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PERFORMANCE IMPROVEMENT IN AMPLITUDE SYNTHESIS OF UNEQUALLY SPACED ARRAY USING LEAST MEAN SQUARE METHOD

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Abstract—In this paper, an efficient method to obtain the elements current distribution for a non uniformly spaced array is presented. For a given far field pattern, after sampling the array factor the proposed method uses the least mean square error technique to solve the system of equations rather than solving the previously published Legendre function method. It's shown that the average side lob level obtained by this proposed method is some 5 dB lower in comparison with the existing Legendre function method of solution. If the Legendre function method published in the literature is to be used to solve for the current distribution, in the final part of this paper, a criteria on how to choose suitable vectors that would result in a 3 dB lower side lobe level performance will be provided.

1. INTRODUCTION

Over the last decade non uniformly spaced arrays have received a lot of attention. This is mainly due to the fact that the degree of freedom that such arrays provide results in reduction in array size, weight and number of elements.

The original work done on non uniformly spaced array pattern synthesis goes back to Unz [1]. In that paper the current distribution for the array geometry was obtained with matrix formulation. Following that, Harrington, [2], worked on displacement of elements from the uniformly spaced case and finds the appropriate displacement by minimizing the SLL. In this way the side lobe level reduces to about $2/N$ times the intensity of the main lobe (where N is the total number of elements) without increasing the main lobe beam widths. Ishimaro [3], used the Poisson sum expansion to design an unequally spaced uniform amplitude array with any desired SLL along with grating lobe suppression.

Besides the analytical methods, several techniques based on optimization and iterative processes for minimizing SLL have also been reported. In [4] the density of elements located within a given array length is made proportional to the amplitude distribution of the conventional equally spaced array. [5] used statistical thinning of arrays with quantized element weights to reduce side lobe level considerably in large circular arrays. Another algorithm that has been used recently is the Genetic Algorithm for optimizing the array spacing [6, 7]. An optimization method based on real-coded genetic algorithm (GA) with elitist strategy is represented in [7]. This method is used for thinning a large linear array of uniformly excited isotropic antennas to yield the maximum relative sidelobe level (SLL) equal to or below a fixed level and the percentage of thinning is always kept equal to or above a fixed value.

An analytic method that enables a designer to determine for a given pattern the appropriate element spacing and a given array current distribution is by means of a Legendre transformation of the array factor, as given by Kumar [8]. [9] presents a new pattern synthesis algorithm for arbitrary arrays based on adaptive array theory. With this algorithm, the designer can efficiently control both main lobe shaping and side lobe levels. In comparison to Olen and Compton's method [10], the new algorithm provides a great improvement in main lobe shaping control.

An efficient method based on bees algorithm (BA) for the pattern synthesis of linear antenna arrays with the prescribed nulls is presented in [11]. Nulling of the pattern is achieved by controlling only the amplitude of each array element and numerical examples of Chebyshev pattern with the single, multiple and broad nulls imposed at the directions of interference are given to show the accuracy and flexibility of the BA in this paper.

Another method for the pattern synthesis of the linear antenna arrays with the prescribed null and multi-lobe beam-forming is based on controlling of phase. In this method multi-lobe pattern and adaptive nulling of the pattern is achieved by controlling only the phase of each array element. The method is based on the Sequential Quadratic Programming (SQP) algorithm and the linear antenna array synthesis was modeled as a multi-objective optimization problem [12]. In [13], the synthesis of linear arrays that produce radiation patterns with arbitrary envelopes is considered. A newly developed point-matching method is used to obtain a set of excitation coefficients for a linear array with nonisotropic elements and with nonuniform spacing between elements, that generates a desired radiation pattern.

In this paper, based on the work of Kumar, for non uniformly spaced arrays with $2N + 1$ elements, an analytic method in matrix form to obtain the elements current distribution is presented. For a given pattern, after sampling the array factor at M points the proposed method uses the Least Mean Square error technique to solve the system of M equations in N variables. The technique results in a better pattern synthesis with lower side lobe level performance in comparison with the existing Legendre function method of solution. Also, consideration to choose suitable vectors for the Legendre function method of solution will be provided.

