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RECONSTRUCTION OF INHOMOGENEOUS DIELECTRICS AT MICROWAVE FREQUENCIES

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Abstract—The transmission line techniques are generalized to measure the permittivity function of one-dimensional inhomogeneous dielectrics. A rectangular waveguide is used and the dielectric slab under test is placed at the input flange of it. Then the measured scattering parameters or the short-circuited reflection coefficient are used to extract the permittivity function of dielectric. To solve the problem an optimization-based procedure is used. The usefulness of the proposed method is verified using a comprehensive example.

1. INTRODUCTION

Many techniques have been presented to measure the dielectric constant of dielectrics. The lumped circuit techniques [1], the cavity perturbation technique [2–4], the free-space techniques [5] and the transmission line techniques [6–9] are the most important techniques. However these techniques are presented only for homogeneous dielectrics. In this paper we generalize the transmission line techniques to measure the permittivity function of one-dimensional inhomogeneous dielectrics. A rectangular waveguide is used and the dielectric slab under test is placed at the input flange of it. Then the measured scattering parameters or the short-circuited reflection coefficient are used to extract the permittivity function of dielectric. In fact, we have to solve a one-dimensional inverse scattering problem [10]. To solve this problem the electric permittivity function is expanded in a truncated Fourier series, first. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method is verified using a comprehensive example.

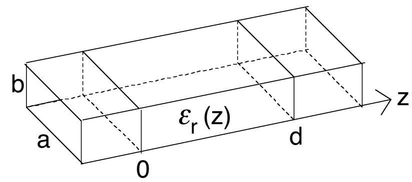

Figure 1. The structure uses to measure the dielectric constant of an inhomogeneous dielectric.

2. DIRECT PROBLEM

Fig. 1 shows the structure, calling it Longitudinally Inhomogeneous Waveguide (LIW), which is used to measure the dielectric constant of an inhomogeneous dielectric. There is a rectangular waveguide with dimensions a and b, filled by an inhomogeneous dielectric with the electric permittivity function $\varepsilon_r(z)$ and length d. It is assumed that only the dominant mode TE_{10} exists in the waveguide. In this section the frequency domain analysis (Direct Problem) of this structure is reviewed. The differential equations describing LIWs have non-constant coefficients and so except for a few special cases no closed form analytic solution exists for them. There are some methods to analyze LIWs such as finite difference [11], Taylor's series expansion [12], Fourier series expansion [13] and the method of Moments [14]. Of course, the most straightforward method is subdividing LIWs into K homogeneous electrically short segments with length

$$\Delta z = d/K \ll \lambda_{\min} \cong \frac{c}{f_{\max}\sqrt{\max(\varepsilon_r(z))}} \tag{1}$$

in which c is the velocity of the light and $f_{\max}$ is the maximum frequency of the analysis. The $ABCD$ parameters of the LIW is obtained from those of its segments as follows

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{2}$$

where the $ABCD$ parameters of the k-th segment are as follows

$$A_k = D_k = \cos(\Delta\theta_k) \tag{3}$$

$$B_k = Z_c^2((k-0.5)\Delta z, f)C_k = jZ_c((k-0.5)\Delta z, f)\sin(\Delta\theta_k) \tag{4}$$

In (3) and (4),

$$\Delta\theta_k = \frac{2\pi f}{c}\sqrt{\varepsilon_r((k-0.5)\Delta z) - (f_c/f)^2}\Delta z \tag{5}$$

is the electrical length of the k-th segment and $Z_c(z, f)$ is the characteristic impedance of the LIW, defined as the ratio of the transverse electric field to the transverse magnetic field, given by

$$Z_c(z, f) = \frac{\sqrt{\mu_0/\varepsilon_0}}{\sqrt{\varepsilon_r(z) - (f_c/f)^2}} \tag{6}$$

