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GROUND CLEARANCE FOR HF AND LOWER FREQUENCY ANTENNA INSTALLATIONS

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Abstract—Setting up ground based antennas for operation in the HF and longer wavelength bands frequently involves clearing large areas of land for the installation of ground mats to provide a high conductivity return path for the displacement currents. In moving from the cleared area to the virgin scrubland beyond, which is assumed to be forested with bushes and small trees, there is the possibility of an abrupt change in surface properties at the boundary resulting from the discontinuity in the vegetation which at longer wavelengths can be modelled as a change in surface impedance. By modelling the trees and bushes as point dipole moments, the aim of this paper is to estimate the significance of any such effect in terms of the gross physical properties of the vegetation. The result is to show that in normal circumstances the effect can be expected to be slight. A solution to this problem has application in helping determine the environmental footprint of the antenna installation, the amount of land that needs to be cleared to satisfactorily accommodate it.

1. INTRODUCTION

Frequently in the semi-arid areas of Australia the ground cover consists of large numbers of non-contiguous bushes or small trees a metre or two or maybe a little more in height, probably also the case in similar areas in other regions of the world. When this scrubland is cleared for the installation of antennas, not unusually the result is to create a more or less well defined line of demarcation between the cleared area and the hinterland beyond. An example is Fig. 1 which shows an aerial view of one of the transmitting sites for the Jindalee over-the-horizon radar which makes use of the ionosphere as a means of detecting targets

Figure 1. Jindalee transmitter site at Hart's Range, near Alice Springs, Australia.

at extended distances [1]. Viewed in the more integrated way that is possible from the air, the boundary is clearly visible.

A question that arises is whether this boundary has any effect on the radiation pattern of the antenna erected on the cleared area and, to any extent that it does, how much land needs to be cleared to acceptably minimise it. This is important both in terms of land clearance costs and the environmental impact that the facility to be built on the site will have. Taken in the context of an RF environment using decametric or longer wavelengths (i.e. frequencies up to and including the HF band), the result of clearing can be modelled as creating an abrupt change in surface impedance at the cleared boundary that opens the possibility of generating a diffraction field that will produce interference ripples in the primary antenna pattern.

One way of making an estimate of the strength of any diffracted ray is to treat the ground as a 180 degree impedance wedge, the "edge" of which coincides with the line of surface impedance discontinuity [2]. If extending out from the cleared area, the ground in its native state, being of the same material as the cleared area, is not rough in terms of the Rayleigh criterion [3], then any change in surface impedance at this edge would be expected to be slight. If in addition the discontinuity in the vegetation also proved to be of little or no account, then the effect of the boundary in a radiation sense might be reckoned as negligible. The aim here is to test this proposition in relation to the vegetation.

2. THEORETICAL DEVELOPMENT

The bushes comprising the scrub grow at random and are not of uniform size. Having to take this into account leads to a very complex problem that there is every incentive to simplify. Here we will assume that the bushes are all set equal at their average size, a term to which we will give more precise meaning shortly, and grow at the nodes of a square grid with a spacing chosen to ensure that the number of bushes per unit area is set at the average for the landscape involved. In this connection it is easy to show that this implies a mesh size

$$a = \frac{1}{\sqrt{\langle N \rangle}} \tag{1}$$

where $\langle N \rangle$ is the average number of bushes per unit area. If for example $\langle N \rangle = 100,000/\mathrm{km}^2$ (equivalent to 1,000/hectare) then $a \approx 3\,\mathrm{m}$, or about 100 bushes per square wavelength at 10 MHz. This suggests that our proposed model will be descriptive of the scrubland up to and including the HF band, all that was of interest in the original motivation for this work.

Such regularity will produce only coherent scatter that may be thought of as the coherent component of what in practice will be a mixture of coherent and incoherent scatter, the latter being an outcome of the randomness that lies at the root of the problem [3]. Inasmuch as averaged over a statistically significant ensemble of such cases it is only the coherent component that has the ability to produce a net phase interference with the primary antenna pattern, this may be a simplification bought at no great price. Moreover, since the aim is to decide whether or not the vegetation is a factor to be reckoned with, we will consider only normal incidence.

These simplifications allow us to cut a section from an incident plane polarised, plane wave and bound it in a TEM-mode waveguide having pairs of perfect electric and magnetic conductors on opposite faces, the dimensions of the waveguide being chosen to coincide with unit cell in the bush matrix. We then have the situation shown in Fig. 2 where the waveguide is terminated in a single symmetrically placed bush growing from the interface of a semi-infinite dielectric plug. It is this case that we will now analyse.

