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AN ANALYTIC FORMULA OF THE CURRENT DISTRIBUTION FOR THE VLF HORIZONTAL WIRE ANTENNA ABOVE LOSSY HALF-SPACE

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Abstract—An analytical formula of the current distribution for the VLF horizontal wire antenna located above the ground is presented in this paper. This formula is suitable for the VLF horizontal antenna which is fed at arbitrary position and with arbitrary loaded impedance at the end. In order to validate the analytical formula, a numerical code based on MoM is also developed. The comparison between the results obtained by two methods proves the validity of the analytical formula proposed in this paper.

1. INTRODUCTION

The horizontal wire antenna located above the earth has been used widely in VLF electromagnetic system [1–7]. Compared to the vertical monopole antenna [8, 9] which is typical of the most VLF transmitting antennas, the advantage of the horizontal antenna includes easy to be constructed and repaired, low cost, requiring no ground plane and so on. However, the radiation efficiency of this antenna is very low because of the negative image under the ground. So the horizontal antenna is more often used in receive system or used as the lash-up transmitting antennas. Seeley had investigated the radiation efficiency of a single horizontal VLF antenna and the array [10]. In his research, it had been indicated that the radiation efficiency can be increased by using closely spaced parallel array. In order to calculate the radiation efficiency, the current distribution is necessary. In Seeley's work, two special cases were considered. In the first case, the dipole was terminated at each end with its characteristic impedance. In this case, the current distribution was a traveling wave. In the second case, the resonant dipole fed at a current maximum was considered. In this case, the current distribution can be approximated with sinusoid function. The antenna fed at arbitrary position and with arbitrary end-loaded impedance was not discussed in Seeley's paper. Although the numerical methods, such as MoM, FDTD, FEM, can be used to model the VLF horizontal antenna, it is still useful and interesting to obtain an analytical formula of the current distribution. However, few studies were made in this field.

The purpose of this paper is to establish an analytical formula of the current distribution for VLF horizontal antenna. This new formula can be applied for the antenna which is fed at arbitrary position and with arbitrary end-loaded impedance. In order to verify the analytical formula, a numerical code based on MoM is also developed in this work to model the VLF horizontal antenna.

2. FORMULATION

Consider a horizontal wire antenna located above a planar ground as shown in Fig. 1. The earth is considered as an isotropic lossy medium with conductivity σ. The loaded impedance at the end of the antenna is denoted as Z_{L1} and Z_{L2} respectively. The values of the loaded impedance depend on the realistic status of the antenna. If the end is open, the loaded impedance is infinite. If the end is grounded well, the loaded impedance is zero. And if there is a grounding resistance at the end, the loaded impedance is equal to the grounding resistance. The grounding resistance can be obtained by measurement. The origin point is selected as the feed-position, and the position of the ends of the antenna are denoted as $-L_1$, L_2 respectively. The radius of the antenna is a, and the height of the antenna is h which is much less than the wave-length in free-space.

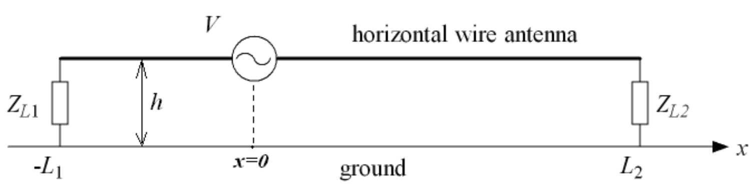


Figure 1. Horizontal wire antenna located above the earth.

Similar to the propagation of plane wave in the layered medium, the current at arbitrary point on the antenna can be considered as the sum of the contribution from excited source and the reflection at each end. Then the current can be written as follow

$$I(x) = I_0 e^{-jk_L|x|} + I_+ e^{-jk_L x} + I_- e^{jk_L x} \quad (1)$$

At the right of Equation (1), the first item I_0 is the contribution from the excited source, and can be considered as the primary part. The second item I_+ and the third item I_- are the contribution of the reflection at left end and right end of the antenna respectively, and can be considered as the scattering part. k_L is the wave number along the antenna. The wave number of the VLF horizontal antenna had been investigated by King in [11] and the k_L can be formulated as

$$k_L = k_0 \left\{ 1 - \frac{1}{\ln\left(\frac{2h}{a}\right)} \left[\ln(k_1 h) + \Upsilon - 0.5 + j\left(\frac{\pi}{2} - \frac{4}{3}k_1 h - \frac{2}{45}(2k_1 h)^3\right)\right]\right\}^{\frac{1}{2}} \tag{2}$$

where $\Upsilon = 0.57721$. k_0 is the wave-number in free-space and k_1 is the wave-number in the earth. The limitation of Equation (2) is that the height of the antenna should be much less than the wave-length.