In (5) and (6) $f_c = c/(2a)$ is the cutoff frequency of the hollow waveguide, which must be less than the frequency f. Finally, the scattering parameters of the structure and the input reflection coefficient of the short-circuited structure can be determined from its $ABCD$ parameters as follows

$$\Gamma_{SC}(f) = \frac{B - Z_{TE}D}{B + Z_{TE}D} \tag{7}$$

$$S_{11}(f) = \frac{(A - Z_{TE}C)Z_{TE} + (B - Z_{TE}D)}{(A + Z_{TE}C)Z_{TE} + (B + Z_{TE}D)} \tag{8}$$

$$S_{21}(f) = \frac{2Z_{TE}}{(A + Z_{TE}C)Z_{TE} + (B + Z_{TE}D)} \tag{9}$$

where Z_{TE} is the known characteristic impedance of the hollow waveguide given by

$$Z_{TE}(f) = \frac{\sqrt{\mu_0/\varepsilon_0}}{\sqrt{1 - (f_c/f)^2}} \tag{10}$$

3. INVERSE PROBLEM

In this section a general method is proposed to reconstruct the electric permittivity function of the LIWs (Inverse Problem). First, we consider the following truncated Fourier series expansion for the electric permittivity function.

$$\ln(\varepsilon_r(z) - 1) = \sum_{n=0}^{N} C_n \cos(n\pi z/d) \tag{11}$$

Using the expansion (11) we need to determine only $N + 1$ coefficients instead of many K parameters used in the direct problem. The values of the unknown coefficients C_n can be obtained through minimizing one of the following error functions corresponding to M measuring frequencies $f_1 < f_2 < \ldots < f_M$.

$$E_1 = \sqrt{\frac{1}{M} \sum_{m=1}^{M} |\Gamma_{SC,sim}(f_m) - \Gamma_{SC,meas}(f_m)|^2} \tag{12}$$

$$E_2 = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left|S_{11,sim}(f_m) - S_{11,meas}(f_m)\right|^2} \tag{13}$$

$$E_3 = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left|S_{21,sim}(f_m) - S_{21,meas}(f_m)\right|^2} \tag{14}$$

$$E_4 = \sqrt{\left(\begin{array}{l}\dfrac{1}{2M}\displaystyle\sum_{m=1}^{M}\Big(\left|S_{11,sim}(f_m) - S_{11,meas}(f_m)\right|^2 \\ \qquad\qquad + \left|S_{21,sim}(f_m) - S_{21,meas}(f_m)\right|^2\Big)\end{array}\right)} \tag{15}$$

The parameter Γ_{SC} in (12) is the input reflection coefficient of the short-circuited structure. Also, the indices "*sim*" and "*meas*" in (12)–(15) represent the simulation (analysis) and measuring ways, to determine the scattering parameters. In fact, the measuring data in the error functions E_1, E_2, E_3 and E_4 are the short-circuited reflection, the matched-end reflection, the transmission and the combination of the matched-end reflection and the transmission, respectively. It is noticeable that if the simulation methods use some approximations or the measuring data are not fully accurate the inverse problems may be diverged because they are ill-posed. So, to stabilize the inverse problems usually various kinds of regularizations are used [10]. To utilize the regularization methods for our problem, a suitable fraction of the energy of the permittivity function or its derivatives should be add to the defined error functions E_1, E_2, E_3 and E_4.

Finally, to investigate the similarity of the reconstructed permittivity function to the exact one, one may define the following error function.

$$E_\varepsilon = \sqrt{\frac{1}{K+1}\sum_{k=0}^{K}\left(\frac{\varepsilon_{r,rec}(kd/K) - \varepsilon_{r,exact}(kd/K)}{\varepsilon_{r,exact}(kd/K)}\right)^2} \tag{16}$$

where the index "*rec*" is the abbreviation of the reconstruction.

