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DIFFRACTION OF ELECTROMAGNETIC PLANE WAVE BY AN IMPEDANCE STRIP

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Abstract—This paper investigates the scattering of electromagnetic plane wave from an impedance strip. Both E- and H-polarizations are considered. The method of analysis is Kobayashi potential, which uses the discontinuous properties of Weber-Schafheitlin's integrals. Imposition of boundary conditions result in dual integral equations. Using the projection, equations reduces to matrix equations. The elements are given in terms of infinite integrals that contains the poles for particular values of surface impedance and these integrals are computed numerically. Far diffracted fields in the upper half space for different angles of incident are computed. To check the validity of the results, we have derived the physical optics (PO) approximate solutions. Numerical results for both the methods are compared. The agreement is good. Current distribution on the strip is also presented.

1. INTRODUCTION

Scattering of electromagnetic waves from a strip is a classic problem in electromagnetics. This has been the subject of many investigations [1–10]. A variety of methods may be used to analyze the problem [11–18]. When the width of the strip is very large as compared to the operating wavelength, high frequency approximate solution may be obtained by using the concept of geometrical theory of diffraction (GTD) [19]. But when the width is not large compared to the wavelength, numerical approaches like the method of moment [8, 9, 20] are more reliable.

Other methods like Wiener-Hopf [18, 21], Maliuzhinet's techniques [22] may be used to solve the problem.

In this paper, we have formulated the problem by applying the Kobayashi potential method [23, 24]. This method has been applied to various kinds of problems, such as the potential problems of electrified circu-lar disks [25, 26], the diffraction of acoustic waves by a circular disk and rectangular plate [27, 28], diffraction of electromagnetic plane wave by rectangular plate and hole, parallel slits, disk and circular hole [29–31]. In Kobayashi potential method, imposition of boundary conditions give us the dual integral equations. These equations are solved using the discontinuous properties of Weber-Schafheitlins integrals [32]. Incorporating the edge conditions, we transform the resulting expressions into the matrix equations. The elements of the matrix are the infinite integrals which are difficult to solve analytically. Numerical computations are con-ducted and results for impedance strip obtained using Kobayashi method are compared with those based on physical optics method.

2. FORMULATION

Consider an impedance strip of width $2a$ as shown in Figure 1. The strip is excited by a uniform electromagnetic plane wave. ϕ_0 is the angle of incidence with the x-axis. Impedances of the upper and lower surfaces of the strip are Z_+ and Z_- respectively. If E_z^i and H_z^i be the incident fields for E- and H-polarization respectively, then

$$\begin{pmatrix} E_z^i \\ H_z^i \end{pmatrix} = \exp\Big[jk(x\cos\phi_0 + y\sin\phi_0)\Big] \tag{1}$$

For simplicity we assumed the amplitude of the incident field is unity. The corresponding diffracted fields may be expressed as

$$\begin{pmatrix} E_z^d \\ H_z^d \end{pmatrix} = \int_0^\infty \Big[g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\Big]\exp\Big[-\sqrt{\xi^2-\kappa^2}\,y_a\Big]d\xi, \quad y_a > 0 \tag{2a}$$

$$\begin{pmatrix} E_z^d \\ H_z^d \end{pmatrix} = \int_0^\infty \Big[h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\Big]\exp\Big[\sqrt{\xi^2-\kappa^2}\,y_a\Big]d\xi, \quad y_a < 0 \tag{2b}$$

Since in 2D problems, the wave in each polarization (E- and H-) does not couple, we use the same symbols $g(\xi)$ and $h(\xi)$ for the unknown functions. In the above equations $x_a = \frac{x}{a}$ and $y_a = \frac{y}{a}$ are the normalized variables and $\kappa = ka$ is the normalized wave number. We consider each polarization separately.

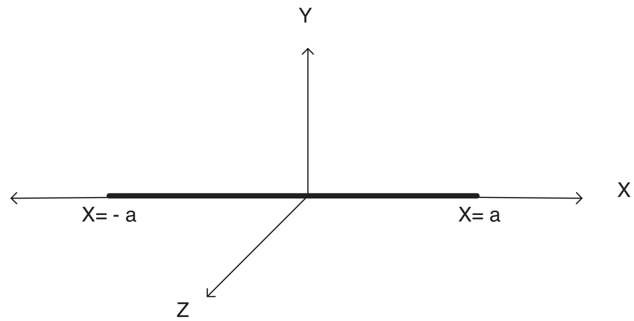

Figure 1. Geometry of the problem.