2. THEORETICAL BACKGROUND

2.1. The LMS Method

The schematic of non uniform linear $2N + 1$ element array is shown in Fig. 1. The goal is to obtain the current distribution from desired pattern for the prescribed geometry.

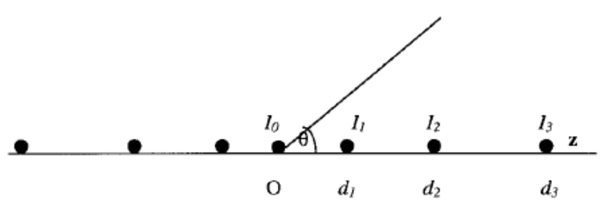


Figure 1. Geometry of non uniformly spaced linear symmetric array.

The theory behind the proposed method is based on the work of Kumar [8] that uses Legendre functions. Initially, the Kumar method modified in matrix form will be presented and show that with solving the system of M equations in N variables by pseudo inverse method leads to a less error and better SLL than that of [8].

The array pattern related to Fig. 1 with uniform quantization is

$$E(u_m) = \sum_{n=0}^{N} I_n \cos(m\beta_n), \quad m = 0, \dots, M - 1 \tag{1}$$

where

$$u = \cos(\theta), \quad 0 \le \theta \le \pi$$

This pattern is equal to the desired pattern at quantized points, i.e.,

$$E(u_m) = E_d(u_m) \tag{2}$$

which in matrix form is represented as

$$\begin{bmatrix} E(u_0) \\ E(u_1) \\ \\ \\ E(u_{M-1}) \end{bmatrix} = \begin{bmatrix} 1 & 1 & & 1 \\ \cos\beta_1 & \cos\beta_2 & & \cos\beta_N \\ & & & \\ & & & \\ \cos(M-1)\beta_1 & & & \cos(M-1)\beta_N \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \\ \\ I_N \end{bmatrix} \tag{3}$$

Kumar [8] obtains the current distribution from the element spacing by transforming the two sides of Equation (1) to Legendre function via

$$\begin{aligned} F(\alpha_p) &= \sum_{m=0}^{M-1} \varepsilon_m E_d(u_m) P_{m-1/2}(\cos\alpha_p) \\ p &= 0, 1, 2, \ldots, N \end{aligned} \tag{4}$$

It is obvious that this is equivalent to the product of the following matrix with that of (3)

$$\begin{bmatrix} P_{-1/2}(\cos\alpha_1) & P_{1-1/2}(\cos\alpha_1) & . & . & P_{M-1/2}(\cos\alpha_1) \\ P_{-1/2}(\cos\alpha_2) & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ P_{-1/2}(\cos\alpha_N) & . & . & . & P_{M-1/2}(\cos\alpha_N) \end{bmatrix} \tag{5}$$

using the limiting relation

$$\begin{aligned} f(\beta, \alpha) &= \sum_{m=0}^{\infty} \varepsilon_m P_{m-1/2}(\cos\alpha) \cos(m\beta) \\ &= \begin{cases} [2/(\cos\beta - \cos\alpha)]^{1/2} & 0 \le \beta < \alpha \\ 0 & \alpha < \beta < \pi \end{cases} \end{aligned} \tag{6}$$

And choosing the α vector inside of the β vector we can get the $I_n's$ from the following Equation (8)

$$\begin{matrix} \beta & \textit{space} & \beta_1 & \beta_2 & \ldots & \beta_N \\ \alpha & \textit{space} & \alpha_1 & \alpha_2 & \ldots & \alpha_N \end{matrix} \tag{7}$$

$$\begin{bmatrix} F(\alpha_1) \\ F(\alpha_2) \\ \\ \\ F(\alpha_N) \end{bmatrix} = \begin{bmatrix} f(\alpha_1,\beta_1) & 0 & 0 & . & 0 \\ f(\alpha_2,\beta_1) & f(\alpha_2,\beta_2) & 0 & . & 0 \\ . & . & & & 0 \\ & & & & 0 \\ f(\alpha_N,\beta_1) & f(\alpha_N,\beta_2) & . & . & f(\alpha_N,\beta_N) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \\ \\ I_N \end{bmatrix} \tag{8}$$

With careful view of expression (3) it is realized that finding of $I_n's$ is same as solving the system with M equations in N variables. Such a system is M super planes in R^n space that do not necessarily intersect at the same point, i.e., finding an N dimension vector that has minimum distance from maximum number of planes, as such the least error can be obtained.

