

## **MODIFIED MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION FOR ELECTROMAGNETIC ABSORBER DESIGN**

**S. Chamaani, S. A. Mirtaheri, M. Teshnehlab  
M. A. Shoorehdeli and V. Seydi**

Faculty of Electrical Engineering  
K.N. Toosi University of Technology  
P.O. Box 16315-1355, Tehran, Iran

**Abstract**—Use of Multi-Objective Particle Swarm Optimization for designing of planar multilayered electromagnetic absorbers and finding optimal Pareto front is described. The achieved Pareto presents optimal possible trade-offs between thickness and reflection coefficient of absorbers. Particle swarm optimization method in comparison with most of optimization algorithms such as genetic algorithms is simple and fast. But the basic form of Multi-objective Particle Swarm Optimization may not obtain the best Pareto. We applied some modifications to make it more efficient in finding optimal Pareto front. Comparison with reported results in previous articles confirms the ability of this algorithm in finding better solutions.

### **1. INTRODUCTION**

Wide spread applications of electromagnetic absorbers, have inspired engineers to explore about optimal design with available algorithms. Ideally a thin, light weight and wideband absorber is an optimum one. But these features are inherently conflicting. For example it is possible to design an absorber with high reflection suppression, but high thickness or weight. On the other hand a thin and light absorber might have low reflection suppression.

Thus for such compulsions of physical realization of absorbers, engineer often deals with many problems that enforce him to search about proper trade-offs between conflicting goals. So if instead of one solution there exist a set of optimal solutions, designer can choose best trade-offs in each case. This set of optimal solutions is known as Pareto front in optimization literatures.

Hitherto, some Pareto optimization methods used to find Pareto front of absorbers [1,2]. These methods were on the base of genetic algorithms (GAs). Recently multi-objective particle swarm optimization (MOPSO) is also applied to solve this problem. But for some cases MOPSO showed slightly worse results than Nondominate Sorting Genetic Algorithm (NSGA-II) [3].

In this study we apply modified MOPSO to multilayer absorbers Pareto design. The proposed method shows better optimization results.

## 2. PARTICLE SWARM OPTIMIZATION

In recent years Particle Swarm Optimization (PSO) was used as an efficient optimization algorithm for various electromagnetic design problems [4–6]. The particle swarm optimization (PSO) is a population based algorithm used to visualize the movement of a bird's flock [7].

PSO is initialized with a population of random solutions (i.e., particles) flown through a hyper dimensional search space. Each particle in PSO has an adaptable velocity. Moreover, each particle has a memory remembering the best position of the search space that has ever been visited. Particles have the tendency to fly towards a better search area over the course of the search process. Thus their movement is an aggregated acceleration towards its best previously visited position and towards the best individual of a particle neighborhood.

Suppose that the search space is D-dimensional, and the  $i$ -th particle of the swarm can be represented by a D-dimensional vector  $X_i = (x_{i1}, x_{i2}, \dots, x_{iD})^T$ . The velocity of this particle is represented by another D-dimensional vector  $V_i = (v_{i1}, v_{i2}, \dots, v_{iD})^T$ . The best experience of  $i$ -th particle (Pbest) is denoted as  $P_i = (p_{i1}, p_{i2}, \dots, p_{iD})^T$ . Let  $g$  be the index of the best particle in the swarm (i.e, the  $g$ -th is the best), and the superscripts denote the iteration number, then in global version the swarm is manipulated according to the following two equations [7]:

$$v_i^{t+1}(d) = \omega * v_i^t(d) + c_1 * rand(p_i^t(d) - x_i^t(d)) + c_2 * rand(p_g^t(d) - x_i^t(d)) \quad (1)$$

$$x_i^{t+1}(d) = x_i^t(d) + v_i^{t+1}(d) \quad (2)$$

where  $v_i^t(d)$  and  $x_i^t(d)$  represent the current velocity and position of the  $d$ -th dimension of the  $i$ -th particle respectively and  $rand$  is a uniform random number in the range  $[0,1]$ ;  $c_1$ ,  $c_2$  are positive constants, called acceleration constants, and  $\omega$  is the inertia weight.

Above equations represent basic form of single objective PSO. In the next section, a fundamental of multi-objective optimization and modifications that must be applied to appropriate PSO for multi objective optimization is expressed.