4. EXAMPLE AND RESULTS

We would like to reconstruct the permittivity function using M frequencies with equal distance in the range of 8.0 to 12.0 GHz (X-Band), assuming $a = 0.9$ inch, $b = 0.4$ inch (WRG-90) $d = 2.0$ cm. Using the proposed reconstruction method considering $M = 5$ measuring frequencies the five defined error functions were obtained versus the number of expansion terms N. Figure 2 shows E_1, E_2, E_3

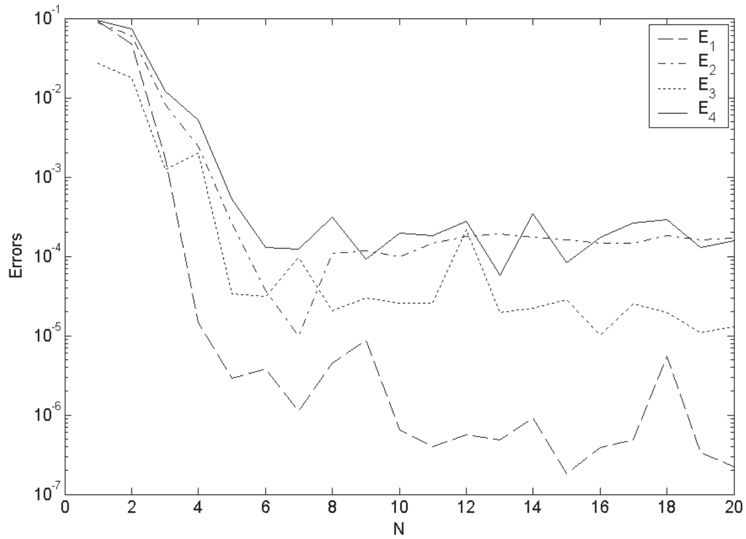

Figure 2. The error functions E_1, E_2, E_3 and E_4 with respect to the number of expansion terms N considering $M = 5$ measuring frequencies.

and E_4 and Figure 3 compares E_ε obtained from E_1, E_2, E_3 and E_4. It is observed from these two figures that as N increases each error function tends to a specified value. Meanwhile, choosing N between 4 and 8 is more suitable than the larger values. Also, it is seen that the reconstruction error E_ε may be sorted from high to low in the cases of using error functions E_3, E_2, E_1 and E_4. So, the worst and the best reconstructions are achieved for the cases of using E_3 and E_4, respectively. Figures 4–7 illustrate the reconstructed permittivity function $\varepsilon_r(z)$ for some M and N using the error functions E_1, E_2, E_3 and E_4, respectively. One sees from these figures that for a fixed N, as the measuring frequencies M is increased the reconstructed permittivity function tends to the exact one. It is seen from Fig. 6 that for some combination of M and N (such as $M = 5$ and $N = 7$) the reverse of the permittivity function may be reconstructed if we use the error function E_3. This is due to the reciprocity principle i.e., $S_{12} = S_{21}$. So, the case of using only transmission data S_{21} has an intrinsic reversing ambiguity. The unknown coefficients of the truncated Fourier series of the reconstructed permittivity function considering $M = 5$ measuring frequencies and $N = 5$ expansion terms have been written in Table 1.