Rewriting (3) as

$$[E] = [A][I] \tag{9}$$

We can obtain the $[I_n]$ vector such that it minimizes the following error

$$[R] = [E] - [A][I] \tag{10}$$

where A is a non square "tall" matrix, the unknown I is a "short" vector, and E is a "tall" vector as stated bellow

$$\begin{aligned} E(u_1) &= I_1 \times 1 + I_2 \times 1 + \ldots + I_N \times 1 \\ E(u_2) &= I_1 \times \cos\beta_1 + I_2 \times \cos\beta_2 + \ldots + I_N \times \cos\beta_N \\ &\cdot \\ &\cdot \\ E(u_M) &= I_1 \times \cos(M-1)\beta_1 + \ldots + I_N \times \cos(M-1)\beta_N \end{aligned} \tag{11}$$

which denotes an over determined system of M equations because $M > N$. Also recall that the $\beta_i's$ and $E(u_i)'s$ are given and that the $I_i's$ are the unknowns. By forcing N of the equations to be exactly satisfied we may cause the others to exhibit large errors. Therefore, we would like to choose unknown values that allow all of the equations to be approximately satisfied instead of forcing $N$of them to be exactly satisfied. The least-square problem, Equation (10) becomes

$$\min_c \|E - AI\|_2 \tag{12}$$

where $\|.\|_2$ represents the norm of the error vector. Representing each column of the $[A]$ matrix as $\{a_1, a_2, a_3, \ldots, a_N\}$ we wish to combine these vectors with those of [In] linearly in order to obtain the best possible approximation to a given vector E. In that case, a matrix A with columns given by the a_i and a vector$I$ whose entries are the unknown coefficients I_i can be defined. There is a geometric interpretation to the general least square problem. We are seeking an element of the subspace$S$spanned by the a_i which is closest to E. The solution is the projection of E on to S. Therefore, the error vector should be orthogonal to$S$which is equivalent to being orthogonal to each of the a_i. Thus, the optimal solution vector$I$must satisfy $a_i.(AI-E) = 0$ for all i or equivalently, in matrix form, $A^T(AI-E) = 0$ or

$$A^T AI = A^T E \tag{13}$$

Note that the independence of the columns of A implies the invertibility of $A^T A$. Now, we have

$$I = (A^T A)^{-1} A^T E \tag{14}$$

$$A^T AI = A^T E \tag{15}$$

$$I = \text{Peseudo Inverse}(A)E \tag{16}$$

2.2. Consideration for Choosing α Vector in Legendre Method for Better SLL

In previous section, Kumar method [8] of Legendre functions was presented in matrix form. In [8] a constant value for α vector has been used and no criteria in that paper was given on how to choose this vector. In this section, we present how to choose the α vector for a better SLL. As given in (13) if we multiply both sides of the matrix Equation (3) with the transpose of matrix A and solve the resulting equation with the pseudo inverse method results in the least error and thus lower SLL.

Therefore in Equation (2) we can choose α values such that the matrix P becomes close to the transpose of matrix A, i.e.,

$$[P] \approx [A]^T \tag{17}$$

Or

$$P_{m-1/2}(\cos\alpha_p) = \cos(m-1)\beta_p \tag{18}$$

If we look carefully at the limiting relation of (3) and from formula bellow for the f function,

$$P_{m-1/2}(\cos\alpha_p) = \frac{1}{\pi}\int_0^{\alpha_p} \frac{\cos(m\theta)}{\sqrt{\cos\theta - \cos\alpha_p}} d\theta \tag{19}$$

realize that the half integer m order Legendre functions are the m'th harmonic polynomials in cosine Fourier series expansion of function f.

