Progress In Electromagnetics Research, PIER 79, 277–290, 2008

ROBUST ADAPTIVE BEAMFORMING FOR STEERING VECTOR UNCERTAINTIES BASED ON EQUIVALENT DOAS METHOD

Y. J. Gu, Z. G. Shi, and K. S. Chen

Department of Information Science and Electronic Engineering
Zhejiang University
Hangzhou 310027, China

Y. Li

Department of Communication and Electronic Engineering
East China University of Science and Technology
Shanghai 200237, China

Abstract—The adaptive beamformers often suffer severe performance degradation when there exist uncertainties in the steering vector of interest. In this paper, we develop a new approach to robust adaptive beamforming in the presence of an unknown signal steering vector. Based on the observed data, we try to estimate an equivalent direction-of-arrival (DOA) for each sensor, in which all factors causing the steering vector uncertainties are ascribed to the DOA uncertainty only. The equivalent DOA of each sensor can be estimated one by one with the assumption that the elements of the steering vector are uncorrelated with each other. Using a Bayesian approach, the equivalent DOA estimator of each sensor is a weighted sum of a set of candidate DOA's, which are combined according to the value of the *a posteriori* probability for each pointing direction. In this way, the signal steering vector and the diagonal loading sample matrix inversion (DL-SMI) version adaptive beamformer can be obtained. Numerical simulations illustrate the robustness of the proposed beamforming algorithm.

1. INTRODUCTION

Adaptive beamforming is widely used in array signal processing for enhancing a desired signal while suppressing interference and noise at the output of an array of sensors [1, 2]. Compared with the data-independent beamformers, the adaptive beamformers have better resolution and much better interference rejection capability. However, the adaptive beamformers are rather sensitive to the steering vector mismatches, which will degrade the performance of the adaptive beamformers severely. The causes of steering vectors mismatches in the practical applications include direction-of-arrival (DOA) errors [3], imperfect array calibration and/or distorted antenna shape [4], array manifold mismodeling due to source wavefront distortions resulting from environmental inhomogeneities [5], near-far problem [6], source spreading and local scattering [7] as well as other effects [8]. Therefore, the robustness against the steering vector uncertainties in adaptive beamformers is highly required [9–11].

Several approaches are known to be able to partly overcome the problem of arbitrary steering vector mismatches. The most popular of them are the diagonal loading approaches [12] and the eigenspace-based beamforming [13]. These pioneer works certainly have some drawbacks, but they ignite the adaptive beamforming as a hot research topic in the following years. Recently, based on the worst-case performance optimization, some new approaches were proposed in [14–16], which ensure that the distortionless response are maintained for all possible steering vector. In essence, this approach belongs to the class of diagonal loading techniques, but they can improve the performances of the adaptive beamformers by finding the optimal diagonal loading factor. In 2004, the authors derived the maximum likelihood (ML) estimator for the deterministic signal, the minimum mean square error (MMSE) estimator and the maximum *a posteriori* (MAP) estimator for the random signal in the presence of the steering vector uncertainties [17]. More recently, an adaptive Bayesian beamforming with order recursive implementation for steering vector uncertainties was proposed in [18], which has the form of a Kalman filter that is recursive in order instead of time.

Though many researchers have made great contributions to the adaptive beamforming, there still exist many problems to be addressed further. For example, in the work of [18], the advantage of reducing computational complexity while maintaining an accurate estimate cannot be held because it is not easy to know *a priori* about which sensors have a small uncertainty level. Moreover, when the steering vector is exactly known, this beamformer is ineffective because it degenerates into a sample matrix inversion (SMI) adaptive version of the minimum variance distortionless response (MVDR) beamformer.

In fact, all the above beamforming algorithms are devoted to finding the optimal weight vector directly without estimating the actual steering vector explicitly. In this paper, we will firstly estimate the equivalent DOA of each sensor from the array observations, which results the steering vector and the robust adaptive beamformer. In a statistical sense, all factors causing the steering vector uncertainties can be ascribed to the DOA uncertainty only. Thus, each sensor will undoubtedly has an unique equivalent DOA, and it is reasonable that the performance of the proposed adaptive beamformer can probably be improved.

