{\bar{Y}}} \qquad (31)$$

The integrals existed in (16) and (17) and (25)–(28) are exactly calculated if the primary parameters are known at all points, continuously. However, these integrals can be approximately calculated if the primary parameters are known only at some points. In this case, which is more practical, we can assume that the primary parameters vary between two adjacent points stepwise, linearly or in other manners.

4. AN ITERATIVE APPROACH

The electric and magnetic field distributions obtained as (25)–(28) require the distributed equivalent sources defined in (20) and (21). On the other hand, these equivalent sources require the electric and magnetic field distributions. To overcome this problem, we can use an iterative approach. At first iteration, we consider the equivalent sources to be zero.

$$M^{(1)}(z)=J^{(1)}(z)=0 \qquad (32)$$

Then, the electric and magnetic field distributions at first iteration are obtained from (25)–(28) as follows

$$E_t^{(1)}(z) = E^{+(1)} \exp(-\bar{\gamma} z) + E^{-(1)} \exp(\bar{\gamma} z) \tag{33}$$

$$H_t^{(1)}(z) = \frac{1}{\bar{Z}_c} \left(E^{+(1)} \exp(-\bar{\gamma} z) - E^{-(1)} \exp(\bar{\gamma} z) \right) \tag{34}$$

in which

$$E^{+(1)} = \frac{\exp(2\bar{\gamma} d)}{\bar{\Gamma}_L} E^{-(1)} = E_S \frac{\bar{Z}_c}{Z_S + \bar{Z}_c} \frac{1}{1 - \bar{\Gamma}_L \bar{\Gamma}_S \exp(-2\bar{\gamma} d)} \tag{35}$$

Now, the equivalent sources are corrected at the second iteration using (20) and (21). Consequently, using (25)–(28) and (20)–(21) alternately, the electric and magnetic field distributions are obtained with a low error.

5. EXAMPLE AND RESULTS

In this section, a comprehensive example is presented to study the validity of the introduced method. Consider an exponential IPL with the following primary parameters

$$\mu_r(z) = \mu_{r0} \tag{36}$$

$$\varepsilon_r(z) = \varepsilon_{r0} \exp(kz/d) \tag{37}$$

$$\sigma(z) = 0 \tag{38}$$

The average of the primary parameters defined in (16) and (17) will be as follows

$$\bar{Z} = j\omega\mu_0 \tag{39}$$

$$\bar{Y} = j\omega\varepsilon_0\varepsilon_{r0} \frac{\exp(k) - 1}{k} \tag{40}$$

We assume that a plane wave with TE polarization is illuminated to the assumed IPL. With this assumption, we will find from (7), (20), (36) and (39) that

$$M(z) = 0 \tag{41}$$

The equivalent electric sources at the second iteration can be obtained using (8), (21), (33), (35), (37) and (40) as follows

$$J^{(2)}(z) = j\omega\varepsilon_0\varepsilon_{r0} E^{+(1)} \left(\frac{\exp(k) - 1}{k} - \exp(kz/d) \right) \left(\exp(-\bar{\gamma} z) + \bar{\Gamma}_L \exp(\bar{\gamma}(z - 2d)) \right) \tag{42}$$

Also, the electric and magnetic field distributions at the second iteration are obtained by substituting (41) and (42) in (25)–(28) as follows

$$E_t^{(2)}(z) = E^{+(2)}\exp(-\bar{\gamma}z)+E^{-(2)}\exp(\bar{\gamma}z)-\frac{1}{2}\bar{Z}_c\exp(\bar{\gamma}z)(A(z)-A(0)) \\ +\frac{1}{2}\bar{Z}_c\exp(-\bar{\gamma}z)\,(B(z)-B(0)) \tag{43}$$

$$H_t^{(2)}(z) = \frac{E^{+(2)}}{\bar{Z}_c}\exp(-\bar{\gamma}z)-\frac{E^{-(2)}}{\bar{Z}_c}\exp(\bar{\gamma}z)+\frac{1}{2}\exp(\bar{\gamma}z)\,(A(z)-A(0)) \\ +\frac{1}{2}\exp(-\bar{\gamma}z)\,(B(z)-B(0)) \tag{44}$$