There are three unknown coefficients I_0, I_+, I_- in Equation (1). They can be solved by utilizing the boundary condition of current at each end of the antenna. At the left end, by using the relation between incident current and reflect current, we can obtain the equation as

$$I_+ e^{jk_L L_1} = -\Gamma_1 \left[I_0 e^{-jk_L L_1} + I_- e^{-jk_L L_1} \right] \tag{3}$$

And at the right end, by using the relation between incident current and reflect current, we can obtain the equation as

$$I_- e^{jk_L L_2} = -\Gamma_2 \left[I_0 e^{-jk_L L_2} + I_+ e^{-jk_L L_2} \right] \tag{4}$$

where Γ_1, Γ_2 are the voltage reflect coefficients at the left end and right end respectively. And it is known that the voltage reflect coefficient and current reflect coefficient are inverse-phase each other.

Using Equations (3) and (4), we can solve the coefficients I_+ and I_- as

$$\begin{aligned} I_+ &= \frac{-\Gamma_1 \left[e^{-2jk_L L_1} - \Gamma_2 e^{-2jk_L (L_1+L_2)} \right]}{1 - \Gamma_1 \Gamma_2 e^{-2jk_L (L_1+L_2)}} I_0 \\ I_- &= \frac{-\Gamma_2 \left[e^{-2jk_L L_2} - \Gamma_1 e^{-2jk_L (L_1+L_2)} \right]}{1 - \Gamma_1 \Gamma_2 e^{-2jk_L (L_1+L_2)}} I_0 \end{aligned} \tag{5}$$

The next task is to solve the coefficient I_0. By using Equations (1), (4) and (5), the input current at the feed-position can be written as

$$I_{in} = I_0 + I_+ + I_- = \frac{\left(1 - \Gamma_1 e^{-2jk_L L_1}\right)\left(1 - \Gamma_2 e^{-2jk_L L_2}\right)}{1 - \Gamma_1 \Gamma_2 e^{-2jk_L (L_1+L_2)}} I_0 \tag{6}$$

The input current can also be calculated by using the input voltage V and the input impedance Z_{in},

$$I_{in} = \frac{V}{Z_{in}} \tag{7}$$

With Equations (6) and (7), the coefficient I_0 can be solved as

$$I_0 = \frac{V}{Z_{in}} \frac{1 - \Gamma_1 \Gamma_2 e^{-2jk_L(L_1+L_2)}}{\left(1 - \Gamma_1 e^{-2jk_L L_1}\right)\left(1 - \Gamma_2 e^{-2jk_L L_2}\right)} \tag{8}$$

The input impedance Z_{in} appearing in Equation (8) can be obtained by using the transmission-line method (TLM)

$$Z_{in} = Z_c \frac{Z_{L1} + Z_c \tan\left(k_L L_1\right)}{Z_c + Z_{L1} \tan\left(k_L L_1\right)} + Z_c \frac{Z_{L2} + Z_c \tan\left(k_L L_2\right)}{Z_c + Z_{L2} \tan\left(k_L L_2\right)} \tag{9}$$

where Z_c is the characteristic impedance of the VLF horizontal antenna and had been formulated in [6] as

$$Z_c = 60 \frac{k_L}{k_1} \ln \frac{2h}{a} \tag{10}$$

The reflect coefficients can be written as

$$\Gamma_1 = \frac{Z_{L1} - Z_c}{Z_{L1} + Z_c} \tag{11}$$

$$\Gamma_2 = \frac{Z_{L2} - Z_c}{Z_{L2} + Z_c} \tag{12}$$

Equation (1) is the analytical formula of current distribution that we want to obtain. And the coefficients in Equation (1) are given in (5), (8) respectively.

