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IDENTIFICATION OF INHOMOGENEOUS OR MULTILAYER DIELECTRIC WALLS

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Abstract—A microwave technique is proposed to measure the permittivity function of inhomogeneous dielectric walls. The measured reflection or transmission coefficients are used to extract the permittivity function of the dielectric walls. To solve the problem an optimization-based procedure is used. The usefulness of the proposed method is verified using some examples.

1. INTRODUCTION

Microwave nondestructive evaluation (NDE) is a widely used method to test the internal integrity of many materials such as concrete (used in bridges, buildings, dams, tunnels, etc.), asphalt (used in pavements), composite materials, plastics, epoxy, and rubber (used in seals) [1]. Microwaves, in contrast to ultrasound, couple very well into these types of materials, and do not require contacting transducers or coupling media. Microwave NDE techniques, when combined with inverse scattering imaging methods, can potentially generate higher resolution images with deeper penetration than the thermal and eddy-current imaging techniques [1]. Many microwave techniques have been presented to measure the dielectric constant of dielectrics. The lumped circuit techniques [2], the cavity perturbation technique [3–5], the free-space techniques [6] and the transmission line techniques [7–9] are the most important techniques. However these techniques are presented only for homogeneous dielectrics. In this paper we generalize the transmission line techniques to measure the permittivity function of inhomogeneous dielectric walls. The measured reflection or transmission coefficients are used to extract the permittivity function of the dielectric wall. In fact, we have to solve a one-dimensional inverse scattering problem [10]. To solve this problem, the electric

permittivity function is expanded in a truncated Fourier series, first. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method is verified using a comprehensive example.

2. ANALYSIS

In this section the frequency domain analysis of the structure (Direct Problem) is reviewed. Fig. 1(a) shows the structure, calling it Inhomogeneous Dielectric Wall (IDW), which we like to measure its non-constant permittivity function. The electric permittivity function is $\varepsilon_r(z)$ and the thickness is d. Also, Fig. 1(b) shows a special case of IDWs, multi-layer dielectric walls, which has more applications than the general cases. The differential equations describing IDWs have non-constant coefficients and so except for a few special cases no closed form analytic solution exists for them. There are some methods to analyze IDWs such as finite difference [11], Taylor's series expansion [12], Fourier series expansion [13], the method of Moments [14] and the equivalent sources method [15]. Of course, the most straightforward method is subdividing IDWs into K homogeneous electrically thin layers with thickness

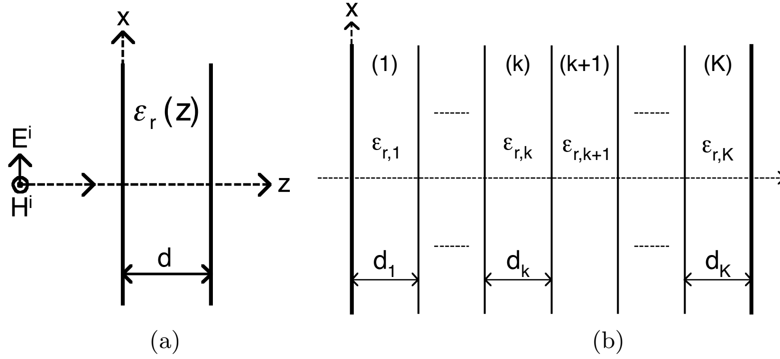

Figure 1. (a) A typical Inhomogeneous Dielectric Wall (IDW) (b) A typical multilayer dielectric wall (a special case of IDWs).

$$\Delta z = d/K \ll \lambda_{\min} \cong \frac{c}{f_{\max}\sqrt{\max(\varepsilon_r(z))}} \tag{1}$$

in which c is the velocity of the light and $f_{\max}$ is the maximum frequency of the analysis. The $ABCD$ parameters of the IDW is

obtained from those of its thin layers as follows

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{2}$$

where the $ABCD$ parameters of the k-th thin layer are as follows

$$A_k = D_k = \cos(\Delta\theta_k) \tag{3}$$

$$B_k = Z_c^2((k-0.5)\Delta z)C_k = jZ_c((k-0.5)\Delta z)\sin(\Delta\theta_k) \tag{4}$$

In (3) and (4),

$$\Delta\theta_k = \frac{2\pi f}{c}\sqrt{\varepsilon_r((k-0.5)\Delta z)} \tag{5}$$

is the electrical thickness of the k-th thin layer. Also, $Z_c(z)$ is the characteristic impedance of the IDW, defined as the ratio of the transverse electric field to the transverse magnetic field, given by

$$Z_c(z) = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\sqrt{\varepsilon_r(z)}} \tag{6}$$

