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SCATTERING OF OBLIQUELY INCIDENT PLANE WAVE BY AN ARRAY OF PARALLEL CONCENTRIC METAMATERIAL CYLINDERS

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Abstract—A rigorous semi-analytical solution is presented for electromagnetic scattering from an array of parallel-coated circular cylinders of arbitrary radii and positions due to an obliquely incident TM_z plane wave excitation. In order to check the validity of this technique, the radar cross-section of a single coated cylinder, a linear array of cylinders, and an arbitrary position array of cylinders are calculated and compared with available data in the literature. Furthermore, the near field is calculated to prove the validity of the boundary conditions on the surface of any cylinder with obliquely incidence wave. As an application, circular metamaterial cylinders are used to show the effect of metamaterial characteristics in altering the forward and backward scattering and in focusing the near field around the objects.

1. INTRODUCTION

The scattering of electromagnetic plane wave from single and array of cylinders for both normal [1–9] and oblique [10, 11] incidence had been studied for many years. In a previous work [12], the principle of equal volume model was used to model any two-dimensional dielectric object of arbitrary cross section by an array of dielectric circular cylinders. A detailed derivation for the problem of scattering of a plane wave from an array of parallel dielectric circular cylinders in case of oblique incidence was presented in [12]. The electromagnetic scattering by metamaterial cylinders was presented in [13, 14], where the scattering of normally incident TM_z plane wave by circular-cylindrical conductor objects coated by metamaterial is only considered. In this paper, the scattering of an obliquely incident plane wave on an array of parallel-coated circular cylinders is considered. The core cylinders and the coating layers can be any of three materials; metamaterial, dielectric, perfect electric conductor or a combination of two of them. The analysis begins by representing each field component by an infinite series of cylindrical harmonic functions with unknown coefficients. Then equations based on the boundary conditions applied on the surface of both the core cylinders and the outer surface of the cylinders are used to deduce the values of the unknown coefficients. To prove the validity of the results, numerical examples are given to compare the scattered RCS based on the deduced coefficients and that based on results reported in [13, 14] for the special case of normal incidence. The emphasis of this paper is to investigate the effect of oblique incidence condition and the metamaterial coating. The developed semi-analytical solution for an array either metamaterial or conductor coated with metamaterial cylinders will be used to show the effect of metamaterial in focusing the field in either the forward or the backward direction.

2. FORMULATION

The scattering from an obliquely incident E-polarized TM_z plane wave from an array of M parallel-coated circular cylinders parallel to each other and to the z-axis is considered in a global coordinate system (ρ, ϕ, z). The incident electric field of a plane wave on cylinder "i" is expressed in the (ρ_i, ϕ_i, z) cylindrical coordinate system for $e^{j\omega t}$ time dependence as

$$
\begin{aligned}
&E_{z_i}^{inc}(\rho_i, \phi_i, z) \\
&= E_0' e^{jk_0 z\cos\theta_0} e^{jk_0\rho_i \sin\theta_0 \cos(\phi_i-\phi_0)} e^{jk_0\rho_i' \sin\theta_0 \cos\left(\phi_i'-\phi_0\right)} \\
&= E_0' e^{jk_0 z\cos\theta_0} e^{jk_0\rho_i' \sin\theta_0 \cos\left(\phi_i'-\phi_0\right)} \sum_{n=-\infty}^{\infty} j^n J_n\left(k_0\rho_i \sin\theta_0\right) e^{jn(\phi_i-\phi_0)}, \quad (1)
\end{aligned}
$$

where $E_0' = E_0 \sin\theta_0$, θ_0 is the oblique incident angle as shown in Fig. 1, and E_0 is the amplitude of the incident electric field component. The parameter k_0 is the free space wave number, ϕ_0 is the angle of incidence of the plane wave in the x-y plane with respect to the negative x-axis, and $J_n(\xi)$ is the Bessel function of order n and argument ξ. The second expression of the incident field component is in terms of the cylindrical coordinate of the i^{th} cylinder, whose center is located at (ρ_i', ϕ_i', z) of the global coordinate (ρ, ϕ, z).

The resulting z component of the scattered electric field from the

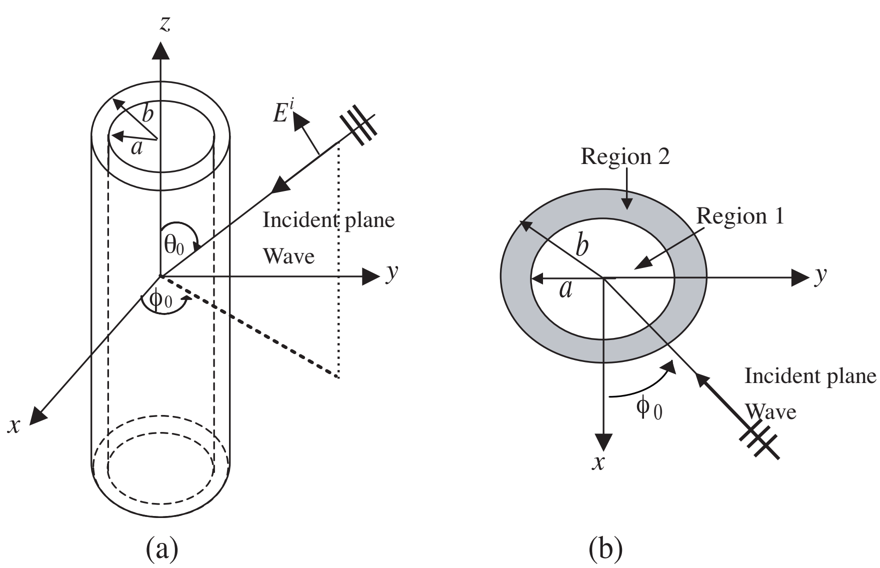


Figure 1. The parameters describing the obliquely incident TM$_z$ plane wave on a coated cylinder, (a) 3D Geometry, (b) x-y, 2D cross-sectional geometry.

i^{th} cylinder can be expressed as

$$E^s_{z_i}(\rho_i, \phi_i, z) = E'_0 e^{jk_0 z \cos\theta_0} \sum_{n=-\infty}^{\infty} A_{in} H_n^{(2)} \left(k_0 \rho_i \sin\theta_0\right) e^{jn(\phi_i - \phi_0)}. \quad (2)$$

where $H_n^{(2)}(\xi)$ is the Hankel function of the second type of order n and argument ξ. The transmitted z component of the field inside the core cylinders and the coating layers can be expressed, respectively, as

$$E^{d1}_{z_i}(\rho_i, \phi_i, z) = E'_0 e^{jk_0 z \cos\theta_0} \sum_{n=-\infty}^{\infty} D_{in} J_n\left(k_0 \rho_i \sqrt{\frac{k_{d1}^2}{k_0^2} - \cos^2\theta_0}\right) e^{jn(\phi_i - \phi_0)}, \quad (3)$$

$$E^{d2}_{z_i}(\rho_i, \phi_i, z) = E'_0 e^{jk_0 z \cos\theta_0} \sum_{n=-\infty}^{\infty} \left[B_{in} J_n\left(k_0 \rho_i \sqrt{\frac{k_{d2}^2}{k_0^2} - \cos^2\theta_0}\right) + C_{in} N_n\left(k_0 \rho_i \sqrt{\frac{k_{d2}^2}{k_0^2} - \cos^2\theta_0}\right)\right] e^{jn(\phi_i - \phi_0)}, \quad (4)$$

