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A PERTURBATIVE APPROACH FOR THE EVALUATION OF EM SCATTERING PROPERTIES FROM ARBITRARY DIELECTRIC BODIES WITH INCLUSIONS

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Abstract—An approximate perturbative technique for the analysis of electromagnetic scattering from dielectric bodies of arbitrary shape containing dielectric inclusions, illuminated by an arbitrarily polarized incident plane wave, is investigated. The perturbative approach here presented allows for the efficient computation of the scattering properties of a given body as the inclusions vary, with a formulation solving only for the inclusion bound field component.

1. INTRODUCTION

In the past few years many efforts have been done in studying and realizing artificial structures to develop composite materials that present new response functions which do not occur in nature. Example of materials which show interesting electromagnetic behaviour in the visible region are plasmonic materials [1]. This is due to the presence of electric resonances in the microscopic molecular domain that induce an overall negative electric permittivity for the bulk medium [2]. Recently, by mimicking the molecular functions that cause these anomalous resonances in a larger scale, metamaterials with non standard values of their constitutive parameters have been proposed and synthesized by properly embedding suitably shaped inclusions in a given host medium. Examples of such engineered materials are double negative, DNG, materials which are artificial materials showing simultaneous negative real permittivity and permeability properties [3–7].

In [8] a composite medium consisting of insulating magneto-dielectric spherical particles embedded in a background matrix has been studied to show that such a material behaves as an effective

DNG. In that work it is shown that the properties of an array of spherical particles can be efficiently exploited to obtain electromagnetic characteristics similar to that ones of an array of more complicated conducting scatterers. A typical geometry of a composite structure consists of a background medium embedding spherical inclusions. The diameter of these magneto-dielectric "atoms" and their spacing are typically large compared to molecular dimensions, but still small with respect to the wavelength in the host material. The electromagnetic behaviour of this composite structures can be investigated by means of an homogenized model with proper effective constitutive parameters [9]. The parameters of the homogenized model depend on those ones characterizing both the inclusions and the host medium, as well as on geometry and spacing of the inclusions [10].

By using the homogenization equation presented in [10], the electromagnetic behaviour of an arbitrary shaped inhomogeneous magneto-dielectric body can be studied by using its homogeneous equivalent. In this case the electromagnetic scattering can be efficiently evaluated by means of a standard Method of Moments, MoM, for the solution of a Surface Integral Equation (SIE) on the surface of the body. If the homogenized model does not provide sufficiently accurate results, one have to characterize the actual body and the numerical procedure becomes very onerous.

A full-wave numerical procedure to determine the electromagnetic field induced inside arbitrary shaped dielectric bodies illuminated by an incident plane wave propagating in arbitrary direction and with arbitrary polarization has been presented in [11]. In that paper the Method of Moments technique is exploited to solve a Volumetric Integral Equation (VIE) within the inhomogeneous material. It is well known that the Method of Moments is very accurate but becomes very onerous especially when it is applied to the solution of a VIE relevant to a body whose dimensions are not small with respect to the wavelength.

On the other hand in [12] an accurate and efficient hybrid technique for the analysis of electromagnetic scattering from an infinite periodic structure containing an impurity is presented. In that contribution a 2D problem is analysed by means of a two-step approach. In the first step the periodic structure is considered without the impurity using a hybrid technique which combines a finite element method (FEM) and a Floquet modes analysis. In the second step the presence of the impurity is taken into account by applying a Method of Moments for the solution of a SIE in a broad region of the lattice containing the impurity and solving for a corrective term to be added to the field of the original problem.

In this contribution an approximate perturbative technique for the analysis of electromagnetic scattering from dielectric bodies of arbitrary shape containing dielectric inclusions is investigated. The perturbative approach here presented has the advantage to efficiently analyse the scattering properties of a dielectric body with different types of inclusions, exploiting the same numerical tool used to characterize the homogeneous bulk.