where

$$E^{+(2)} = \frac{1}{1-\bar{\Gamma}_S\bar{\Gamma}_L\exp(-2\bar{\gamma}d)}\times\left(\frac{\bar{Z}_c}{Z_S+\bar{Z}_c}E_S+\frac{1}{2}\bar{Z}_c\bar{\Gamma}_S\,(A(d)-A(0)) \right. \\ \left. +\frac{1}{2}\bar{Z}_c\bar{\Gamma}_S\bar{\Gamma}_L\exp(-2\bar{\gamma}d)\,(B(d)-B(0))\right) \tag{45}$$

$$E^{-(2)} = \frac{1}{1-\bar{\Gamma}_S\bar{\Gamma}_L\exp(-2\bar{\gamma}d)}\times\left(\bar{\Gamma}_L\exp(-2\bar{\gamma}d)\frac{\bar{Z}_c}{Z_S+\bar{Z}_c}E_S \right. \\ \left. +\frac{1}{2}\bar{Z}_c\,(A(d)-A(0))+\frac{1}{2}\bar{Z}_c\bar{\Gamma}_L\exp(-2\bar{\gamma}d)(B(d)-B(0))\right) \tag{46}$$

in which $A(z)$ and $B(z)$ are two functions defined as

$$A(z) = j\omega\varepsilon_0\varepsilon_{r0}E^{+(1)}\left[\frac{\exp(k)-1}{k}\left(\bar{\Gamma}_L\exp(-2\bar{\gamma}d)z-\frac{1}{2\bar{\gamma}}\exp(-2\bar{\gamma}z)\right) \right. \\ -\frac{1}{k/d-2\bar{\gamma}}\exp((k/d-2\bar{\gamma})z) \\ \left. -\bar{\Gamma}_L\exp(-2\bar{\gamma}d)\frac{1}{k/d}\exp(kz/d)\right] \tag{47}$$

$$B(z) = j\omega\varepsilon_0\varepsilon_{r0}E^{+(1)}\left[\frac{\exp(k)-1}{k}\left(z+\bar{\Gamma}_L\frac{1}{2\bar{\gamma}}\exp(2\bar{\gamma}(z-d))\right) \right. \\ \left. -\frac{1}{k/d}\exp(kz/d)-\frac{\bar{\Gamma}_L\exp(-2\bar{\gamma}d)}{2\bar{\gamma}+k/d}\exp((2\bar{\gamma}+k/d)z)\right] \tag{48}$$

Now, assume that $\varepsilon_{r0} = 4$ and $\mu_{r0} = 1$. A plane wave with TE polarization, the angle of incidence $\theta_i = 60°$, the electric field strength $E^i = 1.0\,\text{V/m}$ and the excitation frequency 1.0 GHz is illuminated to

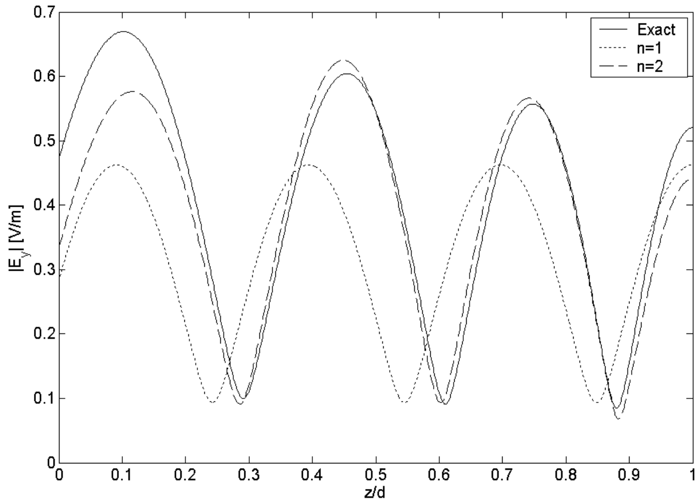

Figure 2. The magnitude of the electric field distribution for $d = 20\,\text{cm}$ and $k = 1$ at frequency $f = 1.0\,\text{GHz}$.