The rest of the paper is organized as follows. Section 2 contains background material. A new robust adaptive beamformer based on equivalent DOA method is developed in Section 3, and performance comparisons are presented in 4. Finally, Section 5 concludes this paper.

2. BACKGROUND

The output of a narrowband beamformer is given by

$$y(k) = \boldsymbol{\omega}^H \mathbf{x}(k) \tag{1}$$

where k is the time index, $\mathbf{x}(k) = [x_1(k), \cdots, x_N(k)]^T \in C^{N\times 1}$ is the complex vector of array observations, $\boldsymbol{\omega} = [\omega_1, \cdots, \omega_N]^T \in C^{N\times 1}$ is the complex vector of beamformer weights, N is the number of array sensors, and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively. The observation vector has the form

$$\mathbf{x}(k) = \mathbf{a}s(k) + \boldsymbol{i}(k) + \boldsymbol{n}(k), \quad k = 1, 2, \cdots \tag{2}$$

where $\mathbf{a}$ is the signal steering vector, $s(k)$, $\boldsymbol{i}(k)$, and $\boldsymbol{n}(k)$ are the desired signal, interference and noise components, respectively. The source, interference and noise are assumed to be zero-mean, complex Gaussian white processes that are statistically independent. The weight vector $\boldsymbol{\omega}$ is chosen to maximize the signal-to-interference-plus-noise ratio (SINR)

$$SINR = \frac{\sigma_s^2 |\boldsymbol{\omega}^H \mathbf{a}|^2}{\boldsymbol{\omega}^H \mathbf{R}_{i+n} \boldsymbol{\omega}} \tag{3}$$

where

$$\mathbf{R}_{i+n} = E\left\{(\boldsymbol{i}(k) + \boldsymbol{n}(k))(\boldsymbol{i}(k) + \boldsymbol{n}(k))^H\right\} \tag{4}$$

is the $N \times N$ interference-plus-noise covariance matrix, and σ_s^2 is the signal power. The most popular approach to solving this problem is to use the MVDR beamforming [19], which maintains a distortionless response toward the desired signal and minimizes the output interference-plus-noise power. Hence, the maximization of (3) is equivalent to

$$\min_{\boldsymbol{\omega}} \ \boldsymbol{\omega}^H \mathbf{R}_{i+n} \boldsymbol{\omega} \quad s.t. \quad \boldsymbol{\omega}^H \mathbf{a} = 1, \tag{5}$$

which can be solved by the Lagrange multiplier method and the solution is given by

$$\boldsymbol{\omega}_{MV} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}}{\mathbf{a}^H \mathbf{R}_{i+n}^{-1} \mathbf{a}}. \tag{6}$$

In practical applications, the exact interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ is unavailable. In general, the sample covariance matrix with K-snapshots [19, 20],

$$\hat{\mathbf{R}}_K = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k) \mathbf{x}(k)^H. \tag{7}$$

is used instead of (4). Replacing $\mathbf{R}_{i+n}$ in (6) with $\hat{\mathbf{R}}_K$, we get a SMI adaptive version of the MVDR beamformer as

$$\boldsymbol{\omega}_{SMI} = \frac{\hat{\mathbf{R}}_K^{-1} \mathbf{a}}{\mathbf{a}^H \hat{\mathbf{R}}_K^{-1} \mathbf{a}}. \tag{8}$$