The paper is organized as follows: the geometry and the mathematical derivation of the problem are described in Section II; then, the iterative numerical procedure is described in detail in Section III. Section IV describes some numerical results obtained through the application of the technique proposed. Finally some conclusions are drawn in Section V.

2. FORMULATION

The problem under investigation consists of a host dielectric medium of arbitrary shape and dielectric constant embedding material inclusions as sketched in Fig. 1. The composite material is illuminated by an incident plane wave propagating in an arbitrary direction and arbitrarily polarized. The medium is assumed to be linear and isotropic with dielectric constant as follows:

$$\epsilon_r(\mathbf{r}') = \begin{cases} \epsilon_h & \text{if } \mathbf{r}' \in V_1 \backslash V_2 \\ \epsilon_i & \text{if } \mathbf{r}' \in V_2 \end{cases} \tag{1}$$

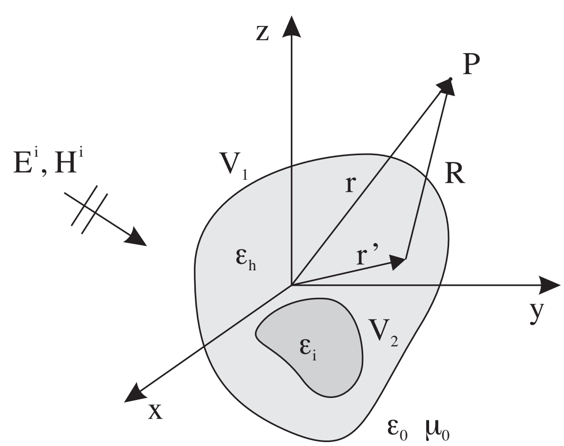

Figure 1. Geometry of the problem under consideration. An arbitrary dielectric bulk contains material inclusions exhibiting a different dielectric constant with respect to the host medium.

where

V_1 is the volume of the host dielectric body;

V_2 is the sub-volume occupied by the inclusion;

ϵ_h is the dielectric constant of the host dielectric body;

ϵ_i is the dielectric constant of the inclusions.

The electric field scattered from the overall structure with the inclusions, $\mathbf{E}_s$, can be recovered by applying the following numerical scheme. The scattered field $\mathbf{E}_s$ at an arbitrary observation point can be written, by means of the volume equivalence theorem, in terms of the electric polarization current $\mathbf{J}$ inside the composite material:

$$\mathbf{E}^s(\mathbf{r}) = \mathcal{L}\{\mathbf{J}(\mathbf{r}')\} \tag{2}$$

with

$$\mathbf{J}(\mathbf{r}') = \tau(\mathbf{r}')\mathbf{E}^{tot}(\mathbf{r}') \tag{3}$$

where $\tau(\mathbf{r}') = j\omega\epsilon_0[\epsilon_r(\mathbf{r}') - 1]$ and the linear integro-differential operator $\mathcal{L}\{\cdot\}$ in (2) has the following expression, [11]:

$$\mathcal{L}\{\cdot\} = -\int_{V_1} j\omega\mu_0\{\cdot\}\left[\frac{e^{-jk_0R}}{4\pi R} + \frac{1}{k_0^2}\nabla\nabla\cdot\frac{e^{-jk_0R}}{4\pi R}\right]dV' \tag{4}$$

In the framework of a perturbative technique, [13], the following series expansion for the total electric field inside the volume can be assumed

$$\mathbf{E}^{tot} = \mathbf{E}^{(0)} + \sum_{k=1}^{K}\mathbf{E}^{(k)} \tag{5}$$

where $\mathbf{E}^{(0)}$ is the total electric field inside the volume for the homogeneous bulk (non perturbed field) and $\mathbf{E}^{(k)}$ are the contributions to the total field which originate from the presence of the inclusions (perturbative contributions); (k) represents the field order, being K the maximum order.