The schematic of the f function is given in Fig. 2. As can be seen from this figure, for $\beta_i's$ values near the α_p limit, the f function is very much like a Dirac delta function.

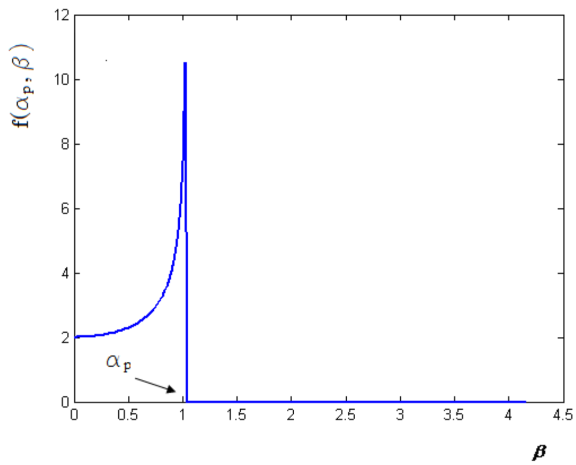


Figure 2. Schematic of the f function.

The m'th harmonic of Dirac delta function in cosine expansion is as follow:

$$\delta(\beta - \alpha) = \sum_{m=0}^{\infty} \cos((m-1)\alpha) \times \cos((m-1)\beta) \tag{20}$$

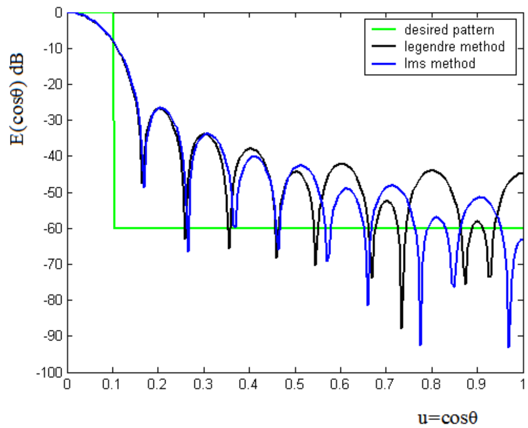
Therefore by choosing the β_p very close to α_p we can get

$$\begin{aligned} &\alpha_p \approx \beta_p \\ &P_{m-1/2}(\cos\alpha_p) = m'\text{th harmonic of } f \text{ function} \approx \\ &m'\text{th harmonic of } \delta \text{ function} \approx \cos((m-1)\alpha_p) \\ &\rightarrow [P] \approx [A]^T \end{aligned} \tag{21}$$

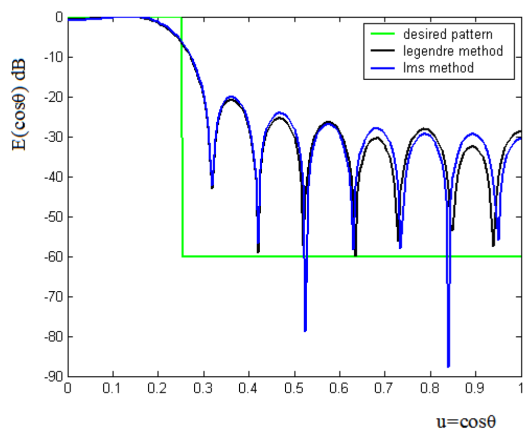
The following figures show the peak side lobe level for several α vector, and as it seen for the β vector near α vector, the peak side lobe level is much better.

3. SIMULATED RESULTS

Based on the above theory, the LMS method, the array of Fig. 1 with $N = 9$ (i.e., 19 elements) and with $N = 19$ (39 elements) have been



(a)



(b)

Figure 3. Desired pattern and the simulated patterns for LMS and Kumar methods $N = 9$. Desired pattern (green), simulated patterns: Legendre method (black), LMS method (blue), (a) narrow pattern and (b) wide pattern.

simulated and results are shown in Figs. 3 and 4, respectively. In each of these figures, the part (a) shows the relevant results for a narrow desired pattern while the part (b) shows the results for a wide desired pattern. Also shown on these figures are the results obtained by the authors based on the Kumar [8] method. It is obvious from these figures that the proposed method gives a better SLL, on average better than 5 dB. If a wider or narrower desired pattern is given it can be shown that the simulated pattern that can be obtained from the LMS method would be closer to the desired pattern in comparison to the Legendre method of Kumar [8].