Finally, the reflection and transmission coefficients of the structure can be determined from its $ABCD$ parameters as follows

$$\Gamma(f) = \frac{(A - C\eta_0)Z_L + (B - D\eta_0)}{(A + C\eta_0)Z_L + (B + D\eta_0)} \tag{7}$$

$$T(f) = \frac{2Z_L}{(A + C\eta_0)Z_L + (B + D\eta_0)} \tag{8}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the equivalent source impedance of the left side and also Z_L defined as the following is the equivalent source impedance of the right side.

$$Z_L = \begin{cases} \eta_0 & \text{right side is matched} \\ 0 & \text{right side is short circuited} \end{cases} \tag{9}$$

3. IDENTIFICATION

In this section a general method is proposed to identify the electric permittivity function of the IDWs (Inverse Problem). First, we consider the following truncated Fourier series expansion for the electric permittivity function.

$$\ln(\varepsilon_r(z) - 1) = \sum_{n=0}^{N} C_n \cos(n\pi z/d) \tag{10}$$

Using the expansion (10) we need to determine only $N+1$ coefficients instead of many, K, parameters used in the direct problem. The values of the unknown coefficients C_n can be obtained through minimizing one of the following error functions corresponding to M measuring frequencies $f_1 < f_2 < \ldots < f_M$.

$$E_1 = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left|\Gamma_{SC,sim}(f_m) - \Gamma_{SC,meas}(f_m)\right|^2} \tag{11}$$

$$E_2 = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left|\Gamma_{m,sim}(f_m) - \Gamma_{m,meas}(f_m)\right|^2} \tag{12}$$

$$E_3 = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\left|T_{sim}(f_m) - T_{meas}(f_m)\right|^2} \tag{13}$$

$$E_4 = \sqrt{\frac{1}{2M}\sum_{m=1}^{M}\left(\left|\Gamma_{m,sim}(f_m) - \Gamma_{m,meas}(f_m)\right|^2 + \left|T_{sim}(f_m) - T_{meas}(f_m)\right|^2\right)} \tag{14}$$

The parameters Γ_{SC} and Γ_m in (11)–(14) are the input reflection coefficient when the right side is short-circuited or matched, respectively. Also, the indices "*sim*" and "*meas*" in (11)–(14) represent the simulation (analysis) and measuring ways, respectively, to determine the scattering parameters. In fact, the measuring data in the error functions E_1, E_2, E_3 and E_4 are the short-circuited reflection, the matched-end reflection, the transmission and the combination of the matched-end reflection and the transmission, respectively. It is noticeable that if the simulation methods use some approximations or the measuring data are not fully accurate, the inverse problems may be diverged because they are ill-posed. So, to stabilize the inverse problems various kinds of regularizations are usually used [10]. To utilize the regularization methods for our problem, a suitable fraction of the energy of the permittivity function or its derivatives should be added to the defined error functions E_1, E_2, E_3 and E_4.

Finally, to investigate the similarity of the identified permittivity function to the exact one, one may define the following identification error function.

$$E_\varepsilon = \sqrt{\frac{1}{K+1}\sum_{k=0}^{K}\left(\frac{\varepsilon_{r,iden}(kd/K) - \varepsilon_{r,exact}(kd/K)}{\varepsilon_{r,exact}(kd/K)}\right)^2} \tag{15}$$

where the index "*iden*" is the abbreviation of the identification.

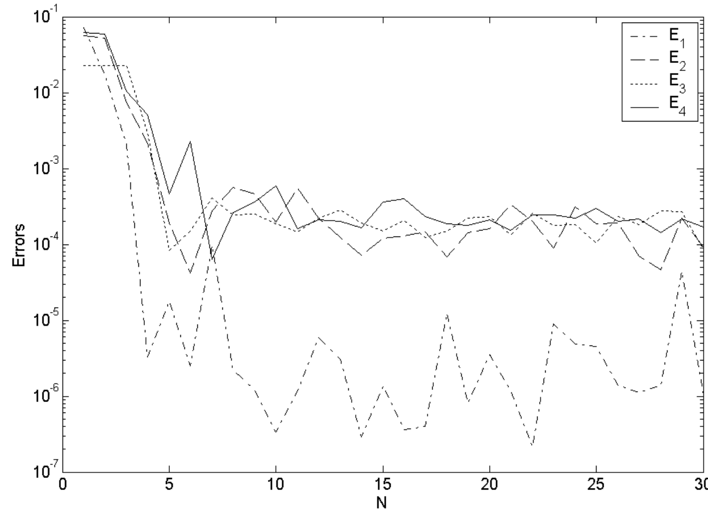

Figure 2. The error functions E_1, E_2, E_3 and E_4 with respect to the number of expansion terms N considering $M = 5$ measuring frequencies.