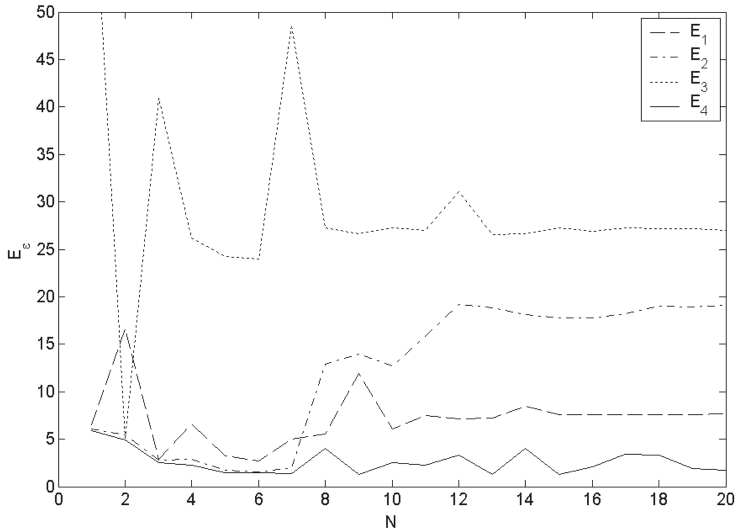

Figure 3. The reconstruction error functions E_ε with respect to the number of expansion terms N considering $M = 5$ measuring frequencies.

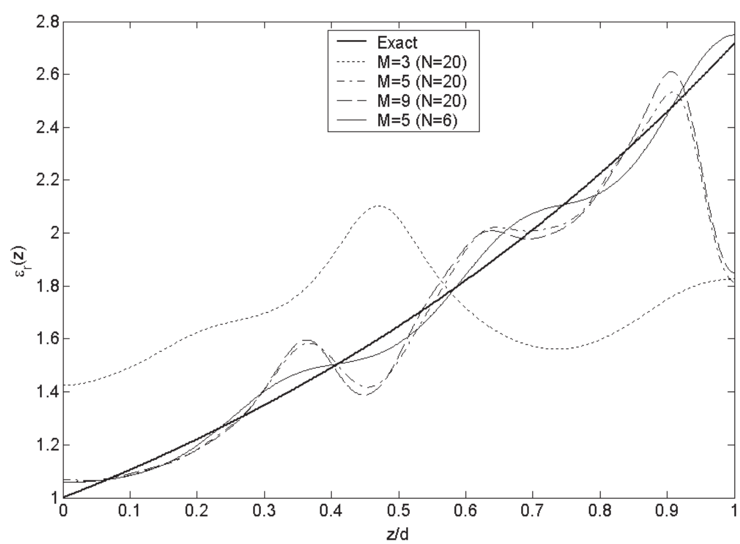

Figure 4. The reconstructed permittivity function $\varepsilon_r(z)$ for some M and N using the error function E_1.

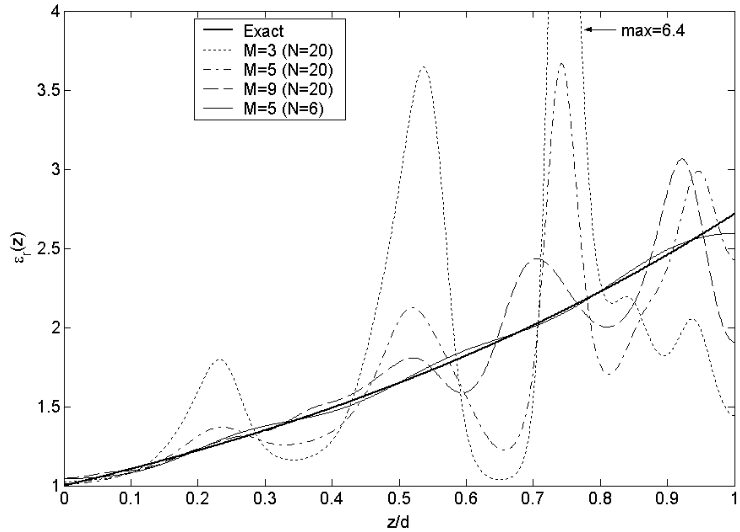

Figure 5. The reconstructed permittivity function $\varepsilon_r(z)$ for some M and N using the error function E_2.