2.1. E-polarization

The required boundary conditions are given by

$$E_z^t\Big|_{y=0_+} = -Z_+ H_x^t\Big|_{y=0_+}, \quad E_z^t\Big|_{y=0_-} = Z_- H_x^t\Big|_{y=0_-} \quad |x_a| \leq 1 \tag{3a}$$

$$E_z^t\Big|_{y=0_+} = E_z^t\Big|_{y=0_-}, \qquad H_x^t\Big|_{y=0_+} = H_x^t\Big|_{y=0_-} \quad |x_a| \geq 1 \tag{3b}$$

where 't' in superscript means total. From the condition (3b) we have

$$\int_0^\infty \Big\{ \Big[g_1(\xi) - h_1(\xi)\Big]\cos(x_a\xi) + \Big[g_2(\xi) - h_2(\xi)\Big]\sin(x_a\xi)\Big\} d\xi = 0 \quad |x_a| \geq 1 \tag{4a}$$

$$\int_0^\infty \sqrt{\xi^2 - \kappa^2} \Big\{ \Big[g_1(\xi) + h_1(\xi)\Big]\cos(x_a\xi) + \Big[g_2(\xi) + h_2(\xi)\Big]\sin(x_a\xi)\Big\} d\xi = 0 \quad |x_a| \geq 1 \tag{4b}$$

With the help of the discontinuous properties of the Weber-Schafheitlin's integrals, we can assume

$$g_1(\xi) - h_1(\xi) = \sum_{m=0}^{\infty} A_m J_{2m+\frac{3}{2}}(\xi)\xi^{-\frac{3}{2}},$$

$$g_2(\xi) - h_2(\xi) = \sum_{m=0}^{\infty} B_m J_{2m+\frac{5}{2}}(\xi)\xi^{-\frac{3}{2}} \tag{5a}$$

$$g_1(\xi) + h_1(\xi) = \sum_{m=0}^{\infty} C_m \frac{J_{2m+\frac{1}{2}}(\xi)}{\sqrt{\xi^2 - \kappa^2}}\xi^{-\frac{1}{2}},$$

$$g_2(\xi) + h_2(\xi) = \sum_{m=0}^{\infty} D_m \frac{J_{2m+\frac{3}{2}}(\xi)}{\sqrt{\xi^2 - \kappa^2}}\xi^{-\frac{1}{2}} \tag{5b}$$

where we have taken into account the edge conditions of E_z^t and H_x^t. From the condition (3a) we have

$$\int_0^\infty \left[1 - j\frac{\zeta_+}{\kappa}\sqrt{\xi^2-\kappa^2}\right]\left[g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\right]d\xi = -2\left[1-\zeta_+\sin\phi_0\right]\exp[j\kappa x_a\cos\phi_0] \quad (6a)$$

$$\int_0^\infty \left[1 - j\frac{\zeta_-}{\kappa}\sqrt{\xi^2-\kappa^2}\right]\left[h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\right]d\xi = -2\left[1+\zeta_-\sin\phi_0\right]\exp[j\kappa x_a\cos\phi_0] \quad (6b)$$

$$\text{for} \qquad |x_a| \le 1$$

In the above expression $\zeta_\pm = Z_\pm/Z_0$ and Z_0 be the impedance of free space. Now we substitute the weighting functions $g_1(\xi) \sim h_2(\xi)$ determined from equations (5) into equations (4), comparing even and odd functions and then project the resulting equations into the functional space with elements $p_n^{\pm\frac{1}{2}}(x_a^2)$. We have

$$\left[K^+_{RE,E}\right]\left[A_m\right] + \left[G^+_{RE,E}\right]\left[C_m\right] = -[1-\zeta_+\sin\phi_0]\left[J_E\right] \quad (7a)$$

$$\left[K^-_{RE,E}\right]\left[A_m\right] - \left[G^-_{RE,E}\right]\left[C_m\right] = [1+\zeta_-\sin\phi_0]\left[J_E\right] \quad (7b)$$

$$\left[K^+_{RE,O}\right]\left[B_m\right] + \left[G^+_{RE,O}\right]\left[D_m\right] = -j[1-\zeta_+\sin\phi_0]\left[J_O\right] \quad (7c)$$

$$\left[K^-_{RE,O}\right]\left[B_m\right] - \left[G^-_{RE,O}\right]\left[D_m\right] = j[1+\zeta_-\sin\phi_0]\left[J_O\right] \quad (7d)$$

where

$$\cos x = \sqrt{\frac{\pi}{2}}J_{-\frac{1}{2}}(x), \quad \sin x = \sqrt{\frac{\pi}{2}}J_{\frac{1}{2}}(x)$$

and

$$x^{-m/2}J_m(\xi\sqrt{x}) = \sum_{n=0}^{\infty} 2(2n+m+1)\frac{\gamma(n+m+1)}{\Gamma(n+1)\Gamma(m+1)}\frac{J_{2n+m+1}(\xi)}{\xi}p_n^m(x)$$

$$p_n^m(x) = \frac{\gamma(n+1)\Gamma(m+1)}{\Gamma(n+m+1)}x^{-m/2}\int_0^\infty J_m(\sqrt{x}\xi)J_{2n+m+1}(\xi)d\xi$$

where the correspondence between the matrices and their elements are given by

$$\left[K^\pm_{RE,E}\right] \Longleftrightarrow K_{RE}\left(2n+\frac{1}{2}, 2m+\frac{3}{2}; \zeta_\pm\right)$$