However, the SMI algorithm has an essential shortcoming that it does not provide sufficient robustness against the mismatch between the presumed and actual signal steering vectors. As we know, the DL-SMI algorithm is robust to the signal steering vector mismatches and small snapshots size [12]. The essence of this approach is to replace the sample covariance matrix $\hat{\mathbf{R}}_K$ by a diagonal loading covariance matrix

$$\hat{\mathbf{R}}_{K,DL} = \hat{\mathbf{R}}_K + \xi \mathbf{I}, \tag{9}$$

where, ξ is a diagonal loading factor, and $\mathbf{I}$ is the identity matrix. Hence, the DL-SMI adaptive version of the MVDR beamformer is

$$\boldsymbol{\omega}_{DL-SMI} = \frac{\hat{\mathbf{R}}_{K,DL}^{-1} \mathbf{a}}{\mathbf{a}^H \hat{\mathbf{R}}_{K,DL}^{-1} \mathbf{a}}. \tag{10}$$

Typically, the diagonal loading factor ξ is chosen to be about $10\sigma_n^2$, where σ_n^2 is the noise power in a single sensor. As a result, the choice in an *ad hoc* way may be not optimal.

3. ROBUST BEAMFORMER BASED ON EQUIVALENT DOAS METHOD

In this section, based on equivalent DOAs method, we develop a new robust adaptive beamforming, which ascribes all the steering vector uncertainties to the DOA uncertainty only.

In general, the steering vector $\mathbf{a}$ can be modeled as an $N \times 1$ complex Gaussian random vector with mean $\bar{\mathbf{a}}$ and covariance matrix $\mathbf{C_a}$, that is,

$$\mathbf{a} \sim \mathcal{CN}(\bar{\mathbf{a}}, \mathbf{C_a}), \tag{11}$$

where $\bar{\mathbf{a}}$ is the presumed steering vector. The covariance matrix $\mathbf{C_a}$ captures the second-order statistics of the uncertainties in the steering vector. By assuming the elements of the steering vector are uncorrelated with each other, the covariance matrix $\mathbf{C_a}$ is diagonal and can be written as

$$\mathbf{C_a} = \begin{pmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_N^2 \end{pmatrix}, \tag{12}$$

where the diagonal element σ_m^2 denotes the variance of the mth element of the steering vector. Even though $\mathbf{C_a}$ may be actually non-diagonal, it typically can be approximated by the scaled identity matrix for the simplicity reason [21].

In a statistical sense, all the factors causing the steering vector uncertainties can be ascribed to the DOA uncertainty only. Consequently, each sensor has an unique equivalent DOA, not a common uniform DOA. And then, the equivalent DOAs of an array sensors can be expressed in a vector form as

$$\boldsymbol{\theta}_{eq} = [\theta_{1_{eq}}, \cdots, \theta_{N_{eq}}]^T. \tag{13}$$

Because the elements of the steering vector are assumed to be uncorrelated with each other, the process of estimating the steering vector can be simplified to estimate the equivalent DOAs one by one. The equivalent DOAs can be estimated as the following process. When an equivalent DOA of one sensor is being estimated, the other sensors' DOAs are assumed to be known and fixed. As soon as the equivalent DOA of one sensor is estimated, it is regarded as known when the others are being estimated. Using this method, the equivalent DOA of each sensor can be estimated one by one.

Using a Bayesian approach, the equivalent DOA $\theta_{n_{eq}}$ can be assumed to be a discrete random variable with *a priori* pdf $p(\theta)$ defined on a discrete set of L points $\boldsymbol{\Theta} = \{\theta_1 \cdots \theta_L\}$. Let $\mathbf{X}$ denote a collection of K consecutive snapshots of the received data vector $\mathbf{x}(k)$. The MMSE estimator of the equivalent DOA $\theta_{n_{eq}}$ is the conditional mean of $\boldsymbol{\theta}$, given $\mathbf{X}$. It can be written as

$$\hat{\theta}_{n_{eq}} = E\{\theta|\mathbf{X}\} = \sum_{i=1}^{L} p(\theta_i|\mathbf{X})\theta_i \tag{14}$$

where $p(\theta_i|\mathbf{X})$ is the *a posteriori* pdf of θ_i given the observations $\mathbf{X}$. Therefore, the Bayesian estimator of the equivalent DOA is a weighted sum of the discrete set $\boldsymbol{\Theta}$, which are combined according to the value of the *a posteriori* probability for each pointing direction.