As a basic step of the solution the field $\mathbf{E}^{(0)}$ inside the bulk can be evaluated in the hypothesis that there are no inclusions (homogeneous non perturbed problem). A VIE is imposed inside the volume of the homogeneous medium:

$$\mathcal{L}\left\{\tau_h\mathbf{E}^{(0)}\right\} - \mathbf{E}^{(0)} = -\mathbf{E}^i \tag{6}$$

where $\tau_h = j\omega\epsilon_0[\epsilon_h - 1]$ is relevant to the homogeneous dielectric body without inclusions and $\mathbf{E}^i$ is the incident field.

Introducing the total field due to the presence of the inclusions and exploiting the linearity of the field operator the new VIE for the composite material can be written as:

$$\mathcal{L}\left\{\tau_h \mathbf{E}^{(0)}\right\} + \sum_{k=1}^{K} \mathcal{L}\{\tau_h \mathbf{E}^{(k)}\} + \mathcal{L}'\{(\tau - \tau_h)\mathbf{E}^{(0)}\} + \\ + \sum_{k=1}^{K} \mathcal{L}'\{(\tau - \tau_h)\mathbf{E}^{(k)}\} - \mathbf{E}^{(0)} - \sum_{k=1}^{K} \mathbf{E}^{(k)} = -\mathbf{E}^{i} \tag{7}$$

where $\mathcal{L}'\{\cdot\}$ represents the perturbed linear operator whose integration is limited to the smaller volume V_2.

Decomposing the two series and exploiting the knowledge of the non perturbed problem, (5), in the hypothesis that the series in (4) is convergent, it is possible to develop an iterative procedure to evaluate every perturbative contribution from the previous one. At k-th step of iteration the perturbative field of k-th order, $1 \leq k \leq K$, is the solution of the approximate VIE

$$\mathcal{L}\left\{\tau_h \mathbf{E}^{(k)}\right\} - \mathbf{E}^{(k)} = -\mathcal{L}'\{(\tau - \tau_h)\mathbf{E}^{(k-1)}\} \tag{8}$$

where the term $\mathcal{L}'\{(\tau - \tau_h)\mathbf{E}^{(k)}\}$ is neglected.

It is worth notice that the convergence of the series in (4) occurs in all practical problems where the volume of the inclusions is small compared to that one of the host material, as in the case of DNG materials realized by embedding small "atoms" in a large background medium. If the volume of the inclusions is comparable to that one of the host medium, the series in (4) does not converge and the iterative procedure fails.

3. NUMERICAL PROCEDURE

The volumetric integral equation (6) can be solved via a standard Method of Moments technique with pulse basis functions defined on N cuboid discretization cells and point matching procedure. In the matrix representation of the VIE, $[G][E^{(0)}] = -[E^i]$, $[G]$ is a $3N \times 3N$ matrix with the following structure:

$$\begin{bmatrix} [G_{xx}] & [G_{xy}] & [G_{xz}] \\ [G_{yx}] & [G_{yy}] & [G_{yz}] \\ [G_{zx}] & [G_{zy}] & [G_{zz}] \end{bmatrix} \begin{bmatrix} [E_x^{(0)}] \\ [E_y^{(0)}] \\ [E_z^{(0)}] \end{bmatrix} = - \begin{bmatrix} [E_x^{i}] \\ [E_y^{i}] \\ [E_z^{i}] \end{bmatrix} \tag{9}$$

while $[E^{(0)}]$ and $[E^i]$ have dimension $3N$ and take the form:

$$[E^{(0)}] = \begin{bmatrix} [E_x^{(0)}] \\ [E_y^{(0)}] \\ [E_z^{(0)}] \end{bmatrix} \qquad [E^i] = \begin{bmatrix} [E_x^i] \\ [E_y^i] \\ [E_z^i] \end{bmatrix} \tag{10}$$

For the evaluation of the $[G]$ matrix elements see (20) and (22) of [11]. The solution $[E^{(0)}]$ can be determined by inverting $[G]$.