$$\left[K^\pm_{RE,O}\right] \Longleftrightarrow K_{RE}\left(2n+\frac{3}{2}, 2m+\frac{5}{2}; \zeta_\pm\right)$$

$$\left[G^{\pm}_{RE,E}\right] \Longleftrightarrow G_{RE}\left(2n+\frac{1}{2}, 2m+\frac{1}{2}; \zeta_{\pm}\right) \tag{8a}$$

$$\left[G^{\pm}_{RE,O}\right] \Longleftrightarrow G_{RE}\left(2n+\frac{3}{2}, 2m+\frac{3}{2}; \zeta_{\pm}\right)$$

$$\left[J_E\right] \Longleftrightarrow 2\frac{J_{2n+\frac{1}{2}}(\kappa\cos\phi_0)}{(\kappa\cos\phi_0)^{\frac{1}{2}}}$$

$$\left[J_O\right] \Longleftrightarrow 2\frac{J_{2n+\frac{3}{2}}(\kappa\cos\phi_0)}{(\kappa\cos\phi_0)^{\frac{1}{2}}}$$

and

$$\begin{aligned} K_{RE}(m,n;\zeta) &= \int_0^\infty \left[1-j\frac{\zeta}{\kappa}\sqrt{\xi^2-\kappa^2}\right]\frac{J_m(\xi)J_n(\xi)}{\xi^2}d\xi \\ &= \int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi^2}d\xi - j\frac{\zeta}{\kappa}K(m,n) \\ &= \frac{4}{\pi}\frac{\sin\left[\frac{1}{2}(m-n+1)\pi\right]}{(m+n+1)(m+n-1)(n-m+1)(m-n+1)} - j\frac{\zeta}{\kappa}K(m,n) \end{aligned} \tag{8b}$$

$$\begin{aligned} G_{RE}(m,n;\zeta) &= \int_0^\infty \left[1-j\frac{\zeta}{\kappa}\sqrt{\xi^2-\kappa^2}\right]\frac{J_m(\xi)J_n(\xi)}{\xi\sqrt{\xi^2-\kappa^2}}d\xi \\ &= \int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi\sqrt{\xi^2-\kappa^2}}d\xi - j\frac{\zeta}{\kappa}\int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi}d\xi \\ &= \int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi\sqrt{\xi^2-\kappa^2}}d\xi - j\frac{\zeta}{\kappa}\frac{\sin\left[\frac{1}{2}(m-n)\pi\right]}{m^2-n^2} \end{aligned} \tag{8c}$$

where

$$K(m,n) = \int_0^\infty \sqrt{\xi^2-\kappa^2}\frac{J_m(\xi)J_n(\xi)}{\xi^2}d\xi$$

Equations (7) may be rewritten as

$$\left\{\left[K^{+}_{RE,E}\right]^{-1}\left[G^{-}_{RE,E}\right] + \left[K^{+}_{RE,E}\right]^{-1}\left[G^{-}_{RE,E}\right]\right\}\left[C_m\right]$$

$$= -\left\{[1-\zeta_+\sin\phi_0]\left[K^{+}_{RE,E}\right]^{-1} + [1+\zeta_-\sin\phi_0]\left[K^{+}_{RE,E}\right]^{-1}\right\}\left[J_E\right]$$

$$\left[A_m\right] = -\left[K^{+}_{RE,E}\right]^{-1}\left[G^{+}_{RE,E}\right]\left[C_m\right] - [1-\zeta_+\sin\phi_0]\left[K^{+}_{RE,E}\right]^{-1}\left[J_E\right]$$

$$\left\{\left[K^{+}_{RE,O}\right]^{-1}\left[G^{-}_{RE,O}\right] + \left[K^{+}_{RE,OE}\right]^{-1}\left[G^{-}_{RE,O}\right]\right\}\left[D_m\right]$$

$$= -j\left\{[1-\zeta_+\sin\phi_0]\left[K^+_{RE,O}\right]^{-1}+[1+\zeta_-\sin\phi_0]\left[K^+_{RE,O}\right]^{-1}\right\}\left[J_O\right]$$

$$\left[B_m\right] = -\left[K^+_{RE,O}\right]^{-1}\left[G^+_{RO,E}\right]\left[D_m\right] - j[1-\zeta_+\sin\phi_0]\left[K^+_{RE,O}\right]^{-1}\left[J_O\right] \tag{9}$$

For the case of $\zeta_+ = \zeta_- = \zeta$, equations (7) or (9) reduce to

$$\left[A_m\right] = \zeta\sin\phi_0\left[K_{RE,E}\right]^{-1}\left[J_E\right],\ \left[B_m\right] = j\zeta\sin\phi_0\left[K_{RE,O}\right]^{-1}\left[J_O\right] \tag{10a}$$

$$\left[C_m\right] = -\left[G^{\pm}_{RE,E}\right]^{-1}\left[J_E\right],\ \left[D_m\right] = -j\left[G^{\pm}_{RE,O}\right]^{-1}\left[J_O\right] \tag{10b}$$