where $N_n(\xi)$ is the Neumann function of order n and argument ξ, k_{d1} is the wave number inside the core cylinders, and k_{d2} is the wave number inside the coating layer material. The corresponding magnetic field components are given as

$$H^s_{z_i}(\rho_i, \phi_i, z) = E'_0 e^{jk_0 z \cos\theta_0} \sum_{n=-\infty}^{\infty} A'_{in} H_n^{(2)} \left(k_0 \rho_i \sin\theta_0\right) e^{jn(\phi_i - \phi_0)}, \quad (5)$$

$$H^{d1}_{z_i}(\rho_i, \phi_i, z) = E'_0 e^{jk_0 z \cos\theta_0} \sum_{n=-\infty}^{\infty} D'_{in} J_n\left(k_0 \rho_i \sqrt{\frac{k_{d1}^2}{k_0^2} - \cos^2\theta_0}\right) e^{jn(\phi_i - \phi_0)}, \quad (6)$$

$$H^{d2}_{z_i}(\rho_i, \phi_i, z) = E'_0 e^{jk_0 z \cos\theta_0} \sum_{n=-\infty}^{\infty} \left[B'_{in} J_n\left(k_0 \rho_i \sqrt{\frac{k_{d2}^2}{k_0^2} - \cos^2\theta_0}\right) + C'_{in} N_n\left(k_0 \rho_i \sqrt{\frac{k_{d2}^2}{k_0^2} - \cos^2\theta_0}\right)\right] e^{jn(\phi_i - \phi_0)}. \quad (7)$$

In this work, we consider only double negative (DNG) metamaterial with negative permittivity and permeability [15, 16]. To show the effect of metamaterial, the wave number and the intrinsic impedance for the i^{th} layer can be expressed as

$$\begin{aligned} k_i &= k_0 n_{ri}, \\ n_{ri} &= \sqrt{\mu_{ri}\varepsilon_{ri}} \qquad \text{for dielectric}, \\ n_{ri} &= -\sqrt{\mu_{ri}\varepsilon_{ri}} \quad \text{for metematerial}, \\ \eta_i &= \eta_0 \sqrt{\mu_{ri}/\varepsilon_{ri}}. \end{aligned} \tag{8}$$

The coefficients A_{in}, B_{in}, C_{in}, D_{in}, A'_{in}, B'_{in}, C'_{in}, and D'_{in} are unknowns to be determined. Using Maxwell's equations, the ϕ components of the magnetic field can be obtained as follow

$$H_\phi = \frac{-1}{j\omega\mu}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}\right),$$

where $E_\rho = \frac{1}{j\omega\varepsilon}\left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)$.

The only dependence on the parameter z is in the exponential term $e^{jk_0 z\cos\theta_0}$, thus the differentiation with respect to z can be expressed in the form of $\frac{\partial}{\partial z} = jk_0\cos\theta_0$, and therefore H_ϕ can be written as

$$H_\phi(\rho,\phi,z)\left[1 - \frac{k_0^2\cos^2\theta_0}{\omega^2\mu\varepsilon}\right] = -\frac{k_0\cos\theta_0}{j\omega^2\mu\varepsilon\rho}\frac{\partial H_z}{\partial \phi} + \frac{1}{j\omega\mu}\frac{\partial E_z}{\partial \rho}. \tag{9}$$

According to Equation (9) and based on the fact that the z component of the incident magnetic field is equal to zero for the TM_z case, then the ϕ component of the incident magnetic field on cylinder "i" can be written as

$$\begin{aligned} H_{\phi_i}^{inc}(\rho_i,\phi_i,z) &= \frac{1}{j\omega\mu_0\sin^2\theta_0}\frac{\partial E_z^{inc}}{\partial \rho_i} \\ &= \frac{E'_0 k_0}{j\eta_0\lambda_0} G P_i \sum_{n=-\infty}^{\infty} j^n J'_n(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)}, \end{aligned} \tag{10}$$

where the prime represents the derivative of the Bessel function with respect to its full argument, $G = e^{jk_0 z\cos\theta_0}$, $h = k_0\cos\theta_0$, $\lambda_0 = k_0\sin\theta_0$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$, and $P_i = e^{jk_0\rho'_i\sin\theta_0\cos\left(\phi'_i-\phi_0\right)}$.

Based on Equation (9), the ϕ components of the scattered magnetic field and the magnetic field transmitted in the core and coating materials of cylinder "i" can be expressed, respectively, as

$$\begin{aligned} H^{s}_{\phi_i}(\rho_i,\phi_i,z) &= E'_0 G \frac{k_0}{j\eta_0\lambda_0}\sum_{n=-\infty}^{\infty} A_{in} H_n^{(2)'}(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)} \\ &\quad - E'_0 G \frac{h}{\lambda_0^2\rho_i}\sum_{n=-\infty}^{\infty} n\, A'_{in} H_n^{(2)}(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)}, \quad (11) \end{aligned}$$

$$\begin{aligned} H^{d1}_{\phi_i}(\rho_i,\phi_i,z) &= \frac{E'_0 k_{d1}}{j\eta_{d1}\lambda_{d1}} G \sum_{n=-\infty}^{\infty} D_{in} J'_n(\lambda_{d1}\rho_i)\, e^{jn(\phi_i-\phi_0)} \\ &\quad - \frac{E'_0 h}{\lambda_{d1}^2\rho_i} G \sum_{n=-\infty}^{\infty} n\, D'_{in} J_n(\lambda_{d1}\rho_i)\, e^{jn(\phi_i-\phi_0)}, \quad (12) \end{aligned}$$

$$\begin{aligned} H^{d2}_{\phi_i}(\rho_i,\phi_i,z) &= \frac{E'_0 k_{d2}}{j\eta_{d2}\lambda_{d2}} G \sum_{n=-\infty}^{\infty} \big[B_{in} J'_n(\lambda_{d2}\rho_i) \\ &\quad + C_{in} N'_n(\lambda_{d2}\rho_i)\big]\, e^{jn(\phi_i-\phi_0)} \\ &\quad - \frac{E'_0 h}{\lambda_{d2}^2\rho_i} G \sum_{n=-\infty}^{\infty} n \big[B'_{in} J_n(\lambda_{d2}\rho_i) \\ &\quad + C'_{in} N_n(\lambda_{d2}\rho_i)\big]\, e^{jn(\phi_i-\phi_0)}, \quad (13) \end{aligned}$$

where $k_d = k_0\sqrt{\mu_{d_r}\varepsilon_{d_r}}$, $\lambda_d = \sqrt{k_d^2 - k_0^2\cos^2\theta_0}$, and $\eta_d = \eta_0\sqrt{\mu_{d_r}/\varepsilon_{d_r}}$.