### 3. MULTI-OBJECTIVE PARTICLE SWAM OPTIMIZATION

The main objective of every multi-objective optimization algorithm is to find Pareto-optimal set. This optimal set balances the trade-offs among the conflicting objectives.

The concept of Pareto optimality was conceptualized by the Italian economist, Vilfredo Pareto, in his work, Manual of Political Economy in 1906 [8]. To define Pareto optimality some basic concepts must be introduced as follow:

Domination: A position vector,  $x_1$  dominates a position vector,  $x_2$  ( $x_1 \prec x_2$ ), if and only if

$$f_k(x_1) \leq f_k(x_2), \quad \forall k = 1, \dots, n_k \quad (3)$$

$$f_k(x_1) < f_k(x_2) \quad \text{for at least one } k. \quad (4)$$

Pareto optimal: A position vector,  $x^* \in F$  is Pareto optimal if there does not exist a position vector,  $x \neq x^* \in F$  that dominates it.

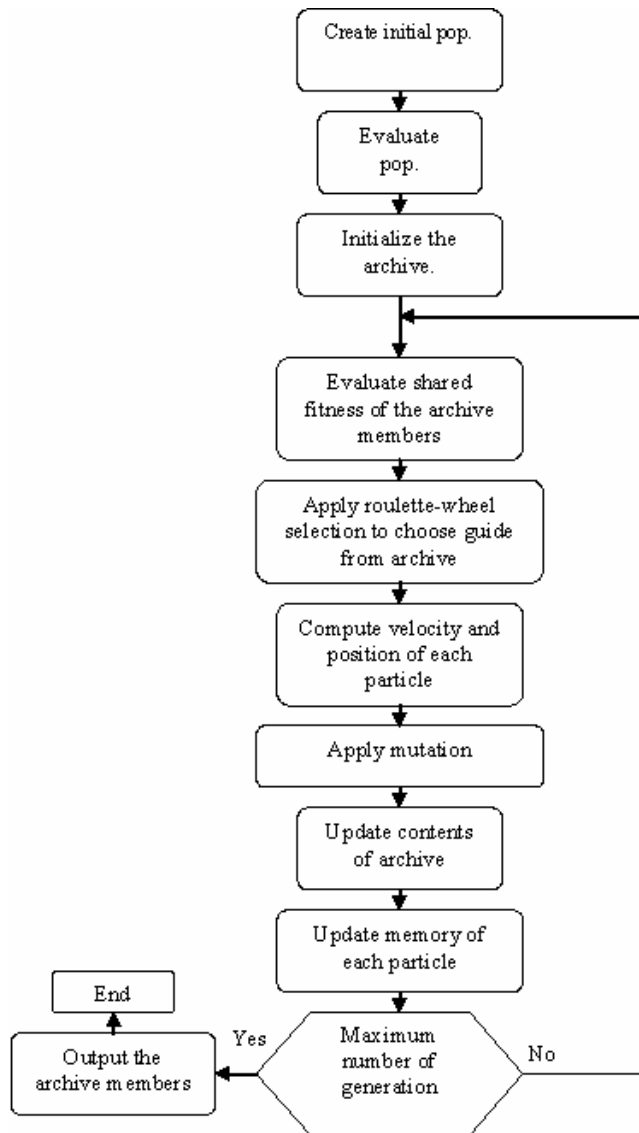
Pareto-optimal set: The set of all Pareto-optimal position vectors form the Pareto optimal set.

Pareto front: All objective vectors corresponding to position vectors of Pareto optimal set.

First step in any multi objective optimization algorithm is to minimize the distance between solutions and the Pareto front. For this objective appropriate fitness functions must be defined.

Traditional type of assigning fitness function is aggregation-based method, where the fitness function is a weighted sum of the objective functions [9].

However this classic approach can be very sensitive to precise aggregation of goals and tend to be ineffective and inefficient [8] where some researchers proposed using very complex methods such as neural network to obtain optimal weights of the objective function [10]. Some newer approaches for fitness assignment are on the base of Pareto dominance, where fitness is proportional to the dominance rank of solutions. MOPSO that used in this study is a dominance-based method that was proposed by Coello Coello et al. [11]. In this algorithm the best nondominated solutions have ever been visited



**Figure 1.** Flowchart of MOPSO.

are stored in a memory space calling archive. The flowchart of this algorithm is shown in Fig. 1.