From [22], for each θ_i, the *a posteriori* pdf is given by

$$p(\theta_i|\mathbf{X}) = \frac{p(\theta_i)p(\mathbf{X}|\theta_i)}{\sum_{j=1}^{L} p(\theta_j)p(\mathbf{X}|\theta_j)}, \quad i = 1, \cdots, L \tag{15}$$

with

$$\begin{aligned} p(\mathbf{X}|\theta_i) &= \prod_{k=1}^{K} \frac{1}{\pi^N |\mathbf{R_x}(\theta_i)|} exp\{-\mathbf{x}(k)^H \mathbf{R_x}^{-1}(\theta_i)\mathbf{x}(k)\} \\ &= \pi^{-NK} |\mathbf{R_x}(\theta_i)|^{-K} exp\left\{-\sum_{k=1}^{K} \mathbf{x}(k)^H \mathbf{R_x}^{-1}(\theta_i)\mathbf{x}(k)\right\}, \end{aligned} \tag{16}$$

where $|\cdot|$ is the determinant operator. Simplification as [23], the approximate *a posteriori* pdf is given by

$$\hat{p}(\theta_i|\mathbf{X}) = c\ p(\theta_i)\ exp\left\{K\gamma \left(\mathbf{a}(\theta_i)^H \hat{\mathbf{R}}_K^{-1} \mathbf{a}(\theta_i)\right)^{-1}\right\}, \tag{17}$$

where c is a normalization factor independent of θ_i, and γ is a constant defined as

$$\gamma \equiv \frac{N}{\sigma_n^2} \frac{N\sigma_s^2/\sigma_n^2}{\sigma_s^2(1 + N\sigma_s^2/\sigma_n^2)}. \tag{18}$$

Substituting (17) in (14), $\hat{\theta}_{n_{eq}}$, the MMSE estimator of the equivalent DOA of the nth sensor, can be obtained. And then, it is regarded as known when the equivalent DOAs of the other sensors are being estimated, till all equivalent DOAs have been estimated. Hence, the steering vector $\mathbf{a}$ is given by

$$\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq}) = [a_1(\hat{\theta}_{1_{eq}}), \cdots, a_N(\hat{\theta}_{N_{eq}})]^T. \tag{19}$$

When the steering vector $\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})$ is determined, based on the DL-SMI adaptive version of the MVDR beamforming method, the proposed robust beamformer has the form of

$$\boldsymbol{\omega}_{EQ} = \frac{\hat{\mathbf{R}}_{K,DL}^{-1}\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})}{\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})^H\hat{\mathbf{R}}_{K,DL}^{-1}\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})}. \tag{20}$$

To summarize, the proposed robust adaptive beamformer based on the equivalent DOAs method performs the following steps to update the beamformer weights.

$$
\begin{aligned}
&1)\ \hat{\mathbf{R}}_{K,DL}^{-1} = \left(\frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}(k)^H + \xi\mathbf{I}\right)^{-1}\\
&2)\ initialize\ \boldsymbol{\theta}_{eq},\ \boldsymbol{\Theta} = \{\theta_1\cdots\theta_L\}\\
&\quad for\ n = 1,\cdots,N\\
&\quad\quad for\ i = 1,\cdots,L\\
&\quad\quad\quad \mathbf{v}_i = \hat{\mathbf{R}}_{K,DL}^{-1}\mathbf{a}(\theta_i)\\
&\quad\quad\quad \hat{P}_{MV,DL}(\theta_i) = (\mathbf{a}(\theta_i)^H\mathbf{v}_i))^{-1}\\
&\quad\quad\quad \hat{p}(\theta_i|\mathbf{X}) = p(\theta_i)exp\{K\gamma\hat{P}_{MV,DL}(\theta_i)\}\\
&\quad\quad end\\
&\quad\quad c = \left\{\sum_{i=1}^{L}\hat{p}(\theta_i|\mathbf{X})\right\}^{-1}\\
&\quad\quad \hat{\theta}_{n_{eq}} = c\sum_{i=1}^{L}\hat{p}(\theta_i|\mathbf{X})\theta_i\\
&\quad end\\
&3)\ \boldsymbol{\omega}_{EQ} = \frac{\hat{\mathbf{R}}_{K,DL}^{-1}\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})}{\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})^H\hat{\mathbf{R}}_{K,DL}^{-1}\mathbf{a}(\hat{\boldsymbol{\theta}}_{eq})}.
\end{aligned}
$$