In the same manner, the ϕ components of the electric field can be derived using Maxwell's equations as follow

$$E_\phi = \frac{1}{j\omega\varepsilon}\left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right),$$

where

$$H_\rho = \frac{-1}{j\omega\mu}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}\right).$$

Thus the ϕ component of the incident electric field on cylinder "i" takes the form

$$E^{inc}_\phi(\rho_i,\phi_i,z) = \frac{-hE'_0}{\lambda_0^2\rho_i} P_i G \sum_{n=-\infty}^{\infty} n j^n\, J_n(\lambda_0\rho_i)\, e^{jn(\phi_i-\phi_0)}, \quad (14)$$

while the ϕ components of the scattered electric field and the electric field transmitted inside the core and coating material of cylinder "i"

can be expressed, respectively, as

$$E^s_{\phi_i}(\rho_i,\phi_i,z) = \frac{-hE'_0}{\lambda_0^2\rho_i}G\sum_{n=-\infty}^{\infty} nA_{in}H_n^{(2)}\left(\lambda_0\rho_i\right)e^{jn(\phi_i-\phi_0)} - \frac{\eta_0 k_0 E'_0}{j\lambda_0}G\sum_{n=-\infty}^{\infty} A'_{in}H_n^{(2)\prime}\left(\lambda_0\rho_i\right)e^{jn(\phi_i-\phi_0)}, \quad (15)$$

$$E^{d1}_{\phi_i}(\rho_i,\phi_i,z) = \frac{-hE'_0}{\rho_i\lambda_{d1}^2}G\sum_{n=-\infty}^{\infty} nD_{in}J_n\left(\lambda_{d1}\rho_i\right)e^{jn(\phi_i-\phi_0)} - \frac{k_{d1}\eta_{d1}E'_0}{j\lambda_{d1}}G\sum_{n=-\infty}^{\infty} D'_{in}J'_n\left(\lambda_{d1}\rho_i\right)e^{jn(\phi_i-\phi_0)}, \quad (16)$$

$$\begin{aligned} E^{d2}_{\phi_i}(\rho_i,\phi_i,z) = &\frac{-hE'_0}{\rho_i\lambda_{d2}^2}G\sum_{n=-\infty}^{\infty} n\left[B_{in}J_n\left(\lambda_{d2}\rho_i\right)\right. \\ &\left.+C_{in}N_n\left(\lambda_{d2}\rho_i\right)\right]e^{jn(\phi_i-\phi_0)} \\ &-\frac{k_{d2}\eta_{d2}E'_0}{j\lambda_{d2}}G\sum_{n=-\infty}^{\infty}\left[B'_{in}J'_n\left(\lambda_{d2}\rho_i\right)\right. \\ &\left.+C'_{in}N'_n\left(\lambda_{d2}\rho_i\right)\right]e^{jn(\phi_i-\phi_0)}. \end{aligned} \quad (17)$$

The expressions of the scattered electric and magnetic field are based on the local coordinates (ρ_i, ϕ_i, z) of cylinder "i". However, the interaction between the M cylinders in terms of multiple scattered fields will require a representation of the scattered field from one cylinder in terms of the local coordinates of another as shown in Fig. 2. Therefore, the addition theorem of Bessel and Hankel functions are used to transfer the scattered field components from one set of coordinates to another. As an example the scattered fields from the "g" cylinder in terms of the "i" cylinder are presented by [6]

$$H_n^{(2)}\left(\lambda_0\rho_g\right)e^{jm\phi_g} = \sum_m J_m\left(\lambda_0\rho_i\right)H^{(2)}_{m-n}\left(\lambda_0 d_{ig}\right)e^{jm\phi_i}e^{-j(m-n)\phi_{ig}} \quad (18)$$

$$d_{ig} = \rho'^2_i + \rho'^2_g - 2\rho'_i\rho'_g\cos\left(\phi'_i - \phi'_g\right),$$

$$\phi_{ig} = \begin{cases} \cos^{-1}\left(\dfrac{\rho'_i\cos\left(\phi'_i\right) - \rho'_g\cos\left(\phi'_g\right)}{d_{ig}}\right) & \rho'_i\sin\left(\phi'_i\right) \geq \rho'_g\sin\left(\phi'_g\right) \\ -\cos^{-1}\left(\dfrac{\rho'_i\cos\left(\phi'_i\right) - \rho'_g\cos\left(\phi'_g\right)}{d_{ig}}\right) & \rho'_i\sin\left(\phi'_i\right) < \rho'_g\sin\left(\phi'_g\right), \end{cases} \quad (19)$$

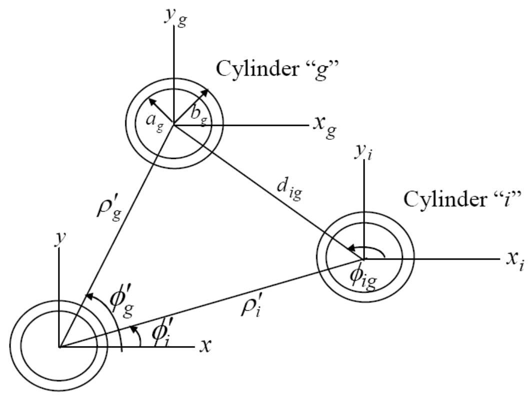


Figure 2. The cross-sectional geometry of the M cylinders in the x-y plane.

where $(\rho'_i,\ \phi'_i,\ z)$ and $(\rho'_g,\ \phi'_g,\ z)$ are the coordinates of the origins of both, the i^{th} and g^{th} coordinate system of cylinder "i" and "g", respectively, in terms of the global coordinate system $(x,\ y)$. The variable d_{ig} represents the distance between the centers of the cylinders, while ϕ_{ig} is the angle between the line joining the centers of the cylinders and the positive x-axis.

The coefficientsA_{in}, B_{in}, C_{in}, D_{in}, A'_{in}, B'_{in}, C'_{in}, and D'_{in} are unknowns to be determined. These unknowns can be obtained by applying the appropriate boundary conditions on the surface of all core cylinders and the coating layered cylinders.

The boundary conditions on the surface of the i^{th} core cylinders with $\rho_i = a_i,\ 0 < \theta < 2\pi$ are given by

$$E_{z_i}^{d1} = E_{z_i}^{d2}, \tag{20}$$
$$H_{z_i}^{d1} = H_{z_i}^{d2}, \tag{21}$$
$$E_{\phi_i}^{d1} = E_{\phi_i}^{d2}, \tag{22}$$
$$H_{\phi_i}^{d1} = H_{\phi_i}^{d2}. \tag{23}$$

Using Equations (20)–(24) and substituting the appropriate field component, the constants D_{in} and D'_{in} can be expressed in terms of B_{in} and B'_{in} as follow

$$B_{in} = D_{in}T_1 + D'_{in}T_2, \tag{24}$$
$$B'_{in} = D_{in}T_3 + D'_{in}T_4, \tag{25}$$

where

$$\begin{aligned}
T_1 &= -\frac{(\alpha_1\alpha_2 + \alpha_4\alpha_5)}{\left(\alpha_2^2 + \alpha_4\alpha_6\right)}, \quad T_2 = \frac{(\alpha_1\alpha_4 - \alpha_2\alpha_3)}{\left(\alpha_2^2 + \alpha_4\alpha_6\right)}, \\
T_3 &= \frac{(\alpha_2\alpha_5 - \alpha_1\alpha_6)}{\left(\alpha_2^2 + \alpha_4\alpha_6\right)}, \qquad T_4 = -\frac{(\alpha_1\alpha_2 + \alpha_3\alpha_6)}{\left(\alpha_2^2 + \alpha_4\alpha_6\right)}, \\
\alpha_1 &= \frac{hn}{a_i}\left[\frac{1}{\lambda_{d2}^2} - \frac{1}{\lambda_{d1}^2}\right] J_n\left(\lambda_{d2}a_i\right), \\
\alpha_2 &= \frac{hn}{a_i}\left[\frac{1}{\lambda_{d2}^2} - \frac{1}{\lambda_{d1}^2}\right] N_n\left(\lambda_{d2}a_i\right), \\
\alpha_3 &= \left(\frac{k_{d2}\eta_{d2}}{j\lambda_{d2}} J_n'\left(\lambda_{d2}a_i\right) - \frac{k_{d1}\eta_{d1}}{j\lambda_{d1}} J_n'\left(\lambda_{d1}a_i\right)\frac{J_n\left(\lambda_{d2}a_i\right)}{J_n\left(\lambda_{d1}a_i\right)}\right), \\
\alpha_4 &= \left(\frac{k_{d2}\eta_{d2}}{j\lambda_{d2}} N_n'\left(\lambda_{d2}a_i\right) - \frac{k_{d1}\eta_{d1}}{j\lambda_{d1}} J_n'\left(\lambda_{d1}a_i\right)\frac{N_n\left(\lambda_{d2}a_i\right)}{J_n\left(\lambda_{d1}a_i\right)}\right), \\
\alpha_5 &= \left(\frac{k_{d2}}{j\eta_{d2}\lambda_{d2}} J_n'\left(\lambda_{d2}a_i\right) - \frac{k_{d1}}{j\eta_{d1}\lambda_{d1}} J_n'\left(\lambda_{d1}a_i\right)\frac{J_n\left(\lambda_{d2}a_i\right)}{J_n\left(\lambda_{d1}a_i\right)}\right), \\
\alpha_6 &= \left(\frac{k_{d2}}{j\eta_{d2}\lambda_{d2}} N_n'\left(\lambda_{d2}a_i\right) - \frac{k_{d1}}{j\eta_{d1}\lambda_{d1}} J_n'\left(\lambda_{d1}a_i\right)\frac{N_n\left(\lambda_{d2}a_i\right)}{J_n\left(\lambda_{d1}a_i\right)}\right).
\end{aligned}$$