Far scattered field in the upper region can be evaluated by applying the saddle point method of integration. The result is given by

$$\begin{aligned}E_z^d &= \frac{1}{2}\sum_{m=0}^{\infty}\int_0^{\infty}\left\{\left[A_m\frac{J_{2m+\frac{3}{2}}(\xi)}{\xi^{\frac{3}{2}}}+C_m\frac{J_{2m+\frac{1}{2}}(\xi)}{\sqrt{\xi(\xi^2-\kappa^2)}}\right]\cos(x_a\xi)\right.\\ &\quad\left.+\left[B_m\frac{J_{2m+\frac{5}{2}}(\xi)}{\xi^{\frac{3}{2}}}+D_m\frac{J_{2m+\frac{3}{2}}(\xi)}{\sqrt{\xi(\xi^2-\kappa^2)}}\right]\sin(x_a\xi)\right\}\\ &\quad\times\exp\left[-\sqrt{\xi^2-\kappa^2}y_a\right]d\xi\\ &= \sqrt{\frac{\pi}{8}}\frac{1}{\sqrt{k\rho}}\exp\left[-jk\rho+j\frac{\pi}{4}\right]\sum_{m=0}^{\infty}\left\{\left[A_mJ_{2m+\frac{3}{2}}(\kappa\cos\phi)\right.\right.\\ &\quad\left.+B_mJ_{2m+\frac{5}{2}}(\kappa\cos\phi)\right]\tan\phi\\ &\quad\left.-j\left[C_mJ_{2m+\frac{1}{2}}(\kappa\cos\phi)+D_mJ_{2m+\frac{3}{2}}(\kappa\cos\phi)\right]\right\}(\kappa\cos\phi)^{-\frac{1}{2}}\end{aligned} \tag{11a}$$

where $\xi = \kappa\cos\phi$. And expansion coefficients A_m, B_m, C_m, D_m can be obtained from equations (10). The current density induced on the impedance strip is obtained as follows.

$$\begin{aligned}J_z &= -\left[H_x^t\Big|_{y=0_+} - [H_x^t\Big|_{y=0_-}\right]\\ &= \frac{jY_0}{\kappa}\sum_{m=0}^{\infty}\int_0^{\infty}\left[C_mJ_{2m+\frac{1}{2}}(\xi)\cos(x_a\xi)+D_mJ_{2m+\frac{3}{2}}(\xi)\sin(x_a\xi)\right]\xi^{-\frac{1}{2}}d\xi\\ &= \frac{jY_0}{\sqrt{2}\kappa}\sum_{m=0}^{\infty}\frac{\Gamma(m+\frac{1}{2})}{\Gamma(m+1)}\left\{C_mp_m^{-\frac{1}{2}}(x_a^2)+x_a(2m+1)D_mp_m^{\frac{1}{2}}(x_a^2)\right\}\end{aligned} \tag{11b}$$

2.2. H-polarization

The required boundary conditions of this problem are given by

$$E_x^t\Big|_{y=0_+} = Z_+ H_z^t\Big|_{y=0_+}, \quad E_x^t\Big|_{y=0_-} = -Z_- H_z^t\Big|_{y=0_-} \quad |x_a| \leq 1 \tag{12a}$$

$$E_x^t\Big|_{y=0_+} = E_x^t\Big|_{y=0_-}, \quad H_z^t\Big|_{y=0_+} = H_z^t\Big|_{y=0_-} \quad |x_a| \geq 1 \tag{12b}$$

The incident wave is given by (1) and the diffracted wave is given by (2). From the condition (12b) we have we have the same equations as (4) and $g_1(\xi) \sim h_2(\xi)$ are given by (5). From the condition (12a) we have

$$\int_0^\infty \left[\sqrt{\xi^2 - \kappa^2} + j\kappa\zeta_+\right]\left[g_1(\xi)\cos(x_a\xi) + g_2(\xi)\sin(x_a\xi)\right]d\xi = j\kappa(\sin\phi_0 - \zeta_+)\exp(j\kappa x_a\cos\phi_0) \tag{13a}$$

$$\int_0^\infty \left[\sqrt{\xi^2 - \kappa^2} + j\kappa\zeta_-\right]\left[h_1(\xi)\cos(x_a\xi) + h_2(\xi)\sin(x_a\xi)\right]d\xi = -j\kappa(\sin\phi_0 + \zeta_-)\exp(j\kappa x_a\cos\phi_0) \quad \text{for } |x_a| \leq 1 \tag{13b}$$