The main parts of this algorithm work as follows:

After initialization of population and archive, we must generate

hypercubes of the search space explored so far, and locate the particles using these hypercubes as a coordinate system where each particle's coordinate are defined according to the values of its objective functions.

Speed of each particle is computed by the following relation:

$$v_i^{t+1}(d) = \omega * v_i^t(d) + c_1 * rand(p_i^t(d) - x_i^t(d)) + c_2 * rand(archive_h^t(d) - x_i^t(d)) \quad (5)$$

The term  $archive_h^t$  is taken from archive. The index  $h$  is selected using following strategy:

Those hypercubes containing more than one particle are assigned a fitness equal to the result of dividing any number  $x > 1$  (we took it 10) by the number of particles that they contain. This aims to decrease the fitness of those hypercubes that contain more particles and it can be seen as a form of fitness sharing. Then we apply roulette wheel selection using these fitness values to select the hypercube from which we will take the corresponding particle. Once the hypercube has been selected, we select randomly a particle within such hypercube.

For avoiding convergence to false Pareto a mutation operator is proposed. The effect of mutation operator decreases with respect the number of iterations. It is controlled with the parameter mutrate [11].

After computation of position, each particle's position must maintain within the valid search space. So when a position variable goes beyond its boundaries, we must do two things: (1) the position variable takes the value of its corresponding boundary (either the lower or the upper boundary), and (2) its velocity is multiplied by  $(-1)$  so that it searches in the opposite direction.

In updating archive, it must remain always dominate free. The size of archive is finite. When the archive reaches to its maximum allowable capacity, those particles located in less populated areas of objective space are given priority over those lying in highly populated regions.

In updating memory of each particle, if the current position dominates the Pbest, Pbest will be updated with current position.

This algorithm presents a good diversity in Pareto front. But some points in edge of Pareto front may not be found. In practice we may be interested in finding these points. For example in some applications, high absorption is very important even if the thickness is high. To obtain these part of Pareto front, we apply some modifications to this algorithm. These modifications are discussed briefly in the next section.

#### 4. MODIFIED MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION

After applying described MOPSO, for design of various absorbers, two serious problems were observed: (1) we could not achieve some points of Pareto front especially in top edge. These points represent absorbers with high reflection suppression that may be practically important. (2) Replacement of position variable with its corresponding boundary (especially lower boundary) to maintain it in the valid space forces the particles to be trapped in down edge of Pareto front. This part of Pareto front does not have any practical importance. On the other hand this dense edge of Pareto front fills the archive with improper solutions. After reaching to maximum capacity of archive, in any iteration we must do extra calculations to decide which member of archive must be removed. And therefore speed of optimizer will be fall down. Thus to eliminate these problems we applied two changes to the original algorithm:

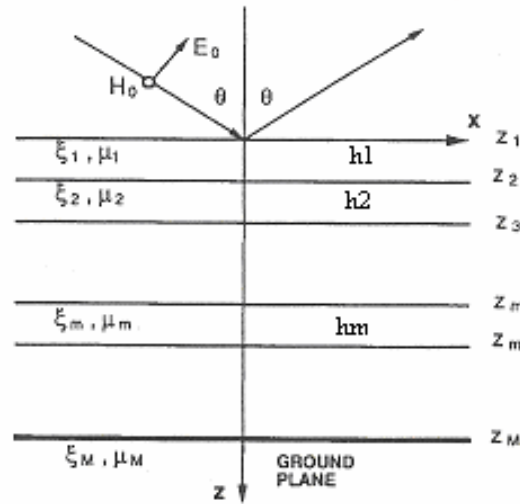
- (1) In order to obtain upper edge of Pareto front, in some of iterations, (for example after N iterations) we run the algorithm just for objective that is important for us (reflection coefficient in this problem).
- (2) To overcome to premature filling of archive with improper particles, we replaced position variables beyond the boundary with a random number in the valid range instead of replacement of them with corresponding boundary.

In applying single objective PSO for first modification, we used a fast and efficient variant of PSO (MLPSO) in those iterations [12]. MLPSO divides total swarm to some neighborhoods without overlap and runs PSO to these sub swarms in parallel. Performance of MLPSO method was confirmed in finding global optimums of absorber design [12]. In the Section 6 simulation results of this algorithm is presented.