Hence, the overall computational complexity of the proposed robust beamformer is $\mathcal{O}(N^3)$. This is the same order of complexity as that of the SMI algorithm.

4. SIMULATIONS

In the simulations, a uniform linear array (ULA) with $N = 10$ omnidirectional sensors spaced half a wavelength apart is considered.

For each scenario, 200 Monte Carlo simulation runs are used to obtain each simulated point. The beampatterns shown are obtained from one Monte Carlo realization only. In all examples, we assume that the DOAs of two interferences are 30° and 50°, respectively. Both interferences are assumed to have the same interference-to-noise ratio (INR), 30 dB. Seven different beamforming algorithms are compared in terms of the mean output SINR: the proposed robust beamformer (20), the SMI beamformer (8), the DL-SMI beamformer (10), the eigenspace-based beamformer [13], the Bayesian beamformer [23], the worst-case based robust beamformer [14], and the Bayesian beamformer with order recursive implementation [18]. In addition, the optimal SINR is also shown in all figures. The diagonal loading factor $\xi = 10\sigma_n^2$ is taken in the proposed robust beamformer and the the DL-SMI beamformer. In the worst-case based robust beamformer, the upper bound of the norm of the steering vector mismatch ϵ is set to be a constant 3, as [14] denotes. In the Bayesian beamformer [18], the order of iteration is assumed to be equal to 10, the number of array sensors, if not otherwise specified. In the proposed robust beamformer, the initialized DOA of each sensor is set to θ_s, and the $\mathbf{\Theta}$ is composed of $L = 11$ evenly spaced points at $\{-5.739, -4.589, -3.440, -2.292, -1.146, 0, 1.146, 2.292, 3.440, 4.589, 5.739\} + \theta_s$, where θ_s is the presumed DOA. The $\mathbf{\Theta}$ is also adopted in the Bayesian beamformer [23]. The *a priori* pdf is uniform with $p(\theta) = \frac{1}{11}$. The parameter γ is set to 0.3, as [23] denotes. In all of the beamformers, $K = 50$ snapshots are used.

4.1. Example 1: Exactly Known Signal Steering Vector

In the first example, we simulate a scenario where the actual signal steering vector is exactly known, hence, the covariance matrix $\mathbf{C_a}$ is an $N \times N$ zero matrix. In this example, the plane-wave signal is assumed to impinge on the array from $\theta_s = 3°$.

Figure 1 compares the performances of different beamforming algorithms in terms of the mean output array SINR versus the input signal-to-noise ratio (SNR). The proposed robust beamformer outperforms the SMI, the DL-SMI and the eigenspace-based beamformers, achieving a performance that is consistently close to the optimal SINR for all values of SNR. Because $\mathbf{C_a} = \mathbf{0}_{N\times N}$ when the signal steering vector is exactly known, the Bayesian beamformer (14) in [18] degenerates into a SMI beamformer by the fact that the desired signal power σ_s^2 can be estimated as the value of the minimum variance