The boundary conditions on the surface of the i^{th} coating layers cylinders with $\rho_i = b_i,\ 0 < \theta < 2\pi$ are given by

$$E_{z_i}^{inc} + \sum_{g=1}^{M} E_{z_g}^{s} = E_{z_i}^{d2}, \tag{26}$$

$$H_{z_i}^{inc} + \sum_{g=1}^{M} H_{z_g}^{s} = H_{z_i}^{d2}, \tag{27}$$

$$E_{\phi_i}^{inc} + \sum_{g=1}^{M} E_{\phi_g}^{s} = E_{\phi_i}^{d2}, \tag{28}$$

$$H_{\phi_i}^{inc} + \sum_{g=1}^{M} H_{\phi_g}^{s} = H_{\phi_i}^{d2}. \tag{29}$$

After some mathematical manipulations and the application of the orthogonality condition on equations (26)–(29), the following equations are obtained

$$\begin{aligned}
&P_i j^l J_l(\lambda_0 a_i) e^{-jl(\phi_0)} + A_{il} H_l^{(2)}(\lambda_0 a_i) e^{-jl(\phi_0)} \\
&+ \sum_{\substack{g=1 \\ g\neq 1}}^{M} \sum_n A_{gn} e^{-jn\phi_0} J_l(\lambda_0 a_i) H_{l-n}^{(2)}(\lambda_0 d_{ig}) e^{-j(l-n)\phi_{ig}} \\
= \ &(B_{il} J_l(\lambda_{d1} a_i) + C_{il} N_l(\lambda_{d1} a_i)) e^{-jl(\phi_0)}, \qquad (30)
\end{aligned}$$

$$\begin{aligned}
&A'_{il} H_l^{(2)}(\lambda_0 a_i) e^{-jl(\phi_0)} \\
&+ \sum_{\substack{g=1 \\ g\neq 1}}^{M} \sum_n A'_{gn} e^{-jn\phi_0} J_l(\lambda_0 a_i) H_{l-n}^{(2)}(\lambda_0 d_{ig}) e^{-j(l-n)\phi_{ig}} \\
= \ &(B'_{il} J_l(\lambda_{d1} a_i) + C'_{il} N_l(\lambda_{d1} a_i)) e^{-jl(\phi_0)}, \qquad (31)
\end{aligned}$$

$$\begin{aligned}
&\frac{hl}{\lambda_0^2 a_i} J_l(\lambda_0 a_i) P_i j^l + \frac{hl}{\lambda_0^2 a_i} H_l^{(2)}(\lambda_0 a_i) A_{il} + \frac{k_0 \eta_0}{j\lambda_0} H_l^{(2)'}(\lambda_0 a_i) A'_{il} \\
&+ \frac{hl}{\lambda_0^2 a_i} J_l(\lambda_0 a_i) \sum_{\substack{g=1 \\ g\neq 1}} \sum_n A_{gn} H_{l-n}^{(2)}(\lambda_0 d_{ig}) e^{-j(l-n)(\phi_{ig}-\phi_0)} \\
&+ \frac{k_0 \eta_0}{j\lambda_0} J'_l(\lambda_0 a_i) \sum_{\substack{g=1 \\ g\neq 1}} \sum_n A'_{gn} H_{l-n}^{(2)}(\lambda_0 d_{ig}) e^{-j(l-n)(\phi_{ig}-\phi_0)} \\
= \ &\frac{hl}{a_i \lambda_{d1}^2} [B_{il} J_l(\lambda_{d1} a_i) + C_{il} N_l(\lambda_{d1} a_i)] \\
&+ \frac{k_{d1} \eta_{d1}}{j\lambda_{d1}} [B'_{il} J'_l(\lambda_{d1} a_i) + C'_{il} N'_l(\lambda_{d1} a_i)], \qquad (32)
\end{aligned}$$

$$\begin{aligned}
&\frac{k_0}{j\eta_0 \lambda_0} J'_l(\lambda_0 a_i) P_i j^l + \frac{k_0}{j\eta_0 \lambda_0} H_l^{(2)'}(\lambda_0 a_i) A_{il} - \frac{hl}{\lambda_0^2 a_i} H_l^{(2)}(\lambda_0 a_i) A'_{il} \\
&+ \frac{k_0}{j\eta_0 \lambda_0} J'_l(\lambda_0 a_i) \sum_{\substack{g=1 \\ g\neq 1}} \sum_n A_{gn} H_{l-n}^{(2)}(\lambda_0 d_{ig}) e^{-j(l-n)(\phi_{ig}-\phi_0)} \\
&- \frac{hl}{\lambda_0^2 a_i} J_l(\lambda_0 a_i) \sum_{\substack{g=1 \\ g\neq 1}} \sum_n A'_{gn} H_{l-n}^{(2)}(\lambda_0 d_{ig}) e^{-j(l-n)(\phi_{ig}-\phi_0)} \\
= \ &\frac{k_{d1}}{j\eta_{d1} \lambda_{d1}} [B_{il} J'_l(\lambda_{d1} a_i) + C_{il} N'_l(\lambda_{d1} a_i)] \\
&- \frac{hl}{\lambda_{d1}^2 a_i} [B'_{il} J_l(\lambda_{d1} a_i) + C'_{il} N_l(\lambda_{d1} a_i)]. \qquad (33)
\end{aligned}$$