We substitute the weighting functions $g_1(\xi) \sim h_2(\xi)$ of (13) determined by (5) and we project the resulting equations into the functional space with elements $p_n^{\pm\frac{1}{2}}(x_a^2)$. Then we have

$$\begin{aligned}
\left[K_{RH,E}^+\right]\left[A_m\right] + \left[G_{RH,E}^+\right]\left[C_m\right] &= j[\sin\phi_0 - \zeta_+]\left[J_E\right], \\
\left[K_{RH,E}^-\right]\left[A_m\right] - \left[G_{RH,E}^-\right]\left[C_m\right] &= j[\sin\phi_0 + \zeta_-]\left[J_E\right]
\end{aligned} \tag{14a}$$

$$\begin{aligned}
\left[K_{RH,O}^+\right]\left[B_m\right] + \left[G_{RH,O}^+\right]\left[D_m\right] &= -[\sin\phi_0 - \zeta_+]\left[J_O\right], \\
\left[K_{RH,O}^-\right]\left[B_m\right] - \left[G_{RH,O}^-\right]\left[D_m\right] &= -[\sin\phi_0 + \zeta_-]\left[J_O\right]
\end{aligned} \tag{14b}$$

where the correspondence between the matrices and their elements are given by

$$\begin{aligned}
\left[K_{RH,E}^\pm\right] &\Longleftrightarrow K_{RH}\left(2n + \frac{1}{2}, 2m + \frac{3}{2}; \zeta_\pm\right) \\
\left[K_{RH,O}^\pm\right] &\Longleftrightarrow K_{RH}\left(2n + \frac{3}{2}, 2m + \frac{5}{2}; \zeta_\pm\right) \\
\left[G_{RH,E}^\pm\right] &\Longleftrightarrow G_{RH}\left(2n + \frac{1}{2}, 2m + \frac{1}{2}; \zeta_\pm\right) \\
\left[G_{RH,O}^\pm\right] &\Longleftrightarrow G_{RH}\left(2n + \frac{3}{2}, 2m + \frac{3}{2}; \zeta_\pm\right)
\end{aligned} \tag{15}$$

$$\left[J_E\right] \iff 2\kappa \frac{J_{2n+\frac{1}{2}}(\kappa\cos\phi_0)}{(\kappa\cos\phi_0)^{\frac{1}{2}}}$$

$$\left[J_O\right] \iff 2\kappa \frac{J_{2n+\frac{3}{2}}(\kappa\cos\phi_0)}{(\kappa\cos\phi_0)^{\frac{1}{2}}}$$

and

$$\begin{aligned} K_{RH}(m,n;\zeta) &= \int_0^\infty \left[j\zeta\kappa + \sqrt{\xi^2-\kappa^2}\right] \frac{J_m(\xi)J_n(\xi)}{\xi^2} d\xi \\ &= j\zeta\kappa \int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi^2} d\xi + K(m,n) \end{aligned} \tag{16a}$$

$$G_{RH}(m,n;\zeta) = j\zeta\kappa \int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi\sqrt{\xi^2-\kappa^2}} d\xi + \int_0^\infty \frac{J_m(\xi)J_n(\xi)}{\xi} d\xi \tag{16b}$$

$$\begin{aligned} &\left\{\left[K^+_{RH,E}\right]^{-1}\left[G^+_{RH,E}\right] + \left[K^-_{RH,E}\right]^{-1}\left[G^-_{RH,E}\right]\right\}\left[C_m\right] \\ &= j\left\{[\sin\phi_0 - \zeta_+]\left[K^+_{RH,E}\right]^{-1} + [\sin\phi_0 + \zeta_-]\left[K^+_{RH,E}\right]^{-1}\right\}\left[J_E\right] \\ \left[A_m\right] &= -\left[K^+_{RH,E}\right]^{-1}\left[G^+_{RH,E}\right]\left[C_m\right] + j[\sin\phi_0 - \zeta_+]\left[K^+_{RH,E}\right]^{-1}\left[J_E\right] \\ &\left\{\left[K^+_{RH,O}\right]^{-1}\left[G^+_{RH,O}\right] + \left[K^-_{RH,O}\right]^{-1}\left[G^-_{RH,O}\right]\right\}\left[D_m\right] \\ &= \left\{-[\sin\phi_0 - \zeta_+]\left[K^+_{RH,O}\right]^{-1} + [\sin\phi_0 + \zeta_-]\left[K^-_{RH,O}\right]^{-1}\right\}\left[J_O\right] \\ \left[B_m\right] &= -\left[K^+_{RH,O}\right]^{-1}\left[G^+_{RH,O}\right]\left[D_m\right] - [\sin\phi_0 - \zeta_+]\left[K^+_{RH,O}\right]^{-1}\left[J_O\right] \end{aligned} \tag{17}$$

For the case of $\zeta_+ = \zeta_- = \zeta$, Equations (17) reduce to

$$\left[A_m\right] = j\sin\phi_0\left[K^\pm_{RH,E}\right]^{-1}\left[J_E\right] \quad \left[B_m\right] = -\sin\phi_0\left[K^\pm_{RH,O}\right]^{-1}\left[J_O\right] \tag{18a}$$

$$\left[C_m\right] = -j\zeta\left[G^\pm_{RH,E}\right]^{-1}\left[J_E\right] \qquad \left[D_m\right] = \zeta\left[G^\pm_{RH,O}\right]^{-1}\left[J_O\right] \tag{18b}$$