In order to verify the analytical formula, a numerical code based on MoM is developed to obtain the numerical results of the current distribution for VLF horizontal antenna. By using the boundary condition on the antenna, the electric field integral equation (EFIE) can be established as

$$V\delta(x) + I\left(-L_1\right) Z_{L1} \delta\left(x + L_1\right) + I\left(L_2\right) Z_{L2} \delta\left(x - L_2\right) = j\omega\mu_0 A_x + \frac{d}{dx}\Phi \tag{13}$$

where the x-component of vector magnetic potential A_x and the scalar electric potential Φ can be formulated as

$$A_x = \int_{-L_1}^{L_2} I\left(x'\right) G_A\left(\boldsymbol{r}, \boldsymbol{r}'\right) dx' \tag{14}$$

$$\Phi = \frac{1}{-j\omega\varepsilon_0} \int_{-L_1}^{L_2} \frac{d}{dx'} I\left(x'\right) G_V\left(\boldsymbol{r}, \boldsymbol{r}'\right) dx' \tag{15}$$

where $G_A(\boldsymbol{r}, \boldsymbol{r}')$ is the Green's function of vector magnetic potential and $G_V(\boldsymbol{r}, \boldsymbol{r}')$ is the Green's function of scalar electric potential. The Green's functions of the horizontal dipole in half-space had been presented in [12–14].

$$G_A = \frac{1}{4\pi}\left[\frac{e^{-jk_0r_0}}{r_0} + \int_{-\infty}^{\infty} \frac{k_\rho}{2jk_{z0}} R_{TE} e^{-jk_{z0}(z+z')} H_0^2(k_\rho \rho)\, dk_\rho\right] \quad (16)$$

$$G_V = \frac{1}{4\pi}\left[\frac{e^{-jk_0r_0}}{r_0} + \int_{-\infty}^{\infty} \frac{k_\rho}{2jk_{z0}} (R_{TE} + R_q)\, e^{-jk_{z0}(z+z')} H_0^2(k_\rho \rho)\, dk_\rho\right] \quad (17)$$

$$R_q = \frac{k_{z0}^2}{k_0^2 - k_{z0}^2}(R_{TE} + R_{TM}) \quad (18)$$

$$R_{TE} = \frac{k_{z0} - k_{z1}}{k_{z0} + k_{z1}} \quad (19)$$

$$R_{TM} = \frac{\varepsilon_r' k_{z0} - k_{z1}}{\varepsilon_r' k_{z0} + k_{z1}} \quad (20)$$

where R_{TE} is the spectrum reflect coefficient of TE mode and R_{TM} is the spectrum reflect coefficient of TM mode. The remainder parameters have the same meaning as defined in [13].

By adopting the discrete complex image theory (DCIM) proposed in [12, 13], the closed form of the Green's function can be obtained as

$$G_A = \frac{e^{-jk_0r_0}}{4\pi r_0} + \frac{1}{4\pi}\sum_{n=1}^{N} a_n \frac{e^{-jk_0r_n}}{r_n} \quad (21)$$

$$G_V = \frac{e^{-jk_0r_0}}{4\pi r_0} + \frac{1}{4\pi}\sum_{m=1}^{M} a_m' \frac{e^{-jk_0r_m'}}{r_m'} \quad (22)$$

where

$$r_n = \sqrt{\rho^2 + (z + z' - jb_n)^2}; \quad r_m' = \sqrt{\rho^2 + (z + z' - jb_m')^2} \quad (23)$$

a_n, b_n and a_m', b_m' are the coefficients obtained by DCIM.

Expand the unknown current with basis functions and use the Galerkin method, the integral Equation (13) can be converted to a matrix equation and the numerical results of the current can be obtained by solving this matrix equation. The process is similar to the analysis of wire antennas in free-space and need not be repeated here. However, it ought to be indicated that two half-basis functions should be added at the end of the antenna in order to model the current at the end points.

3. RESULTS

Several examples are discussed in this section to verify the analytical formula. In these examples, the structure of the antenna had been shown in Fig. 1 and the input voltage is assumed to be 1. It can be seen from Equation (8) that the evaluation of the current relies on the input impedance Z_{in}. So the accurate of the Z_{in} given in (9) by TLM should be verified firstly. In the first example, the input impedance of VLF horizontal antenna with different length is analyzed. The parameters are assumed as follows: frequency $f = 20\,\text{kHz}$; the conductivity and relative permittivity of the earth are $\varepsilon_r = 10$ and $\sigma = 0.0005\,\text{s}$ respectively; the equivalent radius of the antenna is $a = 0.5\,\text{m}$; the height of the antenna is $h = 10\,\text{m}$; the length of the antenna is variational from 5 km to 10 km. In this example, the antenna is fed at the center point.