The integral equation (8) is of the same form of (6) but differs for the forcing term $-\mathcal{L}'\{(\tau - \tau_h)\mathbf{E}^{(k-1)}\}$, thus its matrix representation is formally the same of (9)

$$[G][E^{(k)}] = -[B] \tag{11}$$

where the forcing terms vector is now:

$$[B] = \begin{bmatrix} [B_x] \\ [B_y] \\ [B_z] \end{bmatrix} \tag{12}$$

$$[B_x] = \begin{bmatrix} B_x^1 \\ B_x^2 \\ \vdots \\ B_x^m \\ \vdots \\ B_x^N \end{bmatrix} \quad [B_y] = \begin{bmatrix} B_y^1 \\ B_y^2 \\ \vdots \\ B_y^m \\ \vdots \\ B_y^N \end{bmatrix} \quad [B_z] = \begin{bmatrix} B_z^1 \\ B_z^2 \\ \vdots \\ B_z^m \\ \vdots \\ B_z^N \end{bmatrix} \tag{13}$$

being B_x^m the x component of the scattered electric field in the m-th cell, $m = 1, ..N$, due to the electric field contribution of order $(k-1)$ in all the cells belonging to V_2:

$$B_x^m = \sum_{n=1}^{N} \left\{ [E_x^m(J_x^n)]^{(k-1)} + \left[E_x^m(J_y^n)\right]^{(k-1)} + [E_x^m(J_z^n)]^{(k-1)} \right\} \tag{14}$$

4. NUMERICAL RESULTS

The procedure presented above has been applied to a simple test case. The structure under consideration is a dielectric cube embedding a spherical inclusion, illuminated by an incident z-polarized plane wave propagating in $-\hat{x}$ direction at 1 GHz, Fig. 2.

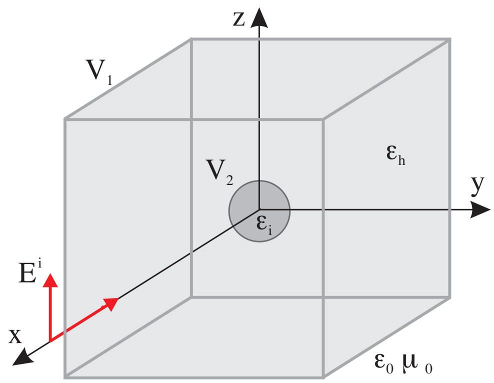

Figure 2. Geometry of the problem under consideration. A dielectric cube embedding a concentric spherical inclusion, illuminated by an incident plane wave.

The side of the cube is $0.7\lambda_0$ in length while the radius of the sphere is $0.2\lambda_0$, being λ_0 the wavelength of the free space. The constitutive parameters of the composite material are $\epsilon_h = 4$ and $\epsilon_i = 6$. In Fig. 3 the amplitude of the electric field contributions in (5) averaged over the N cells of the mesh is reported up to the 5-th order of iteration, to clearly show the convergence of the solution.

Table 1. Computation times.

ϵ_i	MoM [s]	Pert [s]						
		(0)	(1)	(2)	(3)	(4)	(5)	(6)
6	91	91	96	111	116	121	126	131
7	91	-	5	10	15	20	25	30
8	91	-	5	10	15	20	25	30
9	91	-	5	10	15	20	25	30
10	91	-	5	10	15	20	25	30
11	91	-	5	10	15	20	25	30
Total	546	91	121	161	191	221	251	281

Numerical results in Fig. 4 are relevant to the co-polar component of the electric scattered far field on the vertical cut $\phi = 0$. The reference solution as computed via the Method of Moments is compared to the approximate solution provided by the iterative perturbative procedure