Consider then Fig. 2. The longitudinal (z) coordinate is measured from an origin at the centre of the interface with the incoming wave incident from the top. The incident electric and magnetic fields can be written as

$$\boldsymbol{E}_i = V^+ \boldsymbol{e} e^{-jk_0 z} \tag{2a}$$

$$\boldsymbol{H}_i = I^+ \boldsymbol{h} e^{-jk_0 z} \tag{2b}$$

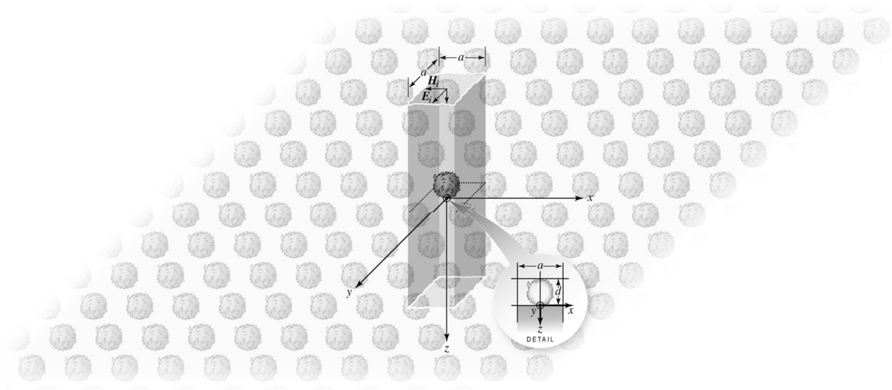

Figure 2. Idealised bushland setting with normally incident wave.

where V^+, I^+ are the mode voltage and current respectively, $\boldsymbol{e}, \boldsymbol{h}$ are the normalised mode functions and k_0 is the free space wave number. These normalised wave functions are

$$\boldsymbol{e} = \hat{y}\frac{\sqrt{\eta}}{a} \tag{3a}$$

$$\boldsymbol{h} = -\hat{x}\frac{1}{a\sqrt{\eta}} \tag{3b}$$

where $\eta = \sqrt{\varepsilon_0/\mu_0}$ is the wave impedance, here the impedance of free space, and ε_0, μ_0 are its permeability and permittivity respectively. Note that these wave functions have been normalised so that, taken over the cross-section of the guide, $\int\limits_S \boldsymbol{e} \cdot \boldsymbol{e} ds = \eta$, $\int\limits_S \boldsymbol{e} \times \boldsymbol{h} \cdot ds = 1$ [4].

The bush will be modelled as a dielectric sphere osculating with the interface. Provided that in all of its dimensions it remains small compared with the wavelength, the exact shape and interior homogeneity of the bush is unimportant and an average bush is chosen to be one such that the sphere has the same volume as the average of the envelopes of the actual bushes. Let its diameter be d. Electromagnetically then the bush can be represented as a point dipole moment located $z = -d/2$ on the incidence side of the interface.

The electric dipole moment that the incident wave will induce in the bush is thus [5]

$$\boldsymbol{p} = \varepsilon_0 \overline{\boldsymbol{A}}_e \cdot \boldsymbol{E}_i \tag{4}$$

so that in terms of $\overline{\boldsymbol{A}}_e = A_{exx}\hat{x}\hat{x} + A_{eyy}\hat{y}\hat{y} + A_{ezz}\hat{z}\hat{z}$, a dyadic determined by the properties of the bush and the proximity of its neighbours,

$$\boldsymbol{p} = V^+ A_{eyy} \boldsymbol{e} e^{j\frac{1}{2}k_0 d} \tag{5}$$

By application of the Lorentz reciprocity theorem, we then have for the reflected wave

$$V^{-} = -\frac{1}{2} j\omega \boldsymbol{e} \cdot \boldsymbol{p} \tag{6}$$

giving as the reflection coefficient

$$\Gamma_b = \frac{V^-}{V^+} = -\frac{jk_0 A_{eyy} e^{jk_0 d}}{2a^2} \tag{7}$$

A_{eyy} is a function of the polarisability of the dielectric sphere and a proximity effect due to its neighbours and may be written as [5]

$$A_{eyy} = \frac{\alpha_{yy}}{1 - C\alpha_{yy}} \tag{8}$$

where C is a proximity factor to be determined and α_{yy} is a component of the polarisability dyadic of a dielectric sphere, given in [6, 7] as

$$\alpha_{yy} = \frac{1}{2}\pi d^3 \left(\frac{\langle K_{eb} \rangle - 1}{\langle K_{eb} \rangle + 2} \right) \tag{9}$$

where $\langle K_{eb} \rangle$ is the dielectric constant averaged over its envelope volume for the bush being modelled.

$\langle K_{eb} \rangle$ can be determined in terms of the properties of the bush by considering its mass to be uniformly distributed over its envelope volume, likely to be a good working assumption given that the bush as a whole (and therefore *a fortiori* any element of it, such as a leaf or branch) is small compared with the wavelength. Bushes are generally observed to float when immersed in water, a process that also fills most of the interstitial space, much the greater part of the envelope volume. When then mass is distributed over envelope volume, the result is a medium having an average density of maybe two orders or so less than that of water. The Clausius-Mossotti relation [6] (aka the Lorentz-Lorenz relationship [8]) applies well to such dilute media.