The results of the input impedance with TLM and MoM are shown in Fig. 2 where the x-axis is the length of the antenna and the y-axis is the input impedance. It can be seen that the results of the TLM approach to the MoM well.

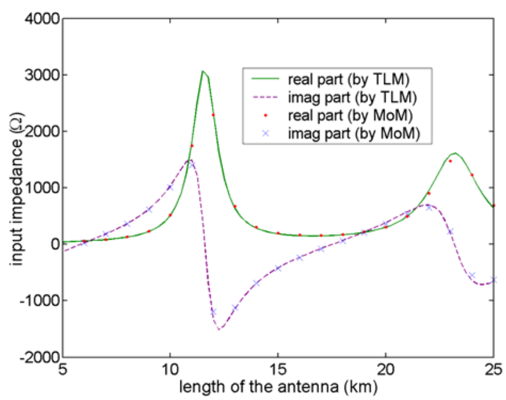
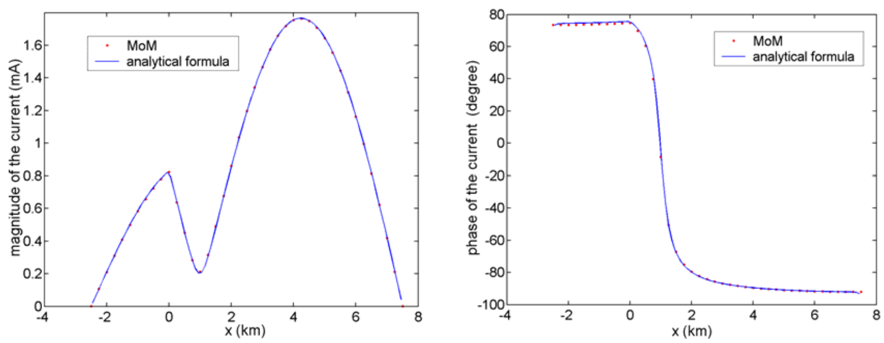


Figure 2. Input impedance of the VLF horizontal antenna with various length.

In the second example, the current distribution of the VLF horizontal antenna with different loaded impedance at its end is considered. The parameters of the antenna are defined as follows: frequency $f = 20\,\text{kHz}$; conductivity of the earth $\sigma = 0.0005\,\text{s}$; relative permittivity of the earth $\varepsilon_r = 10$; length of the antenna $L_1 + L_2 = 10\,\text{km}$; radius of the antenna $a = 0.01\,\text{m}$; height of the antenna $h = 10\,\text{m}$. The distance from the feed-position to the left end is 2.5 km.

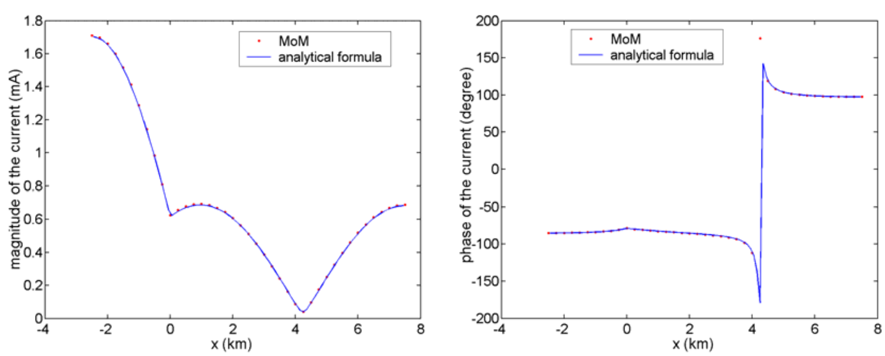
First, each end of the antenna is opened. In order to satisfy this condition, the endmost loaded impedance should be set to infinity. In our work, an adequate large resistance, for example, 100 MΩ, is used to replace the infinity impedance. The results of the analytical formula and the MoM code are shown in Fig. 3 at the same time where (a) is the magnitude of the current and (b) is the phase.