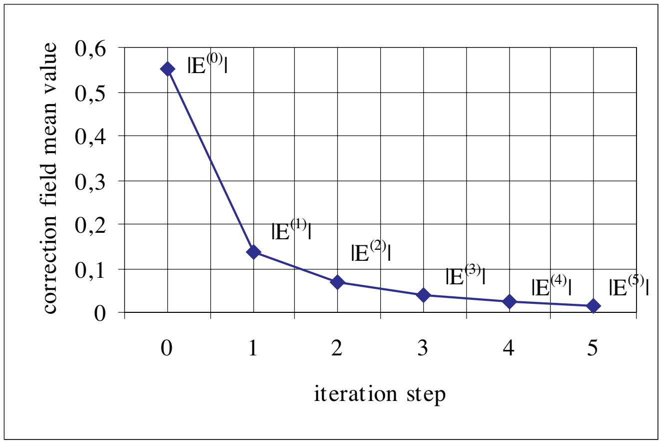

Figure 3. Mean value of the total electric field contributions at different step of iteration.

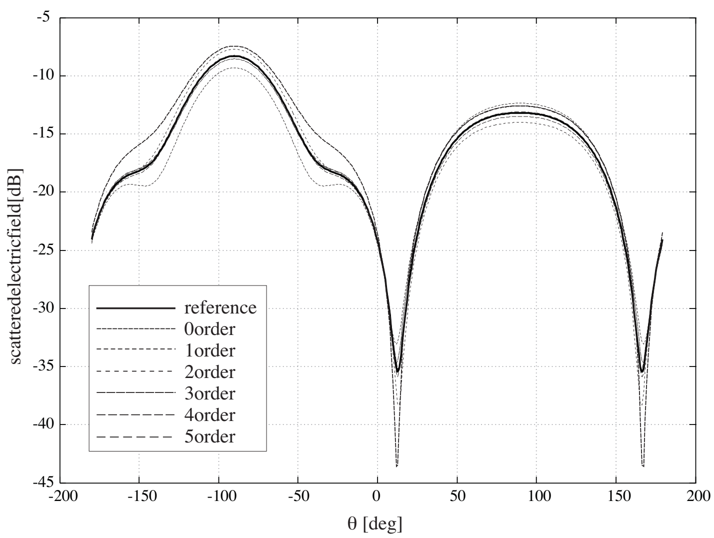

Figure 4. Co-polar scattered electric far field on plane $\phi = 0°$ evaluated by a standard MoM and by the perturbative procedure.

previously discussed. It is evident that the accuracy of the solution improves at each step of iteration and that a good convergence is reached at the 5-th order of approximation.

In Table 1 comparison between computation times relevant to a standard MoM analysis procedure and the perturbative technique presented above is shown. The EM analysis has been performed for the host material of the previous case with six different types of spherical inclusions. For each type of inclusion computation times are reported both for MoM and the perturbative technique up to six order of approximation. It is worth noticing that we can solve the 0-th order problem once and then we can exploit the perturbative approach to compute the correction terms which characterize different inclusions. The last row of the table shows total computation times required for the analysis of all the six configurations, by using MoM and the perturbative technique with different order of approximation.

5. CONCLUSIONS

An approximate MoM based perturbative technique for the evaluation of the electric field induced inside a composite dielectric material and to characterize its scattering behaviour has been presented. The perturbative approach has been exploited for the iterative solution of a volumetric integral equation in the framework of a point-matching, pulse function based Method of Moments technique.

The procedure is particularly advantageous in the aim of investigating the electromagnetic behaviour of a specific host dielectric medium with different types of inclusions. The solution of the volumetric integral equation at each step of iteration actually requires the evaluation of a forcing term which involves integrals over the small volume V_2, while the onerous task of filling and inverting the $[G]$ matrix can be performed only once, exploiting the same numerical tool used for the solution of the non perturbed problem.

The numerical technique here presented provides accurate results for all practical problems where the volume of the inclusions is small with respect to the volume of the host medium. If this is not the case the perturbative approach is not suitable and the iterative procedure may fail.

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