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DESIGN OF CROSS-COUPLED DUAL-BAND FILTER WITH EQUAL-LENGTH SPLIT-RING RESONATORS

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Abstract—A compact dual-band bandpass filter using equal-length split-ring resonators (SRRs) is proposed in this paper. The two-resonance frequency response of equal-length split-ring resonator is described in detail. A cross-coupled dual-band filter with four equal-length split-ring resonators is designed. Several finite out-of-band attenuation poles are realized to improve the selectivity of the proposed filter. The measurement of the filter is in good agreement with the simulation.

1. INTRODUCTION

Dual-band bandpass filters have gained great attention in wireless communication systems recently [5–10]. The initial type of dual-band filter is simply designed by connecting two filter circuits with two different passbands [5]. But this solution suffers from high insertion loss and large overall size. In [6] and [7], making use of the basic topology of a stopband filter, a uniform transmission line is interrupted with multiple shunt stubs to generate dual passbands. However, these solutions increase component amount, circuit size, and power consumption. The importance of keeping filter structures to a minimum size and weight, low insertion loss, high frequency selectivity has been widely recognized. An effective solution is cross-coupled filter.

Split-ring resonators were used as one of the particles for metamaterial construction [13], and have been applied for microstrip band-pass filters to reduce circuit dimensions relying on the fact that these resonators can be designed with dimensions much smaller than signal wavelength at their resonant frequency. However, an interesting character has not been exploited until it was presented in [10] recently. This property is the dual-band frequency response. Some geometrical modifications of the conventional SRRs help to generate two resonant frequencies, both of which can be suitably tuned. The new structure shows great potential in reducing circuit size and simplifying circuit construction.

In this paper, the equal-length SRRs structure is applied to construct the proposed dual-band filter, which has a 2*2 configuration. Cross-coupling between resonator 1 and resonator 4 helps to realize several transmission zeros. The proposed dual-band filter exhibits a good performance including compact size and high selectivity. Details of filter design are presented and measured results are given to demonstrate the performance of the dual-band filter.

2. EQUAL-LENGTH SPLIT-RING RESONATORS

For constructing a cross-coupled dual-band filter, the first thing is to design a resonator as a block of the filter which has two resonant frequencies.

A conventional SRRs is formed by two concentric circular open rings as shown in Fig. 1(a). However, coupling between circular SRRs is not easy to control. The rectangular SRRs in Fig. 1(b) not only provides suitable inner coupling, but also makes coupling between adjacent ones easy to handle. The novel SRRs, as illustrated in Fig. 1(c), has an increased length of the inner conductor by using a meandered line to equalize the lengths of both inner and outer conductors. This is a key factor to generate two resonance frequencies.

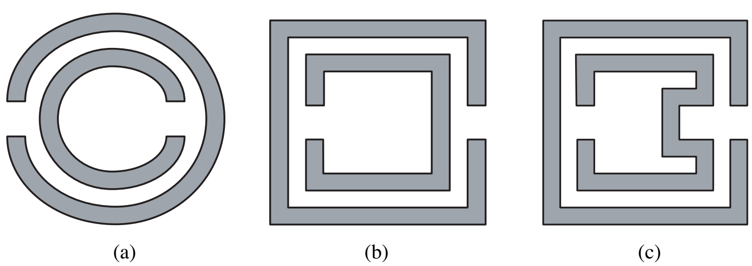

Figure 1. Layout of microstrip split-ring resonators. (a) Circular SRRs. (b) Rectangular SRRs. (c) Equal-length SRRs.