(a) (b)

Figure 3. Current distribution of the end-open VLF horizontal antenna. (a) Magnitude of the current, (b) Phase of the current.

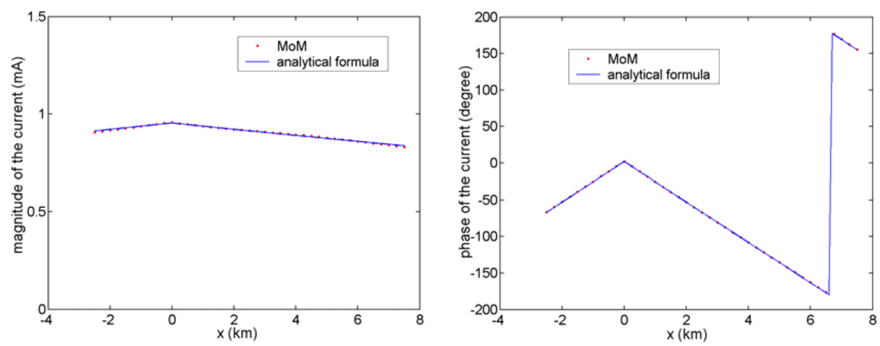
Then, each end of the antenna is grounded well. So the endmost impedance is equal to zero. The results of the analytical formula and the MoM code are shown in Fig. 4 at the same time where (a) is the magnitude of the current and (b) is the phase.



(a) (b)

Figure 4. Current distribution of the end-grounded VLF horizontal antenna. (a) Magnitude of the current, (b) Phase of the current.

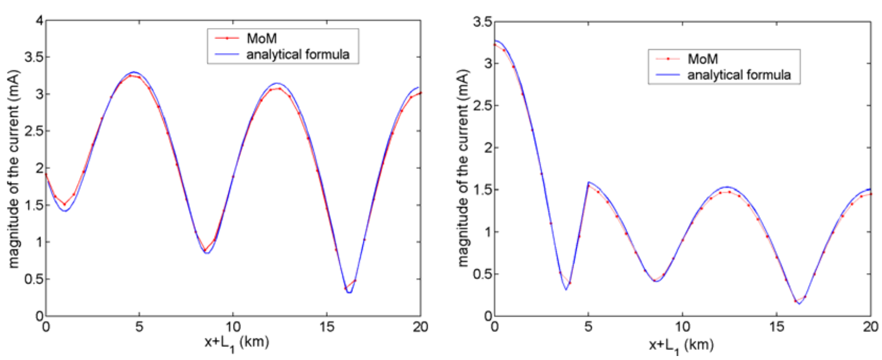
Next, the antenna is terminated at each end with its characteristic impedance. To this example, the characteristic impedance can be calculated by Equation (10) as $Z_c = 524.1 - 18.9j\,\Omega$. Results of the analytical formula and the MoM code are shown in Fig. 5 where (a) is the magnitude of the current and (b) is the phase. It can be seen that the current distribution is a traveling wave.



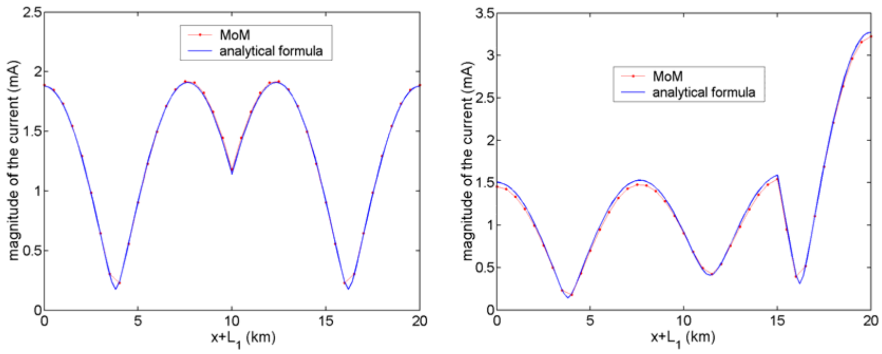
(a) (b)

Figure 5. Current distribution of the antenna terminated at each end with its characteristic impedance. (a) Magnitude of the current, (b) Phase of the current.