The solution for the unknown coefficients A_{in} and A'_{in} can be obtained from equations (30)–(33) and put in the form

$$V_1 = \sum_{g=1}^{M} \sum_{n=-\infty}^{\infty} A_{gn} S_1 + A'_{gn} R_1, \tag{34}$$

$$V_2 = \sum_{g=1}^{M} \sum_{n=-\infty}^{\infty} A_{gn} S_2 + A'_{gn} R_2, \tag{35}$$

where

$$V_1 = \left(\gamma_{12}\gamma_5 + \gamma_{11}\gamma_{10} - \frac{hl}{\lambda_0^2 a_i} J_l(\lambda_0 b_i) \right) P_i j, \tag{36}$$

$$V_2 = \left(\gamma_{14}\gamma_5 + \gamma_{13}\gamma_{10} - \frac{k_0}{j\eta_0\lambda_0} J'_l(\lambda_0 b_i) \right) P_i j^l, \tag{37}$$

$$\begin{aligned} S_1 &= 0 \qquad i = g,\ l \neq n \\ &= \frac{hl}{\lambda_0^2 a_i} H_l^{(2)}(\lambda_0 b_i) - \gamma_{12}\gamma_3 - \gamma_{11}\gamma_8 \qquad i = g,\ l = n \\ &= \left[\frac{hl}{\lambda_0^2 a_i} J_l(\lambda_0 b_i) - \gamma_{12}\gamma_4 - \gamma_{11}\gamma_9 \right] H_{ln} \qquad i \neq g, \end{aligned} \tag{38}$$

$$\begin{aligned} S_2 &= 0 \qquad i = g,\ l \neq n \\ &= \frac{k_0}{j\eta_0\lambda_0} H_l^{(2)'}(\lambda_0 b_i) - \gamma_{14}\gamma_3 - \gamma_{13}\gamma_8 \qquad i = g,\ l = n \\ &= \left[\frac{k_0}{j\eta_0\lambda_0} J'_l(\lambda_0 b_i) - \gamma_{14}\gamma_4 - \gamma_{13}\gamma_9 \right] H_{ln} \qquad i \neq g, \end{aligned} \tag{39}$$

$$\begin{aligned} R_1 &= 0 \qquad i = g,\ l \neq n \\ &= \frac{k_0\eta_0}{j\lambda_0} H_l^{(2)'}(\lambda_0 b_i) - \gamma_{12}\gamma_1 - \gamma_{11}\gamma_6 \qquad i = g,\ l = n \\ &= \left[\frac{k_0\eta_0}{j\lambda_0} J'_l(\lambda_0 b_i) - \gamma_{12}\gamma_2 - \gamma_{11}\gamma_7 \right] H_{ln} \qquad i \neq g, \end{aligned} \tag{40}$$

$$\begin{aligned} R_2 &= 0 \qquad i = g,\ l \neq n \\ &= -\left(\frac{hl}{\lambda_0^2 a_i} H_l^{(2)}(\lambda_0 b_i) + \gamma_{14}\gamma_1 + \gamma_{13}\gamma_6 \right) \qquad i = g,\ l = n \\ &= -\left[\frac{hl}{\lambda_0^2 a_i} J_l(\lambda_0 b_i) + \gamma_{14}\gamma_2 + \gamma_{13}\gamma_7 \right] H_{ln} \qquad i \neq g, \end{aligned} \tag{41}$$

while

$$
\begin{aligned}
\gamma_1 &= \frac{H_l^{(2)}\left(\lambda_0 b_i\right)}{T_5}, \ \gamma_2 = \frac{J_l\left(\lambda_0 b_i\right)}{T_5}, \\
\gamma_3 &= -\frac{H_l^{(2)}\left(\lambda_0 b_i\right) T_3 N_l\left(\lambda_{d2} b_i\right)}{T_5\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)}, \\
\gamma_4 &= -\frac{J_l\left(\lambda_0 b_i\right) T_3 N_l\left(\lambda_{d2} b_i\right)}{T_5\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)}, \ \gamma_5 = \gamma_4, \\
T_5 &= J_l\left(\lambda_{d2} b_i\right)+T_4 N_l\left(\lambda_{d2} b_i\right)-\frac{T_2 T_3 N_l\left(\lambda_{d2} b_i\right) N_l\left(\lambda_{d2} b_i\right)}{\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)}, \\
\gamma_6 &= -T_6 \gamma_1, \quad \gamma_7 = -T_6 \gamma_2, \\
\gamma_8 &= -T_6 \gamma_3+\frac{H_l^{(2)}\left(\lambda_0 b_i\right)}{\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)}, \\
\gamma_9 &= -T_6 \gamma_4+\frac{J_l\left(\lambda_0 b_i\right)}{\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)}, \quad \gamma_{10} = \gamma_9, \\
T_6 &= \left(\frac{T_2 N_l\left(\lambda_{d2} b_i\right)}{\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)}\right), \\
\gamma_{11} &= \left(\frac{hl}{b_i \lambda_{d2}^2}\left(J_l\left(\lambda_{d2} b_i\right)+T_1 N_l\left(\lambda_{d2} b_i\right)\right)+\frac{k_{d2} \eta_{d2}}{j \lambda_{d2}} T_3 N_l'\left(\lambda_{d2} b_i\right)\right), \\
\gamma_{12} &= \left(\frac{hl}{b_i \lambda_{d2}^2} T_2 N_l\left(\lambda_{d2} b_i\right)+\frac{k_{d2} \eta_{d2}}{j \lambda_{d2}}\left(J_l'\left(\lambda_{d2} b_i\right)+T_4 N_l'\left(\lambda_{d2} b_i\right)\right)\right), \\
\gamma_{13} &= \left(\frac{k_{d2}}{j \eta_{d2} \lambda_{d2}}\left(J_l'\left(\lambda_{d2} b_i\right)+T_1 N_l'\left(\lambda_{d2} b_i\right)\right)-T_3 \frac{hl}{\lambda_{d2}^2 b_i} N_l\left(\lambda_{d2} b_i\right)\right), \\
\gamma_{14} &= \left(\frac{k_{d2}}{j \eta_{d2} \lambda_{d2}} T_2 N_l'\left(\lambda_{d2} b_i\right)-\frac{hl}{\lambda_{d2}^2 b_i}\left(J_l\left(\lambda_{d2} b_i\right)+T_4 N_l\left(\lambda_{d2} b_i\right)\right)\right), \\
& \text{and} \\
H_{ln} &= H_{l-n}^{(2)}\left(\lambda_0 d_{ig}\right) e^{-j(l-n)\left(\phi_{ig}-\phi_0\right)}.
\end{aligned}
$$

The integers $n,\ l = 0,\ \pm 1,\ \pm 2,\ \ldots\ \ldots,\ \pm N_i$ and $i, g = 0, 1, 2, \ldots\ldots, M$. Theoretically, N_i is an integer which is equal to infinity; however, it is related to the radius "a_i" of cylinder "i", type of the i^{th} cylinder, and also the distance between the i^{th} cylinder and the surrounding cylinders. Equations (36) and (37) are then cast into a matrix form such as

$$
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} S_1 & R_1 \\ S_2 & R_2 \end{bmatrix} \begin{bmatrix} A \\ A' \end{bmatrix}. \tag{42}
$$

The solution of the above truncated matrix equation yields the unknown scattering coefficients A_{in} and A'_{in}. Hence, the other remaining unknown coefficients can be otained from euqations (30)–(33).