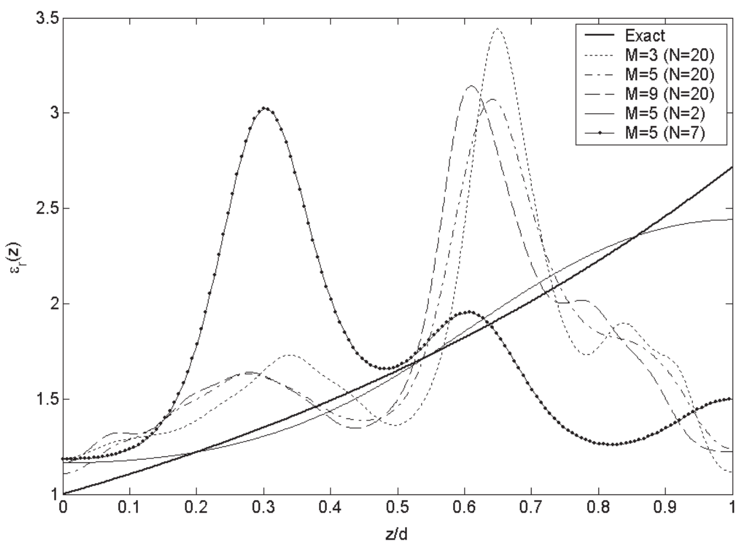

Figure 6. The reconstructed permittivity function $\varepsilon_r(z)$ for some M and N using the error function E_3.

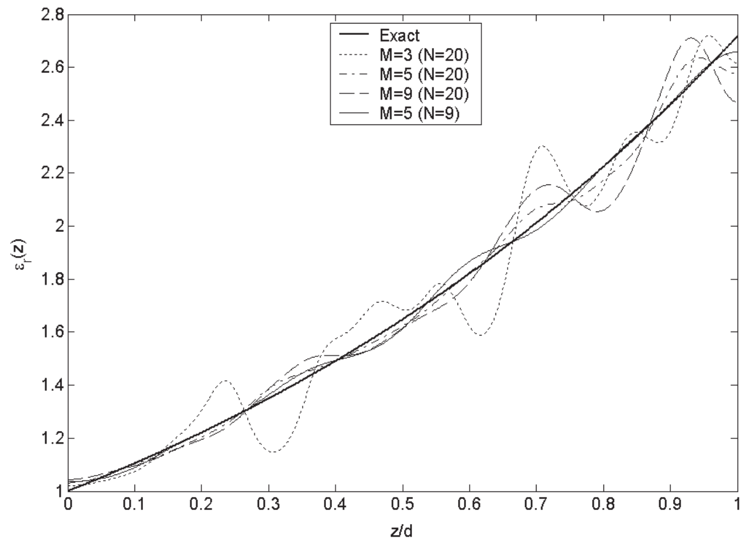

Figure 7. The reconstructed permittivity function $\varepsilon_r(z)$ for some M and N using the error function E_4.

Table 1. The unknown coefficients of the truncated Fourier series of the reconstructed permittivity function considering $M = 5$ and $N = 5$.

	C_0	C_1	C_2	C_3	C_4	C_5
E_1	-0.7126	-1.2893	-0.3981	-0.2692	-0.1799	-0.0674
E_2	-0.7109	-1.3323	-0.3729	-0.3119	-0.1558	-0.1216
E_3	-0.5745	-0.7122	-0.4564	0.1292	-0.5301	-0.3695
E_4	-0.7001	-1.3237	-0.3695	-0.2985	-0.1472	-0.1020

5. CONCLUSION

The transmission line techniques were generalized to measure the permittivity function of one-dimensional inhomogeneous dielectrics. The measured scattering parameters or the short-circuited reflection coefficient are used to extract the permittivity function of dielectric. To solve the problem, the electric permittivity function is expanded in a truncated Fourier series, first. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method was verified using a comprehensive example. It is observed that as the expansion terms increases each error function tends to a low specified value. The worst and the best reconstructions are achieved for the cases of using only transmission and combination of reflection and transmission measuring data, respectively. Also, the case of using only transmission data has an intrinsic reversing ambiguity.

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