4. EXAMPLE AND RESULTS

We would like to identify a permittivity function with exponential variation, $\exp(z/d)$, using M frequencies with equal distance in the range of 8.0 to 12.0 GHz (X-Band) assuming $d = 2.0$ cm. Using the proposed identification method, considering $M = 5$ measuring frequencies, the five defined error functions were obtained versus the number of expansion terms N. Figure 2 shows E_1, E_2, E_3 and E_4 and Figure 3 compares E_ε obtained from E_1, E_2, E_3 and E_4. It is observed from these two figures that as N increases each error function tends to a specified value. Meanwhile, choosing N between 3 and 7 is more suitable than the larger values. Also, it is seen that the identification error E_ε may be sorted from high to low in the cases of using error functions E_3, E_1, E_2 and E_4. So, the worst and the best identifications are achieved for the cases of using E_3 and E_4, respectively. Figures 4–7 illustrate the identified permittivity function $\varepsilon_r(z)$ for some M assuming $N = 5$ using the error functions E_1, E_2, E_3 and E_4, respectively. One sees from these figures that for a fixed N, as the number of measuring frequencies M is increased the identified permittivity function tends to the exact one. It is seen from Figs. 3 and 6 that for some combination of M and N (such as $M = 5$ and

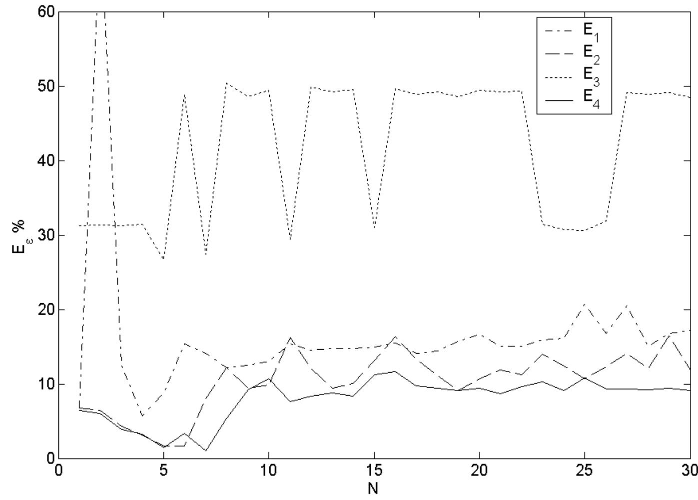

Figure 3. The identified error function E_ε with respect to the number of expansion terms N considering $M = 5$ measuring frequencies.

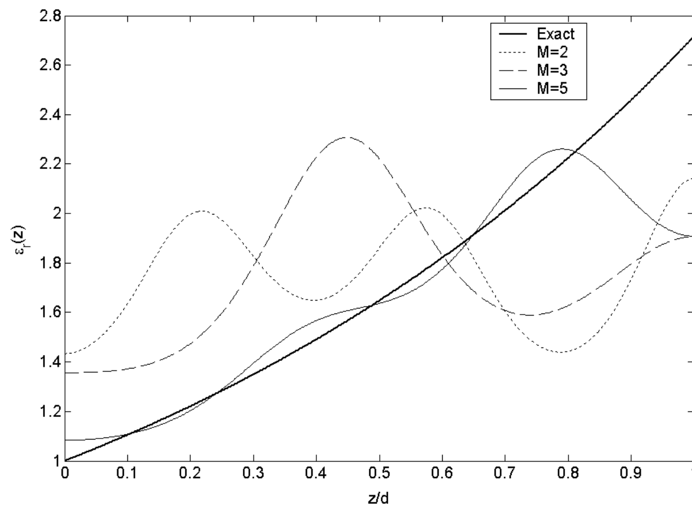

Figure 4. The identified permittivity function $\varepsilon_r(z)$ for some M assuming $N = 5$ using the error function E_1.

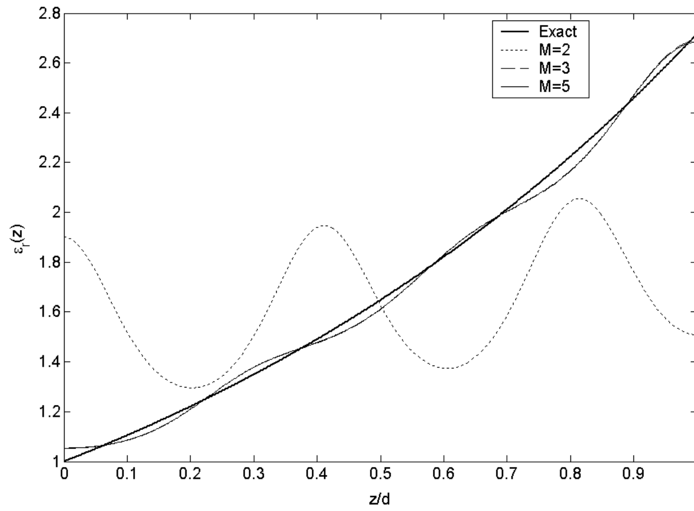

Figure 5. The identified permittivity function $\varepsilon_r(z)$ for some M assuming $N = 5$ using the error function E_2.