Far scattered field in the upper region can be evaluated by applying the saddle point method of integration and the result has the same form as (11a), but the expansion coefficients $A_m \sim D_m$ are given by (17) or (18), instead of (10). The current density induced on the impedance strip is obtained as follows.

$$
\begin{aligned}
J_x &= H_z^t\Big|_{y=0_+} - H_z^t\Big|_{y=0_-} \\
&= \sum_{m=0}^{\infty} \int_0^{\infty} \left[A_m \frac{J_{2m+\frac{3}{2}}(\xi)}{\xi^{\frac{3}{2}}} \cos(x_a \xi) + B_m \frac{J_{2m+\frac{5}{2}}(\xi)}{\xi^{\frac{3}{2}}} \sin(x_a \xi) \right] d\xi \\
&= \frac{1}{\sqrt{2}} \sum_{m=0}^{\infty} \frac{\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(m+1)} \left\{ \frac{A_m}{4m+3} \left[p_m^{-\frac{1}{2}}(x_a^2) + \frac{m+\frac{1}{2}}{m+1} p_{m+1}^{-\frac{1}{2}}(x_a^2) \right] \right. \\
&\quad \left. + x_a \frac{2m+1}{4m+3} B_m \left[p_m^{\frac{1}{2}}(x_a^2) + \frac{m+\frac{3}{2}}{m+1} p_{m+1}^{\frac{1}{2}}(x_a^2) \right] \right\} \qquad (19)
\end{aligned}
$$

3. PHYSICAL OPTICS APPROXIMATE SOLUTIONS

We consider here physical optics solutions for comparison with the previous solutions.

3.1. E-polarization

The total field on the strip is

$$E_z^t = \frac{2\zeta_+ \sin\phi_0}{1+\zeta_+ \sin\phi_0} \exp(jkx\cos\phi_0) \qquad (20a)$$

$$H_x^t = -\frac{2Y_0 \sin\phi_0}{1+\zeta_+ \sin\phi_0} \exp(jkx\cos\phi_0) \qquad (20b)$$

The equivalent currents are

$$M_x = -E_z^t, \qquad J_z = -H_x^t \qquad (20c)$$

Far field expression of the vector potential is given by

$$
\begin{aligned}
A_z &= \frac{\mu Y_0}{j2} Q_0 C(k\rho) \int_{-a}^{a} \exp\Big[jk(\cos\phi + \cos\phi_0)x'\Big] dx' \\
&= \frac{\mu Y_0}{j2} Q_0 C(k\rho) S_0(\phi), \\
F_x &= -\frac{\epsilon\zeta_+}{j2} Q_0 C(k\rho) \int_{-a}^{a} \exp\Big[jk(\cos\phi + \cos\phi_0)x'\Big] dx' \\
&= -\frac{\epsilon\zeta_+}{j2} Q_0 C(k\rho) S_0(\phi) \qquad (21)
\end{aligned}
$$

A_z and F_x are the components of magnetic and electric vector potential respectively.

$$Q_0 = \frac{2\sin\phi_0}{1+\zeta_+\sin\phi_0}, \qquad C(k\rho) = \sqrt{\frac{2}{\pi k\rho}}\exp\left(-jk\rho + j\frac{\pi}{4}\right).$$

Thus far electric field is derived as

$$\begin{aligned} E_z &= -j\omega A_z + \frac{1}{\epsilon}\frac{\partial F_x}{\partial y} = -\frac{k}{4}(1-\zeta_+\sin\phi)Q_0 C(k\rho)S_0(\phi) \\ &= -\frac{1-\zeta_+\sin\phi}{1+\zeta_+\sin\phi_0}ka\sin\phi_0\sqrt{\frac{2}{\pi k\rho}} \\ &\quad\times\exp\left(-jk\rho + j\frac{\pi}{4}\right)\text{sinc}\Big[ka(\cos\phi+\cos\phi_0)\Big] \end{aligned} \tag{22}$$

3.2. H-polarization

The total field on the strip is

$$H_z^t = \frac{2\sin\phi_0}{\zeta_+ + \sin\phi_0}\exp(jkx\cos\phi_0) \tag{23a}$$

$$E_x^t = Z_0\frac{2\zeta_+\sin\phi_0}{\zeta_+ + \sin\phi_0}\exp(jkx\cos\phi_0) \tag{23b}$$