#### 5. THE ABSORBER MODELING

A multilayered coating absorber backed by a perfect electric conductor (PEC) is shown in Fig. 2. The layer number zero is the incident media and layer number M is perfect conductor [13].

Number of layers is fixed in our design. Materials are chosen from an available database with various frequency dependent electromagnetic characteristics. Permittivity and permeability of materials used in our design are listed in Table 1 [14]. For each layer,



**Figure 2.** Multilayered coating absorber [13].

thickness and material type must be specified. Thus for an  $M$ -layer absorber there is a  $2M$  dimensional position vector. Thickness of each layer varies in the range  $[0, 2]$  mm. Two objective functions can be expressed as:

$$f_1 = 20 \log_{10} \{ \max(R(f)), |f \in B \} \tag{6}$$

$$f_2 = \sum_{i=1}^M t_i \tag{7}$$

where  $B$  is the desired bandwidth. And  $R$  is the reflection coefficient of multilayer structure.  $t_i$  is thickness of each layer.

We wish to minimize the maximum of reflection coefficient over the frequency band and total thickness of absorber simultaneously. Since these goals are conflicting we must search about Pareto-optimal front of this problem that represents best trade-offs between these objectives. In the next section, designs resulted from applying Modified MOPSO are presented and compared with results of other algorithms.

## 6. SIMULATION RESULTS

In following simulations we attempt to demonstrate the described algorithm, to design following types of five-layer absorbers in various frequency ranges for normal incidence using materials listed in Table 1.

**Table 1.** Relative permittivity and permeability of the 16 materials in the data base [14].

Lossless Dielectric Materials ( $\mu_r = 1 + j0.$ )		
#	$\varepsilon_r$	
0	10	
1	50	
Lossy Magnetic Materials ( $\varepsilon_r = 15 + j0.$ )		
$\mu_i(f) = \frac{\mu_i(1 \text{ GHz})}{f^\beta},$ $\mu_r(f) = \frac{\mu_r(1 \text{ GHz})}{f^\alpha}, \quad \mu = \mu_r - j\mu_i$		
#	$\mu_r(1 \text{ GHz}), \alpha$	$\mu_i(1 \text{ GHz}), \beta$
2	5,0.974	10,0.961
3	3,1.000	15,0.957
4	7,1.000	12,1.000
Lossy Dielectric Materials ( $\mu_r = 1 + j0.$ )		
$\varepsilon_i(f) = \frac{\varepsilon_i(1 \text{ GHz})}{f^\beta}, \quad \varepsilon_r(f) = \frac{\varepsilon_r(1 \text{ GHz})}{f^\alpha}, \quad \varepsilon = \varepsilon_r - j\varepsilon_i$		
#	$\varepsilon_r(1 \text{ GHz}), \alpha$	$\varepsilon_i(1 \text{ GHz}), \beta$
5	5,0.861	8,0.569
6	8,0.778	10,0.682
7	10,0.778	6,0.861
Relaxation-type magnetic materials		
$\mu = \mu_r - j\mu_i, \quad \mu_r = \frac{\mu_{rm}f_m^2}{f^2 + f_m^2}, \quad \mu_i = \frac{\mu_{rm}f_m f}{f^2 + f_m^2}$ $e_r = 15 + j0$		
#	$\mu_m$	$f_m$
8	35	0.8
9	35	0.5
10	30	1
11	18	0.5
12	20	1.5
13	30	2.5
14	30	2
15	25	3.5



- 1) Low frequency absorber:  $0.2 \text{ (GHz)} < f < 2 \text{ (GHz)}$
- 2) High frequency absorber:  $2 \text{ (GHz)} < f < 8 \text{ (GHz)}$
- 3) Wide band absorber:  $0.2 \text{ (GHz)} < f < 10 \text{ (GHz)}$