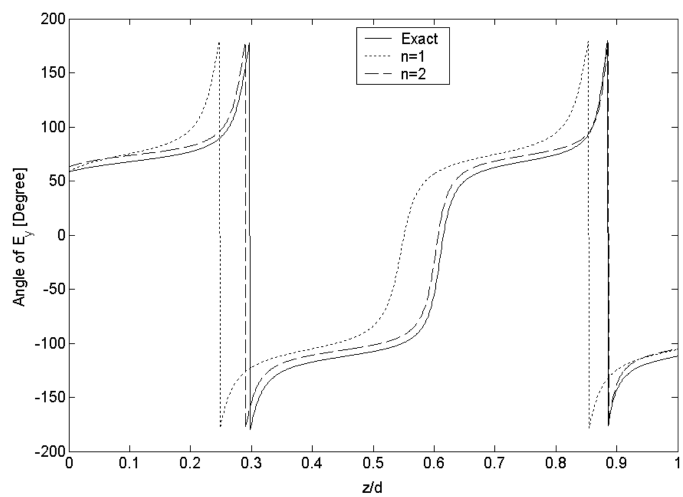

Figure 3. The angle of the electric field distribution for $d = 20\,\text{cm}$ and $k = 1$ at frequency $f = 1.0\,\text{GHz}$.

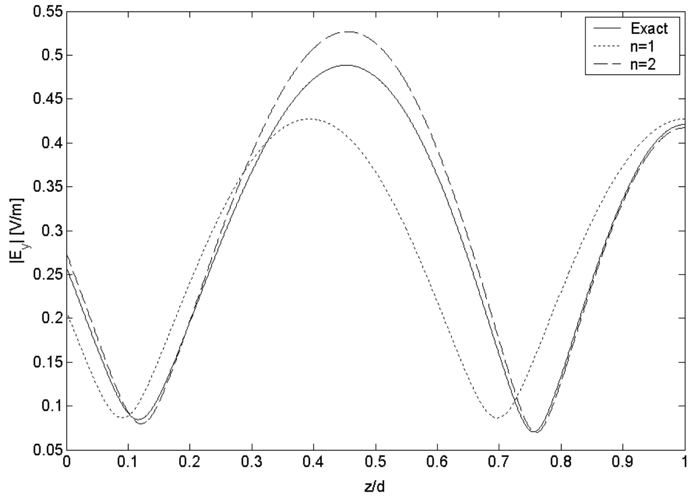

Figure 4. The magnitude of the electric field distribution for $d = 10\,\mathrm{cm}$ and $k = 1$ at frequency $f = 1.0\,\mathrm{GHz}$.

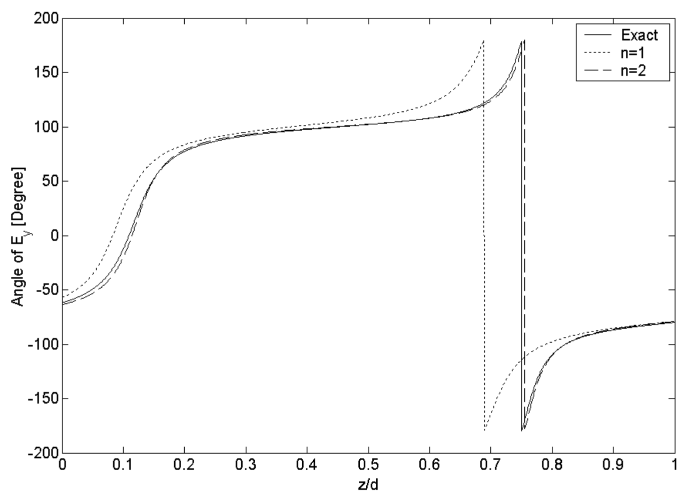

Figure 5. The angle of the electric field distribution for $d = 10\,\mathrm{cm}$ and $k = 1$ at frequency $f = 1.0\,\mathrm{GHz}$.