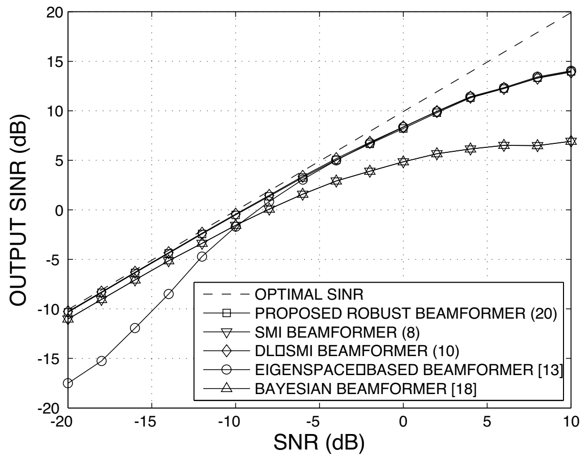

Figure 1. Example 1: Output SINR versus SNR.

(MV) spatial spectral estimate with the presumed steering vector [24],

$$\hat{\sigma}_s^2 = \frac{1}{\bar{\mathbf{a}}^H \hat{\mathbf{R}}_K^{-1} \bar{\mathbf{a}}}, \tag{21}$$

and hence, they have the same performance as Fig. 1 shows.

In addition, the beampatterns of the proposed robust beamformer, DL-SMI beamformer, eigenspace-based beamformer and the Bayesian beamformer [18] are compared in Fig. 2 for SNR $= -10$ dB.

4.2. Example 2: Signal Look Direction Mismatch

In the second example, a scenario with the signal look direction mismatch is considered. We assume that both the presumed and actual signal spatial signatures are plane waves impinging from the DOAs 3° and 5°, respectively. This corresponds to a 2° mismatch in the signal look direction.

The output SINRs of several different beamforming algorithms versus the input SNR are compared in Fig. 3. From Fig. 3, the performance of the proposed robust beamformer is close to the optimal SINR for all values of SNR, which is better than the others. Unlike that of the exactly known signal steering vector, the performance of the DL-SMI beamformer degrades severely especially at high SNR because of DOA mismatch. Because the steering vector uncertainties in this example is caused by the DOA mismatch only, the Bayesian beamformer [23] performs as well as the proposed robust beamformer.

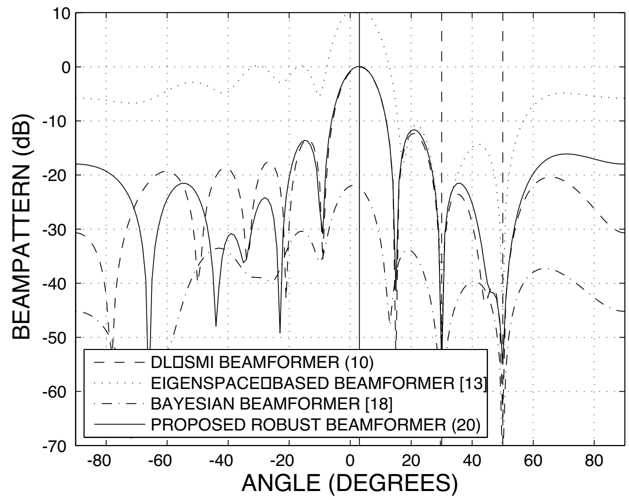

Figure 2. Example 1: Sample beampatterns from one trial. Solid line indicates desired signal location, and dashed line indicate interferer location.

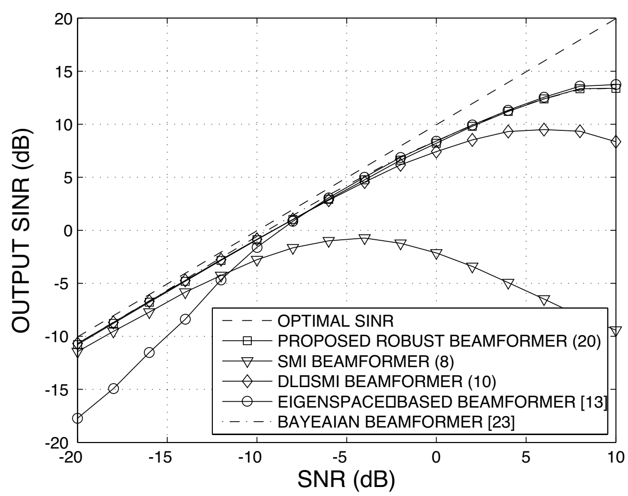

Figure 3. Example 2: Output SINR versus SNR.