In all simulations, for MOPSO and Modified MOPSO we set  $c_1$  and  $c_2$  a random number in  $[1.49, 2]$ ,  $\omega$  a random number in  $[0, 1]$ ,  $\text{mutrate} = 0.5$ . For low frequency absorber both population size and archive size were chosen 100. Maximum number of generation for MOPSO was 1800 and for Modified MOPSO was 1200. The total number of function evaluations in MOPSO was 180000. In Modified MOPSO we set  $N$ , 200 that means every 200 iterations we run single objective PSO for finding high absorption designs (top edge of Pareto front). That is we call single objective PSO, 6 times during the simulation. For this single objective PSO, both population size and maximum number of generation were set 100. Thus total number of function evaluations for Modified MOPSO was:

$$(6 \times 100 \times 100) + (100 \times 1200) = 180000$$

In NSGA-II we set the population size 100 and maximum number of iterations 900. Thus the total number of function evaluations was also:

$$2 \times 100 \times 900 = 180000$$

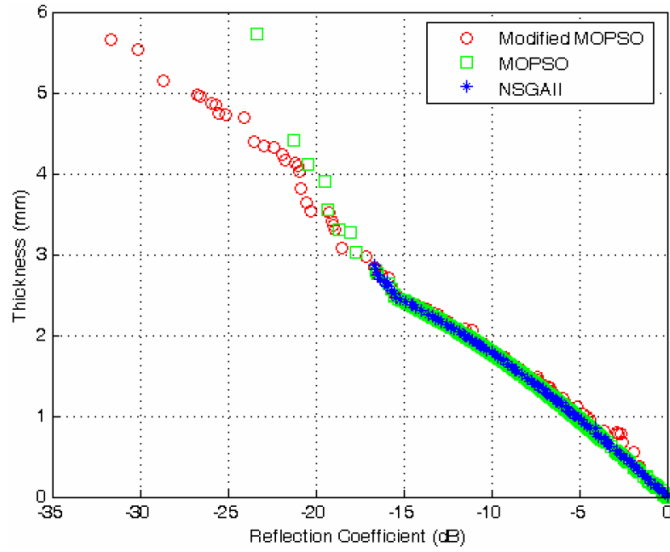
Since the numbers of function evaluations are equal in these three algorithms, we can compare performance of them in finding Pareto front. The Pareto obtained after applying them is shown in Fig. 3.

As it is obvious, Pareto front found with Modified MOPSO represents better solutions (on the base of dominance terminology) and better diversity.

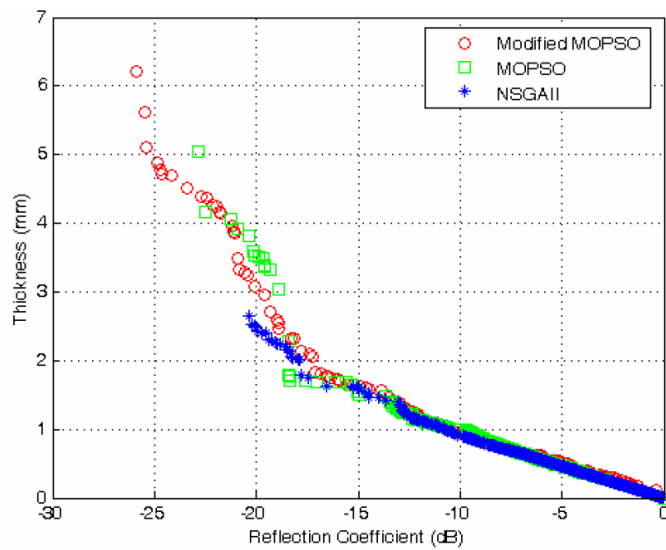
The second example is high frequency absorber. Population size and archive size in MOPSO and Modified MOPSO were 2000 and 200 respectively. Maximum number of iterations for MOPSO and Modified MOPSO was 1560 and 1300 respectively. In Modified MOPSO  $N$  is set to 100. In single objective PSO the swarm size is 100 and maximum number of iterations is 400. For NSGA-II the population size is 1000 and maximum number of function evaluation is 1560. Thus the total number of function evaluations is 3120000 for all algorithms. The obtained Pareto is shown in Fig. 4.

Figure 4 shows that both NSGA-II and MOPSO in some points propose better solutions but diversity of Modified MOPSO is better than other algorithms.