3. NUMERICAL RESULTS

In this section, sample numerical results are presented to proof the validity of the developed formulation for computing the radar cross-section (RCS) of an array of coated cylinders excited by an obliquely incident TM_z plane wave. For all configurations presented in this paper, the frequency of the incident wave was set to 300 MHz, and RCS (alternatively called echo width for 2D scatterers) is defined as

$$\sigma_{2D} = 10 \log \left(\underset{\rho \to \infty}{Lim} \left[2\pi\rho \frac{|E_z^s|^2}{|E_z^i|^2} \right] \right). \tag{43}$$

Figure 3 represents the echo width calculated from a single dielectric cylinder with air core. The inner radius of the cylinder is $a = 0.25\lambda$ and the outer radius is $b = 0.3\lambda$, having relative permittivity $\varepsilon_r = 4$, with incident angel $\theta_i = 90$ and $\phi_i = 180$. The result generated using the presented boundary value solution (BVS)

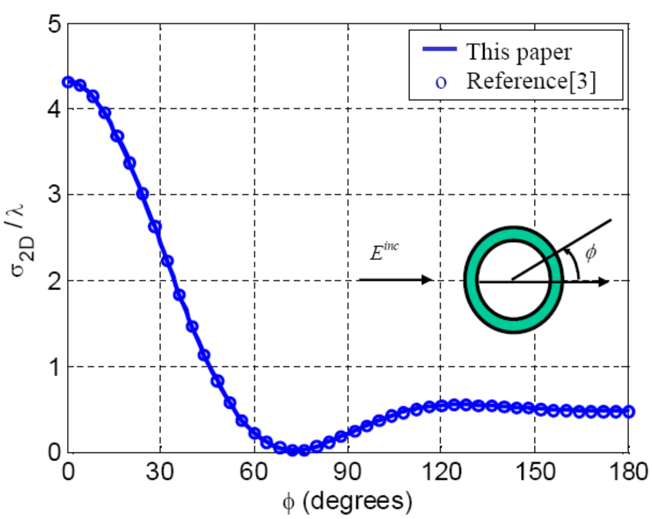


Figure 3. The echo width of a single cylindrical shell of inner radius $a = 0.25\lambda$, outer radius $b = 0.3\lambda$, $\varepsilon_r = 4$, $\theta_i = 90$, and $\phi_i = 180$.

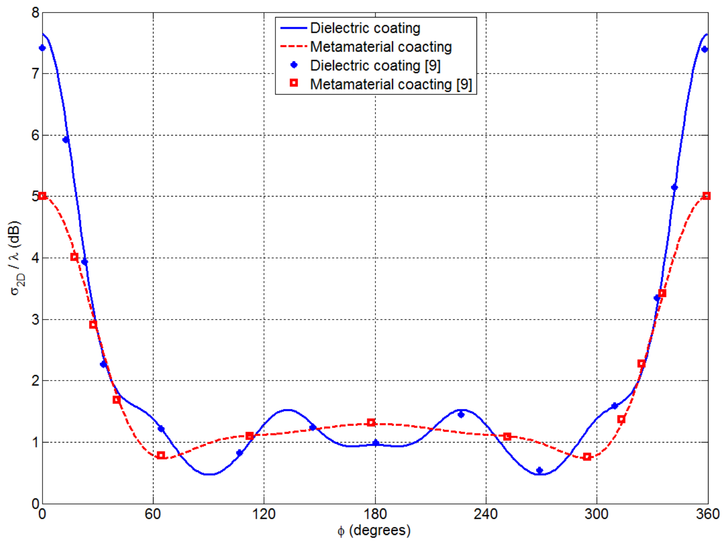
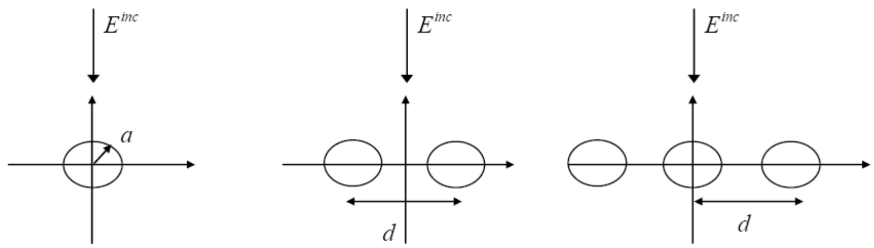


Figure 4. The echo width of a single conducting cylinder of radius $a = 0.05\lambda$ coated with either dielectric or metamaterial of outer radius $b = 0.1\lambda$.

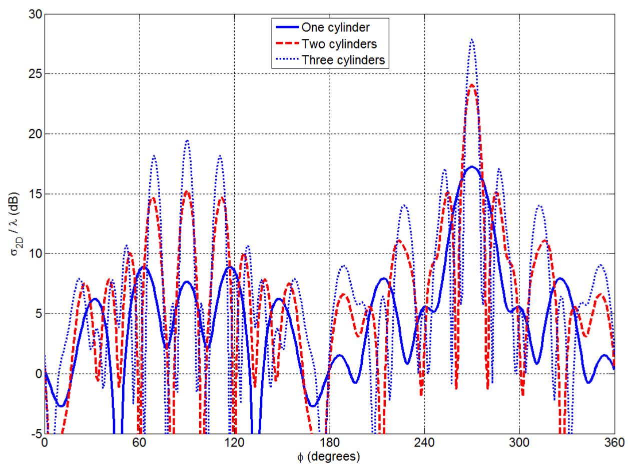
technique is compared to the RCS data based on the method of moment solution presented in [3]. It is clear from the figure that the two solutions are in complete agreement.

The metamaterial parameters used in this paper are selectively used to provide confirmation of the validity of the new formulation by comparison of special cases with published results. Figure 4 represents the RCS calculated from a single coated conducting cylinder, the coating layer was made of a dielectric material having relative permittivity of $\varepsilon_r = 9.8$ or a metamaterial having relative permittivity $\varepsilon_r = -9.8$ and relative permeability $\mu_r = -1$. For comparison, The incident wave frequency is set to 1 GHz, the radius of the conducting cylinder is $a = 0.05\lambda$, the outer radius of the coating cylinder is $b = 0.1\lambda$, and excited by a TM_z plane wave with incident angel $\theta_i = 90$ and $\phi_i = 0$. The results show a complete agreement with the independently reported results in [13].

In order to prove the validity of the presented formulation for multiple cylinders, the scattering echo width of one and two and three metamaterial cylinders in a linear array configuration located along the x-axis is calculated. The metamaterial cylinders have relative permittivity $\varepsilon_r = -4$ and relative permeability $\mu_r = -1$. The radius of the cylinders is $a = 1\lambda$ and the center-to-center separation is $d = 3\lambda$,



(a)



(b)

Figure 5. The echo width of an array of one, two, or three metamaterial cylinders of radius $a = 1\lambda$, (a) the arrangements of the cylinders, (b) The corresponding echo width.

and excited by a TM_z plane wave with incident angel $\theta_i = 90$ and $\phi_i = 90$. The results generated using the developed formulation as shown in Figure 5, show a complete agreement with those given in Figure 10 of [14].

Numerical solution shows that $N_i = 1 + 2k_i a_i$ is sufficient to achieve a converged solution for the scattered field in the case of dielectric cylinders. For DNG metamaterial cylinders, this expression for the number of terms is not enough to achieve convergence and more terms are needed. The distance between the cylinders also greatly affect the needed number of terms for convergence in the case of metamaterial cylinders. Results show that decreasing the distance between the two metamaterial cylinders increases the needed number of terms for convergence. For metamaterial cylinders, the relation $N_i = 1 + 2k_i a_i$ is found to be valid for each cylinder in the case of center-to-center separation three times the radius of each cylinder.

Figure 6 shows the echo width of an arbitrary positioned array of five cylinders. The radius of the cylinders is 0.1λ, and the center-to-center separation is 0.5λ. The graph shows the results for two different cases; dielectric cylinders with $\varepsilon_r = 2.2$, and metamaterial cylinders with $\varepsilon_r = -2.2$ and $\mu_r = -1$. For the two cases the results are in complete agreement with the results published in [14].