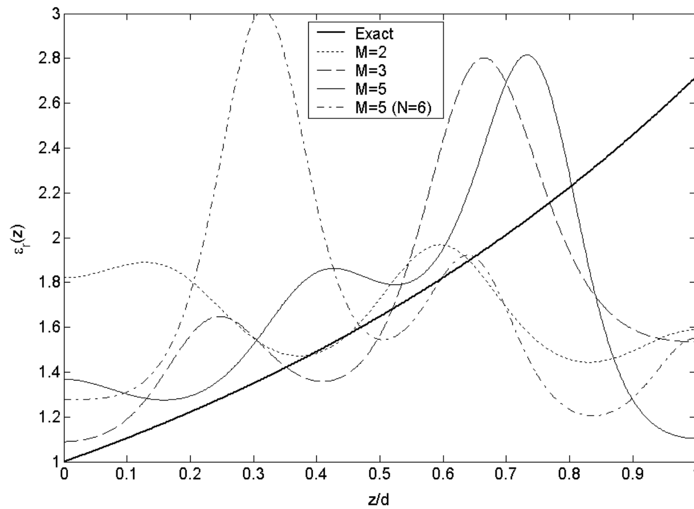

Figure 6. The identified permittivity function $\varepsilon_r(z)$ for some M assuming $N = 5$ using the error function E_3.

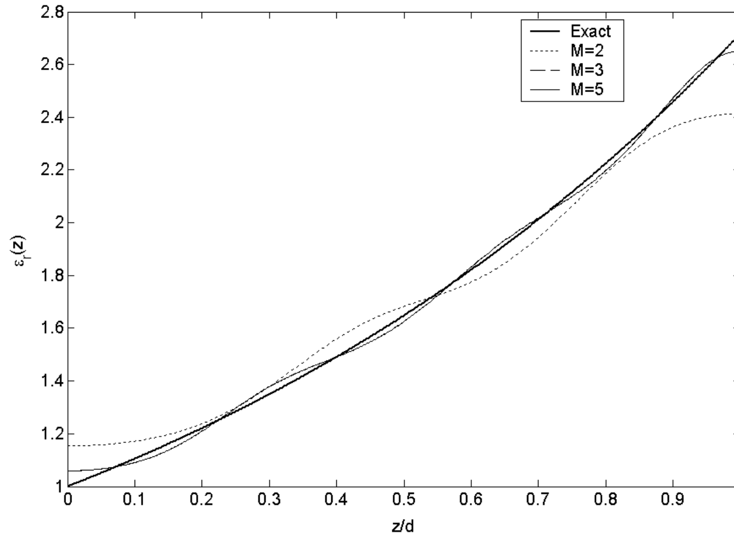

Figure 7. The identified permittivity function $\varepsilon_r(z)$ for some M assuming $N = 5$ using the error function E_4.

$N = 6$) the reverse of the permittivity function may be identified if we use the error function E_3. This is due to the reciprocity principle i.e. $S_{12} = S_{21}$. So, the case of using only transmission data T has an intrinsic reversing ambiguity. The unknown coefficients of the truncated Fourier series of the identified permittivity function considering $M = 5$ measuring frequencies and $N = 5$ expansion terms have been written in Table 1.

Table 1. The unknown coefficients of the truncated Fourier series of the identified permittivity function considering $M = 5$ and $N = 5$.

	C_0	C_1	C_2	C_3	C_4	C_5
E_1	-0.6941	-1.1051	-0.4185	-0.1502	-0.1752	0.0637
E_2	-0.7052	-1.3121	-0.3544	-0.2968	-0.1389	-0.1108
E_3	-0.5674	-0.2357	-0.7045	0.4731	-0.3595	0.3958
E_4	-0.6872	-1.2949	-0.3460	-0.2789	-0.1281	-0.0895

5. CONCLUSION

The transmission line technique, as a microwave technique, is generalized to measure the permittivity function of inhomogeneous dielectric walls. The measured reflection or transmission coefficients are used to extract the permittivity function of the dielectric walls. To solve the problem, the electric permittivity function is expanded in a truncated Fourier series, first. Then, the optimum values of the coefficients of the series are obtained through an optimization approach. The usefulness of the proposed method was verified using a comprehensive example. It is observed that as the expansion terms increases each error function tends to a low specified value. The worst and the best identifications are achieved for the cases of using only transmission and combination of reflection and transmission measuring data, respectively. Also, the case of using only transmission data has an intrinsic reversing ambiguity.

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