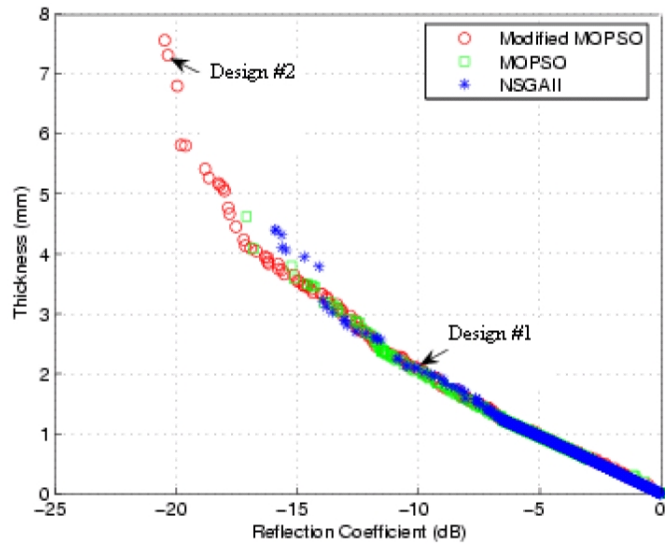
The third example is wide band absorber. Population size for MOPSO, Modified MOPSO and NSGA-II is 1000. Maximum number of iterations for MOPSO, Modified MOPSO and NSGA-II is 1010, 800



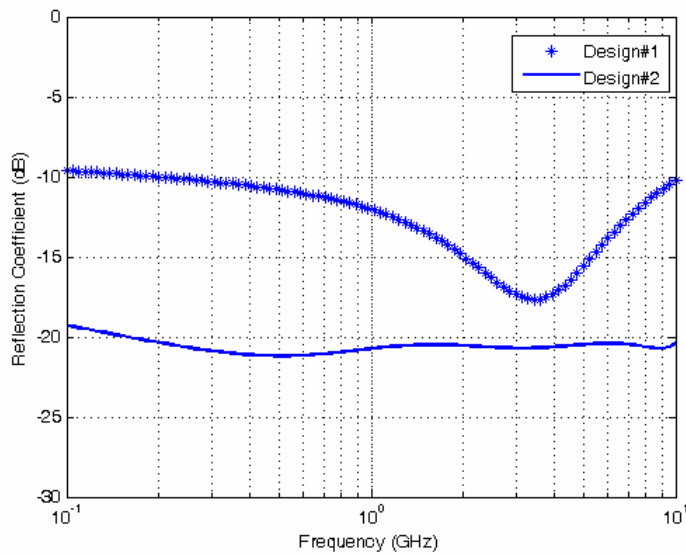
**Figure 3.** Pareto fronts of low frequency absorber found with different algorithms.



**Figure 4.** Pareto fronts of high frequency absorber found with different algorithms.



**Figure 5.** Pareto fronts of high frequency absorber found with different algorithms.



**Figure 6.** The frequency response for the 2 designs highlighted in Fig. 5.

and 505 respectively. In Modified MOPSO we set N 350. Population size and maximum number of iterations in single objective PSO is 100 and 700 respectively. Total number of function evaluations is 10100000 for all of them. The resulted Pareto is shown in Fig. 5.

In wide band absorber design, solutions obtained by Modified MOPSO are better than other algorithms. In this case, such as other examples Modified MOPSO shows better diversity. Since Pareto front is resulted after only one run of a multi-objective algorithm and all of these solutions may be stored in a database and used for future problems with different criteria, the diversity is very important.

High diversity of solutions helps the engineer to have an optimum selection among a wide and well distributed set. For example two different designs of Pareto front are shown in Fig. 6. Design No. 2 is very absorbent but quiet thick. Design No. 1 is very thin but representing lower absorption. Any of them could be proper in a specific case. Parameters of these designs are listed in Tables 2–3.

**Table 2.** Structure of Design No. 1.

Design#1	Material number	Thickness (mm)
Layer1	3	1.2509
Layer2	2	0.0882
Layer3	4	0.1847
Layer4	3	0.3277
Layer5	15	0.2357
Total thickness (mm)	2.09	
Max. Reflectance (dB)	−10	

**Table 3.** Structure of Design No. 2.

Design#2	Material number	Thickness (mm)
Layer1	3	1.5407
Layer2	5	1.9189
Layer3	4	1.943
Layer4	5	1.697
Layer5	13	0.4092
Total thickness (mm)	7.50	
Max. Reflectance (dB)	−20.7	

Among these algorithms, NSGA-II because of inherently sorting in it has the lowest speed. In comparison with NSGA-II, MOPSO and Modified MOPSO show better diversity in Pareto front for all cases. The main reason of this proper diversity is girding strategy. In Modified MOPSO, applying single objective PSO, finds extreme points of Pareto (high absorption points) and girding method fills the gaps during the time. Thus this method in comparison with MOPSO shows better diversity. In other words, Pareto fronts obtained by MOPSO and NSGA-II are subsets of Pareto resulted from Modified MOPSO.