4.3. Example 3: Random Perturbation of the Elements in a Steering Vector

In the third example, the perturbations of the elements in a steering vector are assumed to be random with mean DOA 3°, which is a more practical scenario than the above two examples. In this example, the covariance matrix $\mathbf{C_a}$ is assumed to be diagonal with randomly generated elements and it's trace $tr(\mathbf{C_a})$ is normalized to 1.

In Fig. 4, the mean output SINR of these beamforming algorithms versus the input SNR are compared. The proposed robust beamformer performs almost as well as the former two scenarios, whereas the others are not. Because the equivalent DOAs of the array sensors are not uniform, namely, the steering vector uncertainties are not caused by the DOA uncertainty only, the Bayesian beamformer [23] loses it's robustness, especially at high SNR. Such phenomena also occur in the worst-case based robust beamformer [14] and the DL-SMI beamformer (10).

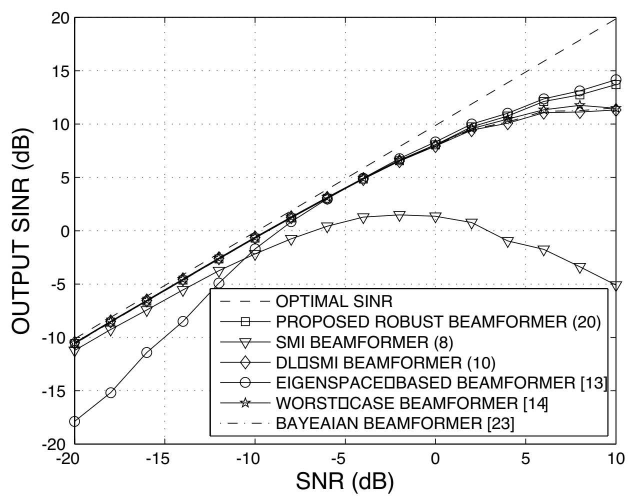

Figure 4. Example 3: Output SINR versus SNR.

5. CONCLUSION

A robust beamforming based on equivalent DOAs method was proposed in this paper. All the factors causing the steering vector uncertainties are regarded as the DOA uncertainty only, and the equivalent DOAs can be estimated one by one under the assumption that the elements of steering vector are uncorrelated with each other. Simulation results show better performance of the proposed beamformer as compared with several popular robust adaptive beamforming algorithms.

ACKNOWLEDGMENT

This work was supported by Natural Science Foundation of China (No. 60604029), Natural Science Foundation of Zhejiang Province (No. Y106384) and China Post Doctoral Science Foundation (No. 20060400313).