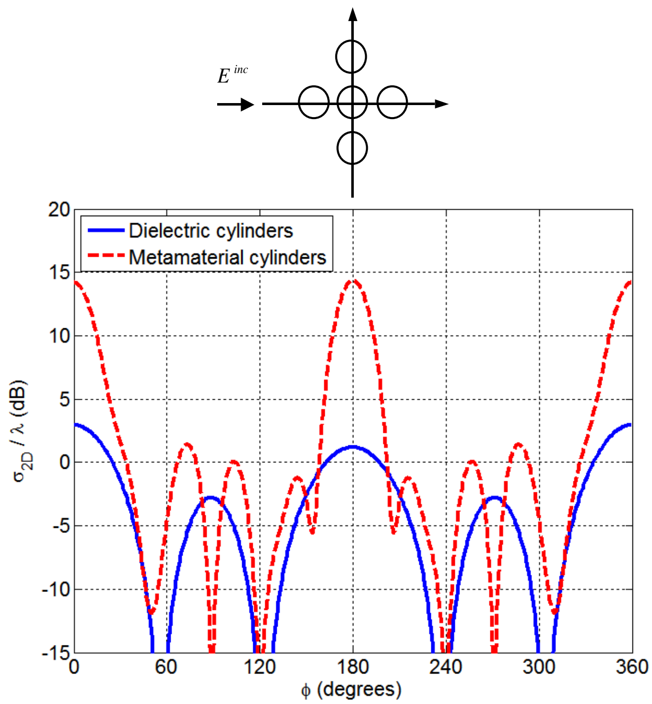


Figure 6. The echo width results of a TM_z plane wave incident on an arbitrary positioned array of five cylinders.

As another method to check the validity of this technique and its accuracy, especially in the near field region, the value of the coefficients of the transmitted field inside both the core cylinders and the coating layers are calculated. Then, the numerical values of these coefficients are used to check the validity of boundary equations from Equation (20) to Equation (23). For simplicity, three cylinders are used. The radius of the core cylinders is 0.1λ, and the radius of the coating layers is 0.2λ.

The cylinders are placed symmetrically around the x-axis with the center of all cylinders located on the y-axis, and the distance between the centers is 0.7λ. Figure 7 shows the numerical value of the transmitted field inside the coating layer of the first cylinder compared with the summation of the incident field and the scattered fields form all cylinders on the outer surface of this cylinder. In Figure 7(b) the continuity of the E_z component is considered in the case of TM_z excitation for dielectric cylinders with $\varepsilon_r = 4$ at an oblique incident angle of $\theta_i = 30$ and $\phi_i = 0$. In Figure 7(c) the continuity of the E_z component is considered in the case of TM_z excitation for metamaterial cylinders with $\varepsilon_r = -4$ and $\mu_r = -1$ at an oblique incident angle of $\theta_i = 30$ and $\phi_i = 0$.

Figure 8 shows the near field distribution resulting from the incidence of a TM_z polarized plane wave on the array of three cylinders shown in Figure 7(a). The graph shows the numerical value of the transmitted field inside the core and coating layer cylinders and the total field outside the cylinders for three different cases; dielectric cylinders with $\varepsilon_r = 4$, metamaterial cylinders with $\varepsilon_r = -4$ and $\mu_r = -1$, and conductor cylinders. All the three cases are excited by an obliquely incident plane wave at angle of $\theta_i = 30$ and $\phi_i = 0$. The graphs show a greet enhancement of the field on the edges of the metamaterial cylinders and fast decay everywhere else.

Finally, to show the effect of metamaterial in forward and backward scattering, Figure 9 shows the near field distribution resulting from the incidence of a TM_z polarized plane wave on an array of five cylinders. The cylinders are placed symmetrically around the x-axis with the center of all cylinders located on the y-axis. The radius of each cylinder is 0.1λ and the distance between the centers is 0.5λ and excited by an incident plane wave at $\theta_i = 90$ and $\phi_i = 180$. Figure 7(a) shows the near field distribution of an array of conducting cylinders while Figure 7(b) shows that of metamaterial cylinders having relative permittivity $\varepsilon_r = -10$ and relative permeability $\mu_r = -1$. A higher isolation in the forward direction was noticed in the case of metamaterial cylinders. Figure 7(c) shows the near field distribution of an array of conducting cylinders with the same radius coated with metamaterial layer of thickness 0.05λ. The relative permittivity of the cylinders change between $\varepsilon_r = -2$ for the center cylinder, $\varepsilon_r = -10$ for the upper and lower cylinders, and $\varepsilon_r = -6$ for the other two cylinders. The result shows a greet focusing of the field in the forward direction.

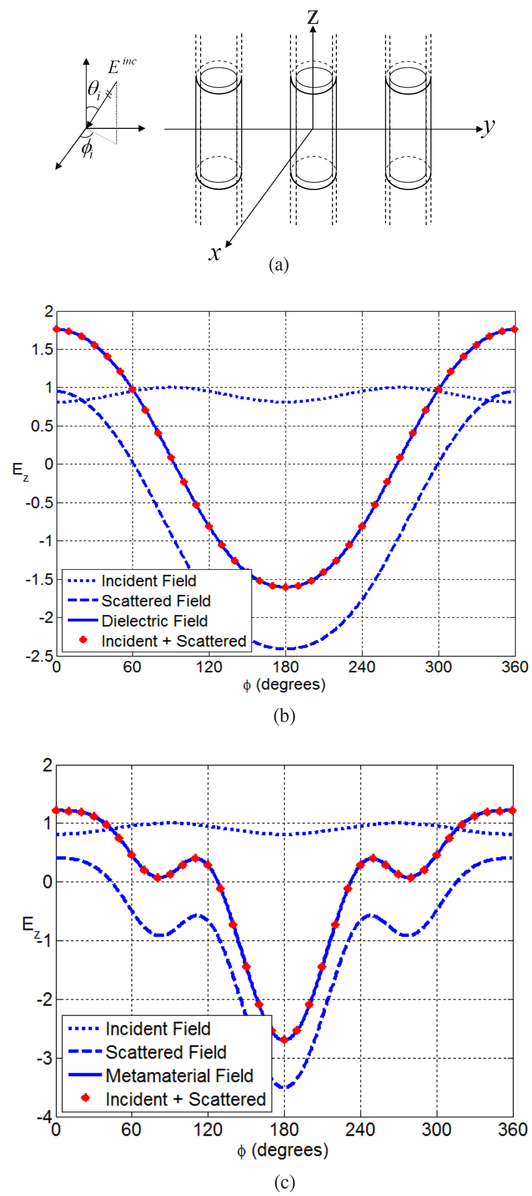
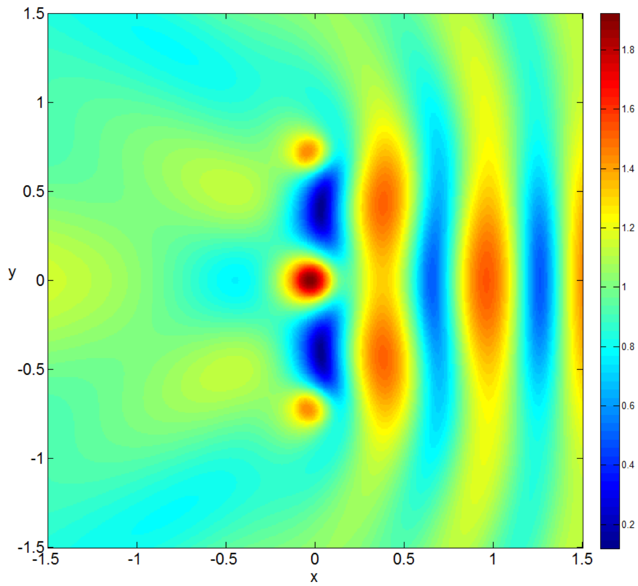
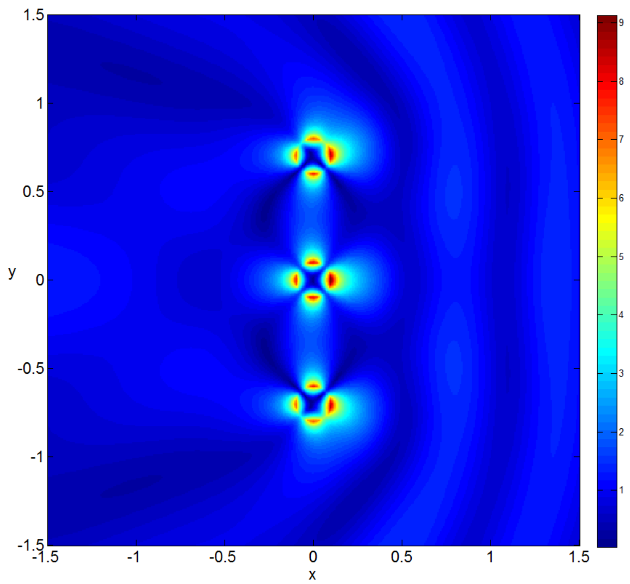