For simplification, let's suppose both the inner and the outer split-ring have equal electrical length 2θ, and neglect the coupling at the opening of each split-ring. The coupling between the inner and the outer is magnetic coupling. Then the equal-length SRRs can be modeled as two identical sections of coupled microstrip lines of electrical length θ and two open ends at crossed locations. The

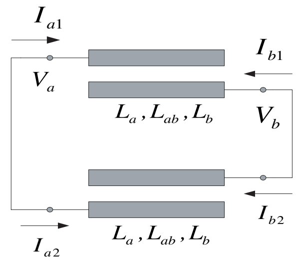

Figure 2. Simplified equivalent circuit of the equal-length SRRs.

remaining terminals are connected as shown in Fig. 2. The open-circuit impedance matrix of the two-port coupled-line network is

$$z = \frac{j\omega}{\beta}\begin{bmatrix} L_a \cot\theta & L_{ab}\csc\theta \\ L_{ab}\csc\theta & L_b\cot\theta \end{bmatrix} \tag{1}$$

Where L_a, L_b are the distributed inductances per unit length of the transmission line L_{ab}, is the distributed mutual inductance between two lines, β is the propagation constant. The equation for the circuit in Fig. 2 can be obtained as

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = Z\begin{bmatrix} I_{a1} \\ I_{b1} \end{bmatrix} = Z\begin{bmatrix} I_{a2} \\ I_{b2} \end{bmatrix} \tag{2}$$

For resonant modes, $I_{a1} = -I_{a2} \neq 0$ and $I_{b1} = -I_{b2} \neq 0$, so Z must have at least one eigenvalue equal to zero while

$$\det Z = -\left(\frac{\omega}{\beta}\right)^2 \left(L_a L_b \cot^2\theta - L_{ab}^2 \csc^2\theta\right) = 0 \tag{3}$$

Since $\omega \neq 0,\ \beta \neq 0$ and $L_a \neq 0,\ L_b \neq 0$, the roots are given by

$$\left(\frac{\cot\theta}{\csc\theta}\right)^2 = \cos^2\theta = \frac{L_{ab}^2}{L_a L_b} \tag{4}$$

$$\theta = \arccos\left(\pm\frac{L_{ab}}{\sqrt{L_a L_b}}\right) \tag{5}$$

Set $\theta_1 = \arccos\left(\frac{L_{ab}}{\sqrt{L_a L_b}}\right)$ and $0 < \theta_1 < \pi,\ \Delta\theta = \frac{\pi}{2} - \theta_1$, the electric length of the SRRs can be expressed as

$$2\theta = (2k+1)\pi \mp 2\cdot\Delta\theta, \quad k = 0, 1\ldots \tag{6}$$

The result shows that the resonance of single split-ring resonator with electric length $2\theta = (2k+1)\pi$ is split into two different resonances at frequencies above and below the resonant frequency of single ring. The separation between the two frequencies is mainly determined by the coupling between the inner and outer ring.

3. DESIGN PROCEDURE

In design of single band bandpass filters, cross-coupled structures are applied to generate finite transmission zeros which greatly improve the selectivity of the filters. Our purpose is to construct a cross-coupled filter with equal-length SRRs mentioned above and realize dual-band filter by suitably tuning the two resonant frequencies. The transfer function of this type of filter is

$$|S_{21}(\Omega)|^2 = \frac{1}{1+\varepsilon^2 F_n^2(\Omega)} \tag{7}$$

$$F_n(\Omega) = \cosh\left[(n-2)\cosh^{-1}(\Omega) + \cosh^{-1}\left(\frac{\Omega_z\Omega - 1}{\Omega_z - \Omega}\right) + \cosh^{-1}\left(\frac{\Omega_z\Omega + 1}{\Omega_z + \Omega}\right)\right] \tag{8}$$

where Ω is the frequency variable that is normalized to the passband cut-off frequency of the lowpass prototype filter, $\Omega = \frac{1}{FBW}\cdot\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$ in which ω is the frequency variable of bandpass filter, ω_0 is the midband frequency and FBW is the fractional bandwidth. ε is a ripple constant related to a given return loss $LR = 20\log|S_{11}|$ in dB, and n is the degree of the filter. It is obvious that $\Omega = \pm\Omega_z (\Omega_z > 1)$ are the frequency locations of a pair of transmission zeros. The locations of two finite frequency transmission zeros of the bandpass filter are given by

$$\begin{aligned} \omega_{z1} &= \omega_0 \frac{-\Omega_z FBW + \sqrt{(\Omega_z FBW)^2 + 4}}{2} \\ \omega_{z2} &= \omega_0 \frac{\Omega_z FBW + \sqrt{(\Omega_z FBW)^2 + 4}}{2} \end{aligned} \tag{9}$$