If we make the further observation that because of the high dielectric constant of water, an order or so larger than most other natural substances, it will be the water content of the plant that contributes most of the polarisation current that gives rise to its dipole moment then we can say that the "participating mass" of the bush is

$$M = p_w M_b \tag{10}$$

where M_b is the superstructure mass (total mass less that of the buried root) of the bush and p_w is the proportion of that that is water. p_w will vary with the season, with whether or not it has rained recently, but in typical semi-arid bushland might be in the order of 0.8.

Then applying these simplifications to the case in point, we have

$$\left(\frac{\langle K_{eb}\rangle - 1}{\langle K_{eb}\rangle + 2}\right) = \frac{p_w M_b}{\frac{1}{6}\pi d^3 \rho_w}\left(\frac{K_{ew} - 1}{K_{ew} + 2}\right) \tag{11}$$

where ρ_w and K_{ew} are respectively the density and dielectric constant of water. Hence we have as the bush polarisability

$$\alpha_{yy} = \frac{3p_w M_b}{\rho_w}\left(\frac{K_{ew} - 1}{K_{ew} + 2}\right) \tag{12}$$

and in view of the fact that $|K_{ew}| \gg 1$ further simplification is possible, leading to

$$\alpha_{yy} = \frac{3p_w M_b}{\rho_w} \tag{13}$$

To proceed further, we need a value for the proximity factor C, a measure of the interaction field produced at the dipole by its images in the waveguide walls and the dielectric plug in which the guide is terminated. The conventional technique for doing this is to use a quasi-static approach that involves solution of an equivalent electrostatics problem. In this we have two infinite arrays of dipoles, one in the transverse plane containing the physical dipole and accounting for imaging in the waveguide walls and the other the mirror image of the first in the extension to a plane interface of the dielectric plug. Doing this, of course, is nothing more than to return to the sanitised version of the original problem in which bushes of equal size are deployed in a regular array above an infinite ground. Provided that the images are closely enough spaced in terms of the wavelength, an electrostatic argument will give a good approximation for the proximity factor.

All dipoles in the first curtain will be of equal strength, positively aligned in the y coordinate direction and arrayed at the nodes of a square grid of size a. Imaging of an electrostatic charge in a plane dielectric interface is treated in [6]. Since dipoles are merely pairs of closely spaced, equal magnitude but oppositely signed charges, the result for isolated charges readily extends to this case. On that basis it is easily shown that in the half-space containing the bushes, the effect of imaging in the interface is accounted for by an infinite array that mirrors the first, parallel to and spaced d from it. However the strength of each of the dipoles in this array is $-(K_{eg} - 1)/(K_{eg} + 1)$ of that of its counterpart in the other curtain, where K_{eg} is the dielectric constant of the plug, the ground beneath the bushes. C will therefore be determined by the superposition of the separate effects of the two arrays and is able to be written as

$$C = \frac{1}{4\pi}\left[S_1 - \left(\frac{K_{eg}-1}{K_{eg}+1}\right)S_2\right] \tag{14}$$

Using the methodology of [5], it is easily shown that

$$S_1 = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\frac{2(ma)^2-(na)^2}{[(ma)^2+(na)^2]^{5/2}} \tag{15}$$

$$S_2 = -\frac{1}{d^3} + \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\frac{2(ma)^2-(na)^2-d^2}{[(ma)^2+(na)^2+d^2]^{5/2}} \tag{16}$$

with the proviso that the term with $n = m = 0$ is excluded from both sums. The $-1/d^3$ term that precedes the double sum in eqn. (16) represents direct imaging of the physical dipole in the dielectric interface, perhaps to be thought of as the dominant image, whereas all other terms contained in both the sums themselves involve interactions between bushes.

The significance of these results to the problem at hand can be assessed by using them to place a bound on C. It is shown in [5] that the first of these series has a positive sum and can be reduced to closed form first by using the Poisson summation formula and then approximating the rapidly convergent double series that constitutes part of the result by its dominant term, a highly accurate approximation in this instance. The outcome is to show that $S_1 = 0.3596/a^3$. Comparing the two series above, it is clear that for any $d > 0$, term for term the second is always smaller than the first and so overall must have a smaller sum. In the calculation of C, they are subtractive, so that taken together the result is a net positive contribution that is less than sum of the first, although due to the multiplier $(K_{eg}-1)/(K_{eg}+1) < 1$, larger than their direct difference. It therefore follows that $C < \frac{1}{4\pi d^3}\left(1 + \frac{0.3596}{(a/d)^3}\right)$. Moreover since the bushes can be at no lesser spacing than their diameter (i.e., $a/d \geq 1$), it must be the case that $C < 1.3596/4\pi d^3$.