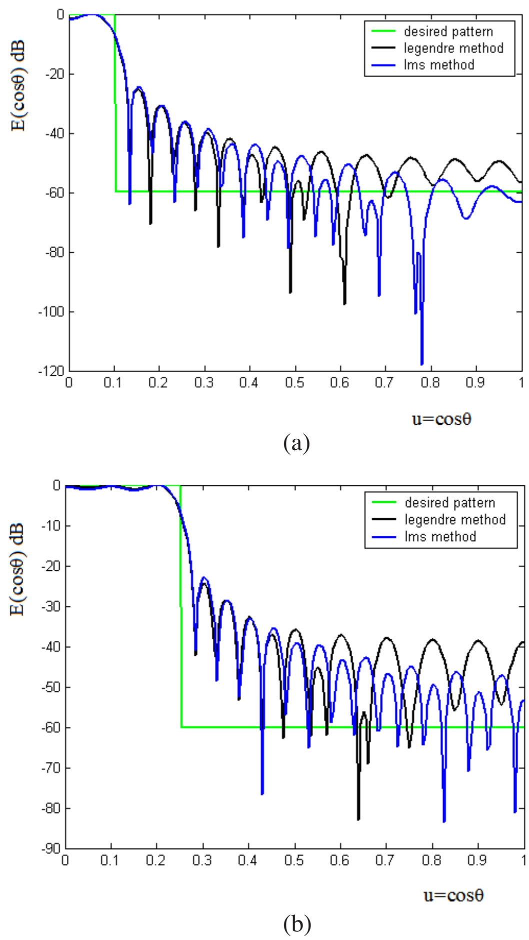


Figure 4. Desired pattern and the simulated patterns for LMS and Kumar's method $N = 19$. Desired pattern (green), simulated patterns: Legendre method (black), LMS method (blue), (a) narrow pattern and (b) wide pattern.

As stated in Section 2.2 above, if the Legendre function method published in [8] is to be used to solve for the current distribution, a set of new α vectors as given by Equation (21) will result in better SLL. Fig. 5 shows the peak side lobe level for several α vectors. It can be

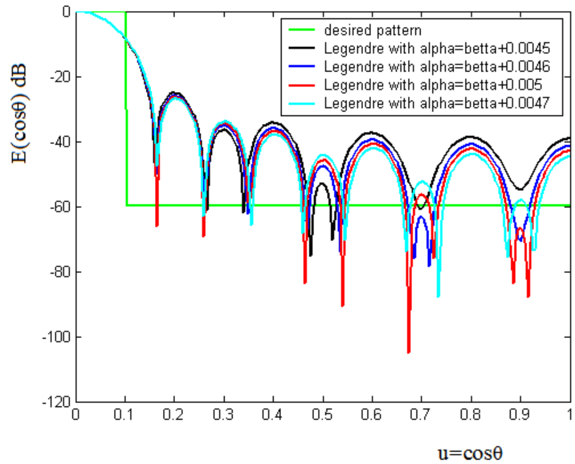


Figure 5. Desired and Simulated patterns for 4 different alphas in legendre method.

seen from this figure that the closer β vector becomes to the α vector, the peak side lobe level reaches that of the desired pattern.

4. CONCLUSION

A simple and more accurate method of determining the current distribution in an unequally spaced linear array is presented in this paper. The proposed method uses the least mean square error technique to solve the system of equations resulting in a better pattern synthesis and result in lower side lobe level about 5 dB in comparison with the existing Legendre function method of solution. Also, if the Legendre function method is used to solve for the current distribution, a criteria on how to choose suitable vectors has been provided that results in a better SLL, about 3 dB.

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