Fig. 3 shows a scheme prototype of the above filter, where $g_1, \ldots, g_{m-1}$, g_m are the standard Chebyshev filter parameter and J_{m-1} and J_m can be found by solving iteratively the equations

$$J_{m-1} = \frac{-J'_m}{(\Omega_z g_m)^2 - J'^2_m} \tag{10}$$

$$J'_m = \frac{J_m}{1 + J_m J_{m-1}} \tag{11}$$

where J'_m is interpreted as the updated J_m. Equations (10) and (11) are solved iteratively with the initial values of J_m and J_{m-1} given by

$$J_m = 1/\sqrt{S}, \quad J_{m-1} = 0, \quad S = \left(\sqrt{1+\varepsilon^2} + \varepsilon\right)^2 \tag{12}$$

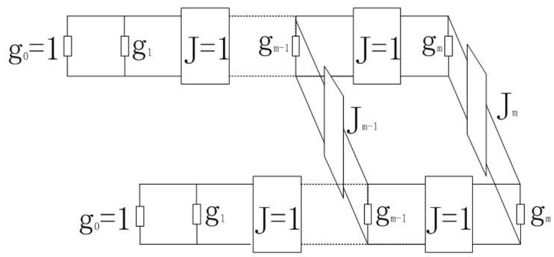

Figure 3. Lowpass prototype of the filter with two finite transmission zeros.

No other elements of the original Chebyshev filter are changed. The design parameters of the bandpass filter, i.e., the coupling coefficients and external quality factors, as referring to the general coupling structure of Fig. 3, can be determined by the formulas

$$\begin{aligned} Q_{er} &= Q_{el} = \frac{g_1}{FBW} \\ M_{i,i+1} = M_{n-i,n-i+1} &= \frac{FBW}{\sqrt{g_i g_{i+1}}} \quad \text{for } i = 1 \text{ to } m-1 \\ M_{m,m+1} &= \frac{FBW \cdot J_m}{g_m} \\ M_{m-1,m+2} &= \frac{FBW \cdot J_{m-1}}{g_{m-1}} \end{aligned} \tag{13}$$

In this work, the cross-coupled dual-band filter is designed by using the above method with the equal-length SRRs as blocks. The degree of the filter is $n = 4$. The frequency locations of the transmission zeros are determined by $\Omega_z = 1.85$. For accurate design, the coefficients g_1, g_2 and J_1, J_2 are obtained from the tables provided by Hong and Lancaster in their book [12]. The center frequencies of the two passbands are designed at 2.4 GHz and 3.1 GHz with fractional bandwidths 6% and 2% respectively. Then the external quality factors and the coupling coefficient matrixes can be calculated. For the first passband:

$$Q_{er1} = Q_{el1} = 15.99$$

$$M_1 = \begin{bmatrix} 0 & 0.0514 & 0 & -0.0132 \\ 0.0514 & 0 & 0.0472 & 0 \\ 0 & 0.0472 & 0 & 0.0514 \\ -0.0132 & 0 & 0.0514 & 0 \end{bmatrix} \tag{14}$$

For the second passband:

$$Q_{er2} = Q_{el2} = 47.98$$

$$M_2 = \begin{bmatrix} 0 & 0.0171 & 0 & -0.0044 \\ 0.0171 & 0 & 0.0157 & 0 \\ 0 & 0.0157 & 0 & 0.0171 \\ -0.0044 & 0 & 0.0171 & 0 \end{bmatrix} \tag{15}$$

By properly placing the four resonators and the tapered feed lines, appropriate external quality factors and coupling coefficients can be obtained for each passband.