Using eqn. (13) then leads straight away to the conclusion that $C\alpha_{yy}$ is of the same order as the ratio of the average density of the bush to the density of water, and so is negligible. We are therefore entitled to set $A_{eyy} = \alpha_{yy}$ and to treat the bushes as though they were totally isolated, neither their interaction with the ground beneath them nor with each other being significant. At this point it can also be remarked that because interaction between the bushes is negligible, provided that on average they remain dense enough for the other assumptions to continue to hold, whatever their actual arrangement, their regularisation onto a square grid, though useful as a crutch to get started, is likely not critical to the final outcome.

In the light of these considerations, it then follows from eqns. (1), (7) and (13) that

$$\Gamma_b = -\frac{j3k_0 p_w \langle N \rangle M_b}{2\rho_w} e^{jk_0 d} \tag{17}$$

If further we note that $\langle N \rangle M_b = M_a$, the mass of vegetation per unit area, then this reduces finally to the particularly simple form

$$\Gamma_b = -\frac{j3k_0 p_w M_a}{2\rho_w} e^{jk_0 d} \tag{18}$$

Inserting as representative a probably rather generous $M_a \approx 10\,\mathrm{kg/m^2}$ (equivalent to 100 tons/hectare), at a frequency of 10 MHz ($\lambda = 10\,\mathrm{m}$), this suggests something like $|\Gamma_b| \approx O(10^{-2})$ for the magnitude of the reflection coefficient.

We are now ready to return to the problem of finding the total reflected wave in the equivalent waveguide. Essentially this has two components, one due to the bush and the other the much larger reflection from the dielectric interface. Solving this problem exactly involves an infinite regress as back and forth reflection between the dipole and the interface is taken into account. While the result is not hard to find - it involves the sum of an infinite geometric series — it is easier to use the foregoing electrostatics problem as a cue to simplification. If the interaction between the bush and the ground is not strong, the total reflection will be well enough approximated as the sum of those of the bush and interface each acting in isolation. At normal incidence the latter is [3, 4]

$$\Gamma_g = \frac{1 - \sqrt{K_{eg}}}{1 + \sqrt{K_{eg}}} \tag{19}$$

Even in semi-arid country it is likely that $|K_{eg}| > 5$ and so Γ_g can be expected to be rather more than an order larger than the bush contribution, allowing the latter to be treated as a differential increment to an overall reflection coefficient that encompasses both.

$z_s = Z_s/\eta$, the surface impedance normalised by the wave impedance of the guide, in this case also the impedance of free space, is related to total reflection coefficient $\Gamma \approx \Gamma_g + \Gamma_b$ by [4]

$$z_s = \frac{1 + \Gamma}{1 - \Gamma} \tag{20}$$

Under these circumstances it is easy to show that the change in surface impedance that results from the presence of the bushes is

$$\Delta Z_s = \eta \frac{dz_s}{d\Gamma}\bigg|_{\Gamma=\Gamma_g} \Delta\Gamma = \frac{2\eta\Gamma_b}{(1-\Gamma_g)^2} \tag{21}$$

Invoking the simplification that comes from assuming that $K_{eg} \gg 1$ and by substitution for the other quantities involved from previous equations then gives the change in surface impedance in terms of the gross physical properties of the bushland as

$$\Delta Z_s = \frac{1}{2}\eta\Gamma_b = -\frac{j3\eta k_0 p_w M_a}{4\rho_w} e^{jk_0 d} \tag{22}$$

On this basis, it is to be expected that $|\Delta Z_s|$ will be small, principally capacitive but probably not more than a few ohms.

In seeking to apply this result to the site clearance problem, it needs to be kept in mind that it has been derived only for the special case of normal incidence. As with the bare ground itself, the surface impedance of the ground cum vegetation can be expected to be a function of the angle of incidence of the incoming wave. However the foregoing would seem to be proof enough that in any circumstances the discontinuity in surface impedance between cleared and uncleared ground is not likely large enough to produce a significant diffracted wave at their boundary. Hence clearance of an area no bigger than required to contain the antenna (which, for vertical polarisation at HF is likely to include a substantial earth mat) plus whatever may be needed for boundary roadways would appear to be sufficient.

3. CONCLUSION

In this paper we have derived a simple formula that relates the gross physical properties of semi-arid bushland, its bio-mass per unit area and the water content of the bio-mass, to the change in surface impedance that results from its presence on otherwise bare ground. On the basis of this we have been able to conclude that the discontinuity in surface impedance that occurs at the boundary between an area cleared for the installation of an HF or longer wavelength antenna is primarily capacitive but likely not of sufficient magnitude to give rise to any significant diffracted wave at the line of demarcation.

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