## 7. CONCLUSION

In this study, multilayered absorbers were optimized for two conflicting objectives: thickness and reflection coefficient. Between optimization methods, PSO based algorithms have been favored because of simplicity and high convergence speed. Recently MOPSO was used to solve Pareto front of multilayered absorbers. For achieving better diversity, we applied some modifications to MOPSO. In Modified MOPSO, in some of iterations a fast and efficient single objective PSO was used to find some extreme points of Pareto that MOPSO could not achieve them. Then in other iterations because of girding used in this algorithm we reach to a high dispersion of solutions. This method was compared with NSGA-II and MOPSO. Results show that in some finite cases MOPSO and NSGA-II demonstrate better solutions but the diversity of Modified MOPSO is always better and proposes a wider set of solutions.

## REFERENCES

1. Weile, D. S., E. Michielssen, and D. E. Goldberg, "Genetic algorithm design of Pareto optimal broadband microwave absorbers," *IEEE Trans. Electromagn. Compat.*, Vol. 38, 518–525, 1996.
2. Cui, S., A. Mohan, and D. S. Weile, "Pareto optimal design of absorbers using a parallel elitist nondominated sorting genetic algorithm and the finite element-boundary integral method," *IEEE Trans. Antennas Propagat.*, Vol. 53, 2099–2107, 2005.
3. Goudos, S. K. and J. N. Sahalos, "Microwave absorber optimal design using multi-objective particle swarm optimization," *Microwave and Optical Technology Letters*, Vol. 48, No. 8, 1553–1558, Aug. 2006.
4. Mahmoud, K. R., M. El-Adawy, and S. M. M. Ibrahim, "A

- comparison between circular and hexagonal array geometries for smart antenna systems using particle swarm optimization algorithm,” *Progress In Electromagnetics Research*, PIER 72, 75–90, 2007.
5. Lee, K.-C. and J.-Y. Jhang, “Application of particle swarm optimization algorithm to the optimization of unequally spaced antenna arrays,” *Journal of Electromagnetic Wave and Applications*, Vol. 20, No. 14, 2001–2012, 2006.
  6. Chen, T. B., Y. L. Dong, Y. C. Jiao, and F. S. Zhang, “Synthesis of circular antenna array using crossed particle swarm optimization algorithm,” *Journal of Electromagnetic Wave and Applications*, Vol. 20, No. 13, 1785–1795, 2006.
  7. Kennedy, J. and R. Eberhart, “Particle swarm optimization,” *Proceedings of IEEE International Conference on Neural Network*, Piscataway, NJ, 1995.
  8. Engelbrecht, A. P., *Fundamentals of Computational Swarm Intelligence*, John Wiley & Sons, 2005.
  9. Lei, J., G. Fu, L. Yang, and D. M. Fu, “Multi-objective optimization design of the Yagi-Uda antenna with an X-shaped driven dipole,” *Journal of Electromagnetic Waves and Applications*, Vol. 21, No. 7, 963–972, 2007.
  10. Lee, Y. H., B. J. Cahill, S. J. Porter, and A. C. Marvin, “A novel evolutionary learning technique for multi-objective array antenna optimization,” *Progress In Electromagnetics Research*, PIER 48, 125–144, 2004.
  11. Coello, C. A. C., G. T. Pulido, and M. S. Lechuga, “Handling multiple objectives with particle swarm optimization,” *IEEE Trans. Evolutionary Computat.*, Vol. 8, 256–279, 2004.
  12. Chamaani, S., S. A. Mirtaheri, and M. A. Shooredeli, “Design of very thin wide band absorbers using particle swarm optimization,” *Proceeding of the 2006 Antem/Ursi*, 629–632, Montreal, QC, Canada, July 2006.
  13. Perini, J. and L. S. Cohen, “Design of broad-band radar-absorbing materials for large angles of incidence,” *IEEE Trans. Electromagn Compat.*, Vol. 35, No. 2, 223–230, May 1993.
  14. Pesque, J. J., D. P. Bouche, and R. Mittra, “Optimization of multilayered antireflection coatings using an optimal control method,” *IEEE Trans. Antennas Propagat.*, Vol. 41, 1789–1796, Sept. 1992.