4. FABRICATED FILTERS AND MEASURED RESULTS

Fig. 4 is the layout and the dimension of the resonator and the proposed filter. The filter is designed on a substrate with a thickness $h = 1$ mm and a dielectric constant $\varepsilon_r = 2.65$.

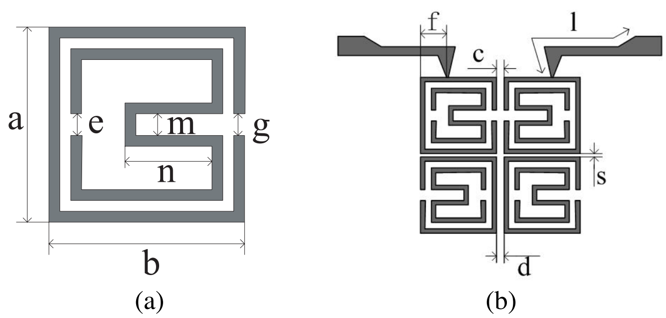

Figure 4. (a) Layout of the resonator of the proposed filter. Dimensions: $a = b = 11$ mm, $e = g = m = 1$ mm, $n = 4.4$ mm, conductor widths and gap between them 0.6 mm. (b) Layout of the proposed dual-band filter. Dimensions: $c = d = 0.9$ mm, $s = 0.3$ mm, $f = 5.4$ mm, $l = 20.1$ mm.

Fig. 5 shows the photograph of the fabricated filter. The simulated and measured results are presented and compared in Fig. 6. There are two transmission zeros out of each passband and the selectivity of the filter is significantly improved. The insertion loss is mainly due to the conductor loss. The frequency shifts are caused by the inaccuracy in fabrication.

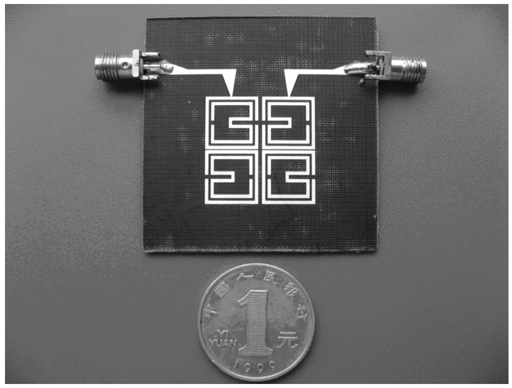

Figure 5. Photograph of the fabricated dual-band filter.

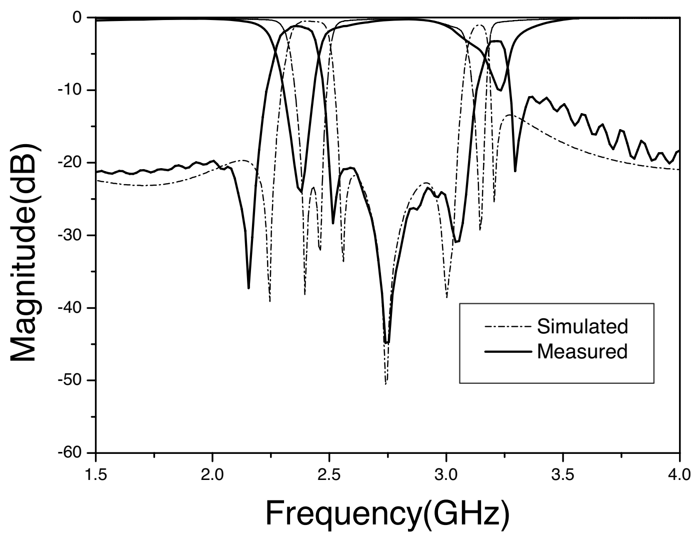

Figure 6. Simulated and measured results of the proposed dual-band filter.

5. CONCLUSION

A novel cross-coupled dual-band bandpass filter is proposed and constructed by using equal- length SRRs. After the descriptions of the frequency response of the equal-length resonators, the filter is designed, fabricated and measured, which is not only compact in size, but also has a advantage of improved selectivity. The good agreement between simulated and measured results validates the proposed structure.

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