REFERENCES

1. Singh, A. K., P. Kumar, T. Chakravarty, G. Singh, and S. Bhooshan, "A novel digital beamformer with low angle resolution for vehicle tracking radar," *Progress In Electromagnetics Research*, PIER 66, 229–237, 2006.
2. Dessouky, M., H. Sharshar, and Y. Albagory, "A novel tapered beamforming window for uniform concentric circular arrays," *J. of Electromagn. Waves and Appl.*, Vol. 20, No. 14, 2077–2089, 2006.
3. Kim, J. W. and C. K. Un, "An adaptive array robust to beam pointing error," *IEEE Trans. Signal Processing*, Vol. 40, No. 6, 1582–1584, 1992.
4. Jablon, N. K., "Adaptive beamforming with the generalized sidelobe canceller in the presence of array imperfections," *IEEE Trans. Antennas Propagat.*, Vol. AP-34, No. 8, 996–1012, 1986.
5. Ringelstein, J., A. B. Gershman, and J. F. Böhme, "Direction finding in random inhomogeneous media in the presence of multiplicative noise," *IEEE Signal Processing Lett.*, Vol. 7, No. 10, 269–272, 2000.
6. Hong, Y. J., C. C. Yeh, and D. R. Ucci, "The effect of a finite-distance signal source on a far-field steering applebaum array — two-dimensional array case," *IEEE Trans. Antennas Propagat.*, Vol. 36, No. 4, 468–475, 1988.
7. Pedersen, K. I., P. E. Mogensen, and B. H. Fleury, "A stochastic model of the temporal and azimuthal dispersion seen at the base station in outdoor propagation environments," *IEEE Trans. Veh. Technol.*, Vol. 49, No. 2, 437–447, 2000.
8. Gershman, A. B., "Robust adaptive beamforming in sensor arrays," *Int. J. Electron. Commun.*, Vol. 53, 305–314, 1999.
9. Mouhamadou, M., P. Vaudon, and M. Rammal, "Smart antenna array patterns synthesis: null steering and multi-user beamforming by phase control," *Progress In Electromagnetics Research*, PIER 60, 95–106, 2006.

10. Guo, B., Y. Wang, J. Li, P. Stoica, and R. Wu, "Microwave imaging via adaptive beamforming methods for breast cancer detection," *J. of Electromagn. Waves and Appl.*, Vol. 20, No. 1, 53–63, 2006.
11. Vescovo, R., "Beam scanning with null and excitation constraints for linear arrays of antennas," *J. of Electromagn. Waves and Appl.*, Vol. 21, No. 2, 267–277, 2007.
12. Cox, H., R. M. Zeskind, and M. H. Owen, "Robust adaptive beamforming," *IEEE Trans. Acoust., Speech, Signal Processing*, Vol. ASSP-35, No. 10, 1365–1376, 1987.
13. Chang, L. and C. C. Yeh, "Performance of DMI and eigenspace-based beamformers," *IEEE Trans. Antennas Propagat.*, Vol. 40, No. 11, 1336–1347, 1992.
14. Vorobyov, S. A., A. B. Gershman, and Z. Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Processing*, Vol. 51, No. 2, 313–324, 2003.
15. Li, J., P. Stoica, and Z. Wang, "On robust capon beamforming and diagonal loading," *IEEE Trans. Signal Processing*, Vol. 51, No. 7, 1702–1715, 2003.
16. Lorenz, R. G. and S. P. Boyd, "Robust minimum variance beamforming," *IEEE Trans. Signal Processing*, Vol. 53, No. 5, 1684–1696, 2005.
17. Besson, O., A. A. Monakov, and C. Chalus, "Signal waveform estimation in the presence of uncertainties about the steering vector," *IEEE Trans. Signal Processing*, Vol. 52, No. 9, 2432–2440, 2004.
18. Lam, C. J. and A. C. Singer, "Adaptive bayesian beamforming for steering vector uncertainties with order recursive implementation," *Proc. ICASSP'06*, 997–1000, Toulouse, France, 2006.
19. Capon, J., "High resolution frequency-wavenumber spectral analysis," *Proc. of the IEEE*, Vol. 57, No. 8, 1408–1418, 1969.
20. Reed, I. S., J. D. Mallett, and L. E. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. AESC10, No. 6, 853–863, 1974.
21. Besson, O. and F. Vincent, "Performance analysis of beamformers using generalized loading of the covariance matrix in the presence of random steering vector errors," *IEEE Trans. Signal Processing*, Vol. 53, No. 2, 452–459, 2005.
22. Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd edition, McGraw Hill Company, 1991.

23. Bell, K. L., Y. Ephraim, and H. L. Van Tress, "A Bayesaian approach to robust adaptive beamforming," *IEEE Transactions on Signal Processing*, Vol. 48, No. 2, 386–398, 2000.

24. Kay, S. M., *Modern Spectral Estimation*, Prentice-Hall, Englewood Cliffs, NJ, 1988.