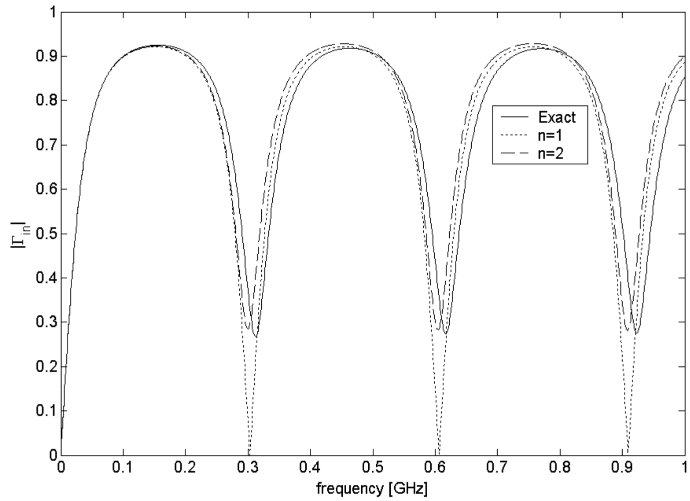

Figure 6. The magnitude of the reflection coefficient versus frequency for $d = 20\,\text{cm}$ and $k = 1$.

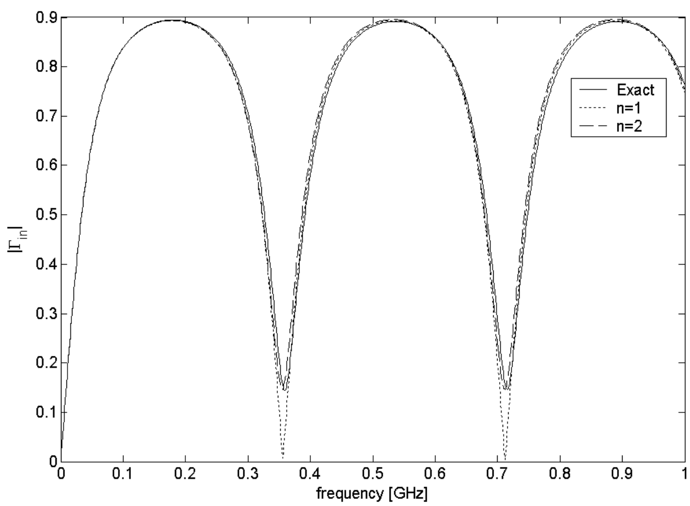

Figure 7. The magnitude of the reflection coefficient versus frequency for $d = 20\,\text{cm}$ and $k = 0.5$.

the assumed structure. Figs. 2–5, compare the magnitude and the angle of the electric field obtained from the exact solutions [9, 10] and from the introduced method in its first and second iterations, for $k = 1$ and $d = 20\,\mathrm{cm}$ and $10\,\mathrm{cm}$. Also, Figs. 6–7 compare the magnitude of the reflection coefficient versus the frequency, obtained from the exact solution and from the presented method for $d = 20\,\mathrm{cm}$ and $k = 1$ and 0.5. One observes a good agreement between the exact solutions and the solutions obtained from the proposed method at the second iteration. It is concluded from Figs. 2–7 that the accuracy of the obtained solutions is increased as the iterations are increased. Also, the error has been spread along the whole thickness of the IPL. Furthermore, the amount of error alters with respect to frequency but is increased altogether. Moreover, the error is increased as the thickness of IPLs or the variations of their primary parameters (k) increases. From the above example, one may satisfy that the introduced method is applicable to all arbitrary IPLs, whose primary parameters are known at all or even at some points along their thickness.

6. CONCLUSION

A new method was introduced for frequency domain analysis of arbitrary lossy and dispersive inhomogeneous planar layers (IPLs). In this method, the equations of IPLs are converted to the equations of homogeneous planar layers, which have been excited by distributed equivalent sources. Then, the electric and magnetic fields are obtained using an iterative approach. The validity of the method was verified using a comprehensive example. It was seen that this method is applicable to all arbitrary IPLs, whose primary parameters are known at all or even at some points along their thickness. It was seen that, the accuracy of the obtained solutions is increased as the iterations are increased. Also, as the excitation frequency, the thickness of IPLs or the variations of their primary parameters (k) increases, the necessary number of iterations increases.

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