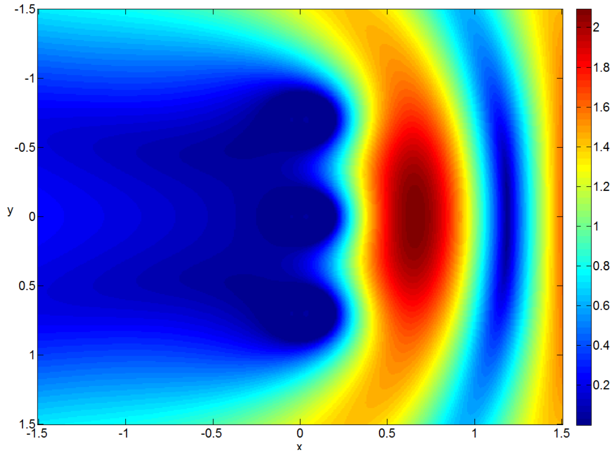
Figure 7. The continuity of the near field components of three parallel coated cylinders excited by an incident plane wave at $\theta_i = 30$, $\phi_i = 0$. (a) The arrangement of the cylinders, (b) Continuity of the E_z for TM_z excitation for dielectric cylinders with $\varepsilon_r = 4$, (c) Continuity of the E_z for TM_z excitation for metamaterial cylinders with $\varepsilon_r = -4$ and $\mu_r = -1$.


1.5
1
0.5
y 0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
x
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2

(a)

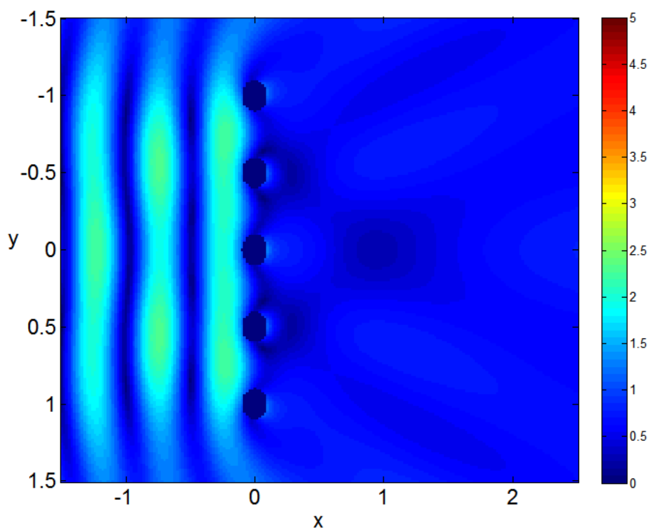

1.5
1
0.5
y 0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
1.5
x
9
8
7
6
5
4
3
2
1

(b)

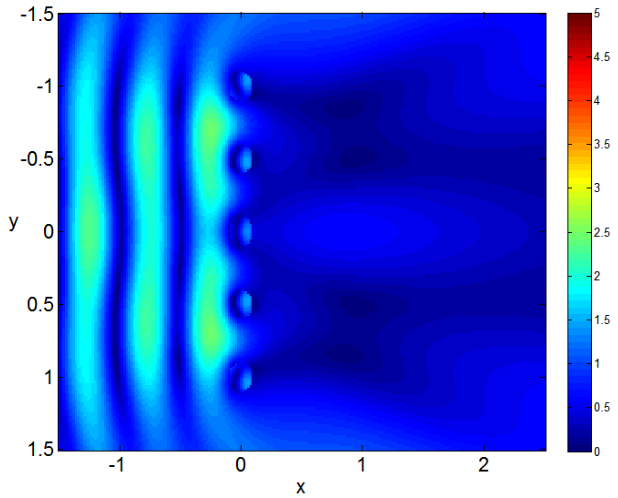


(c)

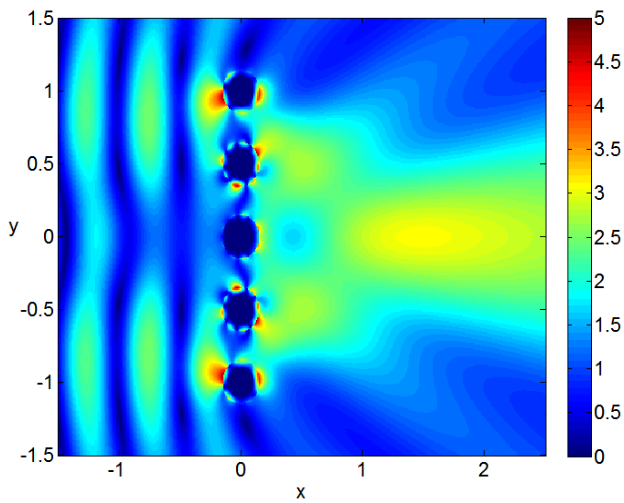
Figure 8. The near field distribution of three parallel coated cylinders excited by an incident plane wave at $\theta_i = 30$ and $\phi_i = 0$. (a) Dielectric cylinders with $\varepsilon_r = 4$, (b) Metamaterial cylinders with $\varepsilon_r = -4$ and $\mu_r = -1$, (c) conductor cylinders.



(a)



(b)



(c)

Figure 9. The near field distribution of an array of five cylinders excited by an incident plane wave at $\theta_i = 90$ and $\phi_i = 180$. (a) conducting cylinders, (b) metamaterial cylinders, and (c) conducting cylinders coated with metamaterial.

4. CONCLUSION

The analyses of an obliquely incident plane wave scattering from an array of parallel-coated circular cross-section cylinders is derived for TMz polarizations. The derivation is based on the application of the boundary conditions on the surface of each core and coated layer cylinder. This solution is verified for metamaterial, dielectric and conductor cylinders, and can be used to study electromagnetic interaction with two-dimensional scattering object that can be constructed from an array of parallel circular cylinders. The effect of metamaterial in enhancing the forward or backward scattering from an array of cylinders was studied, and the effect of isolating or focusing the field in front of the cylinders is also demonstrated. This verified frequency domain technique is thus useful in studying the scattering from practical configurations of cylinders with depressive type material. This verified frequency domain technique is thus useful in studying the scattering from practical configurations of cylinders with depressive type material.

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