In the third example, The VLF horizontal antenna fed at different position is considered. The parameters are defined as follows: frequency $f = 15\,\text{kHz}$; conductivity of the earth $\sigma = 0.0003\,\text{s}$; relative permittivity of the earth $\varepsilon_r = 10$; length of the antenna $L_1 + L_2 = 20\,\text{km}$; equivalent radius of the antenna $a = 0.4\,\text{m}$; height of the antenna $h = 10\,\text{m}$. Each end of the antenna is grounded with



(a) (b)



(c) (d)

Figure 6. Results of the current distribution for the antenna fed at different position. (a) The antenna is fed at the left end, (b) The distance from the feed position to the left end is 5 km, (c) The distance from the feed position to the left end is 10 km, (d) The distance from the feed position to the left end is 15 km.

a grounding resistance $R_g = 2\,\Omega$. In this example, the feed-position is 0 km, 5 km, 10 km and 15 km respectively. The results of the analytical formula and the MoM code are shown in Fig. 6.

4. CONCLUSION

An analytical formula of the current distribution for the VLF horizontal antenna is presented in this paper. This analytical formula can be applied to model the current distribution of the VLF horizontal antenna which is fed at arbitrary position and with arbitrary endmost loaded impedance. In order to check the analytical formula, a numerical code based on MoM is developed. The validity of the analytical formula is proved by several examples.

REFERENCES

1. Wait, J. R., *VLF Radio Engineering*, Pergamon Press, 1967.
2. King, R. W. P., "Directional VLF antenna for communicating with submarines," *Radio Science*, Vol. 32, No. 1, 113–126, 1997.
3. Popovic, B. D. and V. V. Petrovic, "Horizontal wire antenna above lossy half-space: Simple accurate image solution," *International*

Journal of Numerical Modelling: Electronic Networks, Devices and Fields, Vol. 9, No. 3, 191–199, 1996.

4. Chevalier, T. W., U. S. Inan, and T. F. Bell, "Characterization of terminal impedance and radiation properties of a horizontal VLF antenna over Antarctic ice," *Radio Science*, Vol. 46, 2006.
5. Mei, J. P. and K. Li, "Electromagnetic field from a horizontal electric dipole on the surface of a high lossy dielectric coated with a uniaxial layer," *Progress In Electromagnetics Research*, PIER 73, 71–91, 2007.
6. Poljak, D. and V. Doric, "Wire antenna model for transient analysis of simple grounding systems, Part II: The horizontal grounding electrode," *Progress In Electromagnetics Research*, PIER 64, 167–189, 2006.
7. Makki, S. V., T. Z. Ershadi, and M. S. Abrishamian, "Determining the specific ground conductivity aided by the horizontal electric dipole antenna near the ground surface," *Progress In Electromagnetics Research B*, Vol. 1, 43–65, 2008.
8. Liu, C., Q.-Z. Liu, L. Zheng, and W. Yu, "Numeric calculation of input impedance for a giant VLF T-type antenna array," *Progress In Electromagnetics Research*, PIER 75, 1–10, 2007.
9. Xu, X.-B. and Y. Huang, "An efficient analysis of vertical dipole antennas above a lossy half-space," *Progress In Electromagnetics Research*, PIER 74, 353–377, 2007.
10. Seeley, E. W., "High power VLF transmitting antennas using fast wave horizontal dipole arrays," *Radio Science*, Vol. 5, No. 5, 841–852, May 1970.
11. King, R. W. P., "The circuit properties and complete fields of horizontal-wire antennas and arrays over earth or sea," *Journal of Applied Physics*, Vol. 71, No. 3, 1499–1508, Feb. 1992.
12. Fang, D. G., J. J. Yang, and G. Y. Delisle, "Discrete image theory for horizontal electric dipoles in a multilayered medium," *IEE Proc. Pt. H*, Vol. 135, No. 5, 297–303, 1988.
13. Chow, Y. L., J. J. Yang, D. G. Fang, and G. E. Howard, "Closed-form spatial Green's function for the thick substrate," *IEEE Trans. Microwave Theory Tech.*, Vol. 39, No. 3, 588–592, Mar. 1991.
14. Aksun, I. M. and T., Onal, "Critical study of DCIM, and development of efficient simulation tool for 3D printed structures in multilayer media," *PIERS Online*, Vol. 2, No. 1, 35–37, 2006.