The equivalent currents are

$$J_x = -H_z^t, \qquad M_z = E_x^t \tag{23c}$$

Far field expression of the vector potential is given by

$$A_x = \frac{\mu}{j2}Q_1 C(k\rho)S_0(\phi) \tag{24a}$$

$$F_z = \frac{\epsilon\zeta_+}{j2}Z_0 Q_1 C(k\rho)S_0(\phi) \tag{24b}$$

where $Q_1 = \frac{2\sin\phi_0}{\zeta_+ + \sin\phi_0}$, $C(k\rho) = \sqrt{\frac{2}{\pi k\rho}}\exp\left(-jk\rho + j\frac{\pi}{4}\right)$. Thus far magnetic field is derived as

$$\begin{aligned} E_z &= -j\omega F_z - \frac{1}{\mu}\frac{\partial F_x}{\partial y} = \frac{k}{4}(\sin\phi - \zeta_+)Q_1 C(k\rho)S_0(\phi) \\ &= -\frac{\zeta_+ - \sin\phi}{\zeta_+ + \sin\phi_0}ka\sin\phi_0\sqrt{\frac{2}{\pi k\rho}} \\ &\quad\times\exp\left(-jk\rho + j\frac{\pi}{4}\right)\text{sinc}\Big[ka(\cos\phi+\cos\phi_0)\Big] \end{aligned} \tag{25}$$

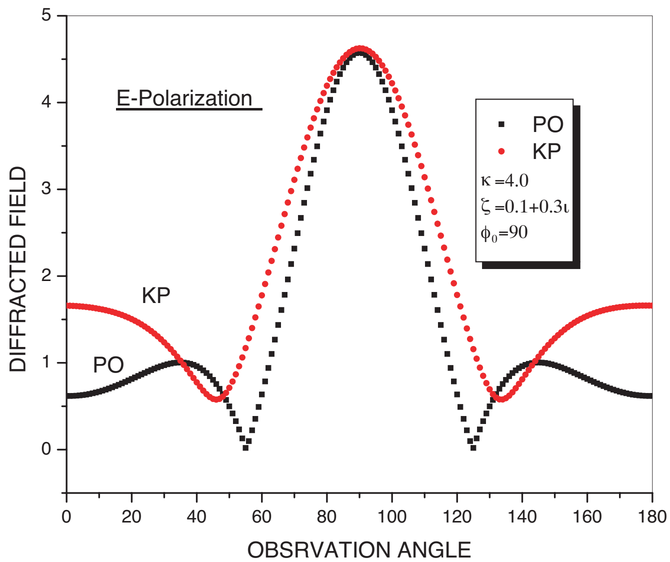

Figure 2. Comparison of diffracted field patterns for normal incidence.

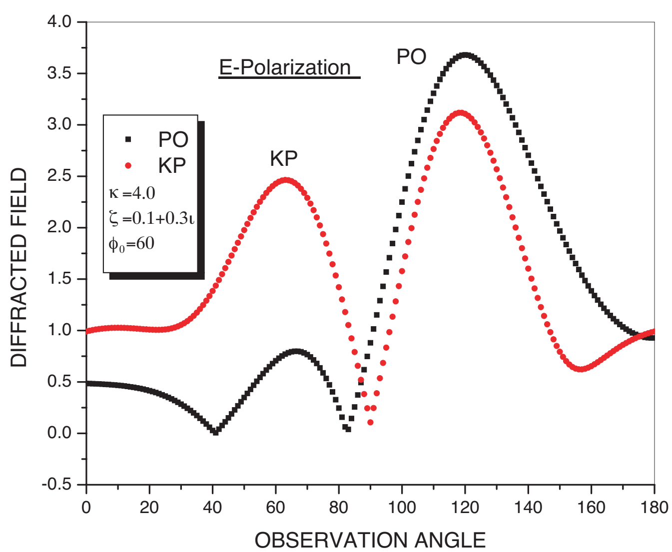

Figure 3. Far diffracted fields in the upper half plane for $\phi_0 = 60$.

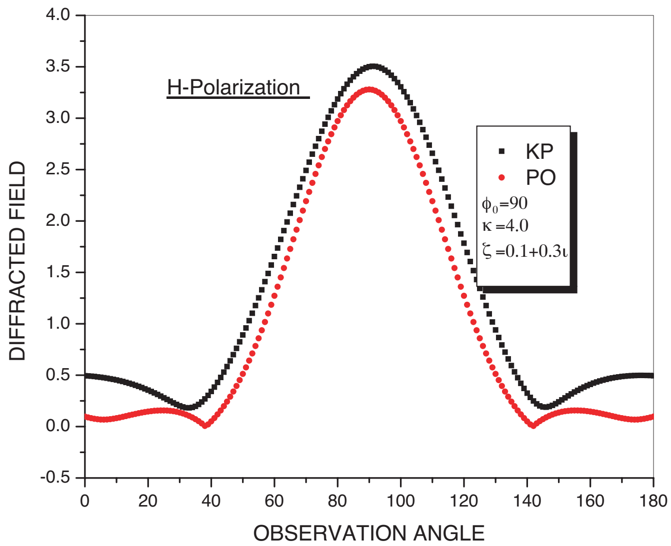

Figure 4. Comparison between the two methods for $\phi_0 = 90$.

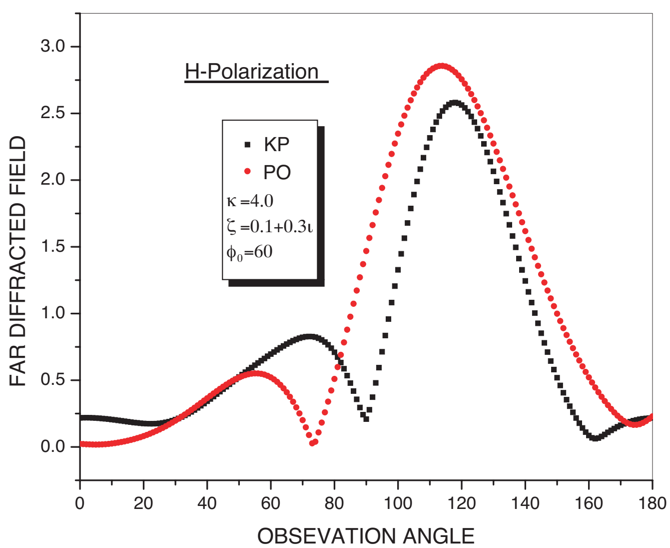

Figure 5. Comparison between PO and KP for H-polarization.

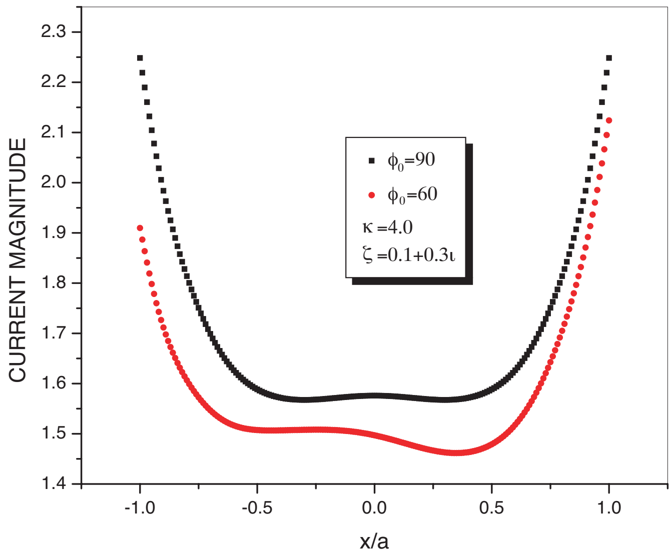

Figure 6. Current distribution on the strip (E-polarization).

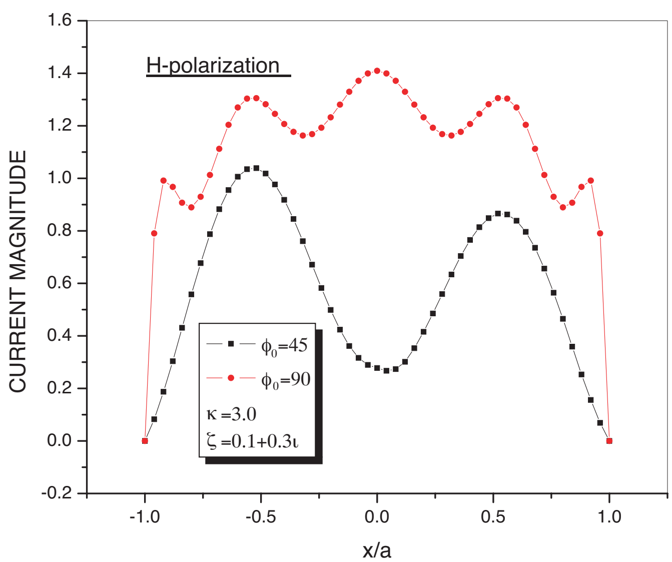

Figure 7. Current distribution H-polarization.

4. NUMERICAL RESULTS AND DISCUSSION

To study the scattering properties of impedance strip, expansion coefficients $A_m \sim D_m$ are computed, for large number of 'm' values, using equations (10) for E-polarization and equations (18) for H-polarization. Far diffracted fields are computed using these expansion coefficients in equation (11a) for both the cases. Fig. 2 and Fig. 3 gives the far field patterns for different values of angle of incidence and $\kappa = 4.0, \quad \zeta = 0.1{+}0.3i$. Line plots with circled symbols corresponds to kobayashi potential while Line plots with blocked symbols corresponds to physical optics. To check the validity of these results, we compared them with those of obtained using physical optics equation (22). Fig. 4 and Fig. 5 give the diffracted patterns and their comparisons for h-polarization case corresponding to $\phi_0 = \pi/2$ and $\phi_0 = \pi/3, \quad \kappa = 4.0, \quad \zeta = 0.1 + 0.3i$ with those obtained using PO equation (25). Fig. 6 and Fig. 7 show the numerical results for current distribution on the strip obtained from equation (11b) for E-polarization and equation (19) for H-polarization